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A nonlinear algorithm for traffic estimation with state constraints

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Abstract

We present a real-time traffic state estimation algorithm for motorways. Natural constraints on the variables, like practical bounds on densities and velocities, are incorporated in the estimation process aiming to obtain better estimation results.
The dynamic equation for the evolution of the traffic is defined by a second order macroscopic model which computes the density, the flow and the mean speed according to several nonlinear equations, but nothing avoids the results being out of those practical bounds. Different extensions of the Kalman method were already applied to this problem, but none of them consider natural constraints in the variables. On the other hand, general filter methods have been designed to cope with a constrained state. In order to incorporate the natural constraints of the traffic model, we adapt one of those methods based on the Unscented Kalman Filter. To validate the approach, many simulation cases over a freeway section were made using a microscopic simulation tool and comparing the Extended Kalman Approach with the proposed one.

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Keywords: state estimation; Kalman filter; second order model

1. Introduction

Undoubtly traffic congestion is a major problem in most cities. A lot of work has been done in order to optimize the performance of traffic networks (Stanciu et al. 2012). In freeway traffic, intelligent vehicle freeway systems (IVHS) can give partial solutions to the traffic congestion problem (Baskar et al. 2011).

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These IVHS must be fed with real-time information observed in the field. As it is stated in Papageorgiou et al. (1990), based on some directly measured traffic variables, the estimation algorithm should calculate and complete all the traffic variables associated to each element of the traffic network for a given time.

Traffic is known to have a highly non-linear behaviour (Treiber and Helbing 2002) with many vehicle interactions, what makes the issue of estimating the average traffic behaviour by means of aggregated variables absolutely challenging. Macroscopic models may be used to describe such behaviour representing the traffic average behaviour in terms of aggregated variables (flow, density and average speed) making use of partial derivation equations Hoogendoorn and Bovy (2001). The model presented in Messner and Papageorgiou (1990) considers that the freeway is divided into consecutively numbered sections, each one with a respective length and a number of lanes. For each section, the model calculates the density, the flow and the average speed according to several equations which accurately model the merging and lane drop phenomena Spiliopoulou et al. (2014). In Wang and Papageorgiou (2005) the extended Kalman filter (EKF) method uses this model for traffic state estimation.

To cope with the nonlinearity in the model, an *unscented* version of the Kalman filter (UKF) applied to the previously mentioned model of Messner and Papageorgiou (1990) is presented in Ngoduy (2011) showing good results. However, the UKF method can fail when the variables are restricted, see the survey Simon (2010). Furthermore, for the traffic model studied here, negative speed or density values could be obtained escaping from the validity domain of the equations.

In order to consider non linear constrained traffic models, some authos have consider the *Particle Filter* approach, for example, the traffic flow of a freeway stretch belonging to the northern of Beijing city in China is estimated in Zhang et al. (2013). In this work a particle filter algorithm based on the second-order macroscopic traffic flow model proposed by Papageorgiou Papageorgiou et al. (1990) is applied. Nevertheless this approach, as it is a sort of Montecarlo method, it has the drawback of the large number of particles needed to make a good estimation.

Considering that the EKF was applied successfully in Wang and Papageorgiou (2005), that there are UKF variants that improve the results of the EKF method as it is demonstrated in other areas (Zhang and Xia, 2011, Risso et al., 2015), and that standard UKF algorithm can give estimations outside the variable domain which make the algorithm to stop (Teixeira et al. 2008), the goal of this work is to develop a constrained filtering method that combining the previous advantages overcomes the difficulties. As it is inspired by the UKF method, it is named *Projected Interval Unscented Kalman Filter* (PIUKF).

The paper is organized as follows. In the first section we present the evolution model. In the second section, we pose the estimation problem, first the UKF method is described and then we present and extension that considers constraints. In the last section, the proposed algorithm is tested using several examples based on the microscopic simulator SUMO. The examples differ in the parameter setting and in the possibility of estimating them.

2. Traffic Model

The second order model proposed in Papageorgiou et al. (1990) is a macroscopic model for traffic simulation. In this model a given freeway stretch is subdivided in N segments of length l_i with λ_i lanes in each segment.

