Brock University Bill Ralph's Last Lecture

Transcript

Brock University Associate Professor of Mathematics Bill Ralph presented his Last Lecture February 5, 2020 in Brock University's Pond Inlet with the company of over 250 colleagues, friends, current, and former students to celebrate his accomplishments, research, teaching and passion for all thing's mathematics.

Bill officially retired from teaching, research and service in the Department of Mathematics & Statistics June 30, 2020. Yet the first slide in his Last Lecture presentation was titled "Bill Ralph Final Lecture (NOT!)" Bill explained that although he was officially retiring, he will continue to lecture, speak at sessions, publish research, and advance his interests in artistic and mathematical pursuits.

Bill's lecture was introduced by Dorothy Levay, Instructor and Manager of Academic Support in Brock University's Department of Mathematics and Statistics.

People mentioned in Bill's presentation:

Professor Sid J. Segalowitz, Professor, Brock University Department of Psychology
Eric Muller, Professor Emeritus, Brock University Department of Mathematics & Statistics
James Stewart, Professor Emeritus, McMaster University
Chantal Buteau, Professor, Brock University Department of Mathematics & Statistics

Brock University people that helped organize this event:

Heather Bellisario, Academic Adviser / Liaison Officer, Faculty of Mathematics and Science
Catherine Brigantino, Experiential Education Coordinator, Faculty of Mathematics and Science
Ian D. Gordon, Liaison Librarian, Brock Libraries
Justin Steepe, Outreach & Communications Officer, Faculty of Mathematics and Science

Bill Ralph

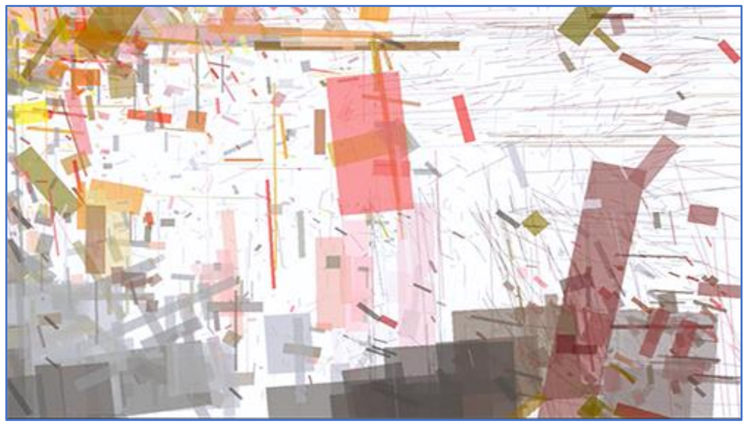
Final Lecture

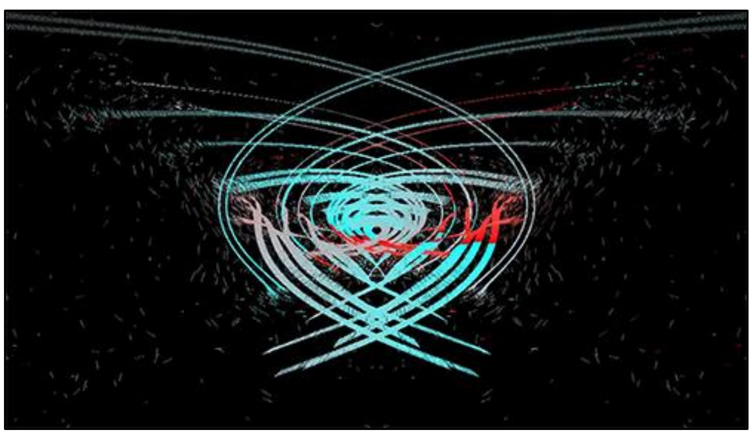
(NOT!)

We join Bill shortly after introductions and Bill's thanks to everyone for attending his last official lecture at Brock. Bill also acknowledged organizers that helped to make this event a success.

…that's been one of the passions of my career - is to visualize mathematical systems.

This visualization is a visualization of data from Sid Segalowitz's lab. This is a human brain thinking. So, I took all the information... I took the readings that Sid was getting off the human participants' scalps and I turned them into this visualization. All the movements, their colors, orientations of the rectangles that all comes from brainwave data, and it's so beautiful...





So how did this all start?

It was the summer between public school and high school and my friend Steve Whitford and I wanted to build rockets. That was my passion. And so, what we would do… Steve was mechanically inclined. I have no mechanical ability hence my PhD in pure mathematics. We would make gunpowder and we took gunpowder and we would shove that into a little CO^2 cartridge, put it on a little cart, and light it with a blowtorch. That was the summer… and that worked okay, but it wasn't good enough.

Up there right now is the formula for gunpowder and in there is potassium nitrate which has three oxygens. I was reading chemistry books and I said, "You know what, this is not enough, we need more oxygen!" I read up and I found that potassium perchlorate, there it is, had four oxygens, and so what I did was I learned how to balance the equations. There had to be the same number of elements on both sides and so I balanced the equations.

$$10(KNO_3) + 8C + 3S$$

$$=2(K_2CO_3) + 3(K_2SO_4) + 6(CO_2) + 5(N_2)+\text{energy}$$

$$KClO_4$$

We weighed it all out and we stuffed it into the CO^2 cartridge. Picture it? It was a bright sunny day in suburban North Bay. The little gang put it down on the road and Steve lit it with a blowtorch. The flame got longer and longer and something in my brain said, "This is impossible." I remember saying "RUN!" I got this far and did a half turn and there was a huge explosion. Huge! Everybody came out of their houses to find out what was going on, but there was nothing more than a cloud of black smoke. We said, "I don't know?" We heard it, but what is it? What I learned was that I get a real bang out of balancing equations. [Laughter]

So, I pressed on. The thing in the chemistry book I couldn't understand… was algebra…

By the way this is the house where I grew up. You can't see it very well, but there's a little window off a basement, here, and when I was in my 40s, I sat my parents down and told them about the explosives …that led me to algebra. When I found algebra, I thought it was so much better than chemistry.

Apologies to the chemists. I thought this is absolutely beautiful.

Algebra comes from an Arabic word meaning restoration. Now we say this was due to Al-Khwarizmi, but of course shortly - a new history of the world will be written in which Al-Khwarizmi will be appointed by Trump to investigate whether we really need equality… (but anyway this is the old version) …I got very interested in algebra.

