DOES CENTRALISATION AFFECT THE NUMBER AND SIZE OF LOBBIES?

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Does Centralization Affect the Number and Size of Lobbies?

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ABSTRACT:

Previous research has shown that if countries "merge", (i.e. move to centralized policy choices) the effect is to reduce lobbying. However empirical evidence suggests that this is not the case. This paper explains the empirical evidence in a two-jurisdiction political economy model of public good provision under policy centralization and policy decentralization, where the policy choice can be affected by the pressure of endogenously formed lobbies. We measure lobbying in three ways:(i) the number of lobbies formed under the two settings, (ii) their impact on policy decisions and (iii) the amount of resources transferred to the policy makers. We show that preference heterogeneity and lobby formation are positively related and that moving from decentralization to centralization can affect both the number and the type of lobbies. We develop some examples; among them: under centralization, compared to decentralization, the size of lobbies can be higher but the impact on policy can be smaller. Moreover we show how the majority groups try to offset lobbying by strategic voting for a candidate of a different group.

KEYWORDS: Lobby Formation, Pressure Group, Centralization.

JEL CLASSIFICATION: H23, H77, D72 .

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1. Introduction

This paper studies the effect that policy centralization has on lobbying. Previous research has shown that if countries "merge", (i.e. move to centralized policy choices) the effect is to reduce lobbying. The reason for this result, known in the literature as *preference dilution effect*, is due to the fact that, given that preference heterogeneity increases under centralization, there is a smaller role in determining policy for politically important groups in each of the countries, and this renders decision making less responsive to factional interests, which dilutes the incentive to lobby, (see among others de Melo, Panagariya, Rodrik (1993)).This kind of arguments were also used by Madison (1787) in The Federalist Papers in support to a "well constructed Union" of American States[2].

However empirical evidence suggests that this is not the case: US has very strong lobby groups at the federal level, and the number of registered lobbies at the European Union level has rapidly increased in the past recent years. Moreover, in a recent empirical study Fishman and Gatti (2002), Root (1999) and Treisman (1999) found evidence of a negative correlation between decentralization and corruption. In Italy, after the political scandal "tangentopoli" (translated: town of bribes) invested national politics in 1992 the favor among voters towards decentralization grew esponentially such that the Northern League, the newly born party whose main political manifesto was the division of Italy into three regions, gained more than 15% of votes in the North.

This paper attempts to provide an explanation for this evidence by developing a two-jurisdiction political economy model of endogenous lobby formation public good provision under policy centralization and policy decentralization,[3] where the public good provision choices can be affected by the pressure of endogenously formed lobbies.[4] In particular we address the following questions. Are citizens more likely to organize a lobby if policy decisions are taken at a central or local level? And, once a lobby exists, in which case does it have more influence on policy? Moreover, is the lobby's size affected by the degree of centralization of the government

[2] The smaller the society, the fewer probably will be the distinct parties and interests composing it; the fewer the distinct parties and interests, the more frequently the majority be found of the same party; and the smaller the number of individuals composing a majority, and the smaller the compass within which they are placed, the more easily will they concert and execute their plans of oppression. Extend the sphere, and you take in a greater variety of parties and interests; you make it less probable that a majority of the whole will have a common motive to invade the rights of the other citizens; or if such a common motive exists, it will be more difficult for all who feel it to discover their own strenght and to act in unison with each other (Hamilton and others, (1787), p.22).

[3] The role played by the level of government responsible for a policy decision in affecting the policy outcome has been broadly studied since Oates (1972). Recent works on centralization and policy outcomes in a political economy framework are due to Besley and Coate (1998) and Lockwood (2002).

[4] The role of lobbies in affecting policy outcome has been recognized both by political scientists and economists and it has lead to the production of a vast literature. Recently economists have started investigating the process of lobby formation (Felli and Merlo (2000), Mitra (1999), Leaver and Makris (2000) and its relationship with the political process (Felli and Merlo, (2000) Besley and Coate (1999)).

in charge of the policy?

As far as we know this is the first attempt to investigate the relationship between centralization and lobby formation despite the fact this is an important political issue.

We model the political process following a simplified version of the citizen-candidate approach due to Besley and Coate(1997), and Osborne Slivinski (1996), where policy makers are elected citizens who select the preferred policy choice that maximizes their utility. We describe lobbies' behavior using the menu-auction[5] model of Bernheim and Whinston (1986) and Dixit, Grossman and Helpman (1997). However this paper differs from these models in few main aspects. First, the citizen-candidate model assumes that the first stage of the political process is the entry of the candidates before elections; we do not model candidate entry but we assume that there is an *exogenous* set of candidates available, second the citizen candidate model assumes that citizens vote strategically for their preferred candidate, here the assumption is that they vote sincerely. Third, the menu auctions approach models the activity of *exogenous* lobby groups that try to influence the policy choice toward their preferred policy choice by offering contributions to the policy-maker; we do not take the lobbies as given but we model a lobby formation stage[6].

