DYNAMIC PRICE COMPETITION WITH PRICE ADJUSTMENT COSTS AND PRODUCT DIFFERENTIATION

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Dynamic Price Competition with Price Adjustment Costs and Product Di¤erentiation[¤]

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Abstract

We study a discrete time dynamic game of price competition with spatially di¤erentiated products and price adjustment costs. We characterise the Markov perfect and the open-loop equilibrium of our game. We …nd that in the steady state Markov perfect equilibrium, given the presence of adjustment costs, equilibrium prices are always higher than prices at the repeated static Nash solution, even though, adjustment costs are not paid in steady state. This is due to intertemporal strategic complementarity in the strategies of the …rms and from the fact that the cost of adjusting prices adds credibility to high price equilibrium strategies. On the other hand, the stationary open-loop equilibrium coincides always with the static solution. Furthermore, in contrast to continuous time games, we show that the stationary Markov perfect equilibrium converges to the static Nash equilibrium when adjustment costs tend to zero. Moreover, we obtain the same convergence result when adjustment costs tend to in…nity.

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Keywords: price adjustment costs, di¤erence game, Markov perfect equilibrium, Open-loop equilibrium.

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JEL:C72, C73, L13

1 Introduction

In this paper we develop a duopolistic dynamic game of price competition, in which products are horizontally di¤erentiated and …rms face adjustment costs every time

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they change their prices. Imposing adjustment costs creates a time dependent structure in our dynamic game and allows us to develop our model as a discrete time di¤erential game, or di¤erence game.[1] The main objective of our analysis is to study the e¤ects of the presence of such adjustment costs on the strategic behaviour of …rms under di¤erent assumptions on the ability of the …rms to commit to price paths in advance. In particular, we focus on two di¤erent classes of strategies that have been widely used in the dynamic competition models, Markov and open-loop strategies.[2] Markov strategies depend only on payo¤ relevant variables that condense the direct e¤ect of the past on the current payo¤.[3] The use of Markov strategies restrict equilibrium evaluation solely to subgame perfect equilibria, which have the desirable property of excluding non-credible threats. In contrast, open-loop strategies are functions of the initial state of the game and of the calendar time, and typically, are not subgame perfect. Thus, Markov and open-loop strategies correspond to extreme assumptions about player's capacities to make commitments about their future actions. Under the open-loop information pattern, the period of commitment is the same as the planning horizon while under Markovian strategies no commitment is possible. The role of price stickiness has been analysed in many theoretical models of business cycles.[4] Very little attention, however, has been paid to strategic incentives associated with price adjustment costs in dynamic oligopoly models.

There are several examples of di¤erential games of Cournot competition with sticky prices, for example, Fershtman and Kamien (1987), Piga (2000) and Cellini and Lambertini (2001.a). The main result of these authors is that the subgame

[1] Di¤erential games are dynamic games in continuous time, in which the di¤erent stages of the game are linked through a transition equation that describes the evolution of the state of the model. Furthermore, the transition equation depends on the strategic behaviour of the players. These kind of games are also called "state-space" games. Di¤erence games are the discrete time counterpart of di¤erential games. See Basar and Olsder (1995) for a detailed analysis. See also De Zeeuw and Van Der Ploeg (1991) for a survey on the use of di¤erence games in economics.

[2] See Fudenberg and Tirole (1991) Ch. 13, for and introduction to Markov and open-loop equilibria and on their use in dynamic games. Amir (2001) provides an extensive survey on the use of these strategies in dynamic economic models.

[3] There is no generally accepted name in di¤erence/di¤erential games theory for such strategies and for the related equilibrium. Basar and Oldser (1995) use the term "Feedback Nash Equilibrium" while Papavassilopoulos and Cruz (1979) use the term "Closed-Loop Memoryless Equilibrium". However, after Maskin and Tirole (1988), the terms Markov strategies and Markov Perfect equilibrium has become standard in economic literature.

[4] Two di¤erent possibilities to model price adjustment costs have been considered in the literature on business cycles. First, there is a …xed cost per price change due to the physical cost of changing posted prices. These …xed costs are called "menu costs". See for example, Akerlof and Yellen (1985) and Mankiw (1985) among the others. Second, there are costs that capture the negative e¤ect of price changes, particularly price increases on the reputation of …rms. These costs are quadratic because reputation of …rms is presumebly more a¤ected by large price changes than by small price changes. See for example, Rotemberg (1982). In our model, we follow the latter approach.

perfect equilibrium quantity is always below the static Cournot equilibrium, even if prices adjust instantaneously. This implies that the presence of price rigidity creates a more competitive market outcome. However, in those models, price rigidity is not modelled using adjustment costs, but instead refers to stickiness in the general price level. This implies that those models deal only with one state variable, that is, the price level given by the inverse demand function, while in our model we have two state variables, the two prices of both …rms. This fact adds considerably to the technical complexity of our problem. The main reason is that dynamic programming su¤ers the "curse of dimensionality", that is the tendency of the state space, and thus computational di¢culties, to grow exponentially with the number of state variables. As far as adjustment costs are concerned, most of the literature on dynamic competition has instead focused on adjustment costs in quantities. However, the same result described above still hold in when these adjustment costs are considered. For example, Reynolds (1989, 1991) and Driskill and McCa¤erty (1989) study a dynamic duopoly with homogenous product and quadratic capacity adjustment costs in continuous time setting. The steady state output in the subgame perfect equilibrium is found to be larger than in the Cournot static game without adjustment costs. The main reason is the presence of intertemporal strategic substitutability in the strategic behaviour of the …rms. A larger output of a …rm today leads the …rm being more aggressive tomorrow. The same result seems to hold independently on the kind of competition that is considered, as showed by Jun and Vives (2001) in their model of Bertrand competition with output adjustment costs.[5]

All the literature described so far shares the common feature of a continuous time setting. An interesting limit result that is common to models in continuous time is that, as adjustment costs or price stickiness tends to zero, the subgame perfect equilibrium approaches a limit that is di¤erent from the Nash equilibrium of the corresponding static game. Discrete time models of dynamic competition have been less developed, since it appears that the discrete time formulation is less tractable than the continuous time formulation. Discrete time models of dynamic competition with adjustment costs have been analysed in Maskin and Tirole (1987), Karp and Perlo¤ (1993) and Lapham and Ware (1994) among others. In the …rst case Cournot competition with quantities adjustment costs is considered, but the equilibrium has been characterised only for the case in which adjustment costs approach in…nity. More interesting for our purposes is the model of Lapham and Ware (1994). They show that the taxonomy of strategic incentives developed by Fudenberg and Tirole (1984) in a two-stage game can be extended to in…nite horizon games with Markov strategies. However, their analysis is limited to the case in which adjustment costs are zero in equilibrium. Nevertheless, they …nd a limit result that contradicts the one of continuous time models. Their steady-state Markov perfect equilibrium when adjustment costs tend to zero is equal to the Nash equilibrium of their static counterpart game

[5] This seems to provide a counterpoint to the idea of Kreps and Scheinkman (1983) that quantity precommitment and price competition yields Cournot outcomes.

without adjustment costs.