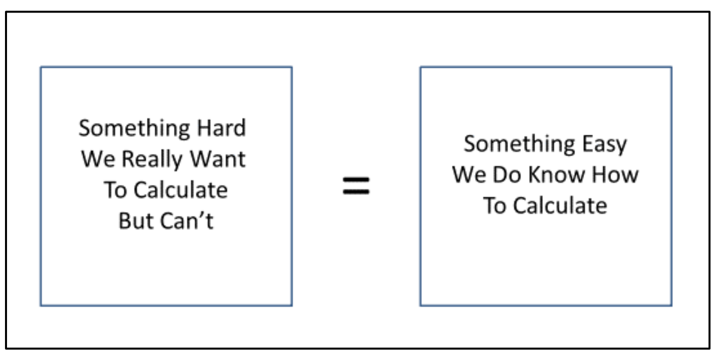
> *al-jabr*, meaning "restoration", referring to adding a number to both sides of the equation to consolidate or cancel terms (Wikipedia)
>
> al-Khwarizmi 780-850

We decided years ago (it was started by Eric Muller) to have a teaching program in mathematics. I got very interested in that program and developed some of the courses. That led me to teach the history of mathematics for years and years which I absolutely loved. Now, at the beginning of that course I had a discussion …an apology that I make to the women in the class. The whole entire course is about men doing mathematics. So, we had to talk about that. And what's the problem of course - that women were not allowed to do mathematics. We had a discussion and I want the class to talk about "What does that mean? What does that mean for us now?" One of the women in the class said this beautiful thing which I'll never forget "Imagine where civilization would be now if women have been allowed to participate in the development of mathematics and science?" Isn't that fantastic! Can you imagine? I mean this is an extraordinary thing.

> "Imagine where civilization would be now if women had been allowed to participate in the development of mathematics and science." (quote from student)

Okay, so I got busy with balancing equations.

Let's talk a little bit about mathematics. I have to do a little bit of math. A typical problem in mathematics is when we have something hard that we really, really, really want to calculate, but we can't. "How many mathematicians have had that problem?" Okay, this is a standard thing. We can't calculate it. What we would like to do is we would like to make it equal to something *easy* that we do know how to calculate. Now, what's a beautiful theorem? A beautiful theorem is something that we really want over here that's maybe very complex and somehow, we see it from just the right angle and suddenly we turn it into something we do know how to calculate.



Just to give you an example of this, as Dorothy said, “I’ve been teaching calculus since the dinosaurs.” So, I have to say something about calculus.

One of the great problems in mathematics is the problem of finding the area under a curve. This problem was around for a couple of thousand years and nobody could do it except for tiny little cases. So, there was a huge question and a lot of people tried to find the area under a curve. What's it equal to?

The Area Under A Curve = ?

Well that symbol there is how we currently write the area under a curve. Newton came along and said “You know what? I know how to do this!” I can tell you how to find the area under a curve in such a simple way that is one of the great contributions to mathematics and here it is…

“What do you think? Everybody happy? Who's unhappy with this? Dorothy?” I put this slide in for Dorothy, and Dorothy “What would you like? Would you like me to fix it [Dorothy responds, “Yes please.”] because I wanted Dorothy to think for a second that I couldn't write it down properly?”

The Fundamental Theorem Of Calculus
(The FTC)

$$\int_a^b f(x)dx = F(a) - F(b)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

So, there it is. This is generally easy to evaluate. That's kind of neat … the idea, we can find the area under curve, but big deal who cares? Right? “Do we really care about that?”

What if I tell you that if you give me this formula - this is the key to unlocking almost every problem in physics, solving every differential equation… with this formula I can tell you the positions how the planets are going to move around the sun and that's exactly what Newton did. He said, “I can take that formula and prove that planets move in ellipses around the sun.”

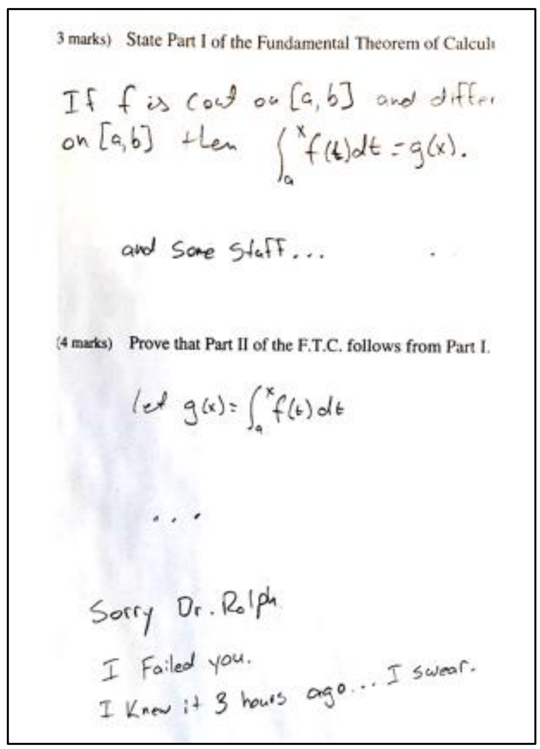
“What am I providing here? Is the formula more interesting now? Right, do you find that it's a little bit more interesting?” Why is it more interesting? ….Because now it's not just an abstract piece of mathematics - now it has a context.

CONTEXT !

For me, the key to mathematics teaching is to create a reason for the formula that you're going to put on the board. Mathematics is so powerful that you can actually just follow the formalism through and never really talk about what it means or how what it is applied. I think it's so important to provide the context. I want to tell my students ahead of time… “This formula unlocks the universe. This formula. Our civilization… is built on that little formula.” It seems so harmless, but there it is.

Creating Context And Motivation That’s Meaningful To My Students Is The Core of My Teaching

Creating content and motivation is meaningful to my students - that's the core of my teaching. That's been the bulk of my work in the last 30 years. I make up every lecture fresh every single time. I don't go in with …so I take a sheet of paper and I write out the entire lecture every time and I think about, “Does this work, is this going to work for my students, is this a good example?” That's a constantly living process. I’m thinking about “Who's my audience, what are we doing and so on and so on…”


3 marks) State Part I of the Fundamental Theorem of Calcul

If f is cont on [a,b] and differ on [a,b] then $\int_a^x f(t)dt = g(x)$.

and some stuff...

(4 marks) Prove that Part II of the F.T.C. follows from Part I.

let $g(x) = \int_a^x f(t)dt$

. . .

Sorry Dr. Ralph

I Failed you.

I Knew it 3 hours ago... I swear.

