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Verlinde gravity effects on the orbits of the planets and the Moon in the Solar System

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Abstract

In this work, we address the effects of a phenomenon known as Verlinde gravity. Here we show that its effect over the planets and the Moon in our solar system is quite negligible. We find that the Verlinde gravity effects on the orbits of planets are at least 10 times smaller than the precision with which we can determine the Sun's mass, and the one on the orbit of the Moon is about 100 times smaller than the precision with which we can determine the Earth's mass. These results let us infer that statements in the literature that Verlinde gravity is ruled out by the observed motion of planets in our solar system aren't correct.

1 Introduction

In 2010, Verlinde proposed what he called “entropic gravity” or “emergent gravity”. This concept states that gravity is an emergent phenomenon due to the entropy change as objects change their distance [1]. It attracted considerable attention in theoretical particle physics community, when the work was first published. He showed how Newton's universal law of gravitation and general relativity can be derived from entropic gravity. In 2016, Verlinde kept developing his theory further and successfully explained Tully-Fisher relation [2]. In other words, his new theory made different predictions than the Newtonian gravity or general relativity. The strict area law for entanglement entropy implies general relativity. However, he suggested that there is a non-zero volume law contribution to the entanglement entropy due to the thermal excitations responsible for the de Sitter entropy. These excitations constitute the positive dark energy that accelerates our Universe, which is similar to de Sitter space. However, his equations are not immediately applicable to more wide range of phenomena, as they are only valid in the presence of spherical symmetry. Thus, in another paper [3] (in

preparation), one of us obtained an expression that is valid in general cases that have no spherical symmetry. We use that expression in this work to show that some claims stating that Verlinde gravity is ruled out by the observed motion of the planets in our solar system are false.

The organization of this paper is as follows. In Section 2, we explain Verlinde gravity. In Section 3, we discuss the strategy developed to calculate the Verlinde gravity effects on the planets in the subsequent sections. In Section 4, we calculate how the Verlinde gravity affects the gravitational forces exerted on the planets. In particular, we will see that the gravitational force on a planet increases by a factor $(1 + \lambda)$, where λ is a constant that differs from planet to planet. Therefore, a non-zero λ can be interpreted as the difference between the gravitational mass and the inertial mass. We will also see that the value of λ depends on the radial density profile of a planet. In Section 5, we will calculate λ for each planet. In Section 6, we will compare the orbit of planets with λ obtained in Section 5. In Section 7, we present our conclusions.

2 Verlinde gravity

Verlinde gravity [2, 3] is given by

$$g = \sqrt{g_B^2 + g_D^2}, \quad (1)$$

where $g = |\vec{g}|$ is the total gravity, $g_B = |\vec{g}_B|$ is the gravity due to the baryonic (i.e., visible) matter, and $g_D = |\vec{g}_D|$ is the gravity due to the apparent dark matter. From the form of the above equation, it is tempting to believe that \vec{g}_B and \vec{g}_D are perpendicular. However, the direction of \vec{g} is the same as the directions of \vec{g}_B and \vec{g}_D . (\vec{g}_B and \vec{g}_D are parallel.) Let's give a brief interpretation of 1. The gravitational energy is proportional to g^2 , and the above equation says that the total gravitational energy is the sum of the gravitational energy due to the visible matter and the one due to the apparent dark matter [3]. Verlinde constructed Verlinde gravity by an analogy with elastic matter, and one can interpret the gravitational energy as the elastic energy of strained matter.

In Eq. (6.11) of [2], Verlinde relates what he calls Σ_{Di} (“surface mass density vector” of apparent dark matter) with g_{Di} (the gravity due to the “apparent” dark matter) as follows:

$$\Sigma_{Di} = - \left(\frac{d-2}{d-3} \right) \frac{g_{Di}}{8\pi G}, \quad (2)$$

where the subscripts i denote the i th vector component. He noted that one could avoid annoying factors by working with Σ_D instead of g_D . But here, we need to restore them. Plugging $d = 4$, we get

$$\Sigma_{Di} = - \frac{2g_{Di}}{8\pi G}. \quad (3)$$

In Eq. (7.37) of [2], Verlinde relates Σ_D with the baryonic variables as follows:

$$\left(\frac{8\pi G}{a_0} \Sigma_D \right)^2 = \left(\frac{d-2}{d-1} \right) \nabla_i \left(\frac{\Phi_B}{a_0} n_i \right), \quad (4)$$

where

$$a_0 = cH_0, \quad \vec{n} = \frac{\vec{g}_B}{g_B}, \quad (5)$$

being H_0 Hubble's constant, $n_i(\vec{n})$ the direction of gravity, and Φ_B is a potential similar to the Newtonian potential; while the Newtonian potential is related to the time time component of the metric, Φ_B is related to the space space component of the metric. In [3], one of us showed that

$$\Phi_B = \frac{2g_B}{\nabla_i n_i}. \quad (6)$$

As an aside, in the presence of spherical symmetry, we have

$$\Phi_B = -rg_B = -\frac{GM(r)}{r}, \quad (7)$$

where $M(r)$ is the mass inside the sphere with radius r . In contrast, the Newtonian potential Φ_N is given by

$$\Phi_N = \int_{\infty}^r g_B(r') dr' = \int_{\infty}^r \frac{GM(r')}{r'^2} dr'. \quad (8)$$

The two potentials coincide when there is a point mass in the origin and no mass elsewhere. Note that Φ_B does not have any a_0 factor, because it is purely "non-Verlinde gravitational" quantity, even though Verlinde gravity uses it. Tolman-Oppenheimer-Volkoff calculated Φ_B in the presence of spherical symmetry, and obtained exactly (7). However, to authors' knowledge, nobody suggested (6) which is our basis of calculation in the absence of spherical symmetry.

Plugging $d = 4$ and (3), we get

$$\left(\frac{8\pi G}{a_0} \left(\frac{-2g_{Di}}{8\pi G} \right) \right)^2 = \frac{2}{3} \nabla_i \left(\frac{\Phi_B}{a_0} n_i \right) \quad (9)$$

$$g_D^2 = \frac{a_0}{6} \nabla_i (\Phi_B n_i). \quad (10)$$

By plugging (6), we get:

$$g_D^2 = \frac{a_0}{6} ((\nabla_i \Phi_B) n_i + 2g_B). \quad (11)$$

Recalling the expression for \vec{n} in (5), we calculate its divergence,

$$\nabla \cdot \vec{n} = \frac{\nabla \cdot \vec{g}_B}{g_B} + \vec{g}_B \cdot \nabla \left(\frac{1}{g_B} \right) = -\frac{4\pi G \rho_B}{g_B} - \frac{\vec{n}}{g_B} \cdot \nabla g_B, \quad (12)$$

where we replaced $\nabla \cdot \vec{g}_B$ in the first term by using Gauss' law, i.e., $\nabla \cdot \vec{g}_B = -4\pi G \rho_B$, where ρ_B is the density of baryonic matter.

Finally, by replacing (12) into (6), we get

$$\Phi_B = -\frac{2g_B^2}{4\pi G \rho_B + \vec{n} \cdot \nabla g_B}. \quad (13)$$

As an aside, we want to mention that one can easily derive Tully-Fisher relation from (11). Let's say the total mass of galaxy is M_{tot} . Then, plugging $g_B = GM_{tot}/r^2$ into (7) (in which case we have $\Phi_B = \Phi_N$ as mentioned), and then into (11) yields

$$(\nabla_i \Phi_B) n_i = -\frac{GM_{tot}}{r^2} = -g_B, \quad g_D^2 = \frac{a_0}{6} g_B, \quad (14)$$

which is exactly Tully-Fisher relation upon the identification $a_0/6$ with Milgrom's constant, which agrees with the one obtained from Hubble's constant within 10%.

3 Our calculation strategy

In this article, we are considering Verlinde gravity in the Solar System, where the typical gravitational acceleration is much bigger than a_M . Therefore, to a very good approximation, (1) can be re-expressed as

$$g = g_B + \frac{g_D^2}{2g_B} \quad (15)$$

Given this, how would the consideration of Verlinde gravity change the orbit of planets? At first glance, it might seem that, in order to calculate the Sun's gravitational attraction of a planet, we compute g_B (i.e., the Sun's Newtonian attraction of the planet) and g_D from quantities that can be obtained from \vec{g}_B , such as ∇g_B , with ρ_B as an additional input.