In our model we characterize the Markov perfect equilibrium for different values of adjustment costs and product differentiation. Given the mathematical complexity involved in the analysis, some qualitative results are derived with numerical simulation. We find that the prices at the steady-state Markov perfect equilibrium are always higher than the prices at the static Nash equilibrium of the corresponding static game but they coincide in two limit cases – when adjustment costs tend to zero and when they tend to infinity. The economic force behind this result is the presence of intertemporal strategic complementarity: a firm, may strategically raise price today, and induce high prices from its rival tomorrow. The presence of adjustment costs enables firms to increase prices today and signal that they plan to keep prices higher next period. However, the magnitude of this strategic complementarity depends on the level of product differentiation. As products become less differentiated, higher equilibrium prices are sustainable only if adjustment costs are sufficiently high. Strategic complementarity is absent in open-loop strategies and the steady-state open-loop equilibrium is in correspondence one-to-one with the static Nash equilibrium. Our analysis is conducted under the assumptions that demand functions are linear function of prices and that profit functions and adjustment costs are quadratic and symmetric between firms. With this structure our model is a linear quadratic game. This kind of game is analytically convenient because the equilibrium strategies are known to be linear in the state variables of the model. Moreover, in some cases linear quadratic models or linear quadratic approximations of non-linear games provide a good representation of oligopolistic behaviour especially around a deterministic steady state.

The paper is organized as follows. In section 2 we describe the model. Section 3 presents the full computation of the Markov perfect equilibrium. In Section 4 we compare the Markov perfect equilibrium with the open-loop equilibrium and then we consider some limit results for these two equilibria. Section 5 concludes.

2 The Model

We start with the derivation of the demand functions. Consider a simple model of spatial competition a' la Hotelling in which there is a continuum of consumers uniformly distributed in the unit interval $[0,1]$, with the position of a firm representing its ideal product. Consumers incur exogenous 'transportation costs' for having to consume one of the available brands instead of their ideal brand. This cost has the form of $(1/2s)$, where $s \in (0,\infty]$ provides a measure of substitutability between both products.[6] In particular, when the transportation (or switching) cost tends to zero,

[6] The formulation of the consumer problem is standard and follows the lines as in Doganouglu (1999). Differently from his analysis, we do not incorporate possible persistent effects in customer tastes.

products become close substitute. In contrast, if $1/2s \to \infty$, a consumer prefers to choose the brand closest to its ideal point, independent of price. Thus, we can think at s as an indicator of market power of firms. Indeed, when products are highly differentiated (s is small), firms can increase prices without significantly affecting their own demand.

The utility function at time t of the consumer located at point α in the unit segment, for product i, is a linear function $U_t(v; \alpha_i; s; p_{it}) = v - \frac{|\alpha_i - F_i|}{2s} - p_{it}$. The term v represents the utility that a consumer derives for consuming her ideal product, that is assumed to be time invariant. Given our specification, the term v represents also an upper boundary level for the price set.[7] If prices plus the exogenous transportation costs are greater than v, then no products will be bought by the consumers. In the following we shall assume that v is sufficiently high so that this does not occur. Finally, $F_i \in [0; 1]$ is the location of firm i; and the term $-p_{it}$ reflects the negative impact of price of product i on the utility of consumers. Following standard procedures, we solve for the demand functions of a consumer $\bar{\alpha}$ who is at the point of indifference. The linear demand function faced by firm i is simply

$$y_{it}(p_{it}; p_{jt}) = \frac{1}{2} + s\,(p_{jt} - p_{it}) \tag{1}$$

with $i; j = 0; 1$ and $i \neq j$. If firms set the same price, both share equally the market. In our duopoly model the two firms are symmetric and they are located at each end of the unit interval. There is no uncertainty. Both firms face fixed quadratic adjustment costs: $\frac{1}{2}\lambda\,(p_{it} - p_{it-1})^2$; where $\lambda \geq 0$ is a measure of the cost of adjusting the price level.[8] This formulation implies that adjustment costs are minimized when no adjustment takes place. The per-period profit function for firm i is the following concave function in its own prices:

$$\pi_{it}(p_{it}; p_{jt}; p_{it-1}) = p_{it} y_{it}(p_{it}; p_{jt}) - \frac{1}{2}\lambda\,(p_{it} - p_{it-1})^2 \tag{2}$$

At time t; firm i decides by how much to change its price $\Delta_{it} = (p_{it} - p_{it-1})$, or equivalently to set the new price level at time t. Thus, using the terminology of the optimal control theory, prices at time t are the control variables for the firms, while prices at time $t-1$ are the state variables.

Firm i maximizes the following discounted stream of future profits over an infinite horizon:

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[7] The fact that the set of prices is bounded assures that instantaneous profit functions are bounded as well, which is a sufficient condition for dynamic programming to be applicable. See Maskin and Tirole (1988.b).

[8] We use convex adjustment costs as in Rotemberg (1982). Quadratic price adjustment costs have been used also by Lapham and Ware (1994) and Jun and Vives (2001) because of their analytical tractability.

$$\Pi_i = \sum_{t=1} \beta^t \pi_{it}(p_{it}, p_{jt}, p_{it-1}) \tag{3}$$

where $\beta \in (0, 1)$ is the time invariant discount factor.[9]

With this structure, our model is a linear quadratic game. This kind of games is analytically convenient because the equilibrium strategies are known to be linear in the state variables.[10]

In our model, we mainly focus on strategic behaviour of the firms based on pure Markov strategies, that is, Markov strategies that are deterministic, in which the past influences current play only through its effect on the current state variables that summarise the direct effect of the past on the current environment. Optimal Markovian strategies have the property that, whatever the initial state and time are, all remaining decisions from that particular initial state and particular time onwards must also constitute optimal strategies. This means that Markov strategies are time-consistent. An equilibrium in Markov strategies is a subgame perfect equilibrium and it is called Markov perfect equilibrium[11].

Following Maskin and Tirole (1988), we define a Markov perfect equilibrium using the game theoretic analogue of dynamic programming[12]:

Definition 1 . Let $V_{it}(p_{it-1}, p_{jt-1})$, with $i, j = 1,2$ and $i \neq j$; be the value of the game starting at period t where both players play their optimal strategy ($\rho_{it}(p_{it-1}, p_{jt-1})$ for $i, j = 1,2$ and $i \neq j$). This pair of strategies constitute a Markov Perfect Equilibrium (MPE) if and only if they solve the following dynamic programming problem:

$$V_{it}(p_{it-1}, p_{jt-1}) = \underset{p_{it}}{Max} \left[\pi_{it}(p_{it}, p_{jt}, p_{it-1})) + \beta V_{it+1}(p_{it}, p_{jt})\right] \tag{4}$$

The pair of equations in 4) are the Bellman's equations of our problem that have to hold in equilibrium. Differentiating the right-hand side of the Bellman's equations

[9] The presence of a discount factor less than one together with per-period payoff functions uniformly bounded assures that the objective functions faced by players are continuous at infinity. Under this condition a Markov perfect equilibrium, at least in mixed strategies, always exist in a game with infinite horizon. See Fudenberg and Tirole p. 515.

[10] For a detailed analysis of this point, see Papavassilopoulos, Medanic and Cruz (1979)

[11] The concept of subgame perfection is much stronger than time consistency, since it requires that the property of subgame perfection hold at every subgame, not just those along the equilibrium path. On the other hand, dynamic (or "time") consistency would require that along the equilibrium path the continuation of the Nash equilibrium strategies remains a Nash equilibrium. For a discussion on the difference between subgame perfection and time consistency see Reinganum and Stokey (1985).