Okay, so the theorem is called the Fundamental Theorem of Calculus (FTC). I want to tell you a story about the FTC. In first your calculus they come in and the first thing I say on the first day is “We are going to prove one of the greatest theorems in mathematics, the FTC.” I build it up week after week, month after month. “Who was in my calculus class? Do you know the FTC? It's gone, okay. [Laughter] Did I talk about the FTC?” Yes, okay, they all agree. All right, and one of the things I say is “You know if you get to the door of the exam room and you don't know the FTC what do you do?” [Laughter] “Did you hear what they said? They said, ‘We go home!’” …and I practiced that with my class. We practiced it! “Do you remember right?”

Everybody here who took calculus from me knows that you get to the door and you don't know it you go home. And you know, they did almost all knew it except this person. This part is not so bad “and some stuff…” but then the second part says, “Sorry Dr Ralph, I failed you. I knew it three hours ago. I swear.” [Laughter]

“Do you understand why they wrote the apology?” You understand? Every day for three months I talked about it! One year …someone said, “I can't remember. I’m going to draw you a picture of a horse” and it was really good. My partner Bruce said, “You give that a mark, that's a beautiful horse, you give them a mark for that.” [Laughter]

Again, just if you don't mind it's just a couple of examination bloopers. State Fermat’s Last Theorem [was answered by a student], “There are no numbers a, b and c in N so that, a plus b equals c.” None! Two plus three doesn’t equal five because there are no numbers so that a plus b equals three? No numbers like that? [Another answer by a student], “The sum of two even numbers will always be a prime number. Proven by a professor.”

State the twin primes conjecture “There are lots of them.” [Laughter]

State the problem and state whether it is proven or unproven.

a) Fermat’s Last Theorem

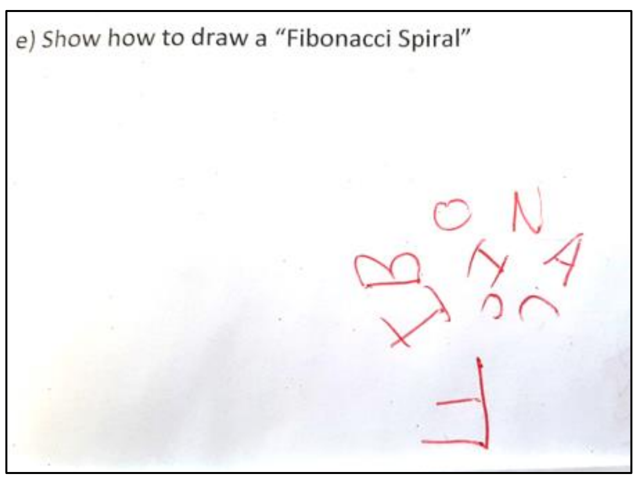
There are no numbers a, b and c in N so that a + b = c.

The sum of any two even numbers will always be a prime number. Proven by a professor.

b) The twin primes conjecture.

There are lots of them.

[A student responded to the challenge to] show how to draw a Fibonacci spiral. F - I - B - O - N - A - C - C - I. [Laughter]

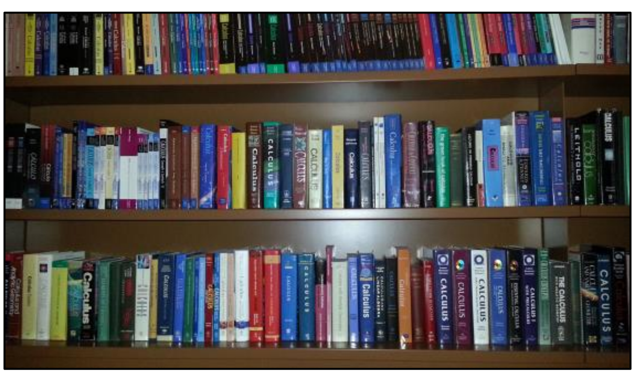


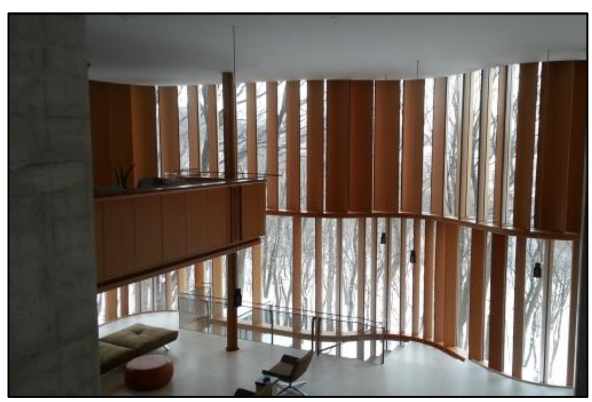
These are my people. This is from a first-year liberal arts mathematics class. I can't tell you how much I love teaching that course. They were the best class. I feel bad even doing bloopers because almost everybody was letter perfect on all of this. It's a bit of fun.

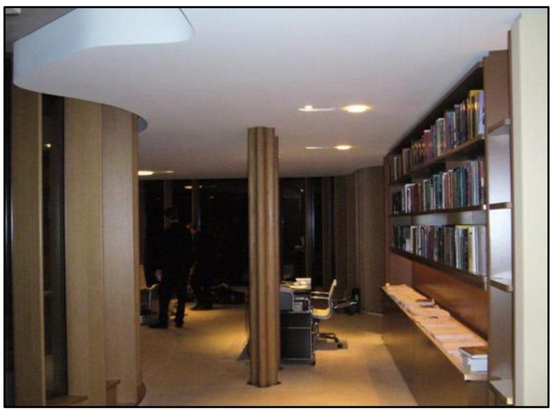
Back to calculus for a second. A lifelong friend of mine was the great mathematics teacher and book writer – my friend Jim Stewart. I’ve known Jim Stewart a very long time. We work together. He plays the violin. I play the piano. We both love calculus. There was one small difference between Jim and I. His royalties on his calculus book were ten million a year. Jim once walked into my house and looked at my couch and said “Bill, if you'll take that couch and put it on the curb, I’ll buy you a brand new one.” But my cat liked it so I wouldn't.

