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Some Topological Indices of Subgroup Graph of Symmetric Group

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Abstract The concept of the topological index of a graph is increasingly diverse because researchers continue to introduce new concepts of topological indices. Researches on the topological indices of a graph which initially only examines graphs related to chemical structures begin to examine graphs in general. On the other hand, the concept of graphs obtained from an algebraic structure is also increasingly being introduced. Thus, studying the topological indices of a graph obtained from an algebraic structure such as a group is very interesting to do. One concept of graph obtained from a group is subgroup graph introduced by Anderson et al in 2012 and there is no research on the topology index of the subgroup graph of the symmetric group until now. This article examines several topological indices of the subgroup graphs of the symmetric group for trivial normal subgroups. This article focuses on determining the formulae of various Zagreb indices such as first and second Zagreb indices and co-indices, reduced second Zagreb index and first and second multiplicatively Zagreb indices and several eccentricity-based topological indices such as first and second Zagreb eccentricity indices, eccentric connectivity, connective eccentricity, eccentric distance sum and adjacent eccentric distance sum indices of these graphs.

Keywords Topological Indices, Zagreb Index, Subgroup Graph, Trivial Normal Subgroup, Symmetric Group

1. Introduction

The topological index of a finite graph is a number associated with the graph and this number is invariant under automorphism [1]. Topological index sometimes called a graph-theoretical descriptor [2–4] or molecular structure descriptor [5] of a graph. Various topological indices have been used to solve problem in biology and chemistry. Three major classifications of the topological index of a graph are based on degree, distance and the eccentricity of vertex in the graph.

The degree-based topological indices for examples Randic index and its variations [6–8], Zagreb index and its variations [6, 9–16], forgotten topological index and its variations [17–21], Narumi-Katayama index [6], atom-bond-connectivity index and its variations [6,7], Harmonic index [22–25], geometric-arithmetic index its variations [26, 27], and sum-connectivity index and its variations [28–31]. The distance-based topological indices for instances Wiener index and its variations [32,33] and Harary index and its variations [34–38]. While the eccentricity-based topological indices for example total eccentricity index [39, 40] and first and second Zagreb eccentricity indices [41]. Based on these three major topological indices, new topological indices are developed for examples Schultz index [42], Gutman index [43–45], additively and multiplicatively Weighted Harary indices [46–49], eccentric connectivity index [50–53], connective eccentricity index [54], eccentric distance sum [3,55–59] and adjacent eccentric distance sum index [5].

Research on the topological index was initially related to graphs of biological activity or chemical structures and reactivity and researches in this regard continue, for example see [59–63]. On the other hand, several studies began to examine the topological index of graphs that are not of chemical structure and reactivity or biological activity, for example [54,64–72]. When several researchers introduced new concepts about graphs obtained from an algebraic structure, research on topological indices on these graphs began to emerge, for example [73].

One concept of graphs obtained from a group is the concept of subgroup graph that was introduced by Anderson, Fasteen and LaGrange [74]. Referring to the definition of the subgroup graph by Anderson et al. [74], let G is a group and H is a normal subgroup in G. The subgroup graph $\Gamma_H(G)$ of group G is a simple and undirected graph with all elements of G as its vertices and $uv \in E(\Gamma_H(G))$ whenever $uv \in H$ for $u, v \in G$ and $u \neq v$. As a result, the complement $\overline{\Gamma_H(G)}$ of the subgroup graph $\Gamma_H(G)$ is also simple and undirected [75]. Several studies related to graphs from a group have been widely reported, mostly of dihedral group [73,76–82] and are still rare from symmetric groups [82–84]. Therefore, this article will examine the formulae of various Zagreb indices and eccentricity-based topological indices of the subgroup graph of the symmetric group.

2. Materials and Methods

All graphs in the present article is finite, simple, undirected and connected. For graph $G = (V(G), E(G))$, the order of G is $p(G) = |V(G)|$ and its size is $q(G) = |E(G)|$. Let $\deg(u)$ denoted the degree of a vertex u in G. If $\deg(u) = 0$, then u is an isolated vertex. If $\deg(u) = 1$, then u is an end-vertex. Let $d(u, v)$ denoted the distance of vertex u and vertex v in G. The eccentricity $e(u)$ of a vertex u is $e(u) = \sup\{d(u, v): v \in V(G)\}$. The total distance $D(u)$ of a vertex u in G is $D(u) = \sum_{v\in V(G)} d(u, v)$.