[12] By definition, Markov strategies are feedback rules. Furthermore, it is well known that dynamic programming produces feedback equilibria by its very construction , thus, it represents a natural tool to analyse Markov perfect equilibria.

) with respect the control variable of each firm, we obtain the following first order conditions:

$$\frac{\partial \pi_{it}(p_{it}, p_{jt}, p_{it-1})}{\partial p_{it}} + \beta \frac{\partial V_{it+1}(p_{it}, p_{jt})}{\partial p_{it}} = 0 \tag{5}$$

Given the linear quadratic nature, the first order conditions of each player completely characterize the Markov perfect equilibrium. To solve this system of equations, we need to guess a functional form for the unknown value functions $V_i(\cdot)$. However, we know that equilibrium strategies in linear quadratic games are linear functions of the state variables and value functions are quadratic. Then, we guess the following functional form for the value functions:[13]

$$V_{it+1}(p_{it}, p_{jt}) = a_i + b_i p_{it} + c_i p_{jt} + \frac{d_i}{2} p_{it}^2 + e_i p_{it} p_{jt} + f_i p_{jt}^2 \quad \text{for } i, j = 1, 2 \text{ and } i \neq j \tag{6}$$

where the parameters a_i, b_i, c_i, d_i, e_i and f_i are unknown parameters. Equation 6) hold for every t.

We are not interested in solving for all these parameters since we are analysing only the derivatives of the value functions that are given by:

$$\frac{\partial V_{it+1}(p_{it}, p_{jt})}{\partial p_{it}} = b_i + d_i p_{it} + e_i p_{jt} \quad \text{for } i, j = 1, 2 \text{ and } i \neq j$$

$$\frac{\partial V_{it+1}(p_{it}, p_{jt})}{\partial p_{jt}} = c_i + e_i p_{it} + f_i p_{jt} \quad \text{for } i, j = 1, 2 \text{ and } i \neq j \tag{7}$$

where the coefficients b_i, c_i, d_i, e_i and f_i are the parameters of interest that are to be determined by the method of undetermined coefficients (See Appendix). A solution for those coefficients, if it exists, will be a function of the structural parameters of the model given by s, β and λ. Substituting the relevant derivatives of the payoff functions and equations 7.1) and 7.2) into 5), we can obtain a solution for the current prices (p_{it}, p_{jt}) as a linear function of the state variables (p_{it-1}, p_{jt-1}). This solution is given by the following pair of linear Markov strategies:

$$p_{it} = F_i + R_i p_{jt-1} + M_i p_{it-1} \quad \text{for } i, j = 1, 2 \text{ and } i \neq j \tag{8}$$

[13] Since the mathematical analysis involved is standard, we follow the lines as in Lapham and Ware (1994).

where the coefficients $F_i; R_i$ and M_i are functions directly and indirectly, through the unknown parameters of the derivatives given by 7.1) and 7.2), of the structural parameters of the model $(s; \beta$ and $\lambda)$: (See Appendix).

We can substitute optimal strategies given by 8), after taking into account for symmetry, into the Bellman's equations 4), and then differentiating 4) for the state variables $(p_{jt-1}; p_{it-1})$. Using the first order conditions 5) to apply the Envelope theorem, we obtain the following system of four Euler equations:

$$\frac{\partial V_{it}}{\partial p_{it-1}} = \frac{\partial \pi^*_{it}}{\partial p_{it-1}} + \left[\frac{\partial \pi^*_{it}}{\partial \hat{p}_{jt}} + \beta \frac{\partial V_{it+1}}{\partial \hat{p}_{jt}}\right] \frac{\partial \hat{p}_{jt}}{\partial p_{it-1}} \quad \text{for } i; j = 1; 2 \text{ and } i \neq j \tag{9.1}$$

$$\frac{\partial V_{it}}{\partial p_{jt-1}} = \frac{\partial \pi^*_{it}}{\partial p_{jt-1}} + \left[\frac{\partial \pi^*_{it}}{\partial \hat{p}_{jt}} + \beta \frac{\partial V_{it+1}}{\partial \hat{p}_{jt}}\right] \frac{\partial \hat{p}_{jt}}{\partial p_{jt-1}} \quad \text{for } i; j = 1; 2 \text{ and } i \neq j \tag{9.2}$$

where the profit functions are given by $\pi^*_{it}(\hat{p}_{it}(p_{it-1}; p_{jt-1}); \hat{p}_{jt}(p_{it-1}; p_{jt-1}); p_{it-1})$: Using equations 7.1) and 7.2) in the above Euler equations, and using the method of undetermined coefficients, we obtain ten non-linear equations in ten unknowns parameters $a_i; b_i; c_i; d_i; e_i$ and f_i for $i = 1; 2$ (see Appendix): If a solutions for these parameters exists, then the linear Markov perfect equilibrium given by 8) exists as well. Given the fact that we have to deal with ten non-linear equations, to simplify the mathematical tractability of the model, we restrict our analysis to a symmetric Markov perfect equilibrium.[14] This implies that from equations 7.1) and 7.2) we have $b_1 = b_2 = b; c_1 = c_2 = c; d_1 = d_2 = d; e_1 = e_2 = e$ and $f_1 = f_2 = f$: Thus, Markov strategies are also symmetric, that is, $F_1 = F_2 = F; R_1 = R_2 = R$ and $M_1 = M_2 = M$: Suppose that a solution for the unknown parameters exists, and denote this solution with $b^*; c^*; d^*; e^*$ and f^*: Using symmetry in the Markov strategies, and using the definitions given in the Appendix for the coefficients $F; R$ and M, we can solve for the symmetric steady state level of the system of first order difference equations given by 8):

$$\bar{p}_i^{mpe} = \frac{1}{2}\frac{1 + 2\beta b^*}{s - \beta(d^* + e^*)} \quad \text{for } i = 1; 2 \tag{10}$$

where we assume $s \neq \beta(d^* + e^*)$. The equilibrium in 10) is the steady-state linear Markov perfect equilibrium of our model. Note that when s tends to infinity the equilibrium prices tend to zero. In the next section we will show how to derive a solution for the unknown parameters $b^*; c^*; d^*; e^*$ and f^*. Then, we will consider in details the properties that the equilibrium given by 10) exhibits.

[14] Strategic symmetric behaviour by firms is a standard assumption in the literature on dynamic competition. See for example, Jun and Vives (2001), Driskill and McAfferty (1989) and Doganouglu (1999) among the others. This assumption can be seen as a natural consequence of the fact that firms face a symmetric dynamic problem. However, we are aware that the presence of ten non-linear equations implies that many equilibria, symmetric and not, can arise from our model.

3 Computation and Properties of the Markov Perfect Equilibrium

In this section we compute more in detail the linear Markov perfect equilibrium of our model. We start with the derivation of the Nash equilibrium of the static game associated with our model without adjustment costs.[15] This particular equilibrium represents a useful benchmark that can be compared with the Markov perfect and the open-loop equilibria of the model and it will be useful when will study the convergence properties of such equilibria when adjustment costs tend to zero. Assuming $\lambda = 0,$ the symmetric static equilibrium of the repeated game without adjustment costs, denoted by $p^s,$ is given by:

$$\bar{p}^s = \frac{1}{2s} \tag{11}$$

As we might expect, without adjustment costs, prices in equilibrium are functions of the measure of substitutability between the two goods. As for the Markov perfect equilibrium, when goods are perfectly homogeneous (s tends to infinity) static equilibrium prices are zero as in the classical Bertrand model.