In any case, Jim wrote the [best] calculus book in the world which we've used here ‘The Stewart Calculus’ for years and years and years. It's used all over the world in every country. It's a stunning book …and Dorothy can do most of the questions. “Dorothy?” [Laughter]

I just thought for fun you'd like to see some of Jim’s house. You can't see it very well here… that's my car park there, that's the cheapest car ever parked at his mansion. Jim built a house called Integral House which is incredibly famous written up in the Wall Street Journal. It is one of the great houses of North America. These are a few shots of it. He has a little room in the house which just has my art in it. There's me, and art …these are his books …this is the pool. I have so many stories, but I’m going to skip over them…



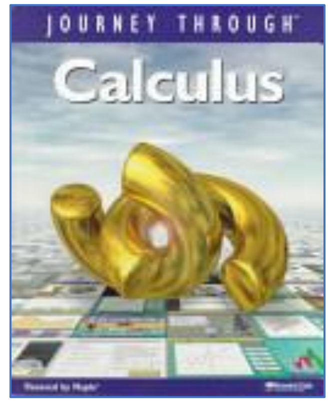




Journey Through Calculus.

Journey Through Calculus

So, Jim and I were friends in the early 90s. Jim said “You know what Bill, we’ve got to start bringing calculus into the teaching. We've got to start. We've got to bring computers into the teaching of calculus.” He said, “I want you to create an interactive computer system that will teach calculus.” So, I was forced to move to San Francisco for three years. I took a leave of absence form Brock. And Brock - thank you for that leave of absence. It was… I had huge fun… I built…



I think I have an example. Maybe I can show you something from Journey Through Calculus just to give you a little sense of it.

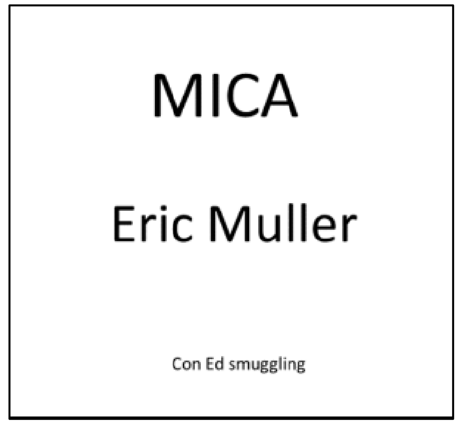
[Bill presents an online digital learning object that simulates a mathematical game to explore principles of calculus].

So, here's a problem. Does the swimmer get in the water and go straight to the person who's drowning or does the swimmer run along the beach and then enter the water? This is a classic calculus problem which you can explore in this little activity. Here we go. “Are you excited?” Come on …the rescue is happening [Music from the movie Jaws] and I rescued them in 95 seconds. Let's see if I can do better. Here we go. I’m running to there …and then I’m going to go to there. I rescued them in 42.8 seconds. You get the idea… and back to my …just one second… What's the significance of this? So, the complete solution is there in interactive form. What else is there? You can change the parameters - the running speed - the swing speed and see what happens. In classical calculus problems what happens? In a classical room you get ‘a’ problem - one speed, and if the data is completely given to you. You solve that problem and that’s it! I wanted it to go better than that. This has affected all of my teaching. I want people to understand systems so you have… you can change this, or change this, how does that change the result, what's the impact on the solution if you change this, this, this, and this. That gave people the ability to do that. It's a vast program. It's filled with little environments to explore calculus problems. Okay, so I have that in my mind. I come home back to St. Catharines.

Embarassing!

I was going to tell you this story so how many okay. I get very nervous the first day of classes. “Has anybody ever experienced that?” Like I’m really nervous the first day of classes. You know I want it to go well. I want them to like me. You want to set the right tone. You know? Friendly, but not that friendly. You know, and so on, and so on. I was particularly nervous this day so I went to lunch and I thought I’m gonna have… I never do this. I’m gonna have a plate of fries with gravy and so I did that and it was really, really good. It was wonderful gravy, big fries, delicious gravy… You're wondering what I’m talking about? And then I go in. I give the class. I could do no wrong. Everything I said landed. I was getting like, you know, wow! I can't believe it? I’ve never been so good. I’m on fire in that room and so that was good and I walked out and I’m… You know, boy, that was great! And you know, I had to go to the bathroom. So I went to the bathroom and I reached down, but I didn't find what I expected because there was a giant french fry covered with gravy and ketchup glued to my zipper. [Laughter]

MICA - so, um “How am I for time? We're good? Yeah we're good okay.”



I come back from California and I’ve been working with Eric Muller. Eric has radical thoughts. He is a quiet unassuming man who has radical thoughts. He does, in that brain of his…Eric said this to me one day, he said, “Bill, why are we teaching people to do all these calculations when computers can do it just as well? Why don't we remove some of those calculations and let's talk about applications and concepts where that space is?” So, I started to do that and that was a wonderful idea and then Eric said this, he said, “Bill, this is the age of computers. We don't have anything at Brock, somebody has got to imagine a brand-new program that fully integrates mathematics and computers.” He said, “You're going to do that!” Eric said that, he, and you know… you don't know Eric, maybe, but you can't deny him. I mean this is… “Dorothy, who could deny Eric?” Eric came to me one day and he said he was doing course loads and he said “Bill, um, so, everybody, so, what are you gonna do? How many tutorials, how many courses and so on, and so on…? And I said, “Uh, I said well, I could take this, and this”. He says, “Yeah that's eight, that's good, a lot of people are doing nine, but that's fine.” [Laughter] He said this “If you're comfortable with eight!” I would have done ten, but that's right, anyways... So, he said, “Make a program.” And so, I thought, and thought, and thought… about what can I do to make a program for our times. What can we do?

Current Culture in Mathematics	Extra Pieces the world now needs
Study particular well-defined beautiful problems	Study systems that may be messy and extremely complex
Analytic Solutions	Approximate Solutions
Visualizing functions with graphs	Visualizing Complex Information
Use technology as sparingly as possible	Leverage technology as much as possible
Data analysis of a handful of variables often with less than a 1000 sample points	Large scale data analysis of hundreds of interconnected variables - millions of sample points
Work alone or in small groups	Large teams of specialists – think tanks
Not very dependent on social and communication skills	Social and communication skills critical to success of project.
Glory of the individual	Glory of the group

MICA stands for Mathematics Integrated with Computing and Applications. And so, here's what I thought. I’ve got a column on the left side that says current culture and mathematics and the on the right-hand side are what I consider the extra pieces the world needs now. What do we need to do? What do we have? The current culture then and it is changing was to study particular well-defined beautiful problems and that has been so successful in mathematics and produced such a beautiful body of work. However, the world is now looking at complex systems that are so diverse and messy and complicated. And they want to know what can a mathematician tell me about this very complicated system? Mathematics wants exact solutions. The world wants you give the solution quickly. They don't care if it's accurate to 10 decimal places. We're done, they're very happy with approximate solutions that computers can give. Early mathematics has been good at visualizing graphs. That's over. Now we have to visualize extremely complex information involving many different parameters. How do we do that? There is a culture in mathematics, and it is a beautiful one. I love it …the idea that we do this by hand we don't use technology. That is a beautiful, beautiful idea. Give it up… [Laughter] It is time to leverage technology as much as possible. We have it available… a data analysis of small samples… Of course, you know now that people are people like Facebook and Google are analyzing millions and millions of sample points. How do you do that? The mathematicians we have are now beginning to catch up.