The complement $\bar{G}$ of graph G is a graph with $V(\bar{G}) = V(G)$ and $uv \in E(\bar{G})$ whenever $uv \notin E(G)$ [85]. Graph $\bar{G}$ is connected if G is disconnected. For $U \subset E(G)$, graph $G - U$ is obtained by erasing all elements of U from G.

For graphs G and H, the union graph $G \cup H$ has $V(G \cup H) = V(G) \cup V(H)$ and $E(G \cup H) = E(G) \cup E(H)$. Graph G is written as $G = mH$ if G consists of m copies of graph H, In this paper, K_p denotes the complete graph of order p. Then, $\overline{K_p} = pK_1$.

Zagreb index is the second oldest of degree-based topological index and Randic index is the first oldest. Firstly, the definition of various Zagreb index of graph G that will be used in this article are presented. After that, the definitions of several eccentricity-based topological indices which will also be used in this article are presented.

The following definitions refer to a graph $G = (V(G), E(G))$.

The first and second Zagreb indices of G are [17]

$$M_1(G) = \sum_{u\in V(G)}(\deg(u))^2 \quad (1)$$

and

$$M_2(G) = \sum_{uv\in E(G)} \deg(u)\deg(v) \quad (2)$$

The first and second Zagreb co-indices of G are [11]

$$\overline{M_1}(G) = \sum_{uv\notin E(G)}[\deg(u) + \deg(v)] \quad (3)$$

and

$$\overline{M_2}(G) = \sum_{uv\notin E(G)} \deg(u)\deg(v) \quad (4)$$

The reduced second Zagreb index of G is [12,14]

$$RM_2(G) = \sum_{uv\in E(G)}[\deg(u) - 1][\deg(v) - 1] \quad (5)$$

The first and second multiplicative Zagreb indices of G are [13,15]

$$\Pi_1(G) = \prod_{u\in V(G)}(\deg(u))^2 \quad (6)$$

and

$$\Pi_2(G) = \prod_{uv\in E(G)} \deg(u)\deg(v) \quad (7)$$

The first and second Zagreb eccentricity indices of G are [16]

$$E_1(G) = \sum_{u\in V(G)} (e(u))^2 \quad (8)$$

and

$$E_2(G) = \sum_{uv\in E(G)} \mathrm{e}(u)\,\mathrm{e}(v) \quad (9)$$

The eccentric connectivity index of G is [86]

$$\xi^c(G) = \sum_{v\in V(G)} e(v)\deg(v) \quad (10)$$

The connective eccentricity index of G is [63]

$$C^{\xi}(G) = \sum_{v\in V(G)} \frac{\deg(v)}{e(v)} \quad (11)$$

The eccentric distance sum index of G is [59]

$$\xi^d(G) = \sum_{v\in V(G)} e(v)D(v) \quad (12)$$

The adjacent eccentric distance sum index of G is [5]

$$\xi^{sv}(G) = \sum_{v\in V(G)} \frac{e(v)D(v)}{\deg(v)} \quad (13)$$

3. Main Result

For a positive integer $n \geq 3$, the symmetric group S_n contains all permutations on the set $\mathbb{N}_n = \{1, 2, 3, \ldots, n\}$ under the composition function operation. The symmetric group S_n is a non-commutative group of order $n!$. For distinct elements $x_1, x_2, \ldots, x_k$ ($k \leq n$) in $\mathbb{N}_n$, a cycle $(x_1x_2 \cdots x_k)$ states a permutation π in S_n such that $\pi(x_i) = x_{i+1}$ and $\pi(x_k) = x_1$ and π maps any other element of $\mathbb{N}_n$ to itself. The cycle $(x_1x_2 \cdots x_k)$ is called k-cycle or cycle with the length k. The 2-cycle is called transposition. The order of a k-cycle is k. The order of a product of disjoint transpositions has order 2. The order of permutation ρ in S_n is 1 if and only if $\rho = (1)$, where (1) is identity element of S_n.

Throughout this paper, let X is the set of permutations in S_n with order less than 3 and let Y is the set of permutations in S_n with order more than 2. Hence, $S_n = X \cup Y$.