In order to derive the properties of the Markov perfect equilibrium given by 10) we need to find a solution for the parameters of the value functions b^*, c^*, d^*, e^* and $f^*.$ Despite the analytical tractability of linear quadratic games, given the analytical difficulty to deal with the highly non-linear system of implicit functions, most of the results that we are going to present are obtained using numerical methods.[16] Numerical techniques are often used in dynamic games in state-space form, as in Judd (1990) or Karp and Perloff (1993). The presence of nonlinearity implies that we must expect many solutions for the unknown parameters, and thus, multiple Markov perfect equilibiria.[17] In order to reduce this multiplicity, we concentrate our analysis to symmetric Markov perfect equilibria that are asymptotically stable as in Driskill and McCafferty (1989) and Jun and Vives (2001). An asymptotically stable equilibrium is one where the equilibrium prices converge to a finite stationary level for every feasible initial condition.[18] The symmetric optimal Markov strategies are

[15] This particular equilibrium is important because it represents a subgame perfect equilibrium of the infinite repeated game without adjustment costs associated with our model.

[16] In particular, we use the Newton's method for nonlinear systems. The numerical analysis is performed with the use of the mathematical software Maple 7. For an introduction to the Newton's method, see Cheney and Kincaid (1999), Ch.3.

[17] Unfortunately, in literature, there are no general results on the uniqueness of Markov perfect equilibria in dynamic games. An interesting exception is Lokwood (1996) that provides sufficient conditions for uniqueness of Markov perfect equilibria in affine-quadratic differential games with one state variable.

[18] Given the lack of a transversality condition in the dynamic problem stated in Definition 1, we focus on asymptotically stable equilibria also because in this case we know that these equilibria will fulfil the tranversality condition even implicitly.

given by:

$$p_{it} = F + Rp_{jt-1} + Mp_{it-1} \quad \text{for } i; j = 1; 2 \text{ and } i \neq j \tag{12}$$

From these strategies we can derive the conditions for stability of the symmetric linear Markov perfect equilibrium :

Proposition 1 Assuming in 12) that $(2s + \lambda - \beta d) \neq (s + \beta e)$; then, this pair of strategies defines a stable equilibrium, if and only if, the eigenvalues of the symmetric matrix $A = \begin{pmatrix} M & R \\ R & M \end{pmatrix}$ are in module less than one. This implies that conditions for the stability of the system given by 12) are: $(d - e) < \frac{3s}{\beta}$ and $(d + e) < \frac{s}{\beta}$:

The result in Proposition 1 is derived from the theory of dynamic systems in discrete time.[19] The condition that $(2s + \lambda - \beta d) \neq (s + \beta e)$ assures that the parameters of the Markov strategies 12) are well defined (see Appendix). Thus, when we look for a solution of the system A3) defined in the Appendix, this solution should respect the conditions in Proposition 1. Furthermore, we impose that at that solution, equilibrium prices cannot be negative. Given these conditions, we now focus on the properties of our Markov perfect equilibrium when adjustment costs are positive and finite. We solve the system A3) in the Appendix for given values of the transportation cost, s, the discount factor β; and the measure of adjustment costs λ:[20] We consider only real solutions for the unknown parameters, and we consider only solutions that are locally isolated, or locally stable.[21] Thus, we construct an asymptotically stable Markov perfect equilibrium numerically instead of analytically as in Driskill and McCafferty (1989) and Jun and Vives (2001). This allows us to obtain a unique solution for the parameters $b^*; c^*; d^*; e^*$ and f^* and thus, a unique Markov perfect equilibrium for each set of values of the structural parameters of the model (s; β and λ): Given the difficult economic interpretation for these parameters, a sample of results is given in the Appendix. Here we report the implications of those solutions for the coefficients of the symmetric Markov strategies F; M and R: We are interested in particular on the coefficient R that represents the effect of one firm's today choice on the rival's choice tomorrow.

Proposition 2 For a plausible range of the structural parameters of the model s; β and λ, we have the following results at the stable solutions of the system A3):

[19] Driskill and McCafferty (1989) and Jun and Vives (2001) construct an asymptotically stable Markov equilibrium using similar conditions for differential equations, called Routh-Hurwitz conditions. For the case of a 2×2 system of difference equations, these conditions imply that $|\det(A)| < 1$ and $|\det(A) + 1| < tr(A)$; where tr is the trace of matrix A.

[20] Most of our results are based on a value of β close to 1, a value that we take from the numerical analysis of Karp and Perloff (1993).

[21] In practice, we want that at one particular solution, if we perturb that solution, the system of implicit functions A3) must remain close to zero.

1) the coe¢cients of the Markov strategies are always positive;

2) R is …rst increasing and then decreasing in ¸: The higher s the higher the persistence of the increasing phase;

3) R is decreasing in s when adjustment costs are small. R …rst increases and then decreases in s

when adjustment costs become large: The higher ¸ the higher the persistence of the increasing

phase;

4) we have always M > R, however, the di¤erence (M ¡ R) is decreasing in s;

5) the coe¢cient M is increasing in ¸ and decreasing in s;

The statements in Proposition 2 are summarised in the following table, in which we report a sample of the results of our numerical analysis made on system A3.

Table 1. Numerical values for the coe¢cients of Markov strategies.

	s = 0:5; ¯ = 0:95			s = 1; ¯ = 0:95			s = 10; ¯ = 0:95		
¸	F	M	R	F	M	R	F	M	R
0.1	0.924	0.104	0.044	0.480	0.057	0.0265	0.049	0.006	0.003
0.5	0.704	0.313	0.099	0.414	0.204	0.0757	0.049	0.031	0.015
0.8	0.567	0.401	0.111	0.374	0.275	0.0919	0.048	0.047	0.022
1	0.541	0.446	0.115	0.352	0.313	0.0991	0.048	0.058	0.026
2	0.362	0.588	0.117	0.270	0.446	0.1154	0.046	0.104	0.044
5	0.174	0.757	0.093	0.154	0.633	0.1145	0.041	0.204	0.075
10	0.092	0.852	0.064	0.087	0.757	0.0931	0.033	0.313	0.099

In Proposition 2, we have identi…ed the strategic behaviour of …rms when adjustment costs are positive and …nite. Not surprisingly, we have that the optimal price choice of a …rm is an increasing function in the rival's price and this e¤ect is captured by the coe¢cient R: Thus, the strategic behaviour of the …rms in our model, is characterised by intertemporal strategic complementarity. Furthermore, the size of the e¤ect of strategic complementarity as a function of ¸ follows a bell-shaped curve. For small adjustment costs, it increases, while, when adjustment costs become higher, it decreases until it reaches zero as ¸ approaches in…nity. This is because, the bene…ts of increasing prices today to a¤ect rival's prices tomorrow, are more than o¤set by the costs of adjusting prices. However, results 2) and 5) in the above Proposition say that this relationship depends on the measure of product di¤erentiation s: When s is high, competition between …rms becomes …ercer, since small changes in prices can have huge e¤ects on consumer's demand. In this case, to sustain a credible strategy of increasing prices it is necessary to have a relatively higher level of adjustment costs than in the case in which s is small and …rms have higher market power. Finally, result 4) in Proposition 2) says that as s increases, …rms tend to assign a relatively higher weight to the rival's price in their strategic behaviour than in the case in which products are more differentiated. From Proposition 2, we can identify the effects of positive adjustment costs on the steady-state Markov perfect equilibrium of our model. The main result is stated in the following Proposition:

Proposition 3 When adjustment costs are positive and finite, prices at the symmetric steady state Markov perfect equilibrium are always higher than prices in the equilibrium of the repeated static game. This is true, even though, no adjustment costs are paid in the steady state.

