Time, Defeasible Logic and Belief Revision: Pathways to Legal Dynamics

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Abstract

In order to properly model norm change in the law, temporal aspects of legal dynamics must be considered. Since there exist several time-based features of law that should be studied, we discuss two interesting approaches: one based on defeasible logic and the other based on belief revision. Each of these makes use of one of the two classic forms of reasoning about time: point-based and interval-based. Both formalisms provide the necessary logical infrastructure to address the characterization of complex behaviour of legal dynamics.

1 Introduction and Background

One peculiar feature of many normative systems, such as the law, is that it necessarily takes the form of a dynamic normative system [24, 23]. Despite the importance of norm-change mechanisms, the logical investigation of legal dynamics is still relatively underdeveloped. However, recent contributions exist and this section is devoted to a brief sketch of this rapidly evolving literature.

Alchourrón and Makinson were the first to logically study the changes of a *legal code* [2, 3, 1]. The addition of a new norm n causes an enlargement of the code, consisting of the new norm plus all the regulations that can be derived from n. Alchourrón and Makinson distinguish two other types of change. When the new norm is incoherent with the existing ones, we have an *amendment* of the code: in order to coherently add the new regulation, we need to reject those norms that conflict with n. Finally, *derogation* is the elimination of a norm n together with whatever part of the legal code that implies n. Alchourrón, Gärdenfors and Makinson [4] inspired by the works above proposed the so called general AGM framework for belief revision. This area has been proved to be a very fertile one and the phenomenon of revision of logical theories has been thoroughly investigated. As is well-known, the AGM framework distinguishes three types of change operation over theories. Contraction is an operation that removes a specified sentence ϕ from a given theory Γ (a logically closed set of sentences) in such a way as Γ is set aside in favour of another theory Γ^-_ϕ which is a subset of Γ not containing ϕ. Expansion operation adds a given sentence ϕ to Γ so that the resulting theory Γ^+_ϕ is the smallest logically closed set that contains both Γ and ϕ. Revision operation adds ϕ to Γ but it is ensured that the resulting theory Γ^*_ϕ be consistent [4]. Alchourrón, Gärdenfors and Makinson argued that, when Γ is a code of legal norms, contraction corresponds to norm derogation (norm removal) and revision to norm amendment.

It is then natural to ask if belief revision offers a satisfactory framework for the problem of norm revision in the law. Some of the AGM axioms seem to be rational requirements in a legal context, whereas they have been criticised when imposed on belief change operators. An example is the *success* postulate, requiring that a new input must always be accepted in the belief set. It is reasonable to impose such a requirement when we wish to enforce a new norm or obligation. However, it gives rise to irrational behaviours when imposed to a belief set, as observed in [14].

The AGM operation of contraction is perhaps the most controversial one, due to some postulates such as recovery [16, 32], and to elusive nature of legal changes such as derogations and repeals, which are all meant to contract legal effects but in remarkably different ways [16]. Standard AGM framework is of little help here: it has the advantage of being very abstract—it works with theories consisting of simple logical assertions—but precisely for this reason it is more suitable to capture the dynamics of obligations and permissions rather than the one of legal norms.

Difficulties behind AGM have been considered and some research has been carried out to reframe AGM ideas within reasonably richer rule-based logical systems able to capture the distinction between norms and legal effects [28, 26]. However, these attempts suffer from some drawbacks: they fail to handle reasoning on deontic effects and are based on a very simple representation of legal systems.

In fact, it is hard in AGM to represent how the same set of legal effects can be contracted in many different ways, depending on how norms are changed. These difficulties have been addressed in logical frameworks combining AGM ideas with richer rule-based logical systems, such as standard or Defeasible Logic [26, 18] or Input/Output Logic [8, 9, 28]. [32] suggested a different route, i.e., employing in the law existing techniques—such as iterated belief change, two-dimensional belief change, belief bases, and weakened contraction—that can obviate problems identified in [16] for standard AGM.

In general, any comprehensive logical model of norm change in the law has to take care of the following aspects:

1. the law usually regulates its own changes by setting specific norms whose peculiar objective is to change the system by stating what and how other existing norms should be modified; for instance, in most countries the Constitution states that only the Congress have powers to lay and regulate taxes. Even more, the Constitution states, by a norm, how to change or amend its own body of norms.

2. since legal modifications are derived from these peculiar norms, they can be in conflict and so are defeasible; for instance, some US states requires non-english foreign driver licenses to be accompanied by the International Drivers Permit. However, in 1989 US and Canada agreed to recognize each other's licenses, even french-written licenses. Hence, norms are contradictory regarding the documentation that a french-canadian driver must show to authorities.

3. legal norms are qualified by temporal properties, such as the time when the norm comes into existence and belongs to the legal system, the time when the norm is in force, the time when the norm produces legal effects, and the time when the normative effects hold. For instance, Belarus established that several laws passed before 1996 ceased to be enforced in the exact moment the President issues the new Constitution. In the United States, the 18th Amendment prohibiting the manufacture of liquor was passed in 1919 and repealed later in 1933. The end of this prohibition was established in turn by another Amendment (the 21st) that also establishes that this amendment "*shall be inoperative unless it shall have been ratified (...) within seven years from the date of the submission*".

To sum up, AGM-like frameworks have the advantage of being very abstract but work with theories consisting of simple logical assertions. For this reason, it is perhaps suitable to capture the dynamics of obligations and permissions, not of norms: the former ones are just possible effects of the application of norms and their dynamics do not necessarily require to remove or revise norms, but correspond in most cases to instances of the notion of *norm defeasibility* [16].

Addressing the above aspects has triggered new research lines in recent years, which break down in the following two approaches:

- Normative dynamics can be modelled by combining logical systems for temporal and defeasible reasoning: previous works [15, 19, 16] have proposed to combine Defeasible Logic with some basic forms of temporal logics;

- Another route is rather to enrich belief revision techniques by adding several temporal dimensions: this has been done in works such as [29, 30].

The layout of the article is as follows. Section 2 introduces the importance of time in legal norms and shows an example to motivate some ideas in the area of legal dynamics. Section 3 summarizes the first approach mentioned above, in which it is described how the Defeasible Logic was extended with temporal parameters to allow for reasoning about the times specified inside norms, and it is described how consider a legal system as a time-series of its versions, where each version is obtained from previous versions by some norm changes. Section 4 summarizes the second approach mentioned above, in which it is proposed a belief revision operator that considers time interval in the revision process. Finally, in Section 5 conclusions are offered and ideas for future work are given.

2 Preamble: Why Does Time Matter?

Legal norms are qualified by temporal properties, such as the time when the norm comes into existence and belongs to the legal system or the time when the norm is in force. Suppose that a municipality establishes that all taxis licensed since 2015 must be all-yellow, and a couple of years later the city adds a new rule establishing that all taxis with license starting in 2018 must be all-black. Hence, the yellow-taxi rule only applies for passenger cars with a valid license from 2015 to 2017. However, this is true only years later, after the introduction of the black-taxi rule.

Since all these properties can be relevant when legal systems change, [17] argued that failing to consider the temporal aspects of legal dynamics poses a serious limit to correctly model norm change in the law.

2.1 The Problem and a Motivating Example

As we have briefly mentioned above, belief revision, and specifically the AGM paradigm, has been advocated to be an elegant and abstract model for legal change.

Its has been, however, argued that standard belief techniques do not capture the following aspects of the law [17]:

1. the law regulates its own changes by issuing norms stating what and how other norms should be modified;
2. legal modifications can be in conflict and so are defeasible;
3. legal norms are qualified by temporal properties, e.g. the time when the norm is in force.

The general temporal model, as proposed in [17] assumes that all legal norms are qualified by different temporal parameters:

- the time when the norm comes into existence and belongs to the legal system,
- the time when the norm is in force,
- the time when the norm produces legal effects (it is applicable), and
- the time when the normative effects (conclusions) hold.

Indeed, it is common legislative practice that, once a legal provision is enacted (for example, the Italian 2018 budget law was enacted on 23 December 2017), its force can for instance be postponed to a subsequent time (for example, the Italian 2018 budget law was in force since 1 January 2018). Similarly, a part of a certain provision, which is in force since a certain time t, can be effective (i.e., can be applied) since a different time t' (for example, the Italian 2018 budget law, which was in force since 1 January 2018, at art. 1, par. 253 states that par. 252 will be applicable since 1 January 2019), or any provision can produce effects that hold retroactively (for example, art. 1 of Italian 2018 budget law, par. 629, states that certain tax effects cover cases since December 2017).