However, this is not the case. When we calculate the (Newtonian or relativistic) gravitational attraction of the Sun towards a planet, we ignore the planet's own gravitational field. A planet receives net zero gravitational force from its own gravitational field. The ground near the North Pole receives the gravitational force toward the Earth's center, which is cancelled by the ground near the South Pole, if we see the Earth as a whole. However, the case is not so when we consider Verlinde gravity.

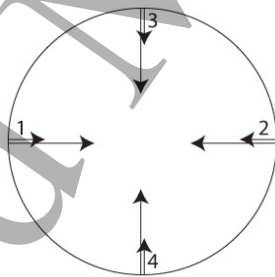


Figure 1: Schematic representation of Earth's gravity on different points (1, 2, 3, 4). Long arrows represent \vec{g}_B , while short arrows represent \vec{g}_D . The figure and the length of the arrows are not to scale.

Figure 1 displays a schematic representation of the Earth, together with some gravitational vectors, depicted as arrows. The long arrows denote \vec{g}_B and the small arrows denote $g_D^2/(2g_B)$. It is worth noting that the picture is only representative and the length of the arrows are not to scale. If \vec{g}_B at every point on the Earth is added, we will get the net gravitational attraction of the Sun GM_S/R_S^2 , and we can forget about planet's own gravitational field. However, it is not so with $g_D^2/(2g_B)$. The vectors \vec{g}_B s at points 1, 2, 3, 4 shown in the Figure are not the same because of the gravitational attraction of the Sun. As an example, if the Sun is positioned far away to the left side of the planet, the long arrow at 1 will be

shorter than the long arrow at 2, because the Sun's gravitational field is directed toward the left. This differences on the values of g_B is responsible for the difference in $g_D^2/(2g_B)$. When $g_D^2/(2g_B)$ is added for all the interior points of the planet by integration (of course, assuming a vector sum by giving $g_D^2/(2g_B)$ the direction of \vec{g}_B), we get an additional non-zero pull towards the Sun. Later, we will calculate this pull, which we call \vec{g}_{Vnet} . It is also important to note that the obtained value of \vec{g}_{Vnet} is very different from (actually much smaller than) $g_D^2/(2g_B)$ without considering the planet's own gravitational field. To repeat, the cancelation effect between the small arrows at 1 and 2 makes the effect of Verlinde gravity on planets much smaller than the one without such a consideration (i.e., without such a cancelation).

4 Verlinde gravity on the Earth and other planets

In this section we will calculate the value of Verlinde gravity for different planets. First, we start computing the value for the Earth as a starting point. Then, by switching the variable, we can easily calculate the Verlinde gravity on other planets.

To calculate g_D , we need to calculate g_B first, which is given by

$$\vec{g}_B = \vec{g}_E + \vec{g}_S. \quad (16)$$

Here, \vec{g}_E is the gravity due to the Earth, and \vec{g}_S is the gravity due to the Sun. Of course, there are additional terms due to the gravity of other planets, but we will ignore them for the moment to simplify our analysis and come back to it again at the end of the section.

Now, to calculate g_B we do the following,

$$\begin{aligned} g_B^2 &= (\vec{g}_E + \vec{g}_S) \cdot (\vec{g}_E + \vec{g}_S) \\ &\approx g_E^2 - 2g_E g_S \cos \theta_S \\ g_B &\approx g_E + \Delta g_B, \end{aligned} \quad (17)$$

where

$$g_E = \frac{GM(r)}{r^2}, \quad \vec{g}_E = -g_E \hat{r}_E, \quad \Delta g_B \equiv -g_S \cos \theta_S, \quad \cos \theta_S = \hat{r}_E \cdot \hat{r}_S. \quad (18)$$

In this expression, \hat{r}_S is the unit vector pointing from the center of the Earth to the Sun.