The equilibrium values of the Markov perfect and the corresponding static Nash equilibrium for the same sample used in Table 1 are reported in the following table.

Table 2. Comparison between the symmetric Markov perfect and the counterpart static Nash equilibrium.

	$s = 0.5; \beta = 0.95$		$s = 1; \beta = 0.95$		$s = 10; \beta = 0.95$	
λ	$\overline{p}^{mpe}$	$\overline{p}^{s}$	$\overline{p}^{mpe}$	$\overline{p}^{s}$	$\overline{p}^{mpe}$	$\overline{p}^{s}$
0.1	1.086	1	0.525	0.5	0.0503	0.05
0.5	1.199	1	0.575	0.5	0.0513	0.05
0.8	1.225	1	0.592	0.5	0.0521	0.05
1	1.234	1	0.599	0.5	0.0525	0.05
2	1.233	1	0.615	0.5	0.0543	0.05
5	1.168	1	0.609	0.5	0.0570	0.05
10	1.106	1	0.582	0.5	0.0573	0.05

From Proposition 3 we can say that in our dynamic game, the presence of adjustment costs and the hypothesis of Markov strategies create a strategic incentive for the firms to deviate from the equilibrium of the repeated static game even if adjustment costs are not paid in steady state. The result is a less competitive behaviour by the firms in the Markov equilibrium than in the static case. The economic force behind this result is the presence of intertemporal strategic complementarity that we have analysed above. A firm by pricing high today will induce high prices from the rival tomorrow, and the cost of adjusting prices lends credibility to this strategy, since it is costly to deviate from that strategy. The result in Proposition 3) contrasts with the one found in dynamic competition models with sticky prices, where general price stickiness creates a more competitive outcome in the subgame perfect equilibrium than in the static Nash equilibrium. The main reason is that in our model, price rigidity is modelled directly in the cost functions of the firms and there is a credibility effect associated with adjustment costs that can sustain high equilibrium price strategies, while, this credibility effect is absent with general price stickiness. We can notice that as products become close substitute (s increases), the higher is the level of adjustment costs the higher is the difference between $\overline{p}^{mpe}$ and $\overline{p}^{s}$ relatively to the case where s is small. The intuition is the same as the one described above to explain results 2) and 3) of Proposition 2). When s is high, firms have a strong incentive to reduce their prices of a small amount in order to capture additional demand. Large adjustment costs can offset this incentive because the credibility of a high price equilibrium strategy will be stronger. Obviously, we know already from Proposition 2) that this credibility effect is not increasing monotonically with λ, because as adjustment costs become extremely high, the coefficient R tends to zero. We shall analyse this aspect in more detail when we will consider the properties of the Markov perfect equilibrium in the limit game. The result in Proposition 3) states that when adjustment costs are positive, firms are better off in the steady state Markov perfect equilibrium than in the static Nash equilibrium of the repeated game. Thus, the presence of positive adjustment costs can induce a tacit collusive behaviour by firms. However, this is true only if products are not homogeneous, that is $s < \infty$. If products are perfect substitute, we already know that the Markov perfect equilibrium given by 10) converges to the static Nash equilibrium independently of the value of λ. Moreover, in this case, both equilibria are equal to zero and we fall into the classical "Bertrand paradox".

3.1 Markov Perfect Equilibrium in the Limit Game

We now consider the properties of the Markov perfect equilibrium of our model in different limit cases. Differently from previous section, the results we are going to show are obtained analytically. We start our analysis evaluating the steady state Markov perfect equilibrium given by 10) in two limit cases, when adjustment costs tend to zero ($\lambda \to 0$), and when they are infinite ($\lambda \to \infty$). This allows us to define the convergence properties of the steady-state Markov perfect equilibrium given by 10) toward the static Nash equilibrium 11), that, as we know from the introduction, is an important issue for the literature on dynamic competition. In order to describe the behaviour of the equilibrium in 10) in the two limit games, we need to evaluate the system A3) in the Appendix in the two extreme assumptions on the adjustment costs parameter λ. The results for our limit games are the following:

Proposition 4 When adjustment costs tends to zero ($\lambda \to 0$), the symmetric solution for the unknown parameters is the following: $b^* = c^* = d^* = e^* = f^* = 0$, and the steady-state Markov perfect equilibrium corresponds with the static Nash equilibrium of the repeated game. When adjustment costs tend to infinity ($\lambda \to \infty$), the symmetric solution for the unknown parameters is: $b^* = \frac{1}{2}$, $c = 0$, $d^* = -2s$, $e^* = s$, $f^* = 0$, and the steady-state Markov perfect equilibrium converges to the static Nash equilibrium given by 10).

Proof (see Appendix). The convergence of the steady state Markov perfect equilibrium toward the static Nash equilibrium when adjustment costs tend to zero, is the same result as in Lapham and Ware (1994), and it contradicts the results found in continuous time models of dynamic competition. In our discrete time model, as in Lapham and Ware (1994), the discontinuity found in continuous time models, when adjustment costs tend to zero, disappears. This result is supported by the numerical analysis developed by Karp and Perloff (1993), that gives also a possible explanation of why discrete and continuous time behave differently in the limit case of zero adjustment costs. In their model, they show that the steady-state Markov perfect equilibrium becomes more sensitive to λ when we pass from discrete time to continuous time. On the other hand, the second result is common in discrete and continuous time models, like in Maskin and Tirole (1987) and Reynolds (1991). The economic intuition behind this result is that: when adjustment costs become very large, the extra costs to change prices strategically outweight the benefits, thus, firms are induced to behave nonstrategically and the steady-state subgame perfect equilibrium converges to the static solution. Of course, the case with infinite adjustment costs is not very practical, especially for our purpose, since we are dealing with price adjustment costs that are normally associated with a small value of λ: However, this limit result is important because it shows that our value functions are continuous at infinity[22]. Finally, we consider what happens to the steady state Markov perfect equilibrium when β tends to zero, that is, when only present matters for firms. Fershtman and Kamien (1987), using a continuous time model with sticky prices, found that when only present matters for firms their stationary Markov perfect equilibrium converges to the competitive outcome. On the other hand, Driskill and McCafferty (1989), using a similar model but with adjustment costs in quantities, found that the Markov perfect equilibrium converges to the static Nash equilibrium of the repeated game without adjustment costs. In our model, we have the following result:

Proposition 5 As β; the discount factor, tends to zero, the steady state Markov perfect equilibrium given by 10) tends to the static Nash equilibrium of the repeated game without adjustment costs.

The proof of this Proposition can be easily seen taking the limit of the Markov perfect equilibrium in 10) for $\beta \to 0$: In this case, it is simple to see that the result is the static Nash equilibrium given by 11).

As we might expect, if future does not matter for firms, the result is the equilibrium of the one shot game. We obtain a result similar to the one of Driskill and McCafferty (1989), however, if we allow products to be homogeneous our equilibrium converges to the competitive outcome as in Fershtman and Kamien (1989), that, in our case, implies equilibrium prices equal to zero.