In [30], for example, the authors concentrate on issue 3 in the list above, i.e., how to integrate belief revision with time in the law. As regards issue 2, in that article, the authors do not work directly on rule-based defeasible reasoning, but they define a revision operator that may remove rules when needed or adapt intervals of time when contradictory norms are introduced in the system: for instance, if n is effective from 2001 to 2008 and a contradictory norm n' is added at 2006, we know that n is still effective from 2001 to 2005.

Let us now present a concrete example that serves to motivate the main ideas proposed in [30], an approach that we will recall in Section 4. It involves information and rules referring to intervals of time in which some taxes applies.

Example 1. *Consider the following pieces of information regarding a legislative attempt to ease tax pressure for people that have been unemployed.*

(a) A citizen was unemployed from 1980 to 1985.

(b) If unemployed from 1980 to 1983, then a tax exemption applies from 1984 to 1986, in order to increase individual savings.

(c) New authorities in government revoke tax exemption for years 1985 and 1986.

(d) Tax exemption reinstated for the year 1985 due to agreements with labor unions. ■

However, later on the legislator approved a new provision establishing that finally there is no tax-exemption for all citizens for the years 1985 and 1986.

Here some rules are produced and, as it happens in legislative bodies, norms change later according to the political and economical context. Rule (a) provides time-bounded information: only between 1980 and 1985 the status of being unemployed holds for a given citizen. Rule (b) states that if some property (unemployed) holds between 1980 and 1983, then other property (tax exemption) holds between 1984 and 1986. Rule (c) establishes that this is no longer valid for a certain interval of time. This means that, from now on, rule (b) of tax exemption should not be applied in its original text. In other words, the intervals of rule (b) are *revised* according to new political positions. Finally, rules are revised again as a consequence of labor unions, only to be revoked later. In this example the general rule of tax-exemption is revised several times. This revision is actually about the moments in which this benefit can be applied. In fact, rule (c) solely demands a revision of the interval for tax exemption. Hence, it cannot be the case that there is a rule in the normative system that entails a tax exemption for 1985 and 1986. From (c) and (b), it can be concluded that the benefit is only applied to 1984. Therefore, (b) should be not used anymore and a new rule for 1984 should be introduced.

3 Defeasible Logic with Time for Modelling Legal Dynamics

Before illustrating in Section 4 how belief revision can be integrated by temporal reasoning, we recall in this section the other approach that we mentioned in the introduction.

In [15, 19, 16] Defeasible Logic was extended with temporal parameters. In particular the authors *temporalised* propositional Defeasible Logic. This means that a temporal parameter is attached to the atomic elements of the logic, i.e., to the atomic propositions. For the logic it is assumed a discrete totally ordered set of instants of time $\mathcal{T} = \{t_0, t_1, t_2, \dots\}$. Based on this we can introduce the notion of *temporalised literals*. Thus if l is a plain literal, i.e., $l \in \text{PlainLit}$, and $t \in \mathcal{T}$ then l^t is a temporalised literals. The intuitive interpretation of l^t is that l is true (or holds) at time t. Lit denotes the set of temporalised literals. Finally, given a time instant t and $y \in \{\mathit{pers}, \mathit{tran}\}$ we call the combination of (t, y) *duration specification*, and literals labelled with a duration specification are called *duration literals*. The labels *pers* and *tran* denotes the quality of being *transient* or *persistent*. A duration literal has the form $l^{(t,y)}$. We denote the set of duration literals DurLit. The reasoning mechanism occurs on a set of *rules*, which are supposed to represent legal rules. The signature of rules is

$$\text{Rule}\colon 2^{\text{Lit}} \times \text{DurLit} \tag{1}$$

this means that a rule has the following form

$$r\colon a_1^{t_1}, \dots, a_n^{t_n} \hookrightarrow c^{(t,y)} \tag{2}$$

where $y \in \{\mathit{tran}, \mathit{pers}\}$ and hence the conclusion of the rule may be *transient* or *persistent*.

The idea behind the distinction between a transient and persistent conclusion is whether the conclusion is guaranteed to hold for a single instant or it continues to hold until it is terminated. This is particular relevant for legal rules, since their conclusions are for example obligations (or, in general deontic effects), and obligations, once triggered, remain in force until they are complied with, violated, or explicitly terminated. Accordingly we can use the duration specification (t, tran) to indicate that an obligation is in force at a specific time t, and must be fulfilled at that time, while the duration specification (t, pers) establishes that a legal effect enters in force at time t.

The inference mechanism extends that of Defeasible Logic taking into account the temporal and durations specification. As in article [6], we equate arguments with rules, thus this is the same as saying that there is a (defeasible) rule such that all the elements in its antecedent are provable and the conclusion is $p^{(t',y)}$. To assert that p holds at time t we have the following steps:

1. Give an argument for p at time t';

2. Evaluate all counterarguments against it. Here, we have a few cases:

    (a) If the duration specification of p is (t, tran) $(t' = t)$, then, the counterargument must be for the same time t given that p is ensured to hold only for t.

(b) If the duration specification of p is $(t', pers)$, then t' can precede t and we can 'carry' over the conclusion from previous times. In this case, the counterarguments we have to consider are all rules whose conclusion has a duration specification (t'', z) such that $t' \leq t'' \leq t$.

3. Rebut the counterarguments. This is the same as the corresponding step of basic defeasible logic, the only thing to pay attention to is that when we rebut with a stronger argument, the stronger argument should have t'' in the duration specification of the conclusion.

The general idea of the conditions outline above is that it is possible to assert that something holds at time t, because it did hold at time t', $t' < t$, by persistence, but there must be no reasons to terminate it. Thus new information defeats previous one.

3.1 From Rules to Meta-Rules

The temporal Defeasible Logic just presented allows for reasoning about the times specified inside norms, but it is not able to capture the natural evolution of legal systems, where new norms are issued, and existing norms are revised or derogated. To obviate this problem [16] proposes to consider a legal system as a time-series of its versions, where each version is obtained from previous versions by some norm changes, e.g., norms entering in the legal system, modification of existing norms, repeals of existing norms, ... This means that we can represent a legal system LS as a sequence

$$LS(t_1), LS(t_2), \dots, LS(t_j) \tag{3}$$

where each $LS(t_i)$ is the snapshot of the rules (norms) in the legal system at time t_i. Graphically it can be represented by the picture in Figure 1.

A *rule* is a relation between a set of premises (conditions of applicability of the rule) and a conclusion. The admissible conclusions are either literals or rules themselves; in addition the conclusions and the premises will be qualified with the time when they hold. Two classes of rules can be considered: *meta-rules* and *proper rules*. Meta-rules describe the inference mechanism of the institution on which norms are formalised and can be used to establish conditions for the creation and modification of other rules or norms, while proper rules correspond to norms in a normative system. In what follows we will use Rule to denote the set of rules, and MetaRules for the set of meta-rules, i.e., rules whose consequent is a rule.

A *temporalised rule* is either an expression $(r\colon \bot)^{(t,x)}$ (the void rule) or $(r\colon \emptyset)^{(t,x)}$ (the empty rule) or $(r\colon A \hookrightarrow B)^{(t,x)}$, where r is a rule label, A is a (possibly empty) set of temporalised literals, B is a duration literal, $t \in \mathcal{T}$ and $x \in \{tran, pers\}$.

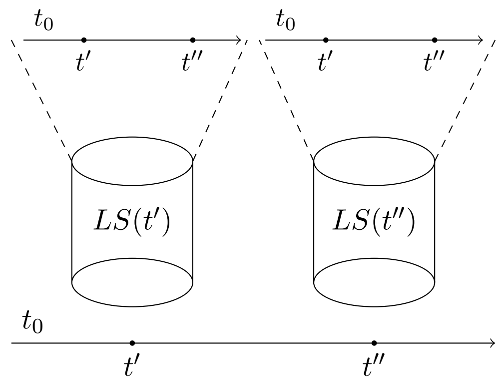


Figure 1: Legal System at t' and t''

We have to consider two temporal dimensions for norms in a normative system. The first dimension is when the norm is in force in a normative system, and the second is when the norm exists in the normative system from a certain viewpoint. So far temporalised rules capture only one dimension, the time of force. To cover the other dimension we introduce the notion of temporalised rule with viewpoint. A *temporalised rule with viewpoint* is an expression

$$(r\colon A \hookrightarrow B)^{(t,x)}@(t', y), \tag{4}$$

where $(r\colon A \hookrightarrow B)^{(t,x)}$ is a temporalised rule, $t' \in \mathcal{T}$ and $y \in \{\mathit{tran}, \mathit{pers}\}$.