Our model is very stylized; there are two jurisdictions, two types of citizens with heterogeneous preferences in each jurisdiction, and one type of citizen is common to both jurisdictions. Policy decisions can be centralized or decentralized. In the first case there exists only one government elected by residents in the two jurisdictions, in the latter case each jurisdiction selects a government which decides the policy independently from the other government. After elections determine the identity of the policy-maker (the government), citizens may form a lobby with citizens of their group and "bribe" the policy maker.

We measure the extent of lobbying in three ways: i) the number of lobbies formed under centralization and decentralization; ii) their impact on the policy decision; and iii) the amount of resources transferred to the policy makers, and we compare the outcomes under policy centralization and policy decentralization.

We show that lobbies are more likely to form when preference heterogeneity among groups is high and when minorities are large. Both results seem to support the idea that lobbying is higher under centralization where we expect both higher preference heterogeneity and groups of bigger size. However, it is also true that lobbying is less likely to form when citizens are divided into many different groups. We think that these aspect should make lobbying less likely to occur

[5] Felli and Merlo (1999) use the citizen- candidate model to explore lobby formation. However the focus of their paper is on the bargaining process between lobbyists and policy maker and its consequences on policy outcomes.

[6] Besley and Coate (1999) study the impact of lobbies on political competition and policy outcome combining the citizen-candidate model with the menu-auction model, but in a model with endogenous entry of candidates and with respect to the central level of government only.

under centralization, where it is more likely that population is more fragmented. Moreover, we show that it is more likely that lobbying will be able to influence policy decisions more under decentralization, where it is easier for an extreme group to be more influent.

Three main results emerge from our analysis. The first result is that lobbying matters. Compared to the benchmark case, the version of the model without lobbying, the identity of the policy maker and the policy outcome can be different from what we observe in a model where lobbying is not taken into account, *even* when we have equilibrium with no lobbies.

The second result is that centralization matters. Lobbying is affected by the level of government who decides the policy. However the effect is ambiguous, contrary to the idea of a preference dilution effect; we present examples where lobbying is higher under centralization and examples where the opposite is true.

The third result is that our conclusions depend on the type of measure used to compare centralization and decentralization. For example we presents scenarios where centralization determines higher lobbying from the point of view of resources spent by the lobbyists, but the effects on policy are smaller, and where the opposite is also true.

The paper is organized as follows. The first section briefly describes the economic environment and the model. The second section presents a simplified version of the model where lobbying is not taken into account, we will refer at this case as the benchmark. In the third section we present and discuss the three definitions of lobbying we use, followed by the analysis. Conclusions and possible extensions are in the last part of the paper.

2. Preference Heterogeneity and Endogenous Lobbying

We develop a political economy model of policy determination and lobby formation, where lobbies are groupings of individual having similar preferences who try to influence policy determination offering contribution to the elected policy maker. This section briefly describes the economic environment and the model. The analysis of the political equilibrium, policy determination, lobby formation and election stage will follow.

2.1. The Economic Environment

Consider two jurisdictions A and B of population size $n(A)$ and $n(B)$ respectively, $n = n(A) + n(B)$. Residents are identical in their income (normalized to unity) and consume a private good and a public good or service but they differ with respect to their preferences over the public good level provided by the government.

Output, Y_k, in each region is produced from labour, which is inelastically supplied by each individual in an amount equal to unity. The production technology is assumed to be linear in total labour inputs, and without loss of generality, units are normalized so that the wage rate is unity:

$$Y_k = n(k),\ k = A, B. \tag{2.1}$$

Output is used for private consumption and for the provision of the public good. The marginal rate of transformation between private consumption and the public good in production is assumed to be, without loss of generality, equal to unity.

Provision of the public good, g_k, is funded by a proportional income tax levied at rate t_k, which is assumed to be the only fiscal instrument available.[7] The level of private consumption for an individual residing in jurisdiction k is then

$$p_k^i = 1 - t_k,\ k = A, B. \tag{2.2}$$

and public good provision

$$g_k = t_k n(k). \tag{2.3}$$

Each citizen has quasilinear preferences over private consumption p_k^i and public good g_k of the form

$$u(p_k^i, g_k) = p_k^i + h(\theta^i, g_k), \quad k = A, B. \tag{2.4}$$

where $h(.)$ is strictly concave and single peaked. The θ term is a public good preference parameter defined in $[0, 1]$, with citizens with higher $\theta's$ having higher valuations of the public good. For ease of exposition, in order to obtain closed form solutions to the model, in what follows we will take

$$h(\theta^i, g_k) = \theta^i \ln g_k, \quad k = A, B. \tag{2.5}$$

There are three *types* of citizens $\omega = L, M, H$. If a citizen i has preference type ω, his valuation of the public good is $\theta^i = \theta_\omega$ with $\theta_L < \theta_M < \theta_H$. Note that we use superscripts to refer to the preference parameters of individuals, and subscripts to refer to the preference parameters of types. Let the set of citizens of preference type ω be N_ω, and let the number be $\#N_\omega = n_\omega$.