What I Wanted From MICA

Students using technology to experiment, to make conjectures and to explore mathematical problems.

This one. One of the things in mathematics is to work alone or in small groups. Everything these days in the outside world is done by large teams of specialists and think tanks. We have to help our students work together and think about working collectively on parts of problems. Mathematical culture is not so dependent on communication skills. People work by themselves and do amazing work, but these days social and communication skills are critical, and on, and on, and on... So, I was thinking about these things and I thought about what do I want? Here is what I wanted. I wanted students using technology, to experiment, to make conjectures, and to explore mathematical problems.

So, I sat down and taught the I taught these courses and developed the templates for what are now MATH 1P40, 2P40, and 3P40 where students do this. Our students in first year make conjectures. They actually have to sit down and imagine something that might be true or not true and then test it. It's a terrible shock for a mathematics student to be said "Okay, go, investigate, think about something that you're interested in, imagine it might be true or false, and now go test it." That's mathematics. What do mathematics students do? Okay, here's how you do it? You do this, this, this, and this. Okay, I'll do that, that, that, and that. That is not MICA at all! MICA is now doing something completely different than that – so, I'm standing back. I think it's one of the hardest things to leave is the MICA program if it weren't for one person named Chantal Buteau.

MICA

Mathematics Integrated With Computing and Applications

The core of the MICA program consists of MATH 1P40, 2P40 and 3P40 in which students confront real world problems requiring them to create mathematical models and run computer simulations. In solving such problems, students are encouraged to develop their own strategies for using the best combination of mathematics and computing.

"Where is Chantal?" Yes! Chantal Buteau is a champion of the MICA program. She's continued to develop it, to study it, and she's taken it around the world. "Am I right Chantal?" [Laughter] So, MICA is alive and well at Brock. This unique philosophy will continue.

Extending The Idea Of Fractal Dimension To Metric Spaces Equipped With Probability Measures

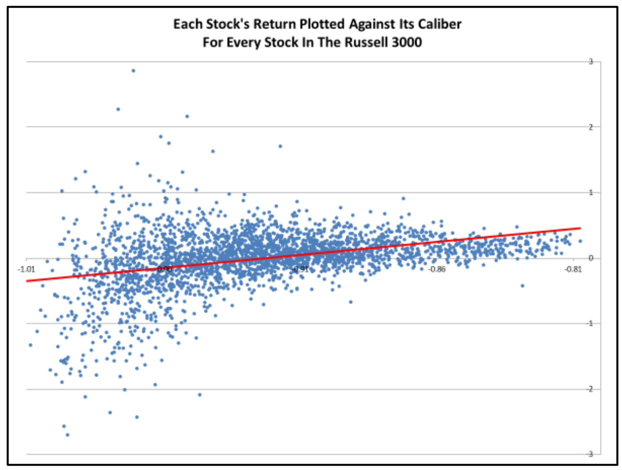
Density	μ_1	σ_1	$1/\mu_1$	$\frac{1}{\rho_1}$ *conjectured*
uniform on $[a,b]$	1.0003	0.0111	0.9997	1
normal	1.2664	0.129	0.7896	$\pi/4 \simeq 0.785$
exponential	1.4590	0.0141	0.6854	?
$1/(2\sqrt{x})$ on $[0,1]$	1.2817	0.0132	0.7802	?
uniform on $[0,1]^2$	0.5023	0.0056	1.9908	2
uniform on $[0,1]^3$	0.3416	0.0037	2.9274	3
uniform on $[0,1]^4$	0.2642	0.0029	3.7850	4
bivariate normal	0.7264	0.0073	1.3766	?
Cantor	1.6014	0.0170	0.6244	$\ln(2)/\ln(3) \approx 0.631$
Sierpinski	0.6344	0.0067	1.57624	$\ln(3)/\ln(2) \approx 1.5849$

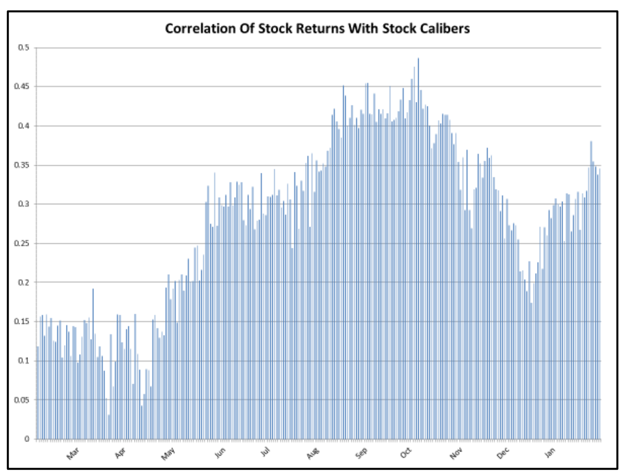
Density	μ_2	σ_2	ρ_2 *conjectured*
uniform on $[a,b]$	-1.6461	.0732	$-\pi^2/6 \approx -1.6449$
normal	-1.0273	0.0860	-1
exponential	-0.7333	0.0920	?
$1/(2\sqrt{x})$ on $[0,1]$	-2.5792	0.1085	?
uniform on $[0,1]^2$	-0.4096	0.0186	$-(\pi^2/6)(1/2)^2 \approx -.4112$
uniform on $[0,1]^3$	-0.1825	0.0083	$-(\pi^2/6)(1/3)^2 \approx -0.1827$
uniform on $[0,1]^4$	-0.1038	0.0049	$-(\pi^2/6)(1/4)^2 \approx 0.1028$
bivariate normal	-0.2004	0.0233	?
Cantor	-4.1464	0.1933	$\frac{(-1)^{2+1}(2-1)!(2-1)\zeta(2)}{(ln(2)/ln(3))^2} \approx -4.132$
Sierpinski	-0.6549	0.0295	$\frac{(-1)^{2+1}(2-1)!(2-1)\zeta(2)}{(ln(3)/ln(2))^2} \approx -0.655$

I was going to talk about my work for a second. Should I skip this bit? The best thing I ever did. I think you've all heard of the idea of fractal dimension? You know the idea of dimension, one-dimensional things, two-dimensional, there's an idea of dimension that has to do with fractional dimensions. Something that could have dimension 1.2, not one, not two, but something in between. So there are lots of ways of calculating these things. I started studying something very crazy…and I realized what my crazy thing gave me was an infinite ladder of statistics. One, two, three, four, five and I started testing the first one the lowest rung of the ladder. I thought, "What is that?" I have no idea what this is. I'm just going to try it on some data. I happen to try it on a uniform distribution, and I got the remarkable. I actually couldn't believe what was happening. The computer said it's one. I said that's okay. Then I rewrote the program because I thought it made a mistake because there's no way it could be a whole number. It couldn't be a number like one, but it was, and then, when I realized that I could extend what people call fractal dimension.

