BAAA, Vol. 58, 2016
P. Benaglia, D. D. Carpintero, R. Gamen & M. Lares, eds.

Asociación Argentina de Astronomía
Boletín de artículos científicos

Black holes: fundamentals and controversies

G.E. Romero[1,2]

[1] *Instituto Argentino de Radioastronomía, CCT-La Plata, CONICET, Argentina*
[2] *Facultad de Ciencias Astronómicas y Geofísicas, UNLP, Argentina*

Contact / romero@iar-conicet.gov.ar

Resumen / Los agujeros negros son objetos totalmente colapsados en su campo gravitacional. Han sido estudiados teóricamente durante más de cuarenta años utilizando la teoría de la relatividad general. Más recientemente, se los ha investigado en el marco de teorías alternativas de la gravitación. En este artículo repasaré las principales propiedades de los agujeros negros y discutiré en forma accesible algunas controversias teóricas recientes sobre su naturaleza.

Abstract / Black holes are fully gravitational collapsed objects. They have been studied from a theoretical point of view during more than 40 years using the theory of General Relativity. Recently they have been also investigated in the context of alternative theories of gravitation. In this paper I review the main properties of black holes and I discuss, in an accesible way, some recent controversies about the nature of these objects.

Keywords / stars: black holes — gravitation

1. Introduction

Black holes are perhaps the most amazing objects thought to exist in the Universe. Their uniqueness is reflected by the huge technical literature devoted to them. As to the end of 2015 there were more than 42 000 articles published in peer reviewed journals with the words "black hole" in the title. In comparison, there were 14 000 and 9000 titles with the words "neutron star" and "white dwarf", respectively. Black holes are also often depicted in the popular press, the news, and even in cartoons and TV shows. They have invaded the popular culture and the media, attracting a lot of attention not only of researchers but also of the general public. This huge interest is up to some extent justified by the peculiarities of black holes and the huge impact that their existence has upon our view of the cosmos and its physical laws. In black holes our most cherished concepts of classical common-sense physics break down beyond repair. The actual nature of black holes and some of the problems they pose for modern physics, nevertheless, remain obscure for a large majority of the public, either scientific or lay. The purpose of the following pages is to present a short review of some basic properties of black holes and the current controversies rised around them. For a longer exposition the readers are referred to Frolov & Novikov (1998) and Romero & Vila (2014).

2. Definitions

Black holes are objects that are gravitationally collapsed and hence, are infinitely redshifted. This means that they are causally disconnected from the rest of the Universe in the following sense: events ocurring inside the black hole can never affect in any way events ocurring outside. A crucial issue, then, is to provide an adequate definition of the boundary between the interior and exterior regions of the black hole. In order to give such a definition, let us introduce first a physical system formed by all events. An event is an occurrence of any type. We call such system *spacetime* and we represent it by a C^{∞}-differentiable, 4-dimensional, real pseudo-Riemannian manifold. A real 4-D manifold is a set that can be covered completely by subsets whose elements are in a one-to-one correspondence with subsets of $\mathbb{R}^4$, the 4-dimensional space of real numbers. We adopt 4 dimensions because it seems enough to give 4 real numbers to localize an event. A metric field $g_{\mu\nu}$ that determines the distance between two events and is locally Minkowskian is introduced on the manifold in accordance to Einstein's field equations: $R_{\mu\nu} - 1/2 g_{\mu\nu} R = \kappa T_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor formed with second order derivatives of the metric, R is the Ricci scalar $g^{\mu\nu} R_{\mu\nu}$, $\kappa = 8\pi G/c^4$ is a constant, and $T_{\mu\nu}$ is a second rank tensor that represents the energy-momentum of all material fields that generate the metric field $g_{\mu\nu}(x^{\mu})$ – here $\{x^{\mu}\}$ is a mathematical coordinate system that corresponds to a physical reference frame. A given spacetime model is specified by a triplet: $ST \equiv (M, g_{\mu\nu}, T_{\mu\nu})$, where M is the manifold, g the metric field, and T the energy-momentum field. Since we will deal with vacuum or electro-vacuum solutions, for simplicity, we will denote a given spacetime by $(M,\ g_{\mu\nu})$.

Because many coordinate systems can be used to describe black holes, it is convenient to give a definition of a black hole that is independent of the choice of coordinates. First, I will introduce some preliminary useful definitions (see, for details, Hawking & Ellis 1973, Wald 1984).

Definition. *A causal curve in a spacetime* $(M,\ g_{\mu\nu})$ *is a curve that is non space-like, that is, piecewise either time-like or null (light-like).*

We say that a spacetime $(M,\ g_{\mu\nu})$ is *time-orientable* if we can define over M a smooth non-vanishing time-like vector field.