Finally, meta-rules are introduced, that is, rules where the conclusion is not a simple duration literal but a temporalised rule. Thus a *meta-rule* is an expression

$$(s\colon A \hookrightarrow (r\colon B \hookrightarrow C)^{(t',x)})@(t, y), \tag{5}$$

where $(r\colon B \hookrightarrow C)^{(t',x)}$ is a temporalised rule, $r \neq s$, $t \in \mathcal{T}$ and $y \in \{\mathit{tran}, \mathit{pers}\}$. Notice that meta-rules carry only the viewpoint time (the validity time) but not the "in force" time. The intuition behind this is that meta-rules yield the conditions to modify a legal system. Thus they specify what rules (norms) are in a normative system, at what time the rules are valid, and the content of the rules. Accordingly, these rules must have an indication when they have been inserted in a normative system, but then they are universal (i.e., apply to all instants) within a particular instance of a normative system.

Every temporalised rule is identified by its rule label and its time. Formally we can express this relationship by establishing that every rule label r is a function

$$r\colon \mathcal{T} \mapsto \text{Rule}. \tag{6}$$

Thus a temporalised rule r^t returns the value/content of the rule 'r' at time t. This construction allows us to uniquely identify rules by their labels[1], and to replace rules by their labels when rules occur inside other rules. In addition there is no risk that a rule includes its label in itself. In the same way a temporalised rule is a function from $\mathcal{T}$ to Rule, we will understand a temporalised rule with viewpoint as a function with the following signature:

$$\mathcal{T} \mapsto (\mathcal{T} \mapsto \text{Rule}). \tag{7}$$

As we have seen above a legal system LS is a sequence of versions $LS(t_0), LS(t_1), \dots$. The temporal dimension of *viewpoint* corresponds to a version of the legal system, while the temporal dimension of a *temporalised rule* corresponds to the time-line inside a version. Thus the meaning of an expression $r^{t_v}@t_r$ is that we take the value of the temporalised rule r^{t_v} in $LS(t_r)$. Accordingly, a version of LS is just a repository (set) of norms (implemented as temporal functions).

Accordingly, given a rule r, the expression $r^t@t'$ gives the value of the rule (set of premises and conclusion of the rule) at time t in the repository t'. The content of a void rule, e.g., $(r\colon \bot)^t@t'$ is $\bot$, while for the empty rule the value is the empty set. This means that the void rule has a value for the combination of the temporal parameters, while for the empty rule, the content of the rule does not exist for the given temporal parameters. Another way to look at the difference between the empty rule and the void rule is to consider that a rule is a relationship between a set of premises and a conclusion. For the void rule this relationship is between the empty set of premises and the empty conclusion; thus the rule exists but it does not produce any conclusion. For the empty rule, the relationship is empty, thus there is no rule. Alternatively, we can think of the function corresponding to temporalised rules as a partial function, and the empty rule identifies instants when the rule is not defined.

For a transient fully temporalised literal $l^{(t,x)}@(t', tran)$ the reading is that the validity of l at t is specific to the legal system corresponding to repository associated to t', while $l^{(t,x)}@(t', pers)$ indicates that the validity of l at t is preserved when we move to legal systems after the legal system identified by t'. An expression $r^{(t,tran)}$ sets the value of r at time t and just at that time, while $r^{(t,pers)}$ sets the values of r to a particular instance for all times after t (t included).

We will often identify rules with their labels, and, when unnecessary, we will drop the labels of rules inside meta-rules. Similarly, to simplify the presentation and when possible, we will only include the specification whether an element is persistent or transient only for the elements for which it is relevant for the discussion at hand.

[1]We do not need to impose that the function is an injective: while each label should have only one content at any given time, we may have that different labels (rules) have the same content.

Meta-rules describe the inference mechanism of the institution on which norms are formalised and can be used to establish conditions for the creation and modification of other rules or norms, while proper rules correspond to norms in a normative system. Thus a temporalised rule r^t gives the 'content' of the rule 'r' at time t; in legal terms it tells us that norm r is in force at time t. The expression

$$(p^{t_p}, q^{t_q} \Rightarrow (p^{t_p} \Rightarrow s^{(t_s,pers)})^{(t_r,pers)})@(t, tran) \tag{8}$$

means that, for the repository at t, if p is true at time t_p and q at time t_q, then $p^{t_p} \Rightarrow s^{(t_s,pers)}$ is in force from time t_r onwards.

A legal system is represented by a temporalised defeasible theory, called *normative theory*, i.e., a structure

$$(F, R, R^{\text{meta}}, \prec) \tag{9}$$

where F is a finite set of facts (i.e., fully temporalised literals), R is a finite set of rules, R^{meta} is a finite set of meta rules, and $\prec$, the superiority relation over rules is formally defined as $\mathcal{T} \mapsto (\mathcal{T} \mapsto \text{Rule} \times \text{Rule})$ accounting that we can have different instances of the superiority relation depending on the legal systems (external time) and the time when the rules involved in the superiority are evaluated[2].

The inference mechanism with meta-rules is essentially an extension of that of temporal defeasible logic, but it involves more steps. Rules are no longer just given, but they can be derived from meta-rules. Thus, to prove a conclusion x the first thing to do is to see if it is possible to derive a rule r supporting x. But we have to derive such rule at the appropriate time. Here, we want to remember that a rule is a function from time (validity time or version of a legal system) to time (when a rule is in force in a version of a legal system) to the content of the rule (relationship between a set of premises and a conclusion). The basic intuition is that a rule corresponds to a norm, and there could be several modifications of a norm, thus deriving a rule means to derive one of such modifications. As we shall see in the next section a meta-rule (or more generally a set of meta-rules) can be used to encode a modification of a norm. In general it is possible to have multiple (conflicting) modifications of a norm. Accordingly, to derive a rule, we have to check that there are no conflicting modifications[3] or the conflicting modifications are weaker than the current modification. The final consideration is that in this case we have two temporal dimensions, and the persistence applies to both.

[2]For instance, if we have $s \prec^{2007}_{Monday} r$ and $r \prec^{2007}_{Tuesday} s$, it means that, according to the regulation in force in 2007, on Monday rule s is stronger than rule r, but on Tuesday r is stronger than s.

[3]Two meta-rules are conflicting, when the two meta-rules have the same rule as their head, but with a different content.

3.2 An Example: Modifications on Norm Validity and Existence – Annulment vs. Abrogation

The expression *repeal* is sometimes used to generically denote the operation of norm withdrawal. However, at least two forms of withdrawal are possible: annulment and abrogation.

An *annulment* makes the target norm invalid and removes it from the legal system. Its peculiar effect applies *ex tunc*: annulled norms are prevented to produce all their legal effects, independently of when they are obtained. Annulments typically operate when the grounds (another norm) for annulling are hierarchically higher in the legal system than the target norm which is annulled: consider when a legislative provision is annulled (typically by the Constitutional Court) because it violates the constitution.

An *abrogation* works differently; the main point is usually that abrogations operate *ex nunc* and so do not cancel the effects that were obtained from the target norm before the modification. If so, it seems that abrogations cannot operate retroactively. In fact, if a norm n_1 is abrogated in 2012, its effects are no longer obtained after then. But, if a case should be decided at time 2013 but the facts of the case are dated 2011, n_1, if applicable, will anyway produce its effects because the facts held in 2011, when n_1 was still in force (and abrogations are not retroactive). Accordingly, n_1 is still in the legal system, even though is no longer in force after 2012. Abrogations typically operate when the grounds (another norm) for abrogating is placed at the same level in the hierarchy of legal sources of the target norm which is abrogated: consider when a legislative provision is abrogated by a subsequent legislative act.

Consider this case:

Example 2 (Abrogation vs Annulment). *[***Target of the modification***]*

> *Legislative Act n. 124, 23 July 2008*
> *Art. 1. With the exception of the cases mentioned under the Articles 90 and 96 of the Constitution, criminal proceedings against the President of the Republic, the President of the Senate, the President of the House of Representatives, and the Prime Minister, are suspended for the entire duration of tenure. [...]*

In case of abrogation, we could have that the legislator enacts the following provision:

> *[***Abrogation enacted and effective at 1 January 2011***] Legislative Act n. 124, 23 July 2008 is abrogated.*

In case of (judicial) annulment, we would rather have

> *[**Annulment enacted and effective at 1 January 2011**] On account of Art. 3 of the Constitution [...] the Constitutional Court hereby declares the constitutional illegitimacy of Art. 1 of the Act n. 124, 23 July 2008.*

As we have recalled, the difference between the two cases is that the annulment has retroactive effects. In particular, let us focus on the following provisions from the Italian penal code:

> Art. 157 Italian of Penal Code – Terms of statute-barred penal provisions.
> When the terms for statute-barred penal effects expire, the corresponding crime is canceled [...]
> Art. 158 Italian Penal Code – Effectiveness of the terms of statute-barred penal provisions
> The effectiveness of terms of statute-barred penal provisions begins starting from the time when the crime was committed.
> Art. 159 Italian of Penal Code – Suspension of time limits for statute-barred penal effects.
> The terms for statute-barred penal effects [...] are suspended whenever the criminal proceedings are suspended under any legislative provisions [...]