[7] Although our model accounts for preference heterogeneity, preferences are unobservable and thus taxes cannot be conditioned on them, even though policymakers may have full information about the distribution of preferences.

In each jurisdiction two types of citizens reside, L and M in A, and, M and H in B. The reason why we introduce this assumption is that we want to pick up the fact that when two jurisdictions decide to form a union there will be an effect not only on the size of the union but also on preference heterogeneity within the union. In jurisdiction A there are therefore n_L citizens of type L and $n_M(A)$ citizens type M and, in jurisdiction B, n_H citizens type H and $n_M(B)$ citizens of type M. If the two jurisdictions decide to form a union, the union will be formed by n_L citizens of type L, n_M =$n_M(A)$+ $n_M(B)$ citizens type M and n_H citizens type H.

The level of the public good is chosen by a policy maker elected by plurality voting over a set of candidates in which each type of citizen is represented. The constitution specifies that all citizens have a vote they may use for one of the candidates and that the candidate who has most votes is the winner. Moreover in the event of ties, the winner is randomly chosen from among the candidates with most votes. In the case where no candidate gets at least a vote the default policy zero public good is selected. Elections are local or central depending on the level of government responsible for the policy. If the policy choice is decentralized each jurisdiction elects a representative; if it is centralized the two jurisdictions elect a common representative.

Once elected, the winning candidate can be lobbied by *endogenously* determined groups formed by citizens with the same preferences. The lobbying activity in this model is the exercise of political influence over government's economic policy decision through contributions after elections.

We now describe the voting and lobbying stages in more detail.

2.2. Voting

In the first stage, prior to lobby formation, there is policy maker selection by plurality voting over the set of candidates; we do not model candidate entry but we assume that there is a single candidate for each type of citizen.[8] Citizens vote sincerely for their preferred candidate anticipating her policy behavior, and anticipating lobbying activities. In particular each citizen simultaneously and independently decides to vote for a candidate or to abstain. Let $v^i = j$ denote citizen i 's decision to vote for citizen of type j, and $v^i = 0$ citizen i's abstention. A vector of voting decisions is denoted by $v = (v^1, v^2, ...v^n)$ under centralization and $v(k) = (v^1, v^2, ...v^{n(k)})$ under decentralization.

The candidate who receives the most votes is elected, and in the event of ties, the policy maker is randomly chosen. We denote by P the preference type of the winner, where $P(A) \in \{L, M\}$

[8] This is possible, for example, if we assume the the cost of becoming a condidate is very low.

and $P(B) \in \{M, H\}$, under decentralization and $P \in \{L, M, H\}$ under centralization. In making their voting decisions, citizen i chooses v^i so as to maximize his expected utility, anticipating lobbying and policy choice.

2.3. Lobbying

After elections, all non-elected citizens simultaneously decide whether or not to become a member of a lobby with citizens having the same preferences i.e. of the same preference type. We can think of this stage as a subscription stage, where each citizen simultaneously offers his contribution schedule and agrees on equally sharing part of the fixed cost K with the other citizens of the same type. At the end of this stage each citizen can observe the subscriptions by the other members and decide accordingly. A lobby is formed if all non-elected citizens of the same group have decided to contribute and not otherwise.

By definition, with centralization, there are three possible lobbies, $l = L, M, H$, and with decentralization, there are two possible lobbies in each jurisdiction, $l = L, M$ in A, and $l = M, H$ in B. Given the lobby formation rule above, the members of the lobby l are the set $S_l = N_l/\{p\}$: so, if a lobby forms, its size $s_l = \#S_l$ is equal to n_l if the policy maker is not a type θ_l and $n_l - 1$ if $P = l$.

At the lobbying stage, every non-elected citizen i can choose to make a contribution to the lobby l that represents his preference. This contribution is the sum of a contribution schedule which depends on the tax, $c^i(t)$, and a share of the fixed cost, K/s_l. The citizen can also choose not to lobby by making an overall contribution of zero. Define the indicator $\lambda^i = 1$ if i chooses to lobby, and $\lambda^i = 0$ otherwise.