Theorem 3.1.

$$|X| = \begin{cases} 1 + \Sigma_{m=1}^{\frac{n-1}{2}} \dfrac{n!}{2^m m!\,(n-2m)!}, & \text{if } n \text{ is odd} \\ 1 + \Sigma_{m=1}^{\frac{n}{2}} \dfrac{n!}{2^m m!\ (n-2m)!}, & \text{if } n \text{ is even} \end{cases}$$

Proof. The set X consists of identity permutation and permutations in term of products of disjoint 2-cycles. The number of permutations in S_n with a given cycle structure is

$$\frac{n!}{\prod_{k=1}^{n}(m_k)!k^{m_k}} \quad (14)$$

where m_k denotes the number of k-cycles. By (14), the number permutation in the term of products of disjoint 2-cycles is

$$\Sigma_{m=1}^{\frac{n-1}{2}} \frac{n!}{2^m m!(n-2m)!}, \text{ if } n \text{ is odd}$$

or

$$\Sigma_{m=1}^{\frac{n}{2}} \frac{n!}{2^m m!\,(n-2m)!}, \text{ if } n \text{ is even.}$$

This proves the theorem. ♦

Theorem 3.2.

$|Y|$ is even and $|Y| = n! - |X|$.

Proof. Let δ is any element in Y. Then δ is a permutation of order more than 2. It implies that δ^{-1} is also a permutation of order more than 2. Hence, Y consists of permutations together with their inverses. This proves that $|Y|$ is even. By Theorem 3.1, $|Y| = |S_n| - |X| = n! - |X|$. ♦

Corollary 3.3.

$|X|$ is even

Proof. Because for $n \geq 3$, $|S_n| = n!$ is even and, by Theorem 3.2, $|Y|$ is even, obviously $|X|$ is even.

The subgroup graph of a group will be simple and undirected if the subgroup is a normal subgroup. For symmetric group S_n, the trivial normal subgroup is {(1)} and S_n. This article only considers these two trivial normal subgroups of S_n.

Theorem 3.4.

$\Gamma_{S_n}(S_n)$ is complete with order n!

Proof. The proof is obvious. ♦

Along this paper, let the set of all permutations of order less than 3 is $X = \{w_1, w_2, \ldots, w_{|X|}\}$ where $w_1 = (1)$ and let the set of all permutations of order more than 2 in S_n is $Y = \{u_1, u_2, \ldots, u_{|Y|/2}, v_1, v_2, \ldots, v_{|Y|/2}\}$ where $v_i = (u_i)^{-1}$.

Theorem 3.5.

$\Gamma_{\{(1)\}}(S_n)$ is disconnected with size

$$q\left(\Gamma_{\{(1)\}}(S_n)\right) = \frac{|Y|}{2}. \tag{15}$$

Proof. By definition of the subgroup graph, the edge set of $\Gamma_{\{(1)\}}(S_n)$ is

$$E\left(\Gamma_{\{(1)\}}(S_n)\right) = \{u_i v_i : 1 \leq i \leq \tfrac{|Y|}{2}\}. \tag{16}$$

Hence, in $\Gamma_{\{(1)\}}(S_n)$, all permutations in Y are end-vertices and all permutations in X are isolated vertices. Thus, $\Gamma_{\{(1)\}}(S_n)$ is disconnected with $q\left(\Gamma_{\{(1)\}}(S_n)\right) = |Y|/2$. ♦

Corollary 3.6.

The number of isolated-vertices in $\Gamma_{\{(1)\}}(S_n)$ is

$$|X| = \begin{cases} 1 + \Sigma_{m=1}^{\frac{n-1}{2}} \dfrac{n!}{2^m m!\,(n-2m)!}, & \text{if } n \text{ is odd} \\ 1 + \Sigma_{m=1}^{\frac{n}{2}} \dfrac{n!}{2^m m!\,(n-2m)!}, & \text{if } n \text{ is even} \end{cases}$$

and the number of end-vertices in $\Gamma_{\{(1)\}}(S_n)$ is

$$|Y| = n! - |X|.$$

Proof. From the proof of Theorem 3.5. ♦

According to Corollary 3.6., then $\Gamma_{\{1\}}(S_n)$ can be written as

$$\Gamma_{\{1\}}(S_n) = \frac{1}{2}|Y|K_2 \cup |X|K_1.$$

with size as presented in (16). Therefore, the complement of $\Gamma_{\{1\}}(S_n)$ is

$$\overline{\Gamma_{\{1\}}(S_n)} = K_{n!} - \left\{u_i v_i : 1 \leq i \leq \tfrac{|Y|}{2}\right\}).$$

Thus, the following two theorems are obtained.