Definition. *If $(M,\ g_{\mu\nu})$ is a time-orientable spacetime, then $\forall\, p \in M$, the causal future of p, denoted $J^+(p)$, is defined by:*

$$J^+(p) \equiv \{q \in M \mid \exists \text{ a future-directed causal curve from } p \text{ to } q\}.$$

Similarly,

Definition. *If $(M,\ g_{\mu\nu})$ is a time-orientable spacetime, then $\forall\, p \in M$, the causal past of p, denoted $J^-(p)$, is defined by:*

$$J^-(p) \equiv \{q \in M \mid \exists \text{ a past-directed causal curve from } p \text{ to } q\}.$$

The causal future $(+)$ and past $(-)$ of any set $S \subset M$ are given by:

$$J^{\pm}(S) = \bigcup_{p\in S} J^{\pm}(p). \tag{1}$$

A set S is said *achronal* if no two points of S are timelike related. A Cauchy surface is an achronal surface such that every non space-like curve in M crosses it once, and only once. A spacetime $(M, g_{\mu\nu})$ is *globally hyperbolic* iff it admits a space-like hypersurface $S \subset M$ which is a Cauchy surface for M.

Causal relations are invariant under conformal transformations of the metric. So, the spacetimes $(M, g_{\mu\nu})$ and $(M, \widetilde{g}_{ab})$, where $\widetilde{g}_{ab} = \Omega^2 g_{ab}$, with Ω a nowhere zero C^r function, have the same causal structure.

Let us now consider a spacetime where all null geodesics start in a region $\mathcal{J}^-$ and end at $\mathcal{J}^+$. Then, such a spacetime, $(M,\ g_{\mu\nu})$, is said to contain a *black hole* if M *is not* contained in $J^-(\mathcal{J}^+)$. In other words, there is a region from where no null geodesic can reach the *asymptotic flat*[⋆] future spacetime, or, equivalently, there is a region of M that is causally disconnected from the global future. The *black hole region*, BH, of such spacetime is $BH = [M - J^-(\mathcal{J}^+)]$, and the boundary of BH in M, $H = J^-(\mathcal{J}^+) \bigcap M$, is the *event horizon*.

Notice that a black hole is conceived as a spacetime *region*, i.e. what characterises the black hole is its metric and, consequently, its curvature. What is peculiar of this spacetime region is that it is causally disconnected from the rest of the spacetime: no events in this region can make any influence on events outside the region. Hence the name of the boundary, event horizon: events inside the black hole are separated from events in the global external future of spacetime. The events in the black hole, nonetheless, as all events, are causally determined by past events. A black hole does not represent a breakdown of classical causality.

[⋆]Asymptotic flatness is a property of the geometry of spacetime which means that in appropriate coordinates, the limit of the metric at infinity approaches the metric of the flat (Minkowskian) spacetime.

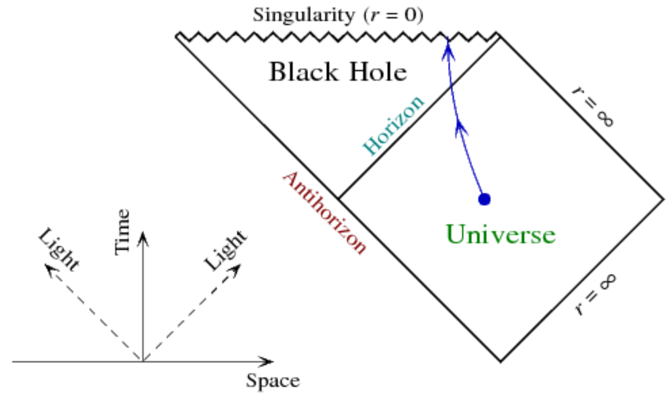

Figure 1: Carter-Penrose diagram of a Schwarzschild black hole.

A useful representation of a black hole is given by a Carter-Penrose diagram. This is a two-dimensional diagram that captures the causal relations between different points in spacetime. It is an extension of a Minkowski diagram where the vertical dimension represents time, and the horizontal dimension represents space, and slanted lines at an angle of 45° correspond to light rays. The biggest difference with a Minkowski diagram (light cone) is that, locally, the metric on a Carter-Penrose diagram is conformally equivalent[⋆⋆] to the actual metric in spacetime. The conformal factor is chosen such that the entire infinite spacetime is transformed into a Carter-Penrose diagram of finite size. For spherically symmetric spacetimes, every point in the diagram corresponds to a 2-sphere. In Fig. 1, I show a Carter-Penrose diagram of a spherically symmetric vacuum (Schwarzschild) spacetime.