Consider a hypothetical case where the Italian Prime Minister is accused in 2007 of accepting bribes at the beginning of 2006. Clearly, if Legislative Act n. 124 is abrogated in 2011, since abrogation has no retroactive effects, art. 159 of Italian Penal Code applied from 2008 to 2011, and so the counting of terms has been suspended between these two years. Hence, from the perspective of 2011 (immediately after the abrogation) the relevant time passed is two years and six months (2006, 2007, and until July 2008). Instead, if the act is annulled in 2011, more time has passed from the perspective of 2011, because it is as if the Legislative Act n. 124 were never enacted: from 2006 until 2011.

As we can see, modeling retroactive legal modifications is far from obvious. The logical model proposed in [16] and recalled in Section 3 offers a solution. In the next section we will illustrate the intuition and apply to the above example of annulment and abrogation.

3.3 Intermezzo – Temporal Dynamics and Retroactivity

As we have previously argued, if $t_0, t_1, \ldots, t_j$ are points in time, the dynamics of a legal system LS can be captured by a time-series $LS(t_0), LS(t_1), \ldots, LS(t_j)$ of its versions. Each version of LS is like a norm repository: the passage from one repository to another is effected by legal modifications or simply by temporal persistence. This model is suitable for modeling complex modifications such as retroactive changes, i.e., changes that affect the legal system with respect to legal effects which were also obtained before the legal change was done.

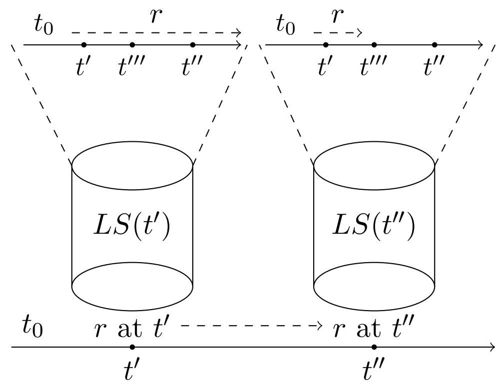


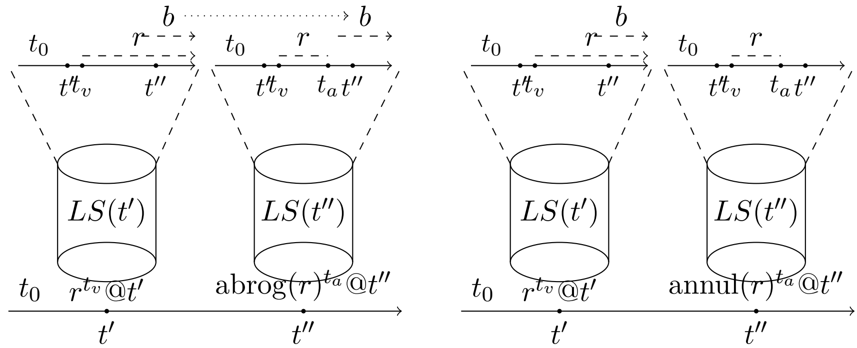
Figure 2: Legal System at t' and t''

The dynamics of norm change and retroactivity need to fully make use of the time-line within each version of LS (the time-line placed on top of each repository in Figure 2). Clearly, retroactivity does not imply that we can really change the past: this is "physically" impossible. Rather, we need to set a mechanism through which we are able to reason on the legal system from the viewpoint of its current version but *as if* it were revised in the past: when we change some $LS(i)$ retroactively, this does not mean that we modify some $LS(k)$, $k < i$, but that we move back from the perspective of $LS(i)$. Hence, we can "travel" to the past along this inner time-line, i.e., from the viewpoint of the current version of LS where we modify norms.

Figure 2 shows a case where the legal system LS and its norm r persist from time t' to time t'' and can have effects immediately from t'. Now, the figure represents the situation where r is retroactively repealed at t'' by stating that the modification applies from t''' (which is between t' and t'') onwards. The difference between abrogation and annulment is illustrated in Figures 3(a) and 3(b).

3.4 Modifications on Norm Validity and Existence: Annulment vs. Abrogation (Cont'd)

On account of our previous considerations, the cases in Example 2 can be reconstructed as follows.



(a) Abrogation. In $LS(t')$ rule r produces a persistent effect b. Literal b carries over by persistence to $LS(t'')$ even if r is no longer in force.

(b) Annulment. In $LS(t')$ rule r is applied and produces a persistent effect b. Since r is annulled in $LS(t'')$, b must be undone as well.

Figure 3: Abrogation and Annulment

Example 3 (Abrogation vs Annulment (cont'd))**.** *First of all, for the sake of simplicity let us*

- *only consider the case of Prime Minister (Legislative Act n. 124 mentions other institutional roles),*
- *assume that the dates of enactment and effectiveness coincide and are generically 2008,*
- *the duration of tenure covers a time span from 2008 to 2012,*

and formalize the corresponding fragment of art. 1 of Legislative Act n. 124 (23 July 2008) as follows:

$$L.\ 124: (Crime^{x}, Tenure^{x+y} \Rightarrow_{\mathsf{O}} Suspended^{(x+y,tran)})^{(2008,pers)})@(2008, pers)$$

The duration of tenure spanning from 2008 to 2012 is represented as follows:

$$r1: (Elected^{2008} \Rightarrow_{\mathsf{O}} Tenure^{(2008,pers)})^{(2008,pers)})@(2008, pers)$$
$$r2: (Elected^{2008} \leadsto_{\mathsf{O}} \neg Tenure^{2012})^{(2008,pers)})@(2008, pers)$$

Arts. 157-159 of the Italian Penal Code state the following:

$$Art.\ 157: (Crime^{x}, Terms^{x+y} \Rightarrow_{\mathsf{O}} CrimeCancelled^{(x+y,pers)})^{(z,pers)})@(z, pers)$$
$$Art.\ 158: (Crime^{x} \Rightarrow_{\mathsf{O}} Terms^{(x,pers)})^{(z,pers)})@(z, pers)$$
$$Art.\ 159: (Crime^{x}, Suspended^{x+y} \Rightarrow_{\mathsf{O}} \neg Terms^{(x+y,tran)})^{(z,pers)})@(z, pers)$$

As proposed by [16], the distinction between abrogation and annulment requires to distinguish between void *rules and* empty *rules. The content of a void rule, e.g.,* $(r\colon \bot)^{t}@t'$ *is* $\bot$*, while for the empty rule the value is the empty set. This means that the void rule has value for the combination of the temporal parameters, while for the empty rule, the content of the rule does not exist for the given temporal parameters.*

Given a rule $(r\colon A \Rightarrow b^{t_b})^{t_r}@t$*, the abrogation of* r *at* t_a *in repository* t' *is basically obtained by having in the theory the following meta-rule*

$$abr_r\colon\ \Rightarrow (r\colon \bot)^{(t_a, pers)})@(t', pers) \tag{10}$$

where $t' > t$*. The abrogation simply terminates the applicability of the rule. More precisely this operation sets the rule to the void rule. The rule is not removed from the system, but it has now a form where no longer can produce effects. In the case of the Legislative Act n. 124 (23 July 2008) we would have*

$$abr_{L.\,124}\colon\ \Rightarrow (L.\ 124\colon \bot)^{(2011, pers)})@(2011, pers)$$

Hence, we can have the following, for example

- *at time* x*, from the viewpoint* x *we derive* $Suspended^{x}$*,* $2008 \le x \le 2010$*;*
- *at time* x*, from the viewpoint* x *we show that we cannot derive* $Terms^{x}$*,* $2008 \le x \le 2010$*;*
- *at time* 2011*, from viewpoint* 2011 *we show that we cannot derive* $Suspended^{2011}$*;*
- *at time* 2011*, from viewpoint* 2011 *we can derive* $Terms^{2011}$*.*

This is in contrast to what we do for annulment where the rule to be annulled is set to the empty rule. This essentially amounts to removing the rule from the repository. From the time of the annulment the rule has no longer any value. All past effects are thus blocked as well.