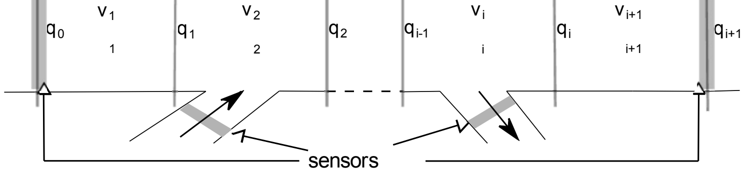

Fig. 1. Variables defined for each segment in the freeway

The length of the segments must satisfy $l_i > Tv_0$, where v_0 is the average free speed. For each segment the following aggregated traffic variables are defined and shown in the fig. 1:

- Traffic flow (veh/h) is the number of vehicles leaving segment i during the time period $[kT,(k+1)T]$, divided by T;
- Traffic density ($veh/km/lane$) is the number of vehicles in segment i at time kT, divided by the segment length l_i and lane number λ_i;
- Mean speed (m/h) is the mean speed oh the vehicles included in segment i at time $t = kT$;
- On-ramp inflow and Off-ramp outflow (veh/h) in segment i.

These variables verify the following equations. First, the traffic flow is related to the density through the fundamental diagram:

$$q_i(k) = \rho_i(k) v_i(k) \lambda_i. \tag{1}$$

The evolution of density is given by the following equation

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{l_i \lambda_i}\left[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)\right], \tag{2}$$

which expresses the vehicle conservation principle, and for the speed we have

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}\left(V(\rho_i(k)) - v_i(k)\right) + \frac{T}{l_i} v_i(k)\left(v_{i-1}(k) - v_i(k)\right) - \frac{\eta T}{\tau l_i}\frac{\rho_{i+i}(k) - \rho_i(k)}{\rho_i(k) + \kappa} - \frac{\delta T}{\lambda_i l_i}\frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa}, \tag{3}$$

which is an equation designed to model the dynamic evolution of the mean speed at each segment. The values $\kappa[veh/km/lane]$, $\tau[h]$, $\eta[km^2/h]$ and δ are model parameters which are given the same values for all the segments. Considering the fundamental diagram used in this work (see Papageorgiou et al. (1990)), the desired speed is related with the segment density as follows

$$V(\rho_i(k)) = v_0 \exp\left(-\frac{1}{a}\left(\frac{\rho_i}{\rho_c}\right)^a\right), \tag{4}$$

where ρ_c is the critical density at which the traffic flow is maximal and a is a model parameter to adjust the shape of the fundamental diagram.

The equations (2)-(3) model the system dynamics and, representing with x_k the vector of all the state variables (density and velocity for each segment) at time k, they can be written in a compact way as $x_{k+1} = f_k(x_k, u_k)$, where u_k is the input that in our case is a vector with the components q_0, v_0, r_i y s_i for time k. The measures are grouped in the variable y_k and the observation is given by $y_k = h_k(x_k, u_k)$.

3. State Estimation

Basically, the estimation problem consists in solving the large scale system of nonlinear equations defined by the values of x that verify given(2)-(3) some initial value for x_0 and the observations y_k, for $k = 1, \ldots, K$. This is an inverse problem not well posed. When the functions f and h are linear, the classical Kalman filter gives an explicit formula to obtain recursively an "optimal" solution to a relaxed version of the equations. When they are nonlinear functions, as in our case, a modified version of the filter called *Extended Kalman Filter* (EKF) can be applied. An example of that appears in the researchwork Wang and Papageorgiou (2005), where the authors present an approach to estimate the state of a freeway applying EKF method and the second order model stated in Papageorgiou et al. (1990).