3. Metrics and properties

Exact solutions of Einstein's field equations representing stationary black holes exist for vacuum and electrovacuum spacetimes. The spherically symmetric solutions are the Schwarzschild and Reisner-Nordström solutions, whereas the axially symmetric solutions are the Kerr and Kerr-Newman solutions. The Kerr-Newman metric of a charged spinning black hole is the most general black hole solution. It was found by Ezra "Ted" Newman and co-workers in 1965 (Newman et al., 1965), and in the appropriate limits allows to recover the other solutions.

The full expression of the interval in the Kerr-Newman spacetime reads (in Boyer-Lindquist coordinates):

[⋆⋆]I remind that two geometries are conformally equivalent if there exists a conformal transformation (an angle-preserving transformation) that maps one geometry onto the other. More generally, two Riemannian metrics on a manifold M are conformally equivalent if one is obtained from the other through multiplication by a function on M.

$$
\begin{aligned}
\mathrm{d}s^2 &= g_{tt}\mathrm{d}t^2 + 2g_{t\phi}\mathrm{d}t\mathrm{d}\phi - g_{\phi\phi}\mathrm{d}\phi^2 - \Sigma\Delta^{-1}\mathrm{d}r^2 \\
&\quad -\Sigma d\theta^2 && (2)\\
g_{tt} &= c^2\left[1-(2GMrc^{-2}-q^2)\Sigma^{-1}\right] && (3)\\
g_{t\phi} &= a\sin^2\theta\,\Sigma^{-1}\left(2GMrc^{-2}-q^2\right) && (4)\\
g_{\phi\phi} &= [(r^2+a^2c^{-2})^2 - a^2c^{-2}\Delta\sin^2\theta]\Sigma^{-1}\sin^2\theta && (5)\\
\Sigma &\equiv r^2 + a^2c^{-2}\cos^2\theta && (6)\\
\Delta &\equiv r^2 - 2GMc^{-2}r + a^2c^{-2} + q^2 && (7)\\
&\equiv (r-r_{\mathrm{h}}^{\mathrm{out}})(r-r_{\mathrm{h}}^{\mathrm{inn}}), && (8)
\end{aligned}
$$

where M is the black hole mass, $a = J/M$ is the specific angular momentum, q is related to the charge Q by

$$q = \frac{GQ^2}{4\pi\epsilon_0 c^4},$$

and the outer horizon is located at

$$r_{\mathrm{h}}^{\mathrm{out}} = GMc^{-2} + [(GMc^{-2})^2 - a^2c^{-2} - q^2]^{1/2}. \quad (9)$$

There is an inner event horizon located at:

$$r_{\mathrm{h}}^{\mathrm{inn}} = GMc^{-2} - [(GMc^{-2})^2 - a^2c^{-2} - q^2]^{1/2}. \quad (10)$$

An essential singularity occurs when $g_{tt} \to \infty$; this happens if $\Sigma = 0$. This condition implies:

$$r^2 + a^2c^{-2}\cos^2\theta = 0. \quad (11)$$

Such a condition is fulfilled only by $r = 0$ and $\theta = \frac{\pi}{2}$. This translates in Cartesian coordinates to:***

$$x^2 + y^2 = a^2c^{-2} \quad \text{and} \quad z = 0. \quad (12)$$

The singularity is a ring of radius ac^{-1} on the equatorial plane. If $a = 0$, then a Schwarzschild's point-like singularity is recovered. If $a \neq 0$ the singularity is not necessarily in the future of all events at $r < r_{\mathrm{h}}^{\mathrm{inn}}$: this means that the singularity can be avoided by some geodesics.

The Kerr-Newman solution is a non-vacuum solution. It shares with the Kerr and Reissner-Nordström solutions the existence of two horizons, and as the Kerr solution it presents an ergosphere (a region where spacetime is dragged around the black hole). At a latitude θ, the radial coordinate for the ergosphere is:

$$r_{\mathrm{e}} = GMc^{-2} + [(GMc^{-2})^2 - a^2c^{-2}\cos^2\theta - q^2]^{1/2}. \quad (13)$$

As the Kerr metric for an uncharged rotating mass, the Kerr-Newman interior solution exists mathematically but is probably not representative of the actual metric of a physically realistic rotating black hole because of stability problems. The surface area of the horizon is:

$$A_{\mathrm{KN}} = 4\pi(r_{\mathrm{h}}^{\mathrm{out}\,2} + a^2c^{-2}). \quad (14)$$

The Kerr-Newman metric represents the simplest stationary, axisymmetric, asymptotically flat solution of Einstein's equations in the presence of an electromagnetic field in four dimensions. Any Kerr-Newman source has its rotation axis aligned with its magnetic axis (Punsly, 1998). Thus, a Kerr-Newman source is different from commonly observed astronomical bodies, for which there might be a substantial angle between the rotation axis and the magnetic moment (as observed in pulsars). In Fig. 2, I present a sketch of the stucture of a theoretical Kerr-Newman black hole.