In our case, (11) and (13) can be re-expressed as follow:

$$g_D^2 = \frac{a_0}{6} (2g_B - \partial_r \Phi_B), \quad (19)$$

with

$$\Phi_B = -\frac{2g_B^2}{4\pi G \rho_B - \partial_r g_B}. \quad (20)$$

By using the definition of $M(r)$ and $g_S/R_S \ll g_E/R_E$, where R_S is the distance to the Sun and R_E is Earth's radius, i.e.,

$$4\pi r^2 \rho_B = \frac{\partial M(r)}{\partial r}, \quad \partial_r g_S \ll \partial_r g_E \quad (21)$$

we get

$$\partial_r g_B \approx \partial_r g_E = \partial_r \left(\frac{GM(r)}{r^2} \right) \quad (22)$$

$$= -\frac{2GM(r)}{r^3} + 4\pi G\rho_B. \quad (23)$$

Thus,

$$\Phi_B = -\frac{2g_B^2}{2g_B/r} = -rg_B, \quad (24)$$

which implies

$$g_D^2 = \frac{a_0}{6}(3g_B + r\partial_r g_B) \approx \frac{a_0}{6}(3g_B + r\partial_r g_E). \quad (25)$$

Therefore, (15) can be re-written as

$$g = g_B + \frac{a_0}{4} + \frac{a_0}{12g_B} r\partial_r g_E \quad (26)$$

Now, we need to integrate this for the whole interior of the Earth. Apparently, to obtain the net Verlinde gravity effect, we need to consider the last term only, because the first term will yield g_S , the Newtonian gravitational pull of the Sun, and the second term cancels out due to the spherical symmetry.

To calculate the third term, notice that

$$\Delta \left(\frac{a_0}{12g_B} \right) = -\frac{a_0}{12g_E^2} \Delta g_B. \quad (27)$$

Thus,

$$M_E \vec{g}_{Vnet} = - \int \rho_B r^2 \sin \theta_S d\phi_S d\theta_S dr \left(\frac{a_0}{12g_E^2} \Delta g_B r \partial_r g_E \cos \theta_S \right) \hat{r}_S, \quad (28)$$

where \vec{g}_{Vnet} is the additional pull toward the Sun due to Verlinde gravity, as we explained in Section 3. The factor $\cos \theta_S$ is due to the fact that only the component of gravity parallel to \hat{r}_S survives due to the spherical symmetry. For example, in Fig. 1, this factor is 0 for the arrows at 3 and 4 of Figure 1, -1 for the arrow at 1, and 1 for the arrow at 2. By plugging Δg_B in (18), we get

$$M_E \vec{g}_{Vnet} = \frac{4}{3} \pi \left(\frac{a_0}{12} \right) (g_S \hat{r}_S) \left(\int_0^{R_E} \frac{\rho_B r^3}{g_E^2} \partial_r g_E dr \right). \quad (29)$$

Summarizing, the total force \vec{F} on the Earth is given by

$$\vec{F} = M_E (\vec{g}_S + \vec{g}_{Vnet}). \quad (30)$$

Notice that (29) implies that

$$\vec{g}_{Vnet} = \lambda_E (g_S \hat{r}_S) = \lambda_E \vec{g}_S, \quad (31)$$

where

$$\lambda_E \equiv \frac{\pi a_0}{9M_E} \int_0^{R_E} \frac{\rho_B r^3}{g_E^2} \partial_r g_E dr. \quad (32)$$

If we consider the effect of the Moon, we have the same expression as the above one, except that \vec{g}_S is replaced by \vec{g}_M . If we consider all the effects, including those of other planets, which we denote by \dots , we have

$$\vec{g}_{Vnet} = \lambda_E(\vec{g}_S + \vec{g}_M + \dots). \quad (33)$$

If we write the total external Newtonian gravity by g_{Newton} as follows

$$\vec{g}_{Newton} = \vec{g}_S + \vec{g}_M + \dots, \quad (34)$$

(30) can be updated to

$$\vec{F} = M_E(\vec{g}_{Newton} + \vec{g}_{Vnet}) = (1 + \lambda_E) M_E \vec{g}_{Newton} \quad (35)$$

If we write

$$\vec{F} = M_E \vec{a} = M_{Egrav} \vec{g}_{Newton}, \quad (36)$$

where M_E is the inertial mass of the Earth, and M_{Egrav} is the gravitational mass of the Earth, we see that our equation can be interpreted as the breakdown of equivalence principle for the Earth and other planets, i.e., $M_E \neq M_{Egrav}$. In particular, we have

$$\vec{a} = (1 + \lambda_E) \vec{g}_{Newton} \quad (37)$$

and

$$\frac{M_{Egrav}}{M_E} = 1 + \lambda_E. \quad (38)$$

In the following section, we will show the results of the calculations of λ for the different planets of our Solar System.