I could talk about fractal dimension on any place where you have a notion of distance and a notion of the chance of something landing in a certain spot. That's all that says. And so I worked on that and published a paper on this. This is certainly the best thing I've done and here's what happened. So what it means is that for any probability distribution "…there'll be a little math just now, okay, pick your favorite fantasy, go there, well not totally there" …so I was able to extend the idea of fractal dimension to any probability distribution …We love beautiful things in mathematics and the first thing that became clear was that for the bell curve the numbers seem to be pi divided by four. How is that possible? This statistic is incredibly complex and yet there it was. A one-dimensional space it gave one. On a two-dimensional space it gave two. On these fractals it gave exactly the right thing. So then I moved on to, oh, I don't go over there… So, I told you that's rank one rank, two of the statistic. The only one I want to show you is this one for a bell curve it gave the number minus one. Again, how is it possible that it gives minus one? In any case I thought okay, I’ve got this new tool nobody's ever had this before. I’m going to apply this to the stock market which is what I did.

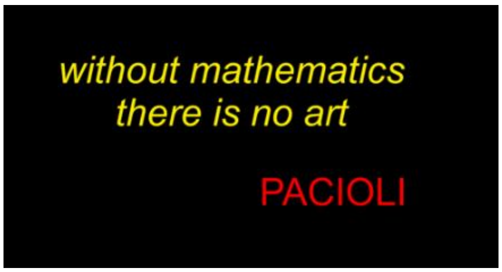
I found that if I applied this to the stock market, I did not get minus one and is still very believed that the returns of the stock market are actually a bell curve. This paper published and showed that it is extremely far from being a bell curve, because this should be minus one, but it was not for that. Well, my interest in the stock market that got me kind of excited about studying the stock market and applying my stuff.



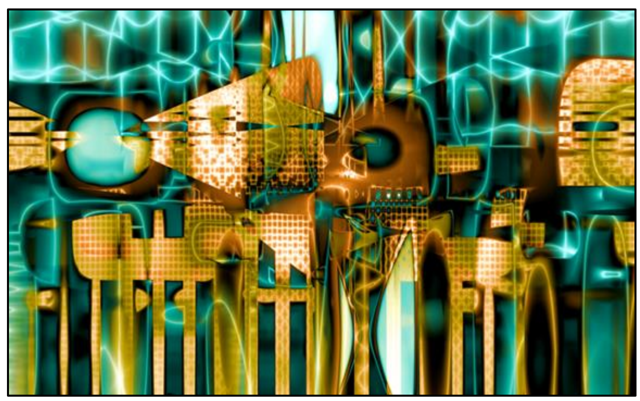


I spent years studying it mathematically and then I developed the financial math course here at Brock. Eventually, too soon, I was walking along the street one day and I had an idea for analyzing data. And the idea was so bizarre, I’ve never seen anything like it. I said, “Oh, that's stupid, that'll never work” and then a few months later I thought I could try it, I could program it, it took a whole day, it was hard, and I saw that it did work. So then, I started applying this idea to the stock market and to this day every weekend I publish a stock market report based on my method. The number on this axis is a number I call caliber. What you're seeing are the returns so individual stock returns plotted against my new measure. What you notice is the slope of the trend line is positive so there seems to be a correlation between returns and my measure. Does that correlation happen all year long? It does. So, over the past year these are the correlations that I’ve received, and you notice that they're all in the black - they're all positive. What you're looking at is only uh three months old. One of the things I wanted to do in retirement was to develop this further and I’m very pleased with where I’ve got this. I’m going to continue to develop this. This is part of my retirement is to work on this stuff right here.

Let's talk about my art. In the 1990s I was trying to visualize dynamical systems and I didn't know I could. I didn't know what was happening. First, I have to talk about this guy. This fellow was Luca Pacioli. He was Leonardo da Vinci’s teacher and you can't see it, but there's a beautiful symmetric object up here. I won't ask you the name of it because you can't see it, but there it is, a rhombic cube octahedron. “How many people knew that?” Quite a few I’m sure, but this is what he said. This is da Vinci’s teacher. “Without mathematics there is no art.”

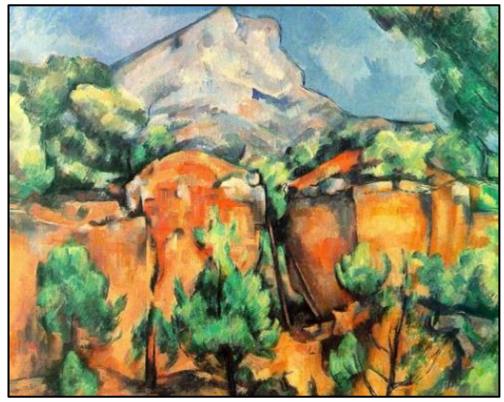


Well for me, there's been a lot of art for mathematics and I really wish you could see it. This particular piece is made from the mathematics of dynamical systems.

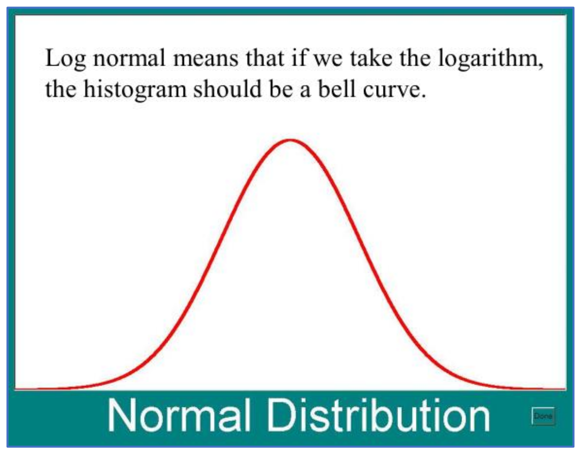


This one is as well. And this work has been shown in galleries in North America. So I’ve shown and sold a lot of this work and it's been huge, huge fun. In fact, I’ve become obsessed with making art.