We can now state the lobby formation rule more formally. If for all $i \in N_L/\{P\}$, $\lambda^i = 1$, then lobby $l = L$ forms, and similarly for lobby H. For lobby $l = M$, we need to distinguish centralization and decentralization; in the first case, citizens type M can form only a lobby, in the second case, citizens type M can form a lobby with citizens of the same type and resident in the same jurisdiction, therefore there will be two possible lobbies M, $i \in N_M(k)/\{P\}, k = A, B$. So, if lobby l forms, we set an indicator variable $\lambda_l = 1$: otherwise, $\lambda_l = 0$. Also, for future reference, define $\Lambda = \{l \in \{L, M, H\} \,|\lambda_l = 1\}$, to be the set of lobbies that form in the case of centralization and similarly, let $\Lambda(A) = \{l \in \{L, M\} \,|\lambda_l = 1\}$, $\Lambda(B) = \{l \in \{M, H\} \,|\lambda_l = 1\}$ be the set of lobbies that form in jurisdictions A, B in the case of decentralization.

If an agent $i \in N_l$ makes contribution $c^i(t) + \frac{K}{s_l}$ to lobby l, his utility is given by

$$H^i(t) = 1 - t + \theta^i \ln tn - \lambda_l[c^i(t) + \frac{K}{s_l}] \tag{2.6}$$

in the case of centralization, and

$$H^i(t) = 1 - t_k + \theta^i \ln t_k N_k - \lambda_l[c_i(t) + \frac{K}{s_l}],\ k = A, B \tag{2.7}$$

in the case of decentralization.

Once formed, the lobby's objective function is the maximization of the utility of its members, which, in the case of centralization, conditional on a given tax, t, is:

$$H_l(t) = \sum_{i \in S_l} H^i = n_l(1-t) + \sum_{i \in S_l} \theta^i \ln tn - C_l(t) - K$$

Where $C_l(t) = \sum_{i \in S_l} c^i(t)$, is the total payment function offered by lobby l. And in the case of decentralization:

$$H_l(t_k) = \sum_{i \in S_l} H^i = n_l(1-t_k) + \sum_{i \in S_l} \theta^i \ln t_k n - C_l(t_k) - K, k = A, B.$$

Finally, in the third stage of the game, the policy maker, given lobbies' contributions, implements her favorite policy choices. In making her policy decision she takes into account the contribution payments offered by the lobby(ies), which are assumed to be credible.

With centralization, her policy choice is therefore the solution of the following maximization problem.

$$t(P, \Lambda) = \arg\max_{t \in \Re} = 1 - t + \theta_P \ln tn + \sum_{l \in \Lambda} C_l(t) \tag{2.8}$$

With decentralization,

$$t_k(P(k), \Lambda(k)) = \arg\max_{t \in \Re} = 1 - t + \theta_{P(k)} \ln tn_k + \sum_{l \in \Lambda_k} C_l(t),\ k = A, B$$

We solve the model when policy decision is decentralized and when it is centralized. We proceed backwards. We first characterize the last stage of the game: policy selection, we then characterize lobbying and election. But before doing that we solve a simple political economy model of policy determination without lobbying possible. We call this the benchmark.

3. The Benchmark: Political Equilibrium without Lobbying

If lobbying is not taken into account our model simply becomes a two stage model of election and policy determination. In the first stage elections determine the identity of the policy maker, who decides the policy in the following stage. We solve this model under decentralization and under centralization and we will use these results to compare the outcome when we introduce lobbying.

The process has two stages. First, an election determines the citizen that represents the jurisdiction. The representative is chosen among the candidates residents in that jurisdiction by plurality voting. We do not model the entry of candidates stage as in Besley and Coate (1997) but we assume that there is a candidate for each type of citizens in each jurisdiction, and that citizens vote sincerely for their preferred candidate. The candidate with most votes is the winner; in the event of ties the policy maker is randomly chosen among the candidates with most votes, if nobody votes no public good is provided.

Second, policies are chosen simultaneously by the policy makers. Since there is no *ex-ante* policy commitment, the preferences of the elected policy makers will determine the policy. A type θ_ω candidate chooses the public good level that maximizes, (2.4) which is easily calculated to be $t = \theta_\omega$.

Moving to the voting stage, we assume that citizens vote sincerely. If each group size differs of more than 1 citizen i.e. $|n_\omega - n_v| > 1$, all $\omega, v \in \{L, M, H\}$, in the case of centralization, and $\omega, v \in \{L, M\}$ in jurisdiction A and $\omega, v \in \{M, H\}$ in jurisdiction B in the case of decentralization. It can be shown that sincere voting is an equilibrium strategy for every citizen in the plurality voting game.[9] So, the outcome must be that in any jurisdiction, the elected policy-maker has the preferences of the largest group in that jurisdiction.