5 The calculation of λ for the planets and the Moon

We present in this Section the results of the calculation of λ for the different planets of our Solar System and for the Moon. First, in the next section, we will compute λ for the terrestrial planets and the Moon. Then, in the following section, we will compute those for the giant planets.

5.1 Terrestrial planets and the Moon

For simplification reasons to our model, we assume that terrestrial planets are comprised by a core and a mantle, each with a constant density. In reality, the density within the core and the mantle depends on its depth. We denote the planet's mass and radius by M and R , respectively. We then define the baryonic density as

$$\rho_B = \begin{cases} \rho_c & , 0 < r < R_c \\ \rho_m & , R_c < r < R, \end{cases} \quad (39)$$

where ρ_c and ρ_m are the planet's core and mantle density, respectively, and R_c is the core radius.

The data considered as physical parameters for the terrestrial planets were taken from [5, 6, 7] and are summarized on Table 1.

Planet	R_c (km)	R (km)	ρ_c (kg/m ³)	ρ_m (kg/m ³)
Mercury	2002	2439	7245	3182
Venus	3228	6052	10600	4300
Earth	3486	6378	10700	4500
Mars	1700	3389	6533	3554

Table 1: Physical parameters for the terrestrial planets. R_c is the planet's core radius, R its radius, and ρ_c and ρ_m are the core and mantle densities, respectively.

To compute the value of λ for the Moon, we followed [8]. From there, we modelled the Moon as an inner core, an outer core, and a mantle. With that consideration, the values adopted were the following:

$$\rho_{Moon}(kg/m^3) = \begin{cases} 8 \times 10^3 & , 0 < r < 240km, \\ 5.1 \times 10^3 & , 240km < r < 330km, \\ 3.3249 \times 10^3 & , 330km < r < 1737.4km, \end{cases} \quad (40)$$

Next we will consider the case of giant planets.

5.2 Giant Planets

Giant planets have an envelope and a solid core. Unlike terrestrial planets, the envelope comprises most of their mass. Because of this, we must consider the following expression:

$$\frac{dP}{dr} = -\rho_B g_E = -\rho \frac{Gm}{r^2}, \quad (41)$$

which implies

$$m = -\frac{r^2}{\rho_B G} \frac{dP}{dr}. \quad (42)$$

If we plug this into Eq. (21), we obtain

$$\frac{d}{dr} \left(\frac{r^2}{\rho_B} \frac{dP}{dr} \right) = -4\pi G \rho_B r^2. \quad (43)$$

Thus, if we know the relation between P and ρ , we can solve the above second order differential equation. Such relations are available for Jupiter and Saturn in [9], and for Juptier in [10]. For our calculations, we used the density profile of Jupiter available in [11], and the density profile of Saturn available in [12].

For the cases of Uranus and Neptune, we considered two scenarios.

- First scenario: the core radius of these planets are one fifth of their radii (i.e., $R_c = 0.2R$). For their density, we used the profile described in [13]. In particular, Ravit Helled kindly provided the following formula [14]. The unit of the densities is in kg/m^3 :

Uranus: $\rho_c = 8231.99$,

$$\rho_{\text{envelope}} = 4039.37 + 20.7051\beta^2 + 3.78416\beta^3 - 38675.2\beta^4 + 53208.8\beta^5 - 18597.6\beta^6 \quad (44)$$

Neptune: $\rho_c = 10598.5$,

$$\rho_{\text{envelope}} = 4614.26 - 23.9265\beta^2 - 4.37289\beta^3 - 29734.9\beta^4 + 32288.5\beta^5 - 7139.54\beta^6, \quad (45)$$

where β is the normalized radius $\beta = r/R$, with R the planet's radius.