The definition of a modification function for annulment depends on the underlying variants of the logic, in particular whether conclusions persist across repositories. Minimally, the operation requires the introduction of a meta-rule setting the rule r *to be annulled to* $\emptyset$*, with the time when the rule is annulled and the time when the meta-rule is inserted in the legal system:*

$$(annul_r\colon\ \Rightarrow (r\colon \emptyset)^{(t_a, pers)})@(t', pers) \tag{11}$$

Hence,

$$(annul_{L.\,124}\colon\ \Rightarrow (L.\ 124\colon \emptyset)^{(2008, pers)})@(2011, pers)$$

If we assume that conclusions persist over repositories we need some additional technical machinery to block pasts effects from previous repositories. In this case, since L. 124 *is modeled as a transient rule, we have basically to add a defeater like the following*[4]*:*

$$((annul_{ef}\colon \leadsto_{\mathsf{O}} \neg Suspended^{2008})^{(2008, pers)})@(2011, pers)$$

Hence, we now have, for example

- *we can show that we cannot derive at x, from viewpoint 2011, $Suspended^x$,* $2008 \leq x$*;*
- *we can prove at x, from viewpoint* 2011 *$Terms^x$,* $2008 \leq x$*.*

As stated before, another approach to address a logical model of norm change in the law is to enrich belief revision techniques by adding several temporal dimensions, as done in [29, 30]. There, techniques from belief revision formalisms are integrated with interval-based logical rules for legal systems, formalizing a revision operator. This operator may remove rules when needed or adapt intervals of time when contradictory norms are added in the system. This is discussed in the following section.

4 Temporalising Belief Revision for the Law

Example 1 involves information and rules referring to intervals of time in which some taxes applies. Cases like this, need to go beyond AGM machinery. Some research has been carried out to reframe AGM ideas, some of these, within richer rule-based logical systems [28, 26], and other, have aimed to study belief revision for situations in which nonmonotonic reasoning is addressed [33, 25]. However, also these attempts suffer from some drawbacks of standard AGM, among them the fact that the proposed frameworks fail to handle the temporal aspects of norm change.

Unlike rich but complex frameworks such as the one of [17]—which we have recalled in Section 3—we claim that belief revision techniques—which are based on an abstract and elegant machinery—can be reconciled with the need to consider several temporal patterns of legal reasoning. In [30] the authors are thus interested in the formalization of a belief revision operator applied to an epistemic model that considers rules and time. They enrich a simple logic language with an interval-based model of time, to represent temporal dimensions such as the effectiveness of norms,

[4]The general procedure to block conclusions when conclusions persist over repositories can be very complex: for all details, see [16].

i.e., when norms are applicable. There, the revision operator may remove rules when needed or adapt intervals of time when newer, contradictory norms are introduced in the system. In particular, the idea is the formalization of a belief revision operator that can address the evaluation of *timed rules* representing legal norms. Technical aspects of temporalised knowledge are considered in the following sections.

4.1 Legal System as Temporalised Belief Base

The problem of representing temporal knowledge and temporal reasoning arises in many disciplines, including Artificial Intelligence. A usual way to do this is to determine a *primitive* to represent time, and its corresponding *metric relations*. There are in the literature two traditional approaches to reasoning with and about time: a point based approach, as in [17], and an interval based approach as in [5, 12]. In the first case, the emphasis is put on *instants* of time (e.g., timestamps) and a relation of precedence among them. In the second case, time is represented as continuous sets of instants in which something relevant occurs. These intervals are identified by the starting and ending instants of time.

The approach introduced in [30], time intervals (like in [7, 12]) are considered. Following the semantics of the temporalised rules proposed in [17] and explained in Section 3 (in an adapted version), the revision operator, in essence, consists in the handling of intervals in order to maintain the consistency.

The above-mentioned temporal machinery is able to explicitly model two temporal dimensions among those mentioned above in Section 2.1, that is the time of norm effectiveness —i.e. when a norm can produce legal effects—and the time when the norm effects hold [17].

In [30], a propositional language $\mathbb{L}$ with a complete set of boolean connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ is adopted and a consequence operator, denoted $Cn(\cdot)$, is used that takes sets of sentences in $\mathbb{L}$ and produces new sets of sentences. In general, in this article, we will write $\alpha \in Cn(A)$ as $A \vdash \alpha$.

Note that the AGM model [4] represents epistemic states by means of belief sets, that is, sets of sentences closed under logical consequence. Other models use belief bases; i.e., arbitrary sets of sentences [13, 20, 31]. In [30], epistemic model is based on an adapted version of belief bases which have additional information (time intervals).

4.2 Time Interval

In [30] a universal finite *set of time labels* $\mathbb{T} = \{t_1, \ldots, t_n\}$ strictly ordered is considered; each time label represents a unique time instant. Simplifying the notation,

$t_i - 1$ is the immediately previous instant to the instant t_i and $t_i + 1$ is the immediately posterior instant to the instant t_i. An interval is considered like a finite ordered sequence of time labels $t_i, \ldots, t_j$ where i, j are natural numbers ($i \leq j$) and $t_i, \ldots t_j \in \mathbb{T}$ denoting instances of time or *timepoints*. The discreteness of the flow of time is appropriate for modelling norms dynamics since norms usually refer to time in the spectrum of hours, days, months and years. Generally speaking, the law itself views time as determined by discrete steps. Thus, let $\alpha \in \mathbb{L}$, we have expressions of the type $\alpha^{interval}$, where *interval* can be as follow:

- $[t_i, t_i]$: meaning that α holds at time t_i. Following [17] α is transient (holding at precisely one instant of time). For simplicity $[t_i, t_i] = [t_i]$.
- $[t_i, \infty]$: meaning that α holds from t_i. Following [17] α is (indefinitely) persistent from t_i.
- $[t_i, t_j]$: meaning that α holds from time t_i to t_j with $t_i < t_j$.

Then a set of time intervals $\mathbb{I}$ contains intervals as those described previously. Thus, for simplicity, we can have expressions like α^J where $J \in \mathbb{I}$. Intervals in $\mathbb{I}$ will be denoted by uppercase Latin characters: $A, B, C, \ldots, Z$. Then, throughout this work α^J is a *temporalised sentence* meaning the sentence α has an effectiveness time indicated by J. Then the semantics of classical propositional logic to a timed context is preserved. A temporalised sentence $\alpha^{[t_a,t_b]}$ is true when its non-temporalised expression α is true in every time point t between t_a and t_b. In other words, α holds at $[t_a, t_b]$.

Naturally, two intervals may not be disjoint, as defined next.

Definition 1 (Contained interval). *Let $R, S \in \mathbb{I}$ be two intervals. R is contained in S, denoted $R \subseteq S$ if and only if for all $t_i \in R$ it holds that $t_i \in S$.*

Definition 2 (Overlapped interval). *Let $R, S \in \mathbb{I}$ be two intervals. R and S are overlapped, denoted $R \top S$ if and only if there exists $t_i \in R$ such that $t_i \in S$.*

Example 4. *Let $R, S, V \in \mathbb{I}$ where $R = [t_3, t_7]$, $S = [t_4, t_6]$ and $V = [t_5, t_9]$ with $t_3, t_4, t_5, t_6, t_7,\ t_9 \in \mathbb{T}$. Then $S \subseteq R$, $R \top V$ and $S \top V$.*

4.3 Temporalised Belief Base

As rules are part of the knowledge, they are subject of temporal effectiveness too. In this perspective, there may be expressions such as

$$\alpha^{[t_a,t_b]} \to \beta^{[t_c,t_d]}$$

meaning that the rule can derive that β holds from time t_c to t_d if it is proved that α holds from time t_a to t_b. The notion of persistence during a given interval could be also applied to rules, although we adopt here a general approach. Note that the above implication itself is not decorated with intervals, but α and β are. This means that the implication always holds at $[-\infty, \infty]$ and hence again the classical semantics of first order logic is preserved. Thus, if the implication holds (since it is not conditioned in time) and α holds at $[t_a, t_b]$ then β holds at $[t_c, t_d]$.

Example 5. *The provision from Example 1 "If unemployed from 1980 to 1983, then a tax exemption applies from 1984 to 1986" can be formalised as follows:*

$$\mathit{Unemployed}^{[1980,1983]} \rightarrow \mathit{Tax\_Exemption}^{[1984,1986]}.$$

Thus, in [29], *temporalised belief base* which will contain temporalised sentences (see Example 6) is defined. This base represents a legal system in which each temporalised sentence defines a norm whose time interval determines the effectiveness time.