Let the type of the majority group be m in the case of centralization, and let the type of the majority group in jurisdictions A, B be $m(A), m(B)$ respectively. For example, if $n_L > n_M, n_H$ then $m = L$. The results are summarized in the following lemma:

Lemma 1. *When lobbying is not possible, each jurisdiction elects the representative having the same preferences as the majority group (m in the case of centralization, $m(A), m(B)$ in the case of decentralization) who implements her favorite policy choice t^m.*

4. Equilibrium with Endogenous Lobbying

In this section, we model citizens' behavior both with respect to voting and lobbying decisions in order to analyze how they are affected by policy centralization or decentralization. We proceed backward. We first characterize the policy choice, followed by the lobbying stage, and finally elections. We solve the model for centralization, and when the same analysis applies for decentralization, in what follows we will omit it.

[9] This guarantees that no citizen's vote can be pivotal, and therefore there is no incentive to deviate from sincere voting.

4.1. Policy Selection

In order to characterize the equilibrium of the policy selection game, in modelling the influence of interests groups in policy decision, we follow Bernheim and Whinston (1986) and Dixit, Grossman and Helpman (1997). We assume that this policy selection game begins with already formed lobbies choosing a payment schedule that maximize the utility of their members, taking as given the payment schedules offered by other lobbies and anticipating policy maker policy choice. Given these payment functions, the elected policy maker selects among the set of feasible policy choices that maximizes her utility. Since we can have multiple equilibria, as in Bernheim and Whinston (1986) and Dixit, Grossman and Helpman (1997), we focus on truthful equilibria.[10], this implies that each lobby offers the policy maker an amount such that she receives the same utility she would have got without lobbies' offer. In other words, truthful contributions reflect the benefits lobbyists will receive from the deviation. If the policy maker deviates toward lobby preferences, the lobby will pay as promised, and she will receive nothing from that lobby otherwise. In taking this decision citizens compare policy outcomes under participation and non-participation, given the choices of the other citizens.

Replacing truthful contributions (11.1), derived in appendix, into (2.8), for any given lobby, the policy maker maximizes

$$1 - t + \theta_P \ln tn + \sum_{l \in \Lambda} s_l (1 - t + \theta_l \ln tn - u_\circ - \frac{K}{s_l}) \tag{4.1}$$

where $u_\circ$ is a given constant. This yields the following policy choice under centralization

$$t(P, \Lambda) = \frac{\theta_P + \sum_{l \in \Lambda} s_l \theta_l}{\sum_{l \in \Lambda} s_l + 1}, \tag{4.2}$$

and under decentralization

$$t\,(k)\,(P, \Lambda) = \frac{\theta_P + \sum_{l \in \Lambda(k)} s_l \theta_l}{\sum_{l \in \Lambda(k)} s_l + 1}, k = A, B. \tag{4.3}$$

When there are no lobbies ($\Lambda = \emptyset$), $t(P, \Lambda) = \theta_P$. When there are one or more lobbies the policy choice is an average between policy maker and lobbyists' preferences.

4.2. Lobby formation

Recall that after elections determine the identity of the policy maker and before the policy choice is implemented, each group of citizens decides whether or not to lobby the policy maker

[10] These equilibria always exist and have the characteristics of being both efficient and coalition proof Dixit, Grossman and Helpman (1997), p.759.

by offering contributions to move her preferred policy choice toward their preferred policy; to form a lobby costs a fixed amount K. Lobbying decisions are based on the comparison of citizens' expected payoffs for any lobbying configuration, given other groups' possible lobbying strategies. If citizens type ω agree on forming a lobby, they will offer the policy maker the following contribution, which depends on the preference type of the policy make (P), and on the other existing lobbies (Λ).

$$\begin{aligned} C_\omega(P,\Lambda) &= \left[t(P,\Lambda) - t(P,\Lambda/\{\omega\}) + \theta_P \ln\left(\frac{t(P,\Lambda/\{\omega\})}{t(P,\Lambda}\right)\right] \\ &\quad + \sum_{l\in\Lambda/\{\omega\}} \left[s_l t(P,\Lambda) - t(P,\Lambda/\{\omega\}) + \theta_l \ln\left(\frac{t(P,\Lambda/\{\omega\})}{t(P,\Lambda)}\right)\right] \end{aligned}$$

A truthful contribution function for principal ω rewards the agent P for every change in the action exactly the amount of change in the principal's welfare, provided that the payment both before and after the change is strictly positive. These contributions compensate the policy maker for the loss of utility due to (i) moving away from her ideal point, and/or (ii) the loss of the contributions paid by the other lobbyists. Then substituting $C_\omega(P,\Lambda)$ into (2.6), we get the payoff to a type ω if lobbies Λ form:

$$H_\omega(P,\Lambda) = 1 - t(P,\Lambda) + \theta_\omega \ln\left(t(P,\Lambda)n\right) - \frac{1}{s_\omega}[C_\omega(P,\Lambda) + K]$$