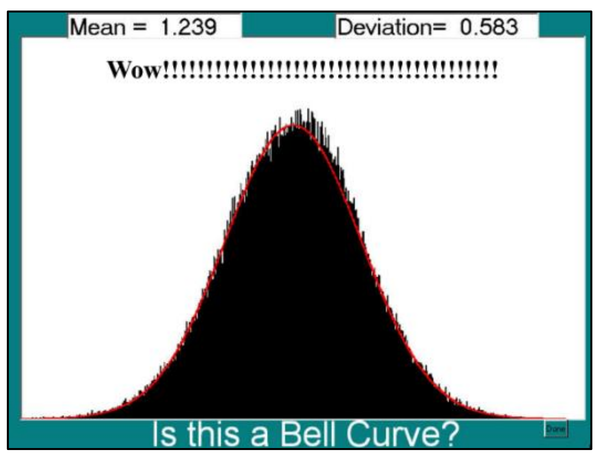
Here's another one of my pieces some people have attributed it to Suzanne, but it's actually mine. [Laughter]



So when you're a mathematician and you're making art what do you ask? What makes good art? Why is this good? Why is this not good? I started analyzing images and I went through a whole lot of measures and tried to figure some stuff out and finally I hit on a statistic. What I found was that for some of the best artists you always get a bell curve.



"Are you ready?" Ok, so I showed you the Cezanne and this is what my statistic did…there's a perfect bell curve.



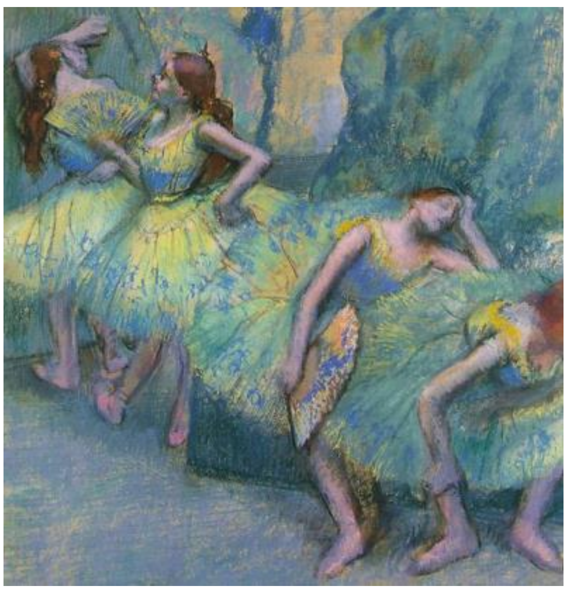
"Are you ready?" Now I'm going to take statistical information from the image and I'm just going to make a histogram and that's what you get. And this is astonishing because it even fits in the tails. Look! Look! Not one of you has ever got a bell curve this good from your research - not one of you. This is the best bell curve ever obtained at this university. Ever! "Look at that curve!"

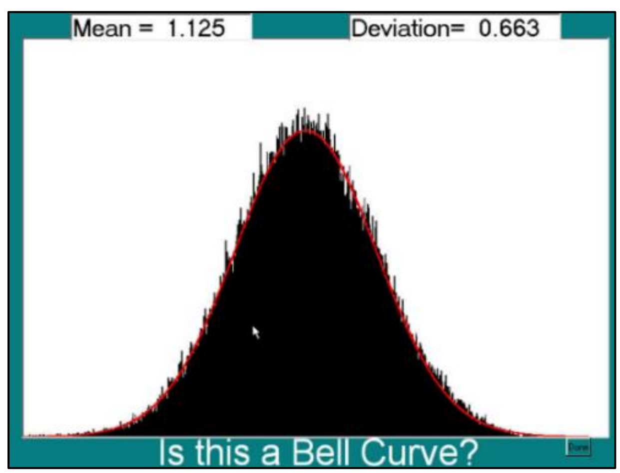
A couple more examples just because I love this.

Degas' dancers. Perfect.

<u>Degas</u>

Ballet Dancers in the Wings

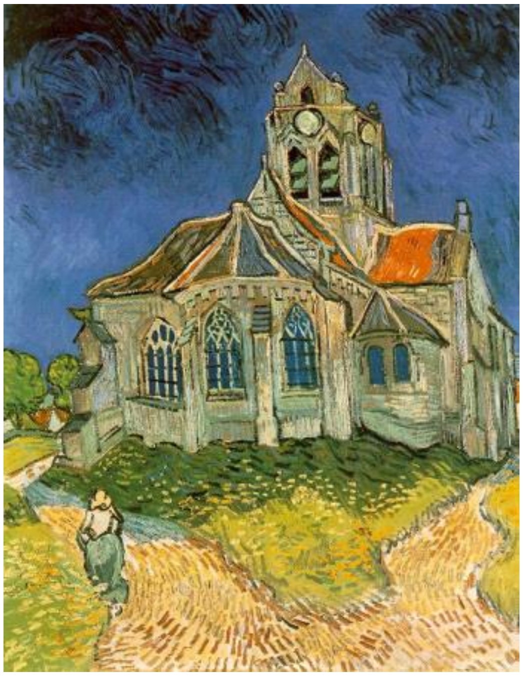


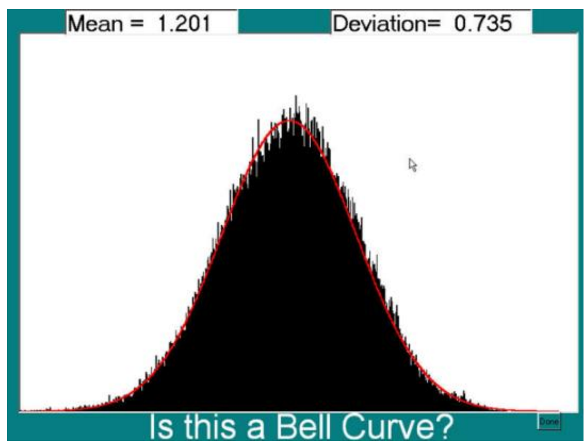


Van Gogh’s church. Perfect.