To apply EKF we consider that the equations are affected by Gaussian errors:

$$x_{k+1} = f_k(x_k, u_k) + \xi_k, \tag{5}$$

$$y_k = h_k(x_k, u_k) + \zeta_k, \tag{6}$$

where ξ and ζ are the error vectors for the model and the observation, assumed to be normally distributed, with 0 mean and known covariance matrices Q and R. The criterium to obtain the optimal estimation is to reduce the mean squared error and this turn to be equivalent to reduce the sum over time of the trace of the covariances matrices of $x_k - \hat{x}_k$ and $y_k - \hat{y}_k$. The EKF linearizes the functions f and h around the current estimation $\hat{x}_k$, and proceeds in two steps:

Prediction step,

$$x_{k+1}^{-} = f(\hat{x}_k, u_k), \tag{7}$$

$$P_{k+1}^{-} = F_k \hat{P}_k F_k^T + Q_k \tag{8}$$

Correction step,

$$\hat{x}_{k+1} = x_{k+1}^{-} + K_{k+1}\left(z_{k+1} - h(x_{k+1}^{-})\right), \tag{9}$$

$$\hat{P}_{k+1} = (I - K_{k+1}H_k)P_{k+1}^{-}, \tag{10}$$

where F_k and H_k are the jacobian matrices obtained by linearizing f and h around $\hat{x}_k$, and the matrix K_k is the Kalman gain computed by

$$K_{k+1} = P_{k+1}^{-} H_k^T (H_k P_{k+1}^{-} H_k^T + R_k)^{-1}. \tag{11}$$

The advantage of the EKF is its recursive formula to compute the estimation, but the validity of the methodology strongly depends on the non linear character of the functions f and h. In fact, the update and correction formulae for the covariance matrix P are exact if the functions are linear and can be considered as good approximations if those functions are aproximatedly linear. This can be explained because of the identity $\text{cov}(Ax) = A\text{cov}(x)A^T$ but there is no similar identity relating $\text{cov}(x)$ and $\text{cov}(f(x))$ for non linear f. When the non linearity affects the performance, a way to circumvect this problem is to estimate the covariance $\text{cov}(f(x))$ by means of the so called *Unscented Tranformation* the resulting method is called *Unscented Kalman Filter* (UKF). As stated in Valverde and Terzija (2011), UKF methodology improves the results obtained from EKF in many different areas of application.

The ***Unscented Transformation*** presented in Julier and Uhlmann (2004), allows estimating the distribution of the random variable $y = f(x)$ knowing the function f and the distribution of the random variable x. Such transformation consists in choosing a set of points $\{X_i\}$ known as *sigma points* in such a way that their mean and covariance coincide with the random variable x. The function f is then applied to the *sigma points* and the mean and covariance of $f(x)$ are estimated from $\{f(X_i)\}$.

More precisely, considering the variable x of dimension n with expectation $E[x] = \hat{x}$ and covariance $\text{cov}(x) = P_x$, the $2n+1$ sigma points $x_0, x_1, \ldots, x_{2n}$ are obtained using the following equations

$$x_0 = \hat{x},\ x_i = \hat{x} + \gamma_i(\sqrt{P_x})_i,\ i = 1,...,n,\ x_i = \hat{x} - \gamma_i(\sqrt{P_x})_{i-n},\ i = n+1,...,2n, \tag{12}$$

where $\gamma_i = \sqrt{n+\lambda},\ \forall i$ and $\lambda = \alpha^2(n+\kappa) - n$. Then, the expectation and covariance are calculated as

$$\overline{x} = \sum_{i=0}^{2n} W_i^{(m)} x_i,\ P_x = \sum_{i=0}^{2n} W_i^{(c)} [x_i - \overline{x}][x_i - \overline{x}]^T, \tag{13}$$

for the weight values

$$W_0^{(m)} = \frac{\lambda}{n+\lambda},\; W_0^{(c)} = \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \beta,\; W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)}. \tag{14}$$

The parameter α determines the spread of the sigma points around $\bar{x}$ and usually takes a value inside the interval $(10^{-4},1)$. The parameter β is used to incorporate the knowledge of the distribution of $\bar{x}$ (for Gaussian distributions $\beta = 2$), κ is usually 0. All of the parameters were obtained from Van der Merwe and Wan (2001).