If lobby ω does not form, a type ω gets

$$H_\omega(P,\Lambda/\{\omega\}) = 1 - t(P,\Lambda/\{\omega\}) + \theta_\omega \ln\left(t(P,\Lambda/\{\omega\})n\right)$$

So, the gain to forming a lobby is

$$\begin{aligned} \Delta_\omega(P,\Lambda) &= H_\omega(P,\Lambda) - H_\omega(P,\Lambda/\{\omega\}) \\ &= t(P,\Lambda/\{\omega\}) - t(P,\Lambda) + \theta_\omega \ln\left(\frac{t(P,\Lambda/\{\omega\})}{t(P,\Lambda)}\right) - \frac{1}{s_\omega}[C_\omega(P,\Lambda) + K] \end{aligned}$$

After simplifications, we get:

$$\Delta_\omega(P,\Lambda) = \frac{\sum_{l\in\Lambda} s_l + 1}{s_\omega}\left(t(P,\Lambda/\{\omega\}) - t(P,\Lambda)\right) + \frac{\sum_{l\in\Lambda} s_l\theta_l + \theta_P}{s_\omega} \ln\left(\frac{t(P,\Lambda)}{t(P,\Lambda/\{\omega\})}\right) - \frac{K}{s_\omega} \tag{4.4}$$

Condition (4.4) represents the payoffs' difference for a citizen type ω if he decides to form a lobby compared to the decision of not lobbying, given the identity of the policy maker, and given a set of lobbies $\Lambda/\{\omega\}$ formed by other citizens So, if

$$\Delta_\omega(P,\Lambda) \geq 0 \tag{4.5}$$

lobbying is preferred to non-lobbying by citizens of type ω, given that other lobbies $\Lambda/\{\omega\}$ have formed. If we set $\Lambda = \{\omega\}$, (4.5) becomes the condition for a citizen of type ω to form a lobby when no other group lobbies. If Λ includes types other than $\{\omega\}$ (4.5) becomes the lobbying condition for a citizen type ω, given that other lobbies exist. On the other hand, if $\Delta_\omega(P, \Lambda \cup \{\nu\}) < 0$ for some type ν, then citizens of that type do not want to form a lobby given that lobbies Λ already exist. This motivates the following definition of an equilibrium set of lobbies, given a policy-maker P.

Definition 1. *With centralization, $\Lambda^*(P)$ is an equilibrium set of lobbies, given P, if (i) $\Delta_\omega(P, \Lambda^*(P)) \geq 0$, all $\omega \in \Lambda^*(P)$, and (ii) $\Delta_\omega(P, \Lambda^*(P) \cup \{\nu\}) < 0$, all $\nu \notin \Lambda^*(P)$. With decentralization, $\Lambda_k^*(P)$ is an equilibrium set of lobbies, given P, if (i) $\Delta_\omega(P, \Lambda^*(P)) \geq 0$, all $\omega \in \Lambda^*(P)$, and (ii) $\Delta_\omega(P, \Lambda^*(P) \cup \{\nu\}) < 0$, all $\nu \notin \Lambda^*(P)$.*

This is, lobbying choices are Nash equilibrium ones: at the equilibrium, those who lobby are better off lobbying than not, given the decisions of others, and vice versa. Ideally, we would like to get a general characterization of $\Lambda^*(P)$ in the case of both centralization and decentralization. This is not possible, but we can establish conditions under which citizens of any given type ω want to lobby given a policy-maker P, given a fixed set of other lobbies. We only present this result for the case of centralization: a very similar result holds for decentralization.

First define $\Delta_\omega(\theta, \Lambda)$ which is the gain to lobbying by preference type ω, in the hypothetical situation that this preference type has a preference parameter θ (in fact, the preference type is fixed at θ_ω). That is:

$$\Delta_\omega(\theta, \Lambda) = \frac{\sum_{l\in\Lambda} s_l + 1}{s_\omega}\left(t(\theta, \Lambda/\{\omega\}) - t(\theta, \Lambda)\right) + \frac{\sum_{l\in\Lambda} s_l\theta_l + \theta}{s_\omega}\ln\left(\frac{t(\theta, \Lambda)}{t(\theta, \Lambda/\{\omega\})}\right) - \frac{K}{s_\omega},\ t(\theta, \Lambda)$$

Also, allow θ to take any value. It is shown in the appendix that this function is convex in θ, and has a minimum value of $-K/s_\omega$ which is less than zero: this minimum value occurs at

$$\theta_{\min}(\omega, \Lambda) = \frac{\theta_P + \sum_{l\in\Lambda/\{\omega\}} s_l\theta_l}{\sum_{l\in\Lambda/\{\omega\}} s_l + 1}.$$