- Second scenario: No core. For this scenario we considered that the giant planets don't have a solid core. As in the case of the first scenario, Ravit Helled kindly provided the following formula [14]. The unit of the densities is in kg/m^3 :

Uranus:

$$\rho_{\text{envelope}} = 4424.91 - 49167.8\beta^4 + 71977.3\beta^5 - 27234.\beta^6 \quad (46)$$

Neptune:

$$\rho_{\text{envelope}} = 5145. - 44515.5\beta^4 + 58559.1\beta^5 - 19188.1\beta^6. \quad (47)$$

5.3 Results

For the sake of precision, there is no need to compute the value of λ with more than two significant digits, as obtained in Section 5.2. The reason for this is that the current measurement of the Hubble's constant has only two significant digits of precision. For this work, we adopted the value for the Hubble's constant of $H_0 = 70 \text{ km/s/Mpc}$. In Table 2, we present the results of the computation of λ for the terrestrial and giant planets and for the Moon. In the table, λ_0 corresponds to λ in the constant density model of planets. The value of λ_0 is given by

$$\lambda_0 = \frac{a_0}{24g_E(R_E)}, \quad (48)$$

where $g_E(R_E)$ is the surface gravity of the Earth (or, correspondingly, each planet). In Appendix A, we prove that λ is always smaller than λ_0 .

In Table 2, we see Jupiter, Uranus and Neptune have two values of λ computed. In the case of Jupiter, both values correspond to the calculation of λ using two different core densities, 10 g/cm^3 (third column) and 100 g/cm^3 (fourth column), as described in [11]. For Uranus and Neptune, the two values were computed using the two considerations described in Section 5.2, i.e: with a core and an envelope (third column) and considering a planet without a core (fourth column).

	$\lambda_0 (\times 10^{-12})$	$\lambda (\times 10^{-12})$	$\lambda (\times 10^{-12})$
Mercury	7.66	4.59	
Venus	3.20	1.01	
Earth	2.89	0.96	
Mars	7.60	3.80	
Jupiter	1.14	-0.10	-0.17
Saturn	2.71	-0.19	
Uranus	3.26	-0.03	0.33
Neptune	2.53	0.00	0.32
Moon	17.47	14.65	

Table 2: Values of λ_0 and λ for the planets in the Solar System and the Moon. The results are expressed in unit of $\times 10^{-12}$. λ_0 is the value of λ in the constant density model of planets. For the calculation of λ_0 , we used the surface gravity available in [15].

6 Comparison with data

In Section 5, we defined and computed the value of λ for the planets in our Solar System. In this section, we will analyze how the inclusion of Verlinde gravity affects different physical phenomena.

6.1 The perihelion precession

The first effect that we analyze is the precession of the perihelion of a planet due to Verlinde gravity. We found out that the effects of the precession of planets caused by Verlinde gravity is small. Moreover, the value of this effect is smaller than the error in the observations. Let's recall that the perihelion precession is present only when the total gravitational force on the planet is not proportional to the inverse square of the distance from the Sun. In other words, it only depends on the gravitational attraction of other planets, and the deviation from the inverse square law from Sun's gravitational force. This concept can be written in the following way:¹

$$\text{precession of perihelion} \propto \frac{\text{gravitational attraction of other planets}}{\text{gravitational attraction of the Sun}} + \text{relativistic effects.} \quad (49)$$

Let's see how these two terms change when we consider Verlinde gravity. In the first term on the right side of the expression, both the numerator and the denominator are multiplied by the factor $(1 + \lambda)$. Thus, as this factor is cancelled out, the first term becomes unaltered

¹More precisely, the first term is given by $(3\pi/2)(m/M)(r/R)^3$ in the lowest approximation. In this expression, m is the mass of the other planet, M is the mass of the Sun, and r is the size of the orbit. See [16].

by the implementation of Verlinde gravity, at least up to order $\mathcal{O}(\lambda)$. The expression for the second term is given by

$$\Delta\theta_{\text{rel}} = \frac{24\pi^3 a^2}{c^2 T^2 (1 - e^2)} = 6\pi \frac{m^2}{l^2}, \quad (50)$$

where $l = r^2 \dot{\theta}$ is the angular momentum divided by the planet's mass. As we include Verlinde gravity, the planet's mass changes, from m to $m(1 + \lambda)$. Since the value for m is squared, we will have a factor $(1 + \lambda)^2$. By developing the power and discarding the quadratic term, we obtain that the expression of $\Delta\theta_{\text{rel}}$ in Eq. (50) will be proportional to 2λ .