Theorem 3.7.

In $\overline{\Gamma_{\{(1)\}}(S_n)}$,

$$\deg(u_i) = \deg(v_i) = n! - 2$$

and

$$\deg(w_j) = n! - 1.$$

Proof. Since $\overline{\Gamma_{\{(1)\}}(S_n)} = K_{n!} - \left\{u_i v_i : 1 \leq i \leq \frac{|Y|}{2}\right\}$, then $\deg(u_i) = \deg(v_i) = n! - 2$ $(1 \leq i \leq |Y|/2)$ and $\deg(w_j) = n! - 1$ $(1 \leq j \leq |X|)$.

Theorem 3.8.

In $\overline{\Gamma_{\{(1)\}}(S_n)}$,

a. $d(u_i, v_i) = 2$ $(1 \leq i \leq |Y|/2)$.
b. $d(u_i, u_s) = d(v_i, v_s) = 1$, for $s \neq i$ $(1 \leq i, s \leq |Y|/2)$.
c. $d(u_i, w_j) = d(v_i, w_j) = 1$ $(1 \leq i \leq |Y|/2$ and $(1 \leq j \leq |X|)$
d. $d(w_i, w_j) = 1$, for $i \neq j$ $(1 \leq i, j \leq |X|)$

As the direct consequences of Theorem 3.8., the following two corollaries are obtained.

Corollary 3.9.

In $\overline{\Gamma_{\{(1)\}}(S_n)}$, $e(u_i) = e(v_i) = 2$ and $e(w_j) = 1$.

Corollary 3.10.

In $\overline{\Gamma_{\{(1)\}}(S_n)}$,

a. $D(u_i) = D(v_i) = n!$
b. $D(w_j) = n! - 1$.

Proof. By Theorem 3.8.

a. $$D(u_i) = \sum_{\substack{z \in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) \\ z \neq v_i}} d(u_i, z) + d(u_i, v_i) = (n! - 2) \cdot 1 + 2 = n!$$

In similar manner, $D(v_i) = n!$

b. Since w_j is adjacent to all other vertices in $\overline{\Gamma_{\{(1)\}}(S_n)}$, it is obvious that $D(w_j) = n! - 1$. ♦

Topological indices of $\Gamma_{S_n}(S_n)$ and $\overline{\Gamma_{\{(1)\}}(S_n)}$ are presented as the following.

Theorem 3.11.

a. $M_1\left(\Gamma_{S_n}(S_n)\right) = n!\,(n!-1)^2$

b. $M_2\left(\Gamma_{S_n}(S_n)\right) = \frac{n!(n!-1)}{2}(n!-1)^2$

c. $RM_2\left(\Gamma_{S_n}(S_n)\right) = \frac{n!(n!-1)}{2}(n!-2)^2$

d. $\Pi_1\left(\Gamma_{S_n}(S_n)\right) = (n!-2)^{2\cdot n!}$

e. $\Pi_2\left(\Gamma_{S_n}(S_n)\right) = (n!-2)^{n!(n!-1)}$

f. $E_1\left(\Gamma_{S_n}(S_n)\right) = n!$

g. $E_2\left(\Gamma_{S_n}(S_n)\right) = \frac{n!(n!-1)}{2}$

h. $\xi^c\left(\Gamma_{S_n}(S_n)\right) = n!\,(n!-1)$

i. $C^\xi\left(\Gamma_{S_n}(S_n)\right) = n!\,(n!-1)$

j. $\xi^d\left(\Gamma_{S_n}(S_n)\right) = n!\,(n!-1)$

k. $\xi^{sv}\left(\Gamma_{S_n}(S_n)\right) = n!$

Proof: By Theorem 3.4. $\Gamma_{S_n}(S_n) = K_{n!}$. Therefore, its order and size are $p\left(\Gamma_{S_n}(S_n)\right) = n!$ and $q\left(\Gamma_{S_n}(S_n)\right) = n!\,(n!-1)$. For any $v \in V\left(\Gamma_{S_n}(S_n)\right)$, $\deg(v) = n! - 1$, $e(v) = 1$ and $D(v) = n! - 1$. These lead to the desired proof. ♦

Theorem 3.12.