Example 6. *The set*

$$\begin{aligned}\mathbb{K} = \{&\alpha^{[t_1,t_3]}, \alpha^{[t_4]}, \alpha^{[t_1,t_4]} \rightarrow \beta^{[t_4,t_6]},\\ &\beta^{[t_5,t_6]}, \beta^{[t_6,t_8]}, \beta^{[t_{10}]}, \delta^{[t_{11}]},\\ &\delta^{[t_{11}]} \rightarrow \beta^{[t_{15},t_{20}]}, \omega^{[t_2,t_8]},\\ &\omega^{[t_4]} \rightarrow \beta^{[t_6,\infty]}, \epsilon^{[t_1,\infty]}\}\end{aligned}$$

is a valid temporalised belief base for a legal system. Note that sentence ϵ is valid (or true) from t_1.

This type of belief base representation implies that a sentence can appear more than once in a temporalised belief base, but from the point of view of the temporalised sentences stored in the temporalised belief base there is no redundancy because each temporalised sentence has different time intervals. For instance, consider Example 6, where α and β appear twice, but with different intervals. Whenever a sentence appears more than once with different intervals, just like $\beta^{[t_5,t_6]}$ and $\beta^{[t_{10}]}$, this sentence is said to be **intermittent**. Also note that if the intervals of a sentence are overlapped or continuous through the knowledge base, like $\beta^{[t_5,t_6]}$, $\beta^{[t_6,t_8]}$ in Example 6, the different occurrences are not collapsed into one, producing $\beta^{[t_5,t_8]}$. This is for two main reasons. First, a knowledge base scanning procedure is needed for identifying overlapped or continuous temporalised sentences, adding extra, yet small, complexity which is not relevant for the belief revision operator discussed here. Second, in law this kind of reiteration of a sentence in different intervals is not uncommon. For instance, the government may decide that there is a tax exemption during quarantine in March, and some weeks later then decide that the same exemption also holds during April. Here there are two legal norms that conform a continuous benefit, but with separate identities that can be revised for different reasons. Even more, the continuity does not need to be so explicit: two different continuous sentences like $\alpha^{[t_1,t_n]}$, $\alpha^{[t_{n+1},t_m]}$ may be derived from different, separate portions of the knowledge base, and even when it is clear that α holds from t_1 to t_m, this wider interval will not be derived as it is. That is, as we will see below, a sentence can be implicitly represented on a belief base by several different derivations that maintain the validity of the sentence at overlapping intervals. In this case, it could not be explicitly represented with a single sentence the validity of it at all times.

4.4 Temporalised Derivation

Note that a norm can explicitly be in a temporalised belief base, as $\alpha^{[t_5]} \in \mathbb{K}$ in Example 6. However, a norm can implicitly be represented in a temporal belief base if some conditions hold. For instance, in Example 6, norm β is implicitly represented with $\omega^{[t_2,t_8]}$, $\omega^{[t_4]} \to \beta^{[t_6,\infty]}$ due to the antecedent of the rule is held in t_4 by the temporalised sentence $\omega^{[t_2,t_8]}$. Next, the notion of temporalised derivation for a sentence is introduced to capture this intuition. To do this, we first give a definition of temporalised derivation in a time instant and then we give a definition of temporalised derivation in time interval.

Definition 3 (Temporalised derivation in a time instant). *Let $\mathbb{K}$ be a set of temporalised sentences and $\alpha^{[t_i]}$ be a temporalised sentence. We say that $\alpha^{[t_i]}$ is derived from $\mathbb{K}$, denoted $\mathbb{K} \vdash^t \alpha^{[t_i]}$, if and only if:*

- $\alpha^J \in \mathbb{K}$ *and* $t_i \in J$*, or*
- $\beta^H \to \alpha^P \in \mathbb{K}$ *and* $t_i \in P$ *and* $\mathbb{K} \vdash^t \beta^{[t_j]}$ *for all* $t_j \in H$.

Definition 4 (Temporalised derivation in a time interval). *Let $\mathbb{K}$ be a set of temporalised sentences and $\alpha^{[t_i,t_j]}$ be a temporalised sentence. We say that $\alpha^{[t_i,t_j]}$ is derived from $\mathbb{K}$ (denoted $\mathbb{K} \vdash^t \alpha^{[t_i,t_j]}$) if and only if $\mathbb{K} \vdash^t \alpha^{[t_p]}$ for all $t_p \in [t_i, t_j]$.*

Computing the temporalised derivation of a sentence through checking each instant of the intervals is useful in special cases where implicit sentences need temporalised sentences with overlapped intervals as antecedents. To determine the time interval of the implicitly derived temporal sentence, the temporal consequence will be defined below.

Definition 5 (Temporalised consequence)**.** *Let* $\mathbb{K}$ *be a set of temporalised sentences and* $\alpha^{[t_i,t_j]}$ *be a temporalised sentence. We say that* $\alpha^{[t_i,t_j]}$ *is a temporalised consequence of* $\mathbb{K}$ *(*$\alpha^{[t_i,t_j]} \in Cn^t(\mathbb{K})$*) if and only if* $\mathbb{K} \vdash^t \alpha^{[t_i,t_j]}$*.*

Example 7. *Consider again the temporalised belief base of Example 6. Then,* $\mathbb{K} \vdash^t \beta^{[t_4,\infty]}$*, that is,* $\beta^{[t_4,\infty]} \in Cn^t(\mathbb{K})$*; and* $\mathbb{K} \vdash^t \alpha^{[t_1,t_4]}$*, that is,* $\alpha^{[t_1,t_4]} \in Cn^t(\mathbb{K})$*.*

Following Definition 4, notice that the **interval of an implicitly derived sentence** will be the interval of the consequent of the rule that derives the conclusion of the proof. For instance, suppose that $\mathbb{K} = \{\gamma^{[t_2,t_5]}, \gamma^{[t_3,t_4]} \rightarrow \epsilon^{[t_6,t_9]}\}$ then the time interval of ϵ is $[t_6, t_9]$. Thus, a temporalised sentence $\alpha^{[t_i,t_j]}$ is **valid** (or true) in $\mathbb{K}$ if $\mathbb{K} \vdash^t \alpha^{[t_i,t_j]}$.

Thus, in [30], a ***contradiction*** arises when two complementary sentences can be derived with time intervals overlapped. For instance, suppose $\mathbb{K} = \{\alpha^{[t_2,t_9]}, \neg\alpha^{[t_1,t_3]}\}$, in this case, there exists a contradiction. However, consider $\mathbb{K} = \{\alpha^{[t_5]}, \neg\alpha^{[t_1,t_3]}\}$, in this case, we will say that $\mathbb{K}$ does not have contradictions. Moreover, a temporalised belief base is ***temporally consistent*** if the base does not have contradictions. The temporalised belief base of Example 6 is temporally consistent.

Remark 1. *If* $\mathbb{K}$ *represents a legal system then* $\mathbb{K}$ *should be temporally consistent.*

4.5 Legal Belief Revision

From a rational point of view, as was mentioned in Remark 1, a legal system should be temporally consistent, i.e., it cannot contain contradictory norms at any time. Hence, in [30], the authors propose a **prioritised legal revision operator** that allows to consistently add a temporalised sentence $\alpha^{[t_i,t_j]}$ to a consistent legal system $\mathbb{K}$.

This special revision operator is inspired by the rule semantics explained above in Section 4.1 (an adapted version from the one proposed in [17]). Thus, following the concept of temporally consistency of Subsection 4.4, the revision operator may remove temporalised sentences or, in some cases, may only modify the intervals to maintain consistency.

To incorporate a norm $\neg\beta^J$ into a legal system, it is necessary to consider all possible contradictions that may arise if the norm is added without checking for consistency. For this reason, it is necessary to compute all proofs of β considering only those temporalised sentences β^P whose effectiveness time is overlapped with the time interval J, that is, $J \top P$. Note that it is optimal to compute all minimal proofs of a temporal sentence considering only those in which the time interval is overlapped with the time interval of the input sentence. Next, a set of minimal proofs for a sentence is defined.

Definition 6 (Minimal proof)**.** *Let $\mathbb{K}$ be a temporalised belief base and α^J a temporalised sentence. Then, $\mathbb{H}$ is a minimal proof of α^J if and only if*

1. $\mathbb{H} \subseteq \mathbb{K}$,
2. $\alpha^P \in Cn^t(\mathbb{H})$ *with* $J \top P$, *and*
3. *if* $\mathbb{H}' \subset \mathbb{H}$, *then* $\alpha^P \notin Cn^t(\mathbb{H}')$ *with* $J \top P$.