In the case of Mercury, the first term in (49) has a value of 532.3 arcsec/century, while the second term is 43.0 arcsec/century. Thus, the observed precession of the perihelion is 574.10 ± 0.65 arcsec/century [17]. Please note that 43.0 arcsec/century multiplied by 2λ is 6.5×10^{-10} arcsec/century, which is much smaller than the error in the observation, which is 0.65 arcsec/century. As we noted at the beginning of this section, the Verlinde gravity effect on the precession of the perihelion is too small to notice.

6.2 Masses of the Sun and planets

At the end of Section 4, we interpreted the Verlinde effect as the breakdown of equivalence principle, i.e., the difference between the inertial mass and the gravitational mass. Another equally valid interpretation is that the masses of the Sun and other planets, which a planet "feels", is multiplied by the factor $(1 + \lambda)$. However, masses of the Sun and planets are known much less precisely than $\mathcal{O}(\lambda)$ for the planets. Even the most precisely known mass is the one of the Sun, which is given by [18],

$$GM_S = (1.32712440018 \times 10^{20} \pm 8 \times 10^9) \text{m}^3/\text{s}^2. \quad (51)$$

Thus, the relative error is around 6×10^{-11} , which is, at least, 10 times larger than λ s.

6.3 Observational uncertainties in the position of planets

The last subsection is enough to prove that Verlinde gravity effects on the orbits of planets are too small to be noticeable, but let us add in this section that observational uncertainties in the position of planets are too big to notice the Verlinde gravity effect either. To obtain the rough estimate in the uncertainties in GM_S , consider Kepler's 3rd law.

$$GM_S \approx r^3 \omega^2 + \text{effects of other planets}. \quad (52)$$

Considering that $r^3 \omega^2$ is much larger than the effects of other planets, we have

$$\frac{\delta(GM_S)}{GM_S} \approx 3 \frac{\delta r}{r} + 2 \frac{\delta \omega}{\omega}. \quad (53)$$

The distance between the Earth and the Sun is estimated by the distance between the Earth and other planets, and is known about by 1 meter uncertainty [19]. Given that the

distance between the Earth and the Sun is about 1.5×10^{11} meters, we obtain

$$3 \frac{\delta r}{r} \approx 2 \times 10^{-11}, \quad (54)$$

which is much larger than λ s.

The uncertainties in the shape of the orbits of Jupiter and Saturn are about 10 meter [19], which yields

$$3 \frac{\delta r}{r} \approx \begin{cases} 4 \times 10^{-11} & \text{(for Jupiter),} \\ 2 \times 10^{-11} & \text{(for Saturn),} \end{cases} \quad (55)$$

which are again much larger than λ s. We do not need to consider Neptune and Uranus, because their uncertainties are much larger as there is no series of spacecraft radio range measurements available from the other planets [19].

Therefore, the uncertainties in r are not small enough to notice the Verlinde gravity effect. Nevertheless, let's consider the uncertainties in ω as well. The uncertainties in ω come mainly from our ability to measure the orbit planes with respect to extra-galactic quasars that astronomers use to define the coordinate system [19]. We can use VLBI observations of spacecraft to measure this to about 0.25 milli-arcseconds, or 1.25 nanoradians [19]. This is both for inner planets in the past and the present and for outer planets in the present. Considering that the Earth orbited around the Sun about 300 radians after the first spacecraft was sent, we can estimate

$$2 \frac{\delta \omega}{\omega} \approx 8 \times 10^{-12}, \quad (56)$$

which is larger than λ .

6.4 The Mass of the Earth

Just as in (6.2), the Moon feels that the mass of the Earth is multiplied by $(1 + \lambda)$. The observed mass of the Earth is given by [18]

$$GM_E = (3.986004418 \times 10^{14} \pm 8 \times 10^5) \text{ m}^3/\text{s}^2. \quad (57)$$

Thus, the relative error is approximately 2×10^{-9} , which is about 100 times larger than λ of the Moon.

6.5 Observational uncertainties in the position of the Moon

Currently, the lunar orbit is known to submeter accuracy [19]. Considering that the distance between the Earth and the Moon is about 380,000 km, we have (assuming $\delta r \approx 0.2 \sim 0.5$ m)

$$3 \frac{\delta r}{r} \approx (2 \sim 4) \times 10^{-9}, \quad (58)$$

which is about 100~200 times larger than λ .