a. $M_1(\overline{\Gamma_{\{(1)\}}(S_n)}) = |Y|(n!-2)^2 + |X|(n!-1)^2$

b. $M_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = \frac{3|Y|}{2}\left(\frac{|Y|}{2}-1\right)(n!-2)^2 + |Y||X|(n!-2)(n!-1) + |X|(|X|-1)(n!-1)^2$

c. $\overline{M_1}(\overline{\Gamma_{\{(1)\}}(S_n)}) = |Y|(n!-2)$

d. $\overline{M_2}(\overline{\Gamma_{\{(1)\}}(S_n)}) = \frac{|Y|}{2}(n!-2)^2$

e. $RM_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = \frac{|Y|}{2}(n!-3)^2$

f. $\Pi_1(\overline{\Gamma_{\{(1)\}}(S_n)}) = (n!-2)^{2|Y|} + (n!-1)^{2|X|}$

g. $\Pi_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = (n!-2)^{3|Y|\left(\frac{|Y|}{2}-1\right)} + [(n!-2)(n!-1)]^{|Y||X|} + (n!-1)^{2|X|(|X|-1)}$

h. $E_1(\overline{\Gamma_{\{(1)\}}(S_n)}) = n! + |Y|$

i. $E_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = 3|Y|^2 + |X|^2 + 2|Y||X| - 6|Y| - |X|$

j. $\xi^c(\overline{\Gamma_{\{(1)\}}(S_n)}) = 2|Y|(n!-2) + |X|(n!-1)$

k. $C^\xi(\overline{\Gamma_{\{(1)\}}(S_n)}) = |Y|\frac{(n!-2)}{2} + |X|(n!-1)$

l. $\xi^d(\overline{\Gamma_{\{(1)\}}(S_n)}) = 2|Y|n! + |X|(n!-1)$

m. $\xi^{sv}(\overline{\Gamma_{\{(1)\}}(S_n)}) = \frac{2|Y|n!}{n!-2} + |X|$

Proof. By using Theorem 3.7., Corollary 3.9 and Corollary 3.10, then

a. $M_1(\overline{\Gamma_{\{(1)\}}(S_n)}) = \sum_{z\in V(\overline{\Gamma_{\{(1)\}}(S_n)})}(\deg(z))^2$

$$= \sum_{z\in Y}(\deg(z))^2 + \sum_{z\in X}(\deg(z))^2$$

$$= |Y|(n!-2)^2 + |X|(n!-1)^2$$

b. $M_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = \sum_{zx\in E(\overline{\Gamma_{\{(1)\}}(S_n)})}\deg(z)\deg(x)$

$$= \sum_{\substack{u_iu_s\\ i\neq s}}\deg(u_i)\deg(u_s) + \sum_{\substack{u_iv_s\\ i\neq s}}\deg(u_i)\deg(v_s) + \sum_{u_iw_j}\deg(u_i)\deg(w_j) + \sum_{\substack{v_iv_s\\ i\neq s}}\deg(v_i)\deg(v_s) + \sum_{v_iw_j}\deg(v_i)\deg(w_j) + \sum_{\substack{w_jw_t\\ j\neq t}}\deg(w_j)\deg(w_t)$$

$$= \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)(n!-2)^2 + \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)(n!-2)^2 + \frac{|Y|}{2}|X|(n!-2)(n!-1) + \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)(n!-2)^2 + \frac{|Y|}{2}|X|(n!-2)(n!-1) + |X|(|X|-1)(n!-1)^2$$

$$= \frac{3|Y|}{2}\left(\frac{|Y|}{2}-1\right)(n!-2)^2 + |Y||X|(n!-2)(n!-1) + |X|(|X|-1)(n!-1)^2.$$

c. $\overline{M_1}(\overline{\Gamma_{\{(1)\}}(S_n)}) = \sum_{u_iv_i}[\deg(u_i) + \deg(v_i)]$

$$= \frac{|Y|}{2}[(n!-2) + (n!-2)]$$

$$= |Y|(n!-2)$$

d. $\overline{M_2}(\overline{\Gamma_{\{(1)\}}(S_n)}) = \sum_{u_iv_i}[\deg(u_i)\deg(v_i)]$

$$= \frac{|Y|}{2}(n!-2)^2$$

e. $RM_2(\overline{\Gamma_{\{(1)\}}(S_n)}) = \sum_{u_iv_i}[\deg(u_i)-1][\deg(v_i)-1]$