Given a temporalised sentence α^J, the function $\Pi(\alpha^J, \mathbb{K})$ returns the set of all the minimal proofs for α^J from $\mathbb{K}$.

Remark 2. *Each set of $\Pi(\alpha^J, \mathbb{K})$ derives α in at least one time instant of J.*

Example 8. *Consider the temporalised belief base of Example 6. Then $\Pi(\beta^{[t_5,t_6]}, \mathbb{K})$ $= \{\mathbb{H}_1, \mathbb{H}_2, \mathbb{H}_3, \mathbb{H}_4\}$ where:*

- $\mathbb{H}_1 = \{\alpha^{[t_1,t_3]}, \alpha^{[t_4]}, \alpha^{[t_1,t_4]} \to \beta^{[t_4,t_6]}\}$,
- $\mathbb{H}_2 = \{\beta^{[t_5,t_6]}\}$,
- $\mathbb{H}_3 = \{\beta^{[t_6,t_8]}\}$,
- $\mathbb{H}_4 = \{\omega^{[t_2,t_8]}, \omega^{[t_4]} \to \beta^{[t_6,\infty]}\}$

Note that $\mathbb{H}_1$ is minimal: α should be derived from t_1 to t_4 to use the rule $\alpha^{[t_1,t_4]} \to \beta^{[t_4,t_6]}$ hence, $\alpha^{[t_1,t_3]}$ and $\alpha^{[t_4]}$ should be in $\mathbb{H}_1$.

The construction of prioritised legal revision by a temporalised sentence is based on the concept of a minimal proof; to complete the construction, an incision function is used which selects in every minimal proof the sentence to be erased later and which can produce legal effects in favour of a possible contradiction with the new norm.

The operator is based on a selection of sentences in the knowledge base that are relevant to derive the sentence to be retracted or modified. In order to perform a revision, following kernel contractions [21], this approach uses *incision functions*, which select from the minimal subsets entailing the piece of information to be revoked or modified. An incision function only selects sentences that can be relevant for α and at least one element from each $\Pi(\alpha^J, \mathbb{K})$:

Definition 7 (Incision function)**.** *Let $\mathbb{K}$ be a temporalised belief base. An incision function σ for $\mathbb{K}$ is a function such that for all $\alpha^J \in Cn^t(\mathbb{K})$:*

- $\sigma(\Pi(\alpha^J, \mathbb{K})) \subseteq \bigcup(\Pi(\alpha^J, \mathbb{K}))$.
- *For each* $\mathbb{H} \in \Pi(\alpha^J, \mathbb{K})$, $\mathbb{H} \cap \sigma(\Pi(\alpha^J, \mathbb{K})) \neq \emptyset$.

In Hansson's approach it is not specified how the incision function selects the sentences that will be discarded of each minimal proof. In this approach, this is solved by considering those sentences that can produce legal effects in favour of a possible contradiction with the new norm. Thus, if the new norm is $\neg\beta^J$ then the incision function selects the temporalised sentences β^P or $\alpha^Q \rightarrow \beta^F$ of each $\Pi(\beta^J, \mathbb{K})$.

Definition 8 (Search consequence function)**.** *Sc: $\mathbb{L} \times \mathbb{K} \mapsto \mathbb{K}$, is a function such that for a given sentence α and a given temporalised base $\mathbb{K}$ with $\mathbb{H} \subseteq \mathbb{K}$,*

$$Sc(\alpha, \mathbb{H}) = \{\alpha^J : \alpha^J \in \mathbb{H}\} \cup \{\beta^P \rightarrow \alpha^Q : \beta^P \rightarrow \alpha^Q \in \mathbb{H} \text{ and } \beta \in \mathbb{L}\}.$$

Definition 9 (Consequence incision function)**.** *Given a set of minimal proofs $\Pi(\alpha^J, \mathbb{K})$, $\sigma^c$is a consequence incision function if it is a incision function for $\mathbb{K}$ such that*

$$\sigma^c(\Pi(\alpha^J, \mathbb{K})) = \bigcup_{\mathbb{H} \in \Pi(\alpha^J, \mathbb{K})} Sc(\alpha, \mathbb{H}).$$

Example 9. *Consider Examples 6 and 8. Then, $Sc(\beta, \mathbb{H}_1) = \{\alpha^{[t_1,t_4]} \rightarrow \beta^{[t_4,t_6]}\}$, $Sc(\beta, \mathbb{H}_2) = \{\beta^{[t_5,t_6]}\}$, $Sc(\beta, \mathbb{H}_3) = \{\beta^{[t_6,t_8]}\}$, and $Sc(\beta, \mathbb{H}_4) = \{\omega^{[t_4]} \rightarrow \beta^{[t_6,\infty]}\}$. Then*

$$\begin{aligned}\sigma^c(\Pi(\beta^{[t_5,t_6]}, \mathbb{K})) &= \bigcup_{\mathbb{H} \in \Pi(\beta^{[t_5,t_6]}, \mathbb{K})} Sc(\beta, \mathbb{H}) \\ &= \{\alpha^{[t_1,t_4]} \rightarrow \beta^{[t_4,t_6]}, \beta^{[t_5,t_6]}, \beta^{[t_6,t_8]}, \omega^{[t_4]} \rightarrow \beta^{[t_6,\infty]}\}\end{aligned}$$

As mentioned before, the revision operator may remove temporalised sentences or, in some cases, may modify the intervals to maintain consistency. Next, a temporal projection will be introduced based on a given time interval. The idea here is, given a temporalised belief base $\mathbb{K}$ and given a time interval $[t_i, t_j]$, to return a temporalised belief base $\mathbb{K}'$ containing those sentences from $\mathbb{K}$ whose time intervals be out of $[t_i, t_j]$.

Definition 10 (Excluding temporal projection)**.** *Let $\mathbb{K}$ be a temporalised belief base and let $[t_i, t_j]$ be a time interval where $t_i, t_j \in \mathbb{T}$. A excluding temporal projection of $\mathbb{K}$ from t_i to t_j, denoted $out(\mathbb{K}, [t_i, t_j])$, is a subset of $\mathbb{K}$ where for all $\alpha^{[t_p,t_q]} \in \mathbb{K}$, $out(\mathbb{K}, [t_i, t_j])$ will contain:*

- $\alpha^{[t_p,t_i-1]}$ *if $t_p < t_i$, $t_q \geq t_i$ and $t_q \leq t_j$,*
- $\alpha^{[t_j+1,t_q]}$ *if $t_p \geq t_i$, $t_q > t_j$ and $t_p \leq t_j$,*

- $\alpha^{[t_p,t_i-1]}$ *and* $\alpha^{[t_j+1,t_q]}$ *if* $t_p < t_i$, $t_q > t_j$,
- $\alpha^{[t_p,t_q]}$ *if* $t_q < t_i$ *or* $t_p > t_j$.

Remark 3. *Note that when* $t_p \geq t_i$ *and* $t_q \leq t_j$, *the temporal sentence is not considered. In this case, this sentence is erased.*

Remark 4. *Note that if* $\delta^{[t_h,t_k]} \in out(\mathbb{K}, [t_i, t_j])$ *and the interval* $[t_h, t_k]$ *is generated through excluding temporal projection of* $\mathbb{K}$ *from* t_i *to* t_j *then there exists a temporal sentence* $\delta^{[t_p,t_q]}$ *in* $\mathbb{K}$ *such that* $[t_h, t_k] \subseteq [t_p, t_q]$.

Example 10. *Consider Example 9 and suppose that* S *is a temporalised belief base and* $S = \sigma^c(\Pi(\beta^{[t_5,t_6]}, \mathbb{K}))$. *Then,* $out(S, [t_5, t_6]) = \{\alpha^{[t_1,t_4]} \to \beta^{[t_4]}, \beta^{[t_7,t_8]}, \omega^{[t_4]} \to \beta^{[t_7,\infty]}\}$.

Following the notion of excluding temporal projection (Definition 10) a norm prioritised revision operator is defined. That is, an operator that allows to *consistently* add temporalised sentences in a temporalised belief base. If a contradiction arises, then the revision operator may remove temporalised sentences or modify the corresponding intervals in order to maintain consistency.