7 Conclusions

In this work, we computed the values for the Verlinde gravity's parameter, λ , for the different planets and the Moon in our Solar System. There, we found that the error bars in the mass of the Sun and the Earth are much larger than the effects of Verlinde gravity on the orbits of planets and the Moon. Furthermore, we found that it's not even possible to observe such effects, if there is no new development to drastically reduce the observation errors in the distance determination of the orbits.

As we mention at the beginning of this work, there are people who claim that Verlinde gravity effects on the planets should be great. In light of the results obtained in this work, we conclude that those asseverations are incorrect. The reason for this discrepancy is that either they don't consider the effect of planets own gravitation nor use our Verlinde gravity expression (11), which is valid beyond spherical symmetry. For example, in [20], when analyzing the Verlinde gravity effects on the planets, the authors considered the following expression from [2]

$$g_D = \sqrt{a_M g_B} = \sqrt{\frac{a_0}{6} g_B}, \quad (59)$$

which is only valid when there is a spherical symmetry. This is a special case of (11), when

$$g_B = \frac{GM_S}{R_S^2}, \quad -\Phi_B = \frac{GM_S}{R_S}, \quad \rho_B = 0, \quad \vec{n} \cdot \nabla g_B = \frac{2GM_S}{R_S^3}. \quad (60)$$

As we have seen in Section 4, none of the above expressions are correct when applied to Verlinde gravitation on the orbits of planets. In the leading order, g_B is given by planet's own Newtonian gravitation, instead of that of the Sun. Φ_B is also determined by each planet's own gravitation instead of the Sun's. ρ_B is not certainly zero, as planets have non-zero mass density. For the derivative of Newtonian gravitational field, planet's own one dominates again.

In Newtonian gravity, we do not need to consider the effects of planets' own gravitation when calculating their motion, but when Verlinde gravity comes in, we need to consider them as it can clearly be seen from our demonstration in this paper; this consideration makes Verlinde gravity effects on planet's motion much smaller.

It is worth to mention that, when we actually perform Verlinde gravity experiments on the Earth, such as the ones described in [3], it is not necessary to consider the test particle's own gravity as it is negligible; when the test particle is just a small metal, its own gravity never dominates, but Earth's gravitation does. The amplitude and the direction of the combined gravity are almost the ones for the gravity of the Earth. However, when the test particle is a planet, its own gravity dominates; the amplitude and the direction of the combined gravity are very close to the ones of the planet's own gravity, not the ones of the Sun's gravity.

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A Proof that λ is smaller than λ_0

The relevant integration we need to consider is the following expression in (32), which we call b .

$$b = \int_0^{R_E} \frac{\rho_B(r)r^3}{g_E^2(r)} \partial_r g_E(r) dr = \int_0^{R_E} \frac{\rho_B(r)r^3}{g_E^2(r)} (-2g_E(r)/r + 4\pi G\rho_B(r)), \quad (61)$$

where we used Eq. (18). Now, if we define $\rho_M(r)$, the average density inside a sphere of radius r as follows

$$\rho_M(r) = \frac{M(r)}{\frac{4}{3}\pi r^3}, \quad (62)$$

we have

$$b = \int_0^{R_E} \frac{\rho_B(r)r}{\frac{4}{3}\pi G\rho_M^2(r)} (3\rho_B(r) - 2\rho_M(r)) dr. \quad (63)$$

Notice that in the constant density model, we have the following:

$$b_0 = \int_0^{R_E} \frac{r}{\frac{4}{3}\pi G} dr. \quad (64)$$

Let's see in which cases we have $b < b_0$ (i.e., $\lambda < \lambda_0$). $b < b_0$ is satisfied if their respective integrands satisfy the same inequality, which implies

$$\frac{\rho_B(r)}{\rho_M^2(r)} (3\rho_B(r) - 2\rho_M(r)) < 1, \quad (65)$$

which is equivalent to

$$\frac{3\rho_B}{\rho_M} - 2 < \frac{\rho_M}{\rho_B}. \quad (66)$$

It is easy to check that the above condition is satisfied if

$$\rho_M(r) > \rho_B(r), \quad (67)$$

which is always satisfied as $\rho_B(r)$ is always decreasing as r is increasing. This completes the proof.

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