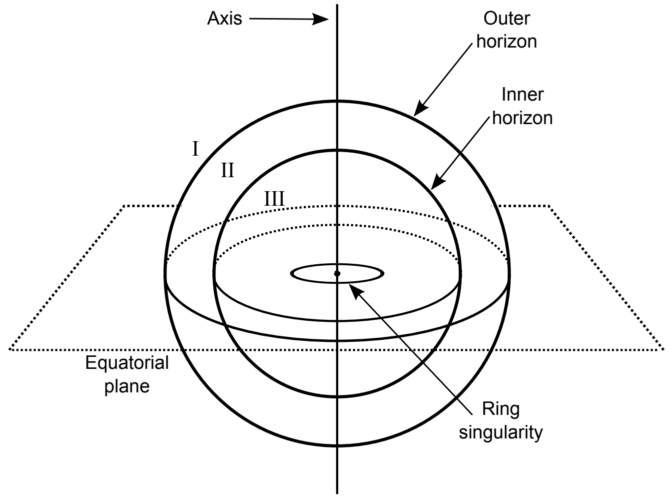

Figure 2: Structure of a Kerr-Newman black hole.

4. Thermodynamics

The area of a Schwarzschild black hole is

$$A_{\mathrm{Schw}} = 4\pi r_{\mathrm{Schw}}^2 = \frac{16\pi G^2M^2}{c^4}. \quad (15)$$

In the case of a Kerr-Newman black hole,

$$
\begin{aligned}
A_{\mathrm{KN}} &= 4\pi\left(\frac{GM}{c^2} + \frac{1}{c^2}\sqrt{G^2M^2 - GQ^2 - a^2}\right)^2 \\
&\quad + 4\pi\frac{a^2}{c^2}. && (16)
\end{aligned}
$$

Notice that expression (16) reduces to (15) for $a = Q = 0$.

When a black hole absorbs a mass δM, its mass increases to $M+\delta M$, and hence, the area increases as well. Since the horizon can be crossed in just one direction, the area of a black hole can only increase. This suggests an analogy with entropy (Bekenstein, 1973). A variation in the entropy of the black hole will be related to the heat (δQ) absorbed through the following equation:

$$\delta S = \frac{\delta Q}{T_{\mathrm{BH}}} = \frac{\delta Mc^2}{T_{\mathrm{BH}}}. \quad (17)$$

Particles trapped in the black hole will have a wavelength:

$$\lambda = \frac{\hbar c}{kT} \propto r_{\mathrm{Schw}}, \quad (18)$$

where k is the Boltzmann constant, and the proportionality requires a constant smaller than 1. Then,

$$\xi\frac{\hbar c}{kT} = \frac{2GM}{c^2},$$

where ξ is the mentioned numerical constant. Hence, we can associate a temperature to the black hole:

***The relation with Boyer-Lindquist coordinates is $z = r\cos\theta$, $x = \sqrt{r^2+a^2c^{-2}}\sin\theta\cos\phi$, $y = \sqrt{r^2+a^2c^{-2}}\sin\theta\sin\phi$.

$$T_{\rm BH} = \xi \frac{\hbar c^3}{2GkM},$$

and

$$S = \frac{c^6}{32\pi G^2 M} \int \frac{dA_{\rm Schw}}{T_{\rm BH}} = \frac{c^3 k}{16\pi\hbar G\xi} A_{\rm Schw} + \text{ constant.}$$

A quantum mechanical calculation of the horizon temperature in the Schwarzschild case leads to $\xi = (4\pi)^{-1}$. So,

$$T_{\rm BH} = \frac{\hbar c^3}{8GMk} \cong 10^{-7}{\rm K}\left(\frac{{\rm M}_\odot}{M}\right). \tag{19}$$

Then, we can write the entropy of the black hole as:

$$S = \frac{kc^3}{4\pi\hbar G} A_{\rm Schw} + \text{ constant} \tag{20}$$

$$\sim 10^{77}\left(\frac{M}{{\rm M}_\odot}\right)^2 k\ {\rm JK}^{-1}. \tag{21}$$

The formation of a black hole implies a huge increase of entropy: a star has an entropy ~ 20 orders of magnitude lower than the corresponding black hole. This tremendous increase of entropy is related to the loss of all the structure of the original system (a collapsing star or a cloud of gas) once the black hole is formed.