Definition 11. *Let* $\mathbb{K}$ *be a temporalised belief base and* α^J *be a temporalised sentence. The operator "* $\otimes$*", called prioritised legal revision operator, is defined as follow:*

$$\mathbb{K} \otimes \alpha^J = (\mathbb{K} \setminus \sigma^c(\Pi(\neg\alpha^J, \mathbb{K}))) \cup out(\sigma^c(\Pi(\neg\alpha^J, \mathbb{K})), J) \cup \ \{\alpha^J\}.$$

Note that, to add α^J to $\mathbb{K}$, all temporized sentences that have $\neg\alpha$ as a consequence and contribute to derive some instant of $\neg\alpha^J$ are erased. Then, these same sentences are added but with their modified intervals (using the excluding temporal projection introduced in Definition 10). Finally, α^J is added.

Example 11. *Consider Example 6 and suppose that a new norm* $\neg\beta^{[t_5,t_6]}$ *it is wished to add. To do this, it is necessary to do* $\mathbb{K} \otimes \neg\beta^{[t_5,t_6]}$. *Consider Examples 8 and 9. Then,* $\mathbb{K} \otimes \neg\beta^{[t_5,t_6]} = \{\alpha^{[t_1,t_3]}, \alpha^{[t_4]}, \alpha^{[t_1,t_4]} \to \beta^{[t_4]}, \beta^{[t_7,t_8]}, \beta^{[t_{10}]}, \delta^{[t_{11}]}, \delta^{[t_{11}]} \to \beta^{[t_{15},t_{20}]}, \omega^{[t_2,t_8]}, \omega^{[t_4]} \to \beta^{[t_7,\infty]}, \epsilon^{[t_1,\infty]}, \neg\beta^{[t_5,t_6]}\}$. *Note that, this new temporalised base is temporally consistent.*

The following example shows how our operator works in a particular situation when a legal system undergoes many changes and has rules that complement each other.

Example 12. *Consider the following temporalised belief base* $\mathbb{K} = \{\beta^{[t_1,t_{10}]}, \beta^{[t_1,t_5]} \to \alpha^{[t_1,t_5]}, \beta^{[t_6,t_{10}]} \to \alpha^{[t_6,t_{10}]}, \delta^{[t_4]}\}$. *Note that,* $\mathbb{K} \vdash^t \alpha^{[t_1,t_{10}]}$ *because* $\mathbb{K} \vdash^t \alpha^{[t_i]}$ *for all* $t_i \in [t_1, t_{10}]$. *Suppose that it is necessary to adopt* $\neg\alpha^{[t_1,t_{10}]}$. *To do this, it is necessary to compute all the minimal proofs of* $\alpha^{[t_1,t_{10}]}$ *in* $\mathbb{K}$. *In this case,* $\Pi(\alpha^{[t_1,t_{10}]}, \mathbb{K}) = \{\{\beta^{[t_1,t_{10}]}, \beta^{[t_1,t_5]} \to \alpha^{[t_1,t_5]}, \beta^{[t_6,t_{10}]} \to \alpha^{[t_6,t_{10}]}\}\}$. *Then,* $S = \sigma^c(\Pi(\alpha^{[t_1,t_{10}]}, \mathbb{K})) = \{\beta^{[t_1,t_5]} \to \alpha^{[t_1,t_5]}, \beta^{[t_6,t_{10}]} \to \alpha^{[t_6,t_{10}]}\}$. *Thus,* $out(S, [t_1, t_{10}]) = \emptyset$. *Therefore,* $\mathbb{K} \otimes \neg\alpha^{[t_1,t_10]} = \{\beta^{[t_1,t_{10}]}, \delta^{[t_4]}, \neg\alpha^{[t_1,t_{10}]}\}$.

4.6 Others works that have discussed the relation between belief revision and temporal reasoning

There are some works in the literature that have discussed the relation between belief revision and temporal reasoning, though none of them addressed the issue in the normative domain. Two prominent lines of investigation are [10, 11] and [27].

[10, 11] address belief revision in a temporal logic setting. These articles consider sets of sentences closed under logical consequence. In contrast to this, the approach proposed in [30] is based on an adapted version of belief bases which have additional information (time intervals). The use of belief bases makes the representation of the legal system state more natural and computationally tractable. That is, following [22, page 24] and [31], it is considered that legal systemsâĂŹ sentences could be represented by a finite number of sentences that correspond to the explicit beliefs on the legal system. The main purpose of [10, 11] is to represent the AGM postulates as axioms in a modal language. The assumption is that belief revision has to do with the interaction of belief and information over time, thus temporal logic seemed a natural starting point. The technical solution is to consider branching-time frames to represent different possible evolutions of beliefs. Hence, belief revision operators are interpreted over possible worlds. Unlike this, the authors in [30] work with legal system in which each temporalised sentence defines a norm whose time interval determines the effectiveness time. Then, the revision process defined in [30] may remove temporalised sentences or, in some cases, may only modify the intervals.

[27] is based on a well-developed theory of action in the situation calculus extended to deal with belief. The authors add to this framework a notion of plausibility over situations, and show how to handle nested belief, belief introspection, mistaken belief, belief revision and belief update together with iterated belief change.

An interesting line of investigation is to study possible correlations with these two last research lines in literature as compared to the system proposed in [30]. Such a comparison cannot be directly done from technical viewpoint for two reasons. First of all, [30] is specifically focused in a propositional language following kernel contraction construction proposed in [21]. Second, the propositional language in

[30] is equipped with explicit time-stamps and with temporal intervals, which allow them for expressing richer temporal specifications in the language.

5 Conclusions

In order to properly model norm change in the law, temporal aspects of legal dynamics must be considered. Several reasons support this idea. The law regulates its own changes by stating, within the system, what and how other existing norms should be modified. The introduced new norms can be in conflict and so norms are defeasible by nature. Even more, legal norms are qualified by diverse temporal properties, such as the time when the norm is added to the legal system, or when the norm is in force and it produces legal effects. Thus, all these aspects may be addressed by two different pathways, as reflected in the literature. First, normative dynamics can be modelled by combining logical systems for temporal and defeasible reasoning [15, 19, 16]. Second, belief revision techniques can be enriched with temporal dimensions: this has been done in works such as [29, 30]. These are two different approaches to the consideration of time within a logical framework for legal dynamics.

Defeasible Logic was extended with temporal parameters to allow for reasoning about time specified inside norms. Two temporal dimensions are considered: the first one is when the norm is in force in a normative system, and the second is when the norm exists in the normative system from a certain viewpoint. Usually only the time of force is considered, but here the notion of *temporalised rule with viewpoint* is introduced, a mechanism through which it is possible to reason on the legal system from the viewpoint of its current version but as if it were revised in the past. This extension increases the expressive power of the logic and it allows us to represent meta-norms describing norm modifications by referring to a variety of possible timelines through which conclusions, rules and derivations can persist over time. This formalism has been shown useful to model retroactive legal modifications, a complex timed behaviour of legal systems that requires special attention. Hence, this model is suitable for modeling changes that affect the legal system with respect to legal effects which were also obtained before the legal change was done. This is not a simple feature and the formalism addresses it properly.

On the other hand, a contrasting approach explores the importance of time in legal dynamics from the point of view of *revision of beliefs* in laws. This make sense since the law is a *dynamic* system of rules. Indeed, a very complex one: as times goes by, rules are introduced in the system, which may be either unexpectedly in conflict with existing rules or be intended to provide new, different norms for society.

This demands a consistent revision of the rules of the system, so an extension of classic belief revision formalism seems to be appropriate. Then, we discussed here the second approach, which proposes a belief revision operator that considers time interval in the revision process. Intervals are used to model a period of time for a piece of knowledge to be effective or relevant, leading to the definition of a new kind of temporal rules. On these interval-decorated rules the corresponding temporalised derivation was defined. The consideration of time requires an adaptation of the notions of contradiction and inconsistency in the classical sense. Temporalised knowledge base is inconsistent only if contradictory information can be derived for the same moment of time. In that approach was defined a novel belief revision operator that allows the consistent addition of temporalised sentences in a temporalised belief base. If a contradiction arises, then the revision operator may either completely remove conflictive temporalised sentences or modify the intervals of some rules. This last action is made because a given consequence a at interval I may fall in contradiction during a sub-interval of I. Thus, a should be a consequence, after the revision, only for the rest of I. Then, intervals in rules should be taken into account for the revision process.

The central idea of this research topic is that formal models of norm change must address the fact that new norms may be elicited and old norms may need to be retracted, with complex consequences. Depending on the particular feature of legal dynamics intended to be modelled, any proposed framework requires an appropriate model of time. There are two mainstream approaches to reasoning with and about time: point based and interval based. Here we explored both flavours, by discussing two different, interesting approaches to the consideration of time within the study of legal dynamics. Both formalisms take time into account and provide the necessary logical infrastructure to address the characterization of complex behaviour of dynamics in normative systems, constituting solid foundations for further research.

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