To estimate the statistics of $f(x)$, the equations in (18) are used with the same weights but applying the function f to the sigma points, i.e., changing x_i for $f(x_i)$:

$$\overline{f(x)} \approx x_+ = \sum_{i=0}^{2n} W_i^{(m)} f(x_i),\; P_{x_+} \approx \sum_{i=0}^{2n} W_i^{(c)} [f(x_i) - x_+][f(x_i) - x_+]^T. \tag{15}$$

The ***Unscented Kalman Filter*** is a combination of the the classical Kalman filter (11-14) with the unscented transformation (12)-(14) to compute the new covariances that in (8) and (10) were computed for a linear f. It was firstly presented in the work Julier and Uhlmann (2004) and the previous (8), (10) and (11) are replaced by (21) and (22).

In our case, two state variables for each segment of the freeway are considered: density ρ and average speed v. When the sigma points are computed considering equation (25), it is possible to obtain values of ρ and v that fall outside the model domain, for instance, a negative speed. What is worse, the dynamic equations of the model (1)-(4) can not be applied to these values. In Teixeira et al. (2008) a method called ***Interval Unscented Kalman Filter*** (IUKF) is presented. This method adds constraints to the *sigma points* fixing the problem of computation of the dynamic equations outside their validity domain.

The IUKF methods assumes that the state variables must verify $d \le x_k \le e$ where d and e are lower and upper bounds of x_k. It redefines the values γ_i and weights in equations (12)-(14). The γ and W values now depend on the time step k and are calculated as $\gamma_i^k = \min_j \Gamma_{ij}^k$, where the matrix Γ is defined by

$$\Gamma_{ij}^k = \sqrt{n+\lambda} - \left(\sqrt{n+\lambda} - S_{ij}^{-1}\left(e_{i,k} - x_{i,k}\right)\right)^+ \Big|_{S_{ij}>0} - \left(\sqrt{n+\lambda} - S_{ij}^{-1}\left(d_{i,k} - x_{i,k}\right)\right)^+ \Big|_{S_{ij}<0} \tag{16}$$

where $i = 1,\dots,n$ corresponds to the sigma point dimension, $j = 1,\dots,2n$ to the sigma point quantity, and the matrix S is given by $S = \left[\sqrt{P} \quad -\sqrt{P}\right]$. Finally the weights assigned to each sigma point now are

$$W_{0,k}^{(m)} = W_{0,k}^{(c)} = b_k,\quad W_{j,k}^{(m)} = W_{j,k}^{(c)} = a_k \gamma_{j,k} + b_k, \tag{17}$$

for each $j = 1,\dots,2n$, where $a_k = \frac{2\lambda - 1}{2\tau_k(n+\lambda)}$, $b_k = \frac{\tau_k - 2\lambda + 1}{2\tau_k(n+\lambda)}$, $\tau_k = \sum_{j=1}^{n} \gamma_{j,k} - (2n+1)\sqrt{n+\lambda}$.

Other extension of UKF that considers constraints is presented in Teixeira et al. (2008) and called ***Projected Unscented Kalman filter*** (PUKF). It makes the same steps as UKF and then projects the estimation $\hat{x}_{k+1}$ to the validity domain solving a quatratic optimization problem (24).

For our traffic model, neither IUKF nor PUKF can be applied because it is possible to obtain non valid values, after the correction in IUKF or after the prediction in PUKF. But it can be combined into a new method called ***Projected Interval Unscented Kalman Filter*** (PIUKF). Precisely, this new method, consists in adding the projection step after the correction step of IUKF. The IUKF part gives valid sigma points and the PUKF part restores the corrected estimation to the validity domain. The k step of the proposed PIUKF method is composed of the following computations:

PIUKF Algorithm

$$X_0 = f_k\left(\hat{x}_k\right);\ X_i = f_k\left(\hat{x}_k + \gamma_i(\sqrt{P_x})_i\right),\ 1 \le i \le n;\ X_i = f_k\left(\hat{x}_k - \gamma_i(\sqrt{P_x})_{i-n}\right),\ n+1 \le i \le 2n,$$

$$\bar{x}_k = \sum_{i=0}^{2n} W_{i,k}^{(m)} X_i, \quad \bar{P}_{x,k} = \sum_{i=0}^{2n} W_{i,k}^{(c)} [X_i - \bar{x}_k][X_i - \bar{x}_k]^T + Q \,(19)\, Y_0 = h_k\left(\bar{x}_k\right)$$