4 The Open-Loop Equilibrium

In this section we will analyse what are the effects of adjustment costs on equilibrium price if we force firms to behave using open-loop strategies. While Markov strategies are feedback rules, open-loop strategies are trajectory, or path, strategies. In

[22] See Jensen and Lokwood (1998) for an analysis of discontinuity of value functions in dynamic games.

particular, open-loop strategies are functions of the initial state of the game (that is known a priori) and of the calendar time. Markov perfect and open-loop strategies correspond to extreme assumptions about player's capacities to make commitments about their future actions. Under the open-loop information pattern, the period of commitment is the same as the planning horizon, that in our case, is infinite. That is, at the beginning of the game, each player must make a binding commitment about the actions it will take at all future dates. Then, in general terms, a set of open-loop strategies constitutes a Nash equilibrium if, for each player, the path to which they are committed is an optimal response to the paths to which the other players have committed themselves. An open-loop strategy for player i is an infinite sequence $p_i^{ol}(p_{i0}, p_{j0}, t) = \{p_{i1}, p_{i2}, ..., p_{it}, ...\} \in \Re^{\infty}$, specifying the price level at every period t over an infinite horizon as a function of the initial price levels (p_{i0}, p_{j0}) and the calendar time. Formally, we define an open-loop equilibrium in the following way:

Definition 2 A pair of open-loop strategies (p_1^{ol}, p_2^{ol}) constitutes an open-loop equilibrium of our game if and only if the following inequalities are satisfied for each player:

$$\Pi_i(p_i^{ol}, p_j^{ol}) \geq \Pi_i(p_i, p_j^{ol}) \qquad \text{with } i, j = 1, 2 \text{ and } i \neq j$$

Typically these equilibria are not subgame perfect by definition, then, they may or not may "time consistent". There are examples of dynamic games in which open-loop equilibria are subgame perfect[23], but in general when closed-loop strategies are feasible, subgame perfect equilibria will typically not be in open-loop strategies. In order to solve for the steady state open-loop equilibrium, we need to use the Pontryagin's maximum principle of the optimal control theory, since it can be shown that there is a close relationship between derivation of an open-loop equilibrium and solving jointly different optimal control problems, one for each player[24].

Proposition 6 The steady-state open-loop equilibrium is in correspondence one-to-one with the Nash equilibrium of the counterpart static game.

Proof. We need to solve a joint optimal control problem for both firms. The Hamiltonians are[25]:

$$H_i = \beta^t \pi_{it}(p_{it}, p_{jt}, p_{it-1}) + \lambda_{ii}(t)(p_{it} - p_{it-1}) + \lambda_{ij}(t)(p_{jt} - p_{jt-1}) \quad \text{with } i, j = 1, 2 \text{ and } i \neq j \tag{13}$$

[23] Cellini and Lambertini (2001.b) show that in a differential oligopoly game with capital accumulation, Markov perfect and open-loop equilibria are the same if the dynamic of the accumulation takes the form a' la Nerlove-Arrow or a' la Ramsey.

[24] For a detailed analysis, see Basar and Olsder (1995), Ch.6.

[25] A formalization of the Maximum Principle in discrete time can be found in Leonard and Van Long (1998), Ch. 4.

The corresponding necessary conditions for an open-loop solution are:

$$\frac{\partial H_i}{\partial p_{it}} = \delta^t \frac{\partial \pi_{it}(p_{it}, p_{jt}, p_{it-1})}{\partial p_{it}} + \mu_{ii}(t) = 0$$

$$\mu_{ii}(t) - \mu_{ii}(t-1) = -\frac{\partial H_i}{\partial p_{it-1}} = -\delta^t \frac{\partial \pi_{it}(p_{it}, p_{jt}, p_{it-1})}{\partial p_{it-1}} + \mu_{ii}(t)$$

$$\mu_{ij}(t) - \mu_{ij}(t-1) = -\frac{\partial H_i}{\partial p_{jt-1}} = \mu_{ij}(t) \tag{14}$$

in steady state we have $\mu_{ii}(t) = \mu_{ii}(t-1)$, $\mu_{ij}(t) = \mu_{ij}(t-1)$, $p_{it} = p_{it-1}$ and $p_{jt} = p_{jt-1}$. Thus, from the last two conditions we obtain $\mu_{ii}(t) = \mu_{ij}(t) = 0$. Using this results, we can see that in steady state, the initial problem reduces to the static maximization problem ($\frac{\partial \pi_{it}(p_{it}, p_{jt})}{\partial p_{it}} = 0$). Q.E.D.

Thus, there is a direct correspondence between the steady state open-loop and the static Nash equilibrium of our model. In a stationary open-loop equilibrium there are no strategic incentives to deviate from the static outcome of the model without adjustment costs. The main reason is that open-loop strategies are independent on state variables and then, there is no way to affect rival's choice tomorrow changing strategy today as in Markov strategies. The same result has been found by Driskill and McCafferty (1989) and Jun and Vives (2001) using different models but it differs from the one in Fershtman and Kamien (1987) and Cellini and Lambertini (2001.a), since in their models, the open-loop solution implies higher output than the static solution, and they coincide only when the price level can adjust instantaneously. This difference is mainly due to the different specification of the transition law attached to the costate variables in the Hamiltonian system. In our model, as in Driskill and McCafferty(1989) and Jun and Vives (2001), this transition law is simply the definition of first difference in prices,[26] while in Fershtman and Kamien (1987) and Cellini and Lambertini (2001.a), this transition law is the difference between current price level and the price on the demand function for each level of output. Finally, from a regulation point of view, if it could be possible to force firms to behave according to open-loop strategies it would be possible to increase the level of competition in equilibrium also with the presence of positive adjustment costs.

5 Conclusion

In this paper we have developed a dynamic duopoly model of price competition over an infinite horizon, with symmetric and convex price adjustment costs and spatially

[26] Obviously, in Driskill and McCafferty (1989) and Jun and Vives (2001) the transition law is the time derivative of prices.

di¤erentiated products. We have concentrated our analysis on the strategic interaction between …rms in a linear quadratic di¤erence game using two di¤erent equilibrium concepts: Markov perfect equilibrium, that has the property of being subgame perfect, and the open-loop equilibrium, that is normally not subgame perfect. Given the existence of adjustment costs in prices, in the steady state Markov perfect equilibrium there is a strategic incentive for …rms to deviate from the repeated static Nash solution even if no adjustment costs are paid in equilibrium. In particular, we have shown that the steady state price equilibrium, and then, the level of competition, is lower in the stationary Markov equilibrium than in the counterpart static Nash solution without adjustment costs. The economic force behind this result is the presence of intertemporal strategic complementarity. A …rm by pricing high today will induce high prices from the rival tomorrow. Moreover, the presence of adjustment costs leads to credibility in strategies that imply higher prices in equilibrium, for each …rm, than in the case of static Nash equilibrium. This implies that …rms are better o¤ when adjustment costs are positive and they behave according to Markov strategies and that the presence of these adjustment costs can sustain collusive behaviour. However, this is true only if products are not homogeneous. If products are perfect substitutes, the steady state Markov perfect equilibrium always coincide with the static Nash equilibrium of the repeated game without adjustment costs and we fall into the classical "Bertrand paradox". The incentive to deviate from the static equilibrium is absent once we consider open-loop strategies. Indeed, the stationary open-loop equilibrium of our model is always in correspondence one-to-one with the Nash equilibrium of the static game. In addition, we have shown that when adjustment costs tend to zero or to in…nity, the limit of our Markov perfect equilibrium converges to the static Nash equilibrium. The former result, con…rmed by Lapham and Ware (1994), seems to be peculiar of discrete time models, since in continuous time models there is a discontinuity in the limit of the Markov perfect equilibrium as adjustment costs tend to zero. A number of extensions can be made in our analysis. Linear quadratic games do not perform well when uncertainty is considered. These particular models allow for speci…c shocks in the transition equation but they cannot deal, for instance, with shocks to the demand function. A natural extension of our analysis could be to relax the hypothesis of linearity of the strategies to allow for demand uncertainty. Another possible extension could be the analysis of the e¤ects of asymmetric adjustment costs between …rms, since this could give rise to a possible set of asymmetric steady state outcomes.