The analogy between area and entropy allows to state a set of laws for black hole thermodynamics (Bardeen et al., 1973):

- First law (energy conservation): $\mathrm{d}M = T_{\rm BH}\mathrm{d}S + \Omega_+\mathrm{d}J + \Phi\mathrm{d}Q + \delta M$. Here, Ω_+ is the angular velocity, J the angular momentum, Q the electric charge, Φ the electrostatic potential, and δM is the contribution to the change in the black hole mass due to the change in the external stationary matter distribution.
- Second law (entropy never decreases): in all physical processes involving black holes the total surface area of all the participating black holes can never decrease.
- Third law (Nernst's law): the temperature (surface gravity) of a black black hole cannot be zero. Since $T_{\rm BH} = 0$ with $A \neq 0$ for extremal charged and extremal Kerr black holes, these are thought to be limit cases that cannot be reached in Nature.
- Zeroth law (thermal equilibrium): the surface gravity (temperature) is constant over the event horizon of a stationary axially symmetric black hole.

5. Quantum fields around black holes

In the current physical view, the world is a collection of quantum fields existing in spacetime. The vacuum state $|0\rangle$ of these fields can be excited to form a Fock basis of the quantized field:

$$|1_k\rangle = a_k^\dagger|0\rangle. \tag{22}$$

Succesive applications of the operator $a_k^\dagger$ yield:

$$a_k^\dagger|n_k\rangle = (n+1)^{1/2}|(n+1)_k\rangle. \tag{23}$$

In Minkowski space, a preferred basis can be constructed using the specific symmetries of this space (the Poincaré group). Then, if $N_k = a_k^\dagger a_k$ is the operator number of particles, we get

$$\langle 0|N_k|0\rangle = 0, \qquad \text{for all } k. \tag{24}$$

This means that the expectation value for all quantum modes of the vacuum is zero: if there are no particles associated with the vacuum state in one reference system, then the same is valid in all of them. In curve spacetime this is not valid any longer: general spaces do not share the Minkowski symmetries, and hence the number of particles is not a relativistic invariant. In particular, the presence of a black hole horizon induces a polarization of the vacuum in such a way that a detector at infinity will measure a net flux of thermal particles:

$$\langle 0|T_{\mu\nu}|0\rangle = \frac{\kappa^2}{48\pi}, \tag{25}$$

where $\kappa = 8\pi G/c^4$, as before. The radiation has a Planckian distribution with a temperature $T_{\rm BH} = \kappa/2\pi k$, in agreement with 19 (see Birrell & Davies 1982 for details). Therefore, quantum field theory reveals the mechanism hidden behind the phenomenological considerations of the previos section. It is not the black hole itself that emits radiation, but the quantum fields in the presence of an event horizon.

6. Controversies

A number of important controversies have arised from recent research on black holes. Below I will comment on some of them.

6.1. Singularities

The term "singularity" is often abused in the context of general relativity. It is usual to see this term as the subject of sentences such as "the singularity is at the center of the black hole" or "the singularity is strong". These expressions and many others found in the literature lead many people to think that singularities are some kind of physical entities where some general physical principles such as causality are not valid any longer. This is incorrect. Singularities are not things or any other kind of existents. Nay, they are features of some mathematical models of spacetime. There are not such a thing as singularities. Rather, there are singular spacetime models in general relativity.

A spacetime is said to be singular if the manifold M that represents the system of all events is incomplete; and a manifold is incomplete if it contains at least one inextensible curve. A curve $\gamma : [0, a) \to M$ is inextensible if there is no point p in M such that $\gamma(s) \to p$ as $a \to s$, i.e. γ has no endpoint in M. So, singularities are defects in our modelling of spacetime with continuous manifolds. And these defects are of different type from the singularities that appear in electromagnetism or Newtonian gravity. For example, the Newtonian potential of a mass m is Gm/r. This potential diverges at $r = 0$. But the location $r = 0$ is well defined in the theory. It just happens that the magnitude of the potential becomes unbounded at that point. The situation in general relativity is quite diferent. Spacetime itself ceases to exist in a singular spacetime model. There is no location where this happens, since the very concept of location requires spacetime to exist. The singularity, then, has no way to interact with the real world. Whatever happens in the pathological region of a singular spacetime model, we cannot say within that model. It is as impossible as to speak without using a language. The reason is simple: to speak *is* to use a language! In a similar way, if you do not have spacetime, you cannot predict the evolution of physical systems using general relativity, because any prediction in this theory is a prediction about the motion of physical systems in spacetime.