So, there must be two values of θ at which $\Delta_\omega(\theta, \Lambda) = 0$. Let these two values be $\underline{\theta}(\omega, \Lambda)$, $\overline{\theta}(\omega, \Lambda)$ respectively with $\underline{\theta}(\omega, \Lambda) < \overline{\theta}(\omega, \Lambda)$. Then we have:

Proposition 1. *Given any policy maker of type P, and other lobbies $\Lambda/\{\omega\}$, citizens of type ω wish to lobby iff $\theta_\omega < \underline{\theta}(\omega, \Lambda)$ or $\theta_\omega > \overline{\theta}(\omega, \Lambda)$. Moreover, $\underline{\theta}(\omega, \Lambda) < \theta_{\min}(\omega, \Lambda) < \overline{\theta}(\omega, \Lambda)$, so if there are no other lobbies, $\underline{\theta}(\omega, \{\omega\}) < \theta_P < \overline{\theta}(\omega, \{\omega\})$.Finally, $\underline{\theta}(\omega, \Lambda) \to \overline{\theta}(\omega, \Lambda)$ as*

$s_\omega \to \infty$, so if $\theta_{\min}(\omega, \Lambda) \neq \theta_\omega$, there exists $\hat{s}_\omega$ high enough so that for all $s_\omega > \hat{s}_\omega$, citizens of type ω wish to lobby.

The first part of the Proposition says lobby formation is determined both by lobbyists and policy maker preferences; the more ω types are far from either of them the more lobbying will occur. Suppose now that in jurisdictions A and B, of identical size, three types of citizens reside - $M, L, H-$. If the elected policy makers are respectively of type M and L, and H types lobby in both jurisdictions, it is more likely that another lobby will form in jurisdiction A.

The second part of Proposition 1 says that as preference heterogeneity between a group and the policy maker increases the likelihood for citizens type ω to form a lobby increases, if no other group lobbies. To give an example; suppose that jurisdictions A and B have the same number of residents, divided into groups in the same proportions, and that preference heterogeneity between groups is higher in A than in B: the group with preference different from the policy maker is more likely to organize in A than in B.

Finally, the third part of the Proposition says that bigger groups are more likely to organize a lobby, because the costs of forming a lobby are shared between more members. Imagine now a situation where there exists two jurisdictions A and B, with the same costs of lobbying and the same preference heterogeneity within jurisdictions, lobbying is more likely to occur in jurisdiction A, for a given policy maker, if : i) the have the same size but A has larger minorities or ii) the ratio majorities/minorities is the same but A is larger.

If we analyze the results the previous Proposition we can draw are the following considerations.

Remark 1. *Lobbies are less likely to form in small jurisdictions.*

This result derives from the last part of Proposition 1, where we have shown that there exists a direct relationship between group size and likelihood for lobbying. For example if compare two jurisdictions, the same two type of individuals residing, the same proportion of majority, but different size, lobbying is more likely to occur in the larger jurisdiction.

Remark 2. *Lobbies are less likely to form in jurisdictions with strong majorities.*

Another example: two jurisdictions of identical size in which the same two types of citizens live, but with different preference distributions, lobbying is more likely to occur in the jurisdiction where preferences are more equally distributed.

Remark 3. *Lobbies are less likely to form in jurisdictions where residents are fragmented in more groups.*

Consider now two jurisdictions of the same size with the same preferences dispersion, i.e. the two extreme types are the same, but in the first one, A, there are only two groups of individual and in the second one, B, the minority group is split into two groups.

If the majority group in B in a citizens of the group in the middle, it is less likely that the extreme groups will lobby, because preferences heterogeneity between the groups and the median group is lower than in B. Note that if the minority group do not form a lobby when the candidate with median preferences is elected, she will be elected, like in the benchmark. If the majority group in B is one of the extreme groups, and the other extreme group will lobby if the extreme candidate is elected, the majority group can offset the lobbying influence by forming a lobby or by voting in the previous stage for a candidate of the group in the middle. In this way it is possible that the same policy outcome, or even a better one, is achieved by the majority group by simply changing their voting strategies.

4.3. Elections

Turning to the first stage of the game, each citizen makes his voting decision voting sincerely among the set of candidates, anticipating lobbying and policy choice.