Appendix

In this section we derive the equations for the unknown parameters of the value functions $b_i; c_i; d_i; e_i$ and f_i that are to solved to compute a Markov perfect equilibrium in our model. We start with the solution of the two first-order conditions given by 5) in the paper. Using the derivatives of the value functions 7.1) and 7.2) into 5) and calculating the relevant derivatives of the payoff functions, we obtain the following system of equations:

$$p_{it} = A_i + B_i p_{jt} + C_i p_{it-1}; \quad \text{for } i; j = 1; 2 \text{ and } i \neq j: \tag{A1}$$

where the coefficients $A_i; B_i$ and C_i have the following functional form:

$$A_i = \frac{\rho + \beta b_i}{(2s + \lambda - \beta d_i)}; \qquad B_i = \frac{(s + \beta e_i)}{(2s + \lambda - \beta d_i)}; \qquad C_i = \frac{\lambda}{(2s + \lambda - \beta d_i)}$$

and where we assume that $2s + \lambda \neq \beta d_i$:

We can solve the system A.1) for p_{it} as a function of the state variables p_{it-1} and p_{jt-1}, for $i; j = 1; 2$ and $i \neq j$: The solution is the pair of linear Markov strategies given by 8) in the paper that we report here for simplicity of exposition:

$$\tilde{p}_{it} = F_i + R_i p_{jt-1} + M_i p_{it-1} \quad \text{for } i; j = 1; 2 \text{ and } i \neq j$$

where

$$F_i = \frac{A_i}{1 - B_i}; \quad R_i = \frac{B_i C_i}{(1 - B_i^2)}; \quad M_i = \frac{C_i}{(1 - B_i^2)}; \quad \text{for } i; j = 1; 2 \text{ and } i \neq j \tag{A2}$$

and where we assume $B_i \neq 1$; that implies that $(2s + \lambda - \beta d) \neq (s + \beta e)$:

Using 7) into the Euler equations 9), and then using 8), we can obtain four equations that depend only on the state variables, p_{it-1}; with $i = 1; 2$: Rearranging and matching the coefficients associated with the state variables as well as for the various constant terms, we obtain the following non-linear system of ten implicit equations in ten unknowns, $b_i; c_i; d_i; e_i$ and f_i :

$$0 = s(R_j - M_i)F_i - b_i + M_j \left[\frac{1}{2} - s(F_i - F_j)\right] - \lambda (M_i - 1) F_i + \beta R_j [F_i (s + e_i + f_i) + c_i]$$

$$0 = s(M_j - R_i)F_i - c_i + \frac{1}{2}R_j - \lambda (F_j -_j) + \beta R_i \left[\frac{1}{2} - F_i (2s + \lambda + d_i) + F_j (s + e_i) + b_i\right]$$

$$0 = s(R_j - M_i)M_i - d_i - sM_j[M_j - R_j] - \lambda(M_i - 1)^2 + \beta R_j[M_i(s + e_i) + f_i R_j]$$

$$0 = s(R_j - M_i)R_i - e_i - sM_j[R_i - M_j] - \lambda(M_i - 1)R_i + \beta R_j[R_j(s + e_i) + f_i M_i]$$

$$0 = s(R_i - M_j)R_i - f_i - sR_i(R_i - M_j) - \lambda R_i^2 + \beta R_i[M_j(s + e_i) - R_i(2s + \lambda - d_1)] \tag{A3}$$

where for all the equations we have that $i, j = 1, 2$ and $i \neq j$. Imposing symmetry, that is $b_1 = b_2 = b$, $c_1 = c_2 = c$, $d_1 = d_2 = d$, $e_1 = e_2 = e$, $f_1 = f_2 = f$, $F_1 = F_2 = F$, $R_1 = R_2 = R$ and $M_1 = M_2 = M$, implies that the steady state associated with the system in 8) is given by:

$$p_i^{mpe} = \frac{F}{1 - (M + R)} \tag{A4}$$

and substituting the definitions for the coefficients F, R and M given above, we find equation 10) in the paper.

We use numerical techniques to solve the system A3). In particular, we adopt the iterative Newton's method. To solve system A3), we first define different set of values for the structural parameters of the model, $(s, \lambda$ and $\beta)$. We evaluate the system A3) for these values and then we look for the corresponding solution for the parameters b, c, d, e and f using the Newton algorithm. Using the restrictions described in the paper we are able to obtain a unique solution for any set of values of the structural parameters. In our analysis the value of β is fixed to 0.95 as in Karp and Perloff (1993). The parameters s and λ varies from 0 to 1000 with different length of variation. A sample of the results is reported in the following tables:

Table A1). Solution for the parameters b, c, d, e and f given values for s, λ and β

	$s = 0.5; \beta = 0.95$					$s = 1; \beta = 0.95$				
λ	b	c	d	e	f	b	c	d	e	f
0.1	0.129	0.049	-0.084	0.006	0.0024	0.072	0.02	-0.09	0.004	0.001
0.5	0.373	0.125	-0.286	0.062	0.0162	0.252	0.09	-0.35	0.052	0.016
1	0.488	0.146	-0.423	0.127	0.0248	0.373	0.12	-0.57	0.125	0.032
2	0.552	0.139	-0.567	0.219	0.028	0.486	0.13	-0.84	0.255	0.049
5	0.554	0.097	-0.740	0.341	0.019	0.556	0.10	-1.22	0.501	0.054
10	0.535	0.063	-0.842	0.409	0.010	0.551	0.07	-1.48	0.683	0.038
25	0.516	0.031	-0.927	0.461	0.003	0.528	0.04	-1.73	0.851	0.016

Table A2). Solution for the parameters b, c, d, e and f given values for s, λ and β

	$s = 10; \beta = 0.95$					$s = 100; \beta = 0.95$				
λ	b	c	d	e	f	b	c	d	e	f
0.1	0.008	0.003	-0.09	0.0005	0.0002	0.0008	0.0003	-0.09	0.00005	0.00002
0.5	0.038	0.015	-0.47	0.0118	0.0046	0.0041	0.0016	-0.49	0.0013	0.0005
1	0.072	0.027	-0.90	0.0416	0.0159	0.0081	0.0032	-0.98	0.0053	0.0021
2	0.12	0.045	-1.68	0.134	0.0487	0.016	0.0063	-1.95	0.0206	0.0082
5	0.24	0.028	-3.53	0.526	0.165	0.0383	0.0118	-4.75	0.118	0.0465
10	0.33	-0.19	-5.72	1.254	0.325	0.071	-0.014	-9.09	0.416	0.159
25	0.30	-1.45	-8.28	3.100	0.536	0.104	-0.816	-20.3	1.913	0.676