We can sum up all this saying that general relativity is incomplete: it cannot describe completely the spacetime inside a black hole.

Now, everyone knows that there are some theorems about gravitational collapse in general relativity that state that even if the collapse is not symmetric, a singular spacetime results if some conditions hold (Penrose, 1965). These singularity theorems are not theorems that imply the physical existence, under some conditions, of spacetime singularities. Material existence cannot be formally implied.

In general, existence theorems imply that under certain assumptions there are functions that satisfy a given equation, or that some concepts can be formed in agreement with some explicit syntactic rules. Theorems of this kind state the possibilities of some formal system or language. Such possibilities, although not obvious in many occasions, are always a necessary consequence of the assumptions of the formal system.

In the case of the singularity theorems of classical field theories like general relativity, what is implied is that under some assumptions the solutions of the equations of the theory are defective beyond repair. The correct interpretation of these theorems is that they point out the incompleteness of the theory: there are some statements that cannot be made within the theory. In this sense (and only in this sense), the theorems are like Gödel's famous theorems of mathematical logic (Romero, 2012).

6.2. Information paradox

"Is information destroyed by black holes?" This is a question often heard in the popular scientific press and even in academic journals. The interest in this supposed problem is additionally sparked by the notorious changes of opinion of Stephen Hawking, a popular persona always at the center of public attention. In 1976, he answered by the positive (Hawking, 1976), recently by the negative (Hawking, 2015).

Most of the discussion of the so-called information paradox is misfocused because of a lack of understanding of the concept of information. What is, exactly, information? This word is a polysemic term. In ordinary usage it designates a property of languages (the propositional content of a signal). Therefore, there is no "law of conservation" of the information, nor it is true that information can never decrease. In fact, it may disappear, as anyone who has lost a hard disk can corroborate.

Some authors confuse "information" with "entropy", which is a thermodynamic concept. Others, with time invariance of the solutions of an equation. So, according to the level of confusion we can differentiate several supposed paradoxes. Let us see.

- **"Entropic paradox"**: The entropy of black holes decreases when they evaporate. This is supposed to be a paradox because of, we are said, black holes would violate the second law of thermodynamics.
  The second law of thermodynamics demands only that the total entropy of a closed system is either maximum or increases. A black hole is not a closed system, then there is no violation if its entropy decreases. A generalized second law is perfectly valid:
  $$\mathrm{d}(S_{\mathrm{BH}} + S_{\mathrm{Universe}})/\mathrm{d}t > 0. \tag{26}$$
- **"Paradox of predictability"**: This is another pseudo-problem. It is a fact that we cannot predict the state of the Universe after the evaporation of the black hole just using general relativity and quantum mechanics. This is, professedly, paradoxical. The answer is trivial: of course we cannot! General relativity is an incomplete theory as the singularity theorems clearly show. There is no paradox, just the need of a better description of nature.
- **The paradox of the loss of unitary evolution**: This is nowadays the most amply discussed paradox. I remind that, in order to say that a system has unitary evolution, the final state must evolve from the initial state and this evolution must be reversible. Black holes seem to be objects that do not behave in this way if they evaporate.
  Let us consider a quantum system in a pure state and let it fall into a black hole. Let us wait a certain amount of time until the hole has evaporated enough to return to its previous mass. First we had a pure state and a black hole of mass M. Afterwards, we have a thermal state and a black hole of the same mass M. Physically, both black holes are indistinguishable. There is, then, a process that (apparently) turns a pure state into a thermal state. But a thermal state is a mixed state, so unitary evolution does not occur. We cannot retrodict the initial state from the final one and the physical laws. In technical jargon, the black hole has performed a non unitary transformation on the state of the system. Standard quantum mechanics is violated.

There are several possible solutions to this problem:

- Quantum mechanics fails at the horizon. This is a heavy hypothesis. Quantum mechanics is a very robust theory and no one, ever, has detected any problem with it.
- Relativity fails at the horizon. This is the favorite option of particle physicists. What is supposed to fail is the equivalence principle at the horizon. The so-called "firewalls" are an example of the proposals put forth by particuleers. A firewall is a chaotically violent surface of highly energetic

quantum states located close to the infinite redshift surface of the black hole. The only way this can happen is if the quantum state in the part of the slice inside the black hole has no dependence on the initial state. This is effectively a "bleaching" of the "information": all distinctions between the initial states of infalling matter are expunged before the system crosses the global event horizon. A regular horizon implies increasing the entanglement. Conversely, if entanglement is to decrease, then the state at the horizon cannot be the vacuum. This is the firewall argument in a nutshell. As a consequence, the equivalence principle is no longer valid.