$$\bar{y}_k = \sum_{i=0}^{2n} W_{i,k}^{(m)} Y_i, \quad P_{y,k} = \sum_{i=0}^{2n} W_{i,k}^{(c)} [Y_i - \bar{y}_k][Y_i - \bar{y}_k]^T + R.$$

$$P_{xy,k} = \sum_{i=0}^{2L} W_{i,k}^{(c)} [X_i - \bar{x}_k][Y_i - \bar{y}_k]^T,\ K_k = P_{xy,k} P_{y,k}^{-1},$$

$$\hat{x}_{k+1} = \bar{x}_k + K_k\left(y_k - \bar{y}_k\right),\ P_{k+1} = \bar{P}_k - K_k P_{yk} K_k^T,$$

$$\hat{x}_{k+1}^P = \underset{d \le x \le e}{\operatorname{argmin}} J_2(x) = \left(\hat{x}_{k+1} - x\right)\left(P_{k+1}\right)^{-1}\left(\hat{x}_{k+1} - x\right).$$

4. Results

For the numerical experiments we used an open source traffic simulation package SUMO, that also has tools to import network topologies and to model demand parameters (Behrisch et al. 2011). It is a purely microscopic traffic simulation. It is important to remark that the simulation methodology of SUMO (microscopic simulation software) is completely independet to the equations used to derive our estimation method and hence the output can be considered very approximated to real observed data. We only compare the performance of EKF and PIUKF due to the limitations that PUKF and IUKF have when the sigma points become negative.

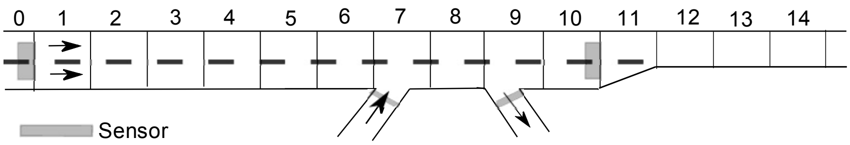

Fig. 2 Network used for simulations

The network example is presented in fig. 2 and has 14 segments. Segments 1 to 11 have two lanes, while segment 12 to 14 have a single lane. The stretch also has one on-ramp and one off-ramp in segment 7 and 9 respectively. The minimum density is $\rho_{min} = 0$ and the maximun is $\rho_{max} = 100$, while the speed constraints are $v_{min} = 0$ and $v_{max} = 130$ for each segment. The standard deviations for the model errors are 0.08 for the density and 9 for the speed, and for the observation errors are 90 for the density and 8 for the speed. Car-following parameters are the default values used by the SUMO simulator.

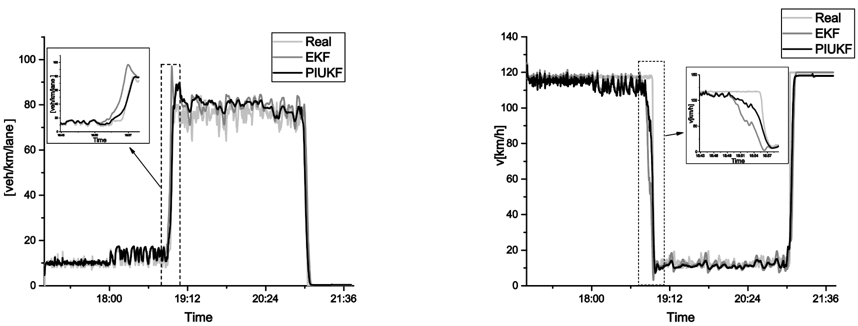

Fig. 3. Estimated density and speed in segment 3

In the Fig. 3, density and speed obtained for segment 3 are shown. In these figures it is noted that when the most important non-linearities occur (periods when congestion begins) the PIUKF method values are closer to real results. This fact is possible because the PIUKF method is based on the UKF method, which has a better approximation of the error covariance matrix when more important non-linearities occur.