Proof of Proposition 4. First of all, we impose symmetry in the system A3). Then, we consider the case in which $\lambda \to 0$. Evaluating the term C at $\lambda = 0$ we clearly obtain that $C_{(\lambda=0)} = 0$, and using this fact into the definitions of R and M, we have that $R_{(\lambda=0)} = 0$; $M_{(\lambda=0)} = 0$. Thus, irrespective of the value of F, using $R_{(\lambda=0)} = 0$; $M_{(\lambda=0)} = 0$ into system A3) we have: $b^* = c^* = d^* = e^* = f^* = 0$. Furthermore, as in Lapham and Ware (1994), the Jacobian matrix associated with that system is an identity matrix, and thus, non singular. This implies, that at λ close to zero, the solution for the unknown parameters is a continuous function of λ. Substituting $b^* = d^* = e^* = 0$ into 10) in the paper, we see that the Markov perfect equilibrium becomes equal to $\frac{1}{2s}$; that is the static Nash equilibrium given by 11). Now consider the case in which $\lambda \to \infty$. Taking the limit for $\lambda \to \infty$ of the coefficients A, B and C, we obtain $\lim_{\lambda\to\infty} A = 0$; $\lim_{\lambda\to\infty} B = 0$ and $\lim_{\lambda\to\infty} C = 1$. Applying the well known rules for limits, we have that $\lim_{\lambda\to\infty} F = 0$; $\lim_{\lambda\to\infty} R = 0$ and $\lim_{\lambda\to\infty} M = 1$. Thus, taking the limit for $\lambda \to \infty$ of the implicit functions in the system A3) gives the following results for the unknown parameters: $b^* = 1/2$; $c^* = 0$; $d^* = -2s$; $e^* = s$ and $f^* = 0$. Again, substituting this fact into the Markov perfect equilibrium given by 10) we can see that it coincides with the static Nash equilibrium 11). Obviously, we can obtain the same limit result for the Markov perfect equilibrium, taking the limit for $\lambda \to \infty$ of A4) and applying the Hospital's rule.

References

[1] Akerlof, G. and J. Yellen (1985), "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?", American Economic Review 75, pp. 708-721.

[2] Amir, R. (2001), "Stochastic Games in Economics and Related Fields: an Overview", CORE Discussion Paper 2001/60.

[3] Basar, T. and G.J. Olsder (1995), Dynamic Noncooperative Game Theory, Academic Press: New York.

[4] Cellini, R. and L. Lambertini (2001.a), "Dynamic Oligopoly with Sticky Prices: Closed-Loop, Feedback and Open-Loop Solutions", Working Paper, Dipartimento di Scienze Economiche, Universita' degli Studi di Bologna.

[5] Cellini, R. and L. Lambertini (2001.b), "Di¤erential oligopoly Games where the Closed-Loop Memoryless and the Open-Loop Equilibria Coincide", Working Paper, Dipartimento di Scienze Economiche, Universita' degli Studi di Bologna.

[6] Cheney W. and D. Kincaid (1999), Numerical Mathematics and Computing, Brooks/Cole Publishing, Paci…c Grove, CA.

[7] Doganouglu, T. (1999), "Dynamic Price Competition with Persistent Consumer Tastes", University of Bonn, mimeo.

[8] Driskill R.A. and S. McCa¤erty (1989), "Dynamic Duopoly with Adjustment Costs: A Di¤erential Game Approach", Journal of Economic Theory, vol. 49, pp. 324-338.

[9] Fershtman, C. and M. Kamien (1987), "Dynamic Duopolistic Competition with Sticky Prices", Econometrica, vol. 55, no. 5, pp. 1151-1164.

[10] Fudenberg, D. and J. Tirole (1984), "The Fat-Cat E¤ect, the Puppy-Dog ploy, and the Lean and Hungry Look", American Economic Review, Papers and Proceedings 74, pp. 361-389.

[11] Fudenberg, D. and J. Tirole (1991), Game Theory, The MIT Press: Cambridge, MA.

[12] Jensen, H. and B. Lokwood (1998), "A Note on Discontinuous Value Functions and Strategies in A¢ne-Quadratic Di¤erential Games", Economic Letters 61, pp. 301-306.

[13] Judd, K.L. (1989), "Cournot vs. Bertrand: A Dynamic Resolution", University of Stanford, mimeo.

[14] Jun, B. and X. Vives (1999), "Strategic Incentive in Dynamic Duopoly", mimeo.

[15] Karp, L. and J. Perlo¤ (1993), "Open-loop and Feedback Models of Dynamic Oligopoly", International Journal of Industrial Organization, vol. 11, pp. 369-389.

[16] Kreps, D. and J. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", Bell Journal of Economics, vol.14, pp.326-337.

[17] Lapham, B. and R. Ware (1994), "Markov Puppy Dogs and Related Animals", International Journal of Industrial Organization, vol. 12, pp. 569-593.

[18] Leonard D. and N. Van Long (1998), Optimal Control Theory and Static Optimization in Economics, Cambridge University Press: Cambridge, UK.

[19] Lockwood, B. (1996), "Uniqueness of Markov Perfect Equilibrium in Infinite Time Affine-Quadratic Differential Games", Journal of Economics Dynamics and Control, vol.20, pp. 751-765.

[20] Mankiw, G. (1985), "Small Menu Costs and Large Business Cycles: Macroeconomic Model of Monopoly", Quarterly Journal of Economics 100, pp. 529-539.

[21] Maskin, E. and J. Tirole (1987), "A Theory of Dynamic Oligopoly III", European Economic Review 31, pp. 947-968.

[22] Maskin, E. and J. Tirole (1988), "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles", Econometrica, vol.56, no. 3, pp. 571-599.

[23] Maskin, E. and J. Tirole (2001), "Markov Perfect Equilibrium: I. Observable Actions", Journal of Economic Theory, vol.100, pp. 191-219.

[24] Papavassilopoulos G.P., J.V. Medanic, and J.B. Cruz (1979), "On the Existence of Nash Strategies and Solutions to Coupled Riccati Equations in Linear-Quadratic Games", Journal of Optimization Theory and Applications, vol.28, no. 1, pp.49-76.

[25] Piga, C. (2000), "Competition in a Duopoly with Sticky Prices and Advertising", International Journal of Industrial Organization 18, pp. 595-614.

[26] Reinganum, J. and N. Stokey (1985), "Olygopoly Extraction of a Common Property Natural Resource: The Importance of the Period of Commitment in Dynamic Games", International Economic Review, vol. 26, pp. 161-173.

[27] Reynolds, S.S. (1991), "Dynamic Oligopoly with capacity Adjustment Costs", Journal of Economic Dynamics and Control, vol. 15, no. 3, pp. 491-514.

[28] Rotemberg, J.J. (1982), "Sticky Prices in the United States", Journal of Political Economy, vol. 90, no. 6, pp. 1187-1211.

[29] Slade, M.E. (1999), "Sticky Prices in a Dynamic Oligopoly: An Investigation of $(s; S)$ Thresholds", International Journal of Industrial Organization, vol. 17, pp. 477-511.

[30] Tirole, J. (1988), The Theory of Industrial Organization, MIT Press: Cambridge, MA.

[31] Zeeuw,A. and J. Van der Ploeg (1991), "Di¤erence Games and Policy Evaluation: A Conceptual Framework", Oxford Economic Papers, vol. 43, pp. 612-636.