$$= \frac{|Y|}{2}(n!-3)^2$$

f. $\Pi_1(\overline{\Gamma_{\{(1)\}}(S_n)}) = \prod_{z\in V(\overline{\Gamma_{\{(1)\}}(S_n)})}(\deg(z))^2$

$$= \prod_{z\in Y}(\deg(z))^2 + \prod_{z\in X}(\deg(z))^2$$
$$= (n!-2)^{2|Y|} + (n!-1)^{2|X|}$$

g. $\Pi_2\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \prod_{zx\in E\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)}(\deg(z))^2(\deg(x))^2$

$$= \prod_{\substack{u_iu_s\\ i\neq s}}\deg(u_i)\deg(u_s) + \prod_{\substack{u_iv_s\\ i\neq s}}\deg(u_i)\deg(v_s) + \prod_{u_iw_j}\deg(u_i)\deg(w_j) + \prod_{\substack{v_iv_s\\ i\neq s}}\deg(v_i)\deg(v_s) + \prod_{v_iw_j}\deg(v_i)\deg(w_j)$$

$$+ \prod_{\substack{w_jw_t\\ j\neq s}}\deg(w_j)\deg(w_t)$$

$$= (n!-2)^{3|Y|\left(\frac{|Y|}{2}-1\right)} + [(n!-2)(n!-1)]^{|Y||X|} + (n!-1)^{2|X|(|X|-1)}$$

h. $E_1\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{z\in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)}(e(z))^2$

$$= \sum_{z\in Y}(e(z))^2 + \sum_{z\in X}(e(z))^2$$
$$= 2|Y| + |X|$$
$$= n! + |Y|$$

i. $E_2\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{zx\in E\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)} e(z)e(x)$

$$= \sum_{\substack{u_iu_s\\ i\neq s}} e(u_i)\,e(u_s) + \sum_{\substack{u_iv_s\\ i\neq s}} e(u_i)\,e(v_s) + \sum_{u_iw_j} e(u_i)\,e(w_j) + \sum_{\substack{v_iv_s\\ i\neq s}} e(v_i)\,e(v_s) + \sum_{v_iw_j} e(v_i)\,e(w_j) + \sum_{\substack{w_jw_t\\ j\neq t}} e(w_j)\,e(w_t)$$

$$= \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)4 + \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)4 + \frac{|Y|}{2}|X|2 + \frac{|Y|}{2}\left(\frac{|Y|}{2}-1\right)4 + \frac{|Y|}{2}|X|2 + |X|(|X|-1)$$

$$= 6|Y|\left(\frac{|Y|}{2}-1\right) + 2|Y||X| + |X|(|X|-1)$$

$$= 3|Y|^2 + |X|^2 + 2|Y||X| - 6|Y| - |X|$$

j. $\xi^c\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{z\in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)} e(z)\deg(z)$

$$= \sum_{z\in Y} e(z)\deg(z) + \sum_{z\in X} e(z)\deg(z)$$
$$= |Y|2(n!-2) + |X|1(n!-1)$$
$$= 2|Y|(n!-2) + |X|(n!-1)$$

k. $C^\xi\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{z\in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)} \frac{\deg(z)}{e(z)}$

$$= \sum_{z\in Y}\frac{\deg(z)}{e(z)} + \sum_{z\in X}\frac{\deg(z)}{e(z)}$$
$$= |Y|\frac{(n!-2)}{2} + |X|(n!-1)$$

l. $\xi^d\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{z\in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)} e(z)D(z)$

$$= \sum_{z\in Y} e(z)D(z) + \sum_{z\in X} e(z)D(z)$$
$$= 2|Y|n! + |X|(n!-1)$$

m. $\xi^{sv}\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right) = \sum_{z\in V\left(\overline{\Gamma_{\{(1)\}}(S_n)}\right)} \frac{e(z)D(z)}{\deg(z)}$

$$= \sum_{z\in Y}\frac{e(z)D(z)}{\deg(z)} + \sum_{z\in X}\frac{e(z)D(z)}{\deg(z)}$$
$$= \frac{2|Y|n!}{n!-2} + \frac{|X|(n!-1)}{n!-1}$$
$$= \frac{2|Y|n!}{n!-2} + |X|. \ \blacklozenge$$

4. Conclusions

This article has presented the formulae of some degree-based and eccentric-based topological indices of the subgroup graphs of the symmetric group. The discussion in this article limited on trivial normal subgroups of symmetric group. For further research, examining on non-trivial normal subgroup needed to be studied.

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