In order to assess the performance of the method, the following indexes were taken from the work Wang and Papageorgiou (2005). They represent for each time step, the mean square error for the estimated density and speed:

$$PI_{\rho,k} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\rho_i(k) - \bar{\rho}_i(k)\right)^2},\ PI_{v,k} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(v_i(k) - \bar{v}_i(k)\right)^2}, \tag{18}$$

where N is the number of segments, v_i is the average speed of segment i and ρ_i is the density in segment i. The variables $\bar{v}_i$ and $\bar{\rho}_i$ are the estimated speed and density in segment i. In the figure 4 the indexes defined in (18) are shown, there it can be seen that PIUKF has a lower error than EKF.

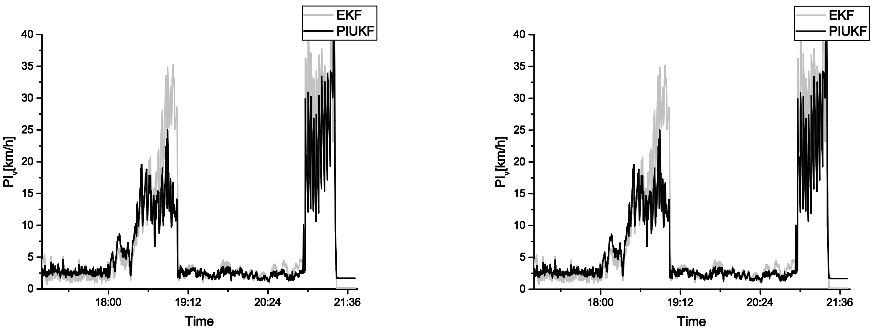

Fig. 4. PI_ρ (left) and PI_v (right) obtained for EKF and PIUKF methods simulating with SUMO

Finally, to compare the overall performance of the methods we defined seven cases and we used the indexes in **Hiba! A hivatkozási forrás nem található.** averaged in time. The first case (E1) is the previously defined example. The other cases are ment to study the performance when estimating also the parameters of the fundamental diagram (FD). In practical applications those parameters should also be estimated. The values used for SUMO simulations of all the examples are $a = 2$ and $\rho_c = 35$. The cases E2, E4 and E6 were estimated with different values for a and ρ_c without estimating them, and in cases E3, E5 and E7 thoses values were estimaded. The values of the coefficients and the preformance of the methods all these cases are shown in Table 1.

Table 1 Values of the estimation errors in the different cases

Test Case	EKF		PIUKF		Improvement EKF	
	PI_v	PI_ρ	PI_v	PI_ρ	PI_v	PI_ρ
S1 SUMO Reference case	12.66	7.15	9.85	5.35	22.2%	25.2%
S2 wrong a = 2.1	12.73	7.13	9.78	5.46	23.2%	23.4%
S3 FD parameter estimation	15.26	8.96	9.77	6.6	36.2%	26.3%
S4 wrong a = 1.8	14.07	7.97	10.03	6.44	28.7%	19.2%
S5 FD parameter estimation a	16.33	8.82	9.79	6.61	40.0%	25.0%
S6 wrong a = 1.8 and ρ_c=37	13.93	8.55	10.05	7.95	27.8%	7.0%
S7 FD parameter estimation	17.85	11.07	9.78	6.61	45.2%	40.3%

As can be seen in the estimations, the PIUKF method behaves better than the EKF method estimating both, speed and density. The improvement is more important in the cases where the FD parameters are estimated (and in the real implementation this should be the case) obtaining improvements ranging from 25% to 40% for density and from 36% to 45% for velocity.

5. Conclusion

In this work we showed that a constrained version of the Unscented Kalman filter applied to a second order model, further improves the numerical results obtained with Extended Kalman filter. This better results are explained due to the restriction of the estimated values to be inside the natural domain of the variables.

The numerical simulations made with a microscopic simulator showed an improvement in the performance index as much as 45% in speed and 40% in density. It was also shown that the estimation of hidden parameters can be done better with the UKF, and this also has to do with the non linearity of the equations where those parameters appear.

Other investigations are being done in real situations using other kinds of observations like video streaming, probe vehicles and wireless sensors. This will be the subject of a future publication.

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