- Hawking radiation does not exist. This solution suggests that there is something wrong with the application of quantum field theory to curved spacetime. But nobody knows what.

- Black holes do not exist. Several authors have suggested that there is not such a thing as a black hole in the Universe. Several alternative objects like fuzzy balls, gravastars, boson stars, and taquionic condensates have been proposed. These objects are a lean medicine: they are far more complicated than black holes and are plagued with problems of their own, from instabilities to the invocation of unknown fields or states of matter.

- A final option, suggested by Roger Penrose, is that, indeed, the evolution of the quantum system is not unitary and there is no problem. This is the "accept the reality as it is" solution.

Whatever is going on here, it is likely that the actual situation will become clear only when a quantum theory of gravity be available.

6.3. Cosmological black holes

Black hole solutions as those described in Sect. 3. represent stationary regions in a static background spacetime. The real Universe, however, is expanding. Moreover, it seems to expand in an accelerated way. Since both the black hole and the global spacetime itself are expanding, this expansion should be taken into account in the description of the black hole, at least on long timescales.

McVittie (1933) was the first to combine a Schwarzschild solution with a Friedman-Lemaître-Robertson-Walker (FLRW) background metric to find the effects of the expanding Universe on a massive object. McVittie metric is based on the following assumptions: 1) at large distances from the compact object the metric is given approximately by the FLRW expression; 2) when the expansion is ignored (i.e. when the scale factor $a(t) = a_0$ is constant), Schwarzschild metric is recovered; 3) the metric must be a consistent solution to Einstein's field equations with a perfect fluid energy-momentum tensor, and 4) there is no radial matter infall.

McVittie metric, with the assumption that the mass of the black hole increases with the scale factor $a(t)$ in the form $M_{\rm BH}(t) = M_0 a(t)$, reads (for a flat universe):

$$\mathrm{d}s^2 = -\frac{\left\{1 - \dfrac{[M_0 a(t)]}{2[ra(t)]}\right\}^2}{\left\{1 + \dfrac{[M_0 a(t)]}{2[ra(t)]}\right\}^2}\mathrm{d}t^2 + a(t)^2\left\{1 + \frac{[M_0 a(t)]}{2[ra(t)]}\right\}^4\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right); \quad (27)$$

whereas for the open and closed Friedmann models, the corresponding metrics take the form:

$$\mathrm{d}s^2 = -\frac{\left\{1 - \dfrac{[M_0 a(t)]}{2[ra(t)]}\left(1 \pm \dfrac{r^2}{4R_0^2}\right)^{1/2}\right\}^2}{\left\{1 + \dfrac{[M_0 a(t)]}{2[ra(t)]}\left(1 \pm \dfrac{r^2}{4R_0^2}\right)^{1/2}\right\}^2}\mathrm{d}t^2 + \frac{\left\{1 + \dfrac{[M_0 a(t)]}{2[ra(t)]}\left(1 \pm \dfrac{r^2}{4R_0^2}\right)^{1/2}\right\}^4}{\left(1 \pm \dfrac{r^2}{4R_0^2}\right)^2} a(t)^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right). \quad (28)$$

It is not trivial to show that these metrics represent a black hole. Actually, the validity of the metrics 28 as a correct description of a compact object embedded in a curve FLRW spacetime has been recently questioned (Nandra et al., 2012). In the flat metric there are 2 horizons; one is an event horizon, and the other a cosmological horizon. Contrary to Scharzschild metric, however, the horizon at $r = M_0/2a(t)$ is singular, corresponding to a divergent pressure. The interpretation of this singularity has been under debate for some time, but it is clear that it corresponds to the event horizon from which the background fluid cannot escape. Most authors agreee on that McVittie solution is only valid for $> M_0/2a(t)$.

So far no solution for a Kerr metric embedded in a FLRW spacetime is known. In addition, little is known of the properties of black hole spherically symmetric solutions in evolving universes. This is an open topic that certainly deserves further attention.

6.4. Mimickers

Black holes are characterised by horizons, which are null surfaces of infinite redshift. This is what makes these regions of spacetime "black". In practice, however, a surface of infinite redshift is almost impossible to differentiate from a surface of *almost* infinite redshift. This fact has been used to create models of compact objects other than black holes, such as gravastars. Such objects have problems of stability and their existence in the real Universe seems implausible. The problem of stability can be circumvented if the object of extremely high redshift is itself dynamical. One possibility are Dark Stars (e.g. Barceló et al. 2008): ever collapsing stars that, nonetheless, never develop horizons. To achieve this, quantum effects are usually invoked. Another possibility is to explore gravitational collapse of matter with an equation of state such that a smooth transition is allowed from a polytropic state to a state of infinite rigidity in order to enforce a bounce.