Van Gogh

The Church at Auvers-Sur-Oise

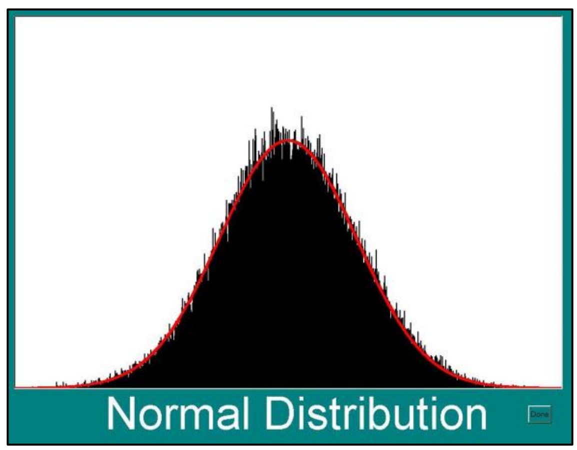




Emily Carr, the great Canadian artist. Perfect.

Emily Carr

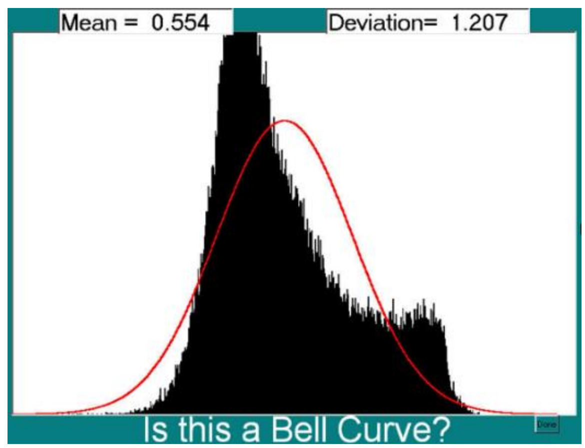
Untitled



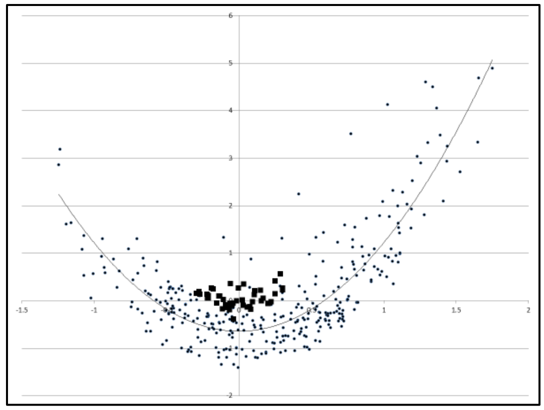
Marilyn Monroe. Andy Warhol. Not perfect. Check that out. It's not worth anything.

Warhol

Marilyn Monroe

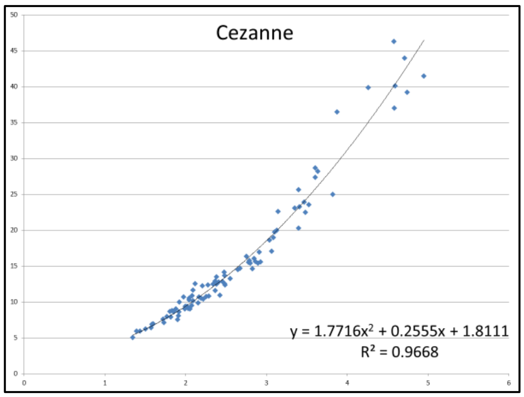


The little dots that you see here are all the pictures from my Florida trip. They're all here.

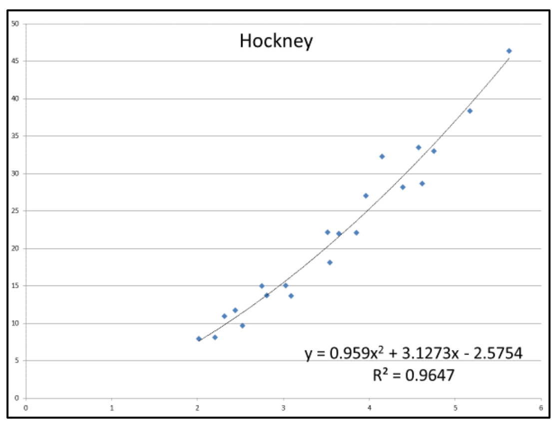


The big squares in the middle - this is all the work of great artists. I just wanted you to see the difference in how that sits in there.

The blue dots are my Florida pictures on a trip. This is where the great artists are situated. If it's a perfect bell curve. Everything would be right here in the middle. So, I'm just writing this all up into a paper now.



Another thing I'm interested in is when you look at Cezanne you can sort of tell that these are all Cezanne's. Is there a signature? Can I somehow say these are all from the same artist? So, this is a statistic I'm using and you notice how beautifully this is fit by a polynomial and the polynomial has coefficients as polynomials often do, but now look at…



Let's look at David Hockney's work. Again, it slides nicely along a curve, but the coefficients are completely different. So again, this is something that I'm working on all the time.

People ask, "What are you doing in retirement?" I am busy every second. I'm working on my art. I'm working on this stuff, on my stock stuff all the time, but what I'm what's really excited about - these numbers are so different than Cezanne's numbers. I'm hoping what I've got here is a kind of thumb print for an artist. This is being worked on right now.

This is going to be a really effective part!

I'll just splash through.

This is a very bad seagull picture, and this is what I did to it. Right now, I'm working on changing photographs.

Here's a coffee table, some coffee cups, and this is what it became.

Lately I've been working with what are called vector fields and so what I take an image and I put on it. I tell you which direction to go at every point based on the color information. I take the picture like that, and it becomes this.

This is my partner Bruce. I'm not allowed to show the before picture apparently, that would cost me money.

There's a kind of ordinary sunset and if I let the vector field go wild - I get a piece like that.

This is another thing because you know I work, and I create art, and I sell my work.

Here's a little final thought. This is from a student. “You shouldn't worry that you're not as smart as the other professors…it's because you're at our level that you can teach us so well.” [Laughter]

“You shouldn’t worry that you’re not as smart as the other professors.”

“It’s because you’re at our level that you can teach us so well.”

(a well meaning student)

It's been wonderful talking to you. I think if I can on the teaching side. I would think what is teaching to me. Teaching to me is high standards. I think I’ve had pretty high standards and a great deal of kindness. Kindness mixed with high standards. We have to be kind when we teach students. It's not easy being a student and we have to remember that and be aware of that. We also of course have to keep our standards.

Thank you everyone it's been wonderful to see you all.

I think that's it!

[Applause]