From Lemma 1, if citizens vote sincerely for their preferred candidate, the candidate of the majority group is always elected. However, when lobbying is possible ordinary citizens and candidates of the same type do not have necessary the same preferences over a policy choice. Once elected, the policy maker can indeed earn a rent from choosing a policy choice close to lobbies preference, if lobbies are formed. Citizens vote for the candidate whose policy choice will be closer to their ideal point and/or who will be cheaper to pay in case they will form a lobby. Suppose, for example, that if citizens of type ω vote for a candidate of type l in case of election, she will implement a policy choice t^*, which is a combination of her preferences and lobbying preferences. Moreover, suppose that in order to achieve this result citizens of type ω will have to lobby together with another opposite lobby. But, if they vote instead for a candidate type ω', who will implement a policy choice t^{**}, which is less preferred by citizens type ω than t^*, they will not have to lobby. The final voting decision will therefore depend on the comparison between policy choices and costs of lobbying.

$$H(\omega, P) = 1 - t(P, \Lambda^*(P)) + \theta_\omega \ln\left(t(P, \Lambda^*(P))n\right) - \frac{1}{s_\omega}[C(\omega, P, \Lambda^*(P)) + K]$$

So, now sincere voting means that a citizen of type ω will vote for the P that maximises $H(\omega, P)$: let this be $P(\omega)$. In general, $P(\omega)$ need not be the same as ω. Recall that $m \in \{L, M, H\}$ is

the majority preference group in the population i.e. $n_m > n_\omega,\ \omega \neq m$. Then it is clear that a citizen of preference type $P(m)$ is elected.

When the possibility of forming lobbies is introduced, the majority group is still able to elect its preferred candidate, but this candidate is not necessarily the candidate having their *a priori* preferences. In other words, $P(m)$ may not be equal to m. Allowing for lobbying gives citizens, who are not happy with the electoral result, another instrument: to form a lobby and "pay" the policy maker in order to move her policy choice closer to their ideal point. However this action is anticipated by the other citizens who can react in different ways: i) to form a lobby themselves, and/or ii) vote for another candidate in the first stage of the game. This means that the majority group is still able to elect its preferred candidate, who is not necessarily the one sharing their *ex ante* policy preferences.

We are now in a position to define an equilibrium in the game as a whole.

Definition 2. *A political equilibrium is a* $\omega^*,\ \Lambda^*(\omega^*)$ *such that (i)* $\omega^* = P(m)$ *is the type of the elected policy-maker; (ii)* $\Lambda^*(\omega^*)$ *is the set of lobbies that form, given the policy-maker's type.*

We have now succeeded in defining an equilibrium with endogenous lobbying for our model. In the next section, we characterise this equilibrium. Before we do so, however, we should note a possible complication in the definition: that is, conditional on ω^*, it is possible to have several equilibrium sets of lobbies $\Lambda^*(\omega^*)$.

Example of multiple equilibrium.

We now give a numerical example of multiple equilibrium where there are two Nash equilibria in pure strategy, either to form two lobbies by the minority groups or not to lobby at all. Consider a jurisdiction in which the two minority groups ω and v , with preferences parameter $\theta_\omega = 0.3$ and $\theta_v = 0.9$, have a size of $n_\omega = 150$ and $n_v = 100$ respectively. Moreover, the majority group m has preference parameter $\theta_\omega = 0.5$. To form a lobby costs an amount $K = 15$ unit for each lobby. Consider now the lobbying decision by type ω : if no other lobby exists: their gain is $\Delta_\omega(M,(\omega)) = -0.0536$, therefor they do not want to lobby alone. However if v types form a lobby, ω types gain becomes $\Delta_\omega(M,(\omega,v)) = 0.0383$. Now let us turn to the lobbying decisions by v types; their gain to lobby alone is $\Delta_v(M,(v)) = -0.0220$ and with lobby ω is $\Delta_\omega(M,(\omega,v)) = 0.0414$. Therefore it is clear that there exists two Nash equilibria of the lobbying subgame $\Delta(M) = \{\emptyset\},\{\omega,v\}$.♡

5. Lobbying and Political Equilibrium

We begin by establishing conditions for a political equilibrium not to be affected by the introduction of a lobby formation stage. Say that the equilibrium is *lobby-free* if in equilibrium, (i) no lobby forms, and (ii) and the elected policy-maker is type m, i.e. the same type in the model without lobbying. Then, it is clear that the policy choice is also the same as in the benchmark, and so in every possible sense, the equilibrium is unaffected by lobbying.

Proposition 2. *Under decentralization, the equilibrium is lobby-free iff* $\Delta_\omega(m,\{\omega\}) < 0$ *for* $\omega \neq m$. *Under centralization, if (i)* $\Delta_\omega(m,\{\omega\}), \Delta_\omega(m,\{\omega,v\}) < 0$, *(ii)* $\Delta_v(m,\{v\}) < 0$ *for* $\omega,\nu \neq m$, (iii) $\Delta_n(m,\{\omega,v,m\}) < 0$, some $n \in \{\omega,v,m\}$, *the equilibrium is lobby-free.*