An equation of state of this type has been recently proposed by Mbonye & Kazanas (2005) and used to describe a regular black hole interior. This interior, however, was demonstrated to be dynamically and thermodynamically unstable by Pérez et al. (2013). Nevertheless, Pérez & Romero (2016) have recently shown that a black hole can be mimicked by a bouncing system described by the Mboyne-Kazanas equation of state if the bounce occurs on timescales longer than the Hubble time. The bounce occurs when the coasting matter reaches a regime of full rigidity, located well beyond the nuclear density, but below the densities necessary to produce a stellar-mass black hole.

Properties of dynamical mimickers such as Hawking radiation remain mostly unexplored, as well as cosmological effects in their evolution.

7. The importance of black holes: philosophical remarks

Black holes are the most extreme objects known in the Universe. Our representations of physical laws reach their limits in them. The strange phenomena that occur around black holes put to the test our basic conceptions of space, time, determinism, irreversibility, information, and causality. It is then not surprising that the investigation of black holes has philosophical impact in areas as diverse as ontology, epistemology, and theory construction. In black holes, in a very definite sense, we can say that philosophy meets physics, and, hopefully, experiment. Philosophers have almost paid no attention to the problems raised by the existence of black holes in the real world. For a notable and solitary exception see Weingrad (1979); a recent discussion of some ontological implications of black holes can be found in Romero & Pérez (2014); for a review see Romero (2014).

Among other philosophically important topics, the existence of black holes can be invoked to argue for substantivalism (the doctrine that spacetime is a physical entity, see Romero 2015), for the existence of discrete spacetime (Romero 2015), and to refute presentism, the idea that only the present is real (Romero & Pérez 2014). Many other issues remain to be studied.

8. Final comments

Black holes are the key ingredient in the mechanisms producing the most violent phenomena in the Universe, from gamma-ray bursts to active galactic nuclei. They are also essential for galaxy formation and evolution. These strange objects hide in their interior ultra compact remnants of collapsed stars and gas clouds. Our current knowledge of the laws of physics is not enough to explain these eerie entities. Black holes offer a unique framework where both large scale and microscopic interactions interplay in different regimes of the gravitational field. Research in both astrophysics and theoretical physics is necessary to shed some light upon the dark nature of these objects.

Acknowledgements: This work is supported by ANPCyT under grant PICT 00878 and by grant AYA2013-47447-C3-1-P (Spain).

References

Barceló C., et al., 2008, Physical Review D, 77, 044032
Bardeen J. M., Carter B., Hawking S. W., 1973, Communications in Mathematical Physics, 31, 161
Bekenstein J. D., 1973, Physical Review D, 7, 2333
Birrell N. D., Davies P. C. W., 1982, Quantum fields in curved space. Cambridge University Press
Frolov V. P., Novikov I. D., 1998, Black hole physics : basic concepts and new developments. Kluwer Academic Publishers
Hawking S. W., 1976, Physical Review D, 14, 2460
Hawking S. W., 2015, ArXiv e-prints
Hawking S. W., Ellis G. F. R., 1973, The large-scale structure of space-time. Cambridge University Press
Mbonye M. R., Kazanas D., 2005, Physical Review D, 72, 024016
McVittie G. C., 1933, MNRAS, 93, 325
Nandra R., Lasenby A. N., Hobson M. P., 2012, MNRAS, 422, 2931
Newman E. T., et al., 1965, Journal of Mathematical Physics, 6, 918
Penrose R., 1965, Physical Review Letters, 14, 57
Pérez D., Romero G. E., 2016, General Relativity and Gravitation, in preparation
Pérez D., Romero G. E., Perez-Bergliaffa S. E., 2013, International Journal of Theoretical Physics, 53, 734
Punsly B., 1998, ApJ, 498, 640
Romero G. E., 2012, Foundations of Science, 18, 297
Romero G. E., 2014, Philosophical Issues of Black Holes. Nova Science Publishers, NY, pp 27–58
Romero G. E., 2015, Foundations of Science, in press, pp 1-19
Romero G. E., Pérez D., 2014, European Journal for Philosophy of Science, 4, 293
Romero G. E., Vila G. S., 2014, Introduction to Black Hole Astrophysics. Vol. 876 of Lecture Notes in Physics, Berlin Springer Verlag
Wald R. M., 1984, General relativity. The University of Chicago Press
Weingrad R., 1979, Synthese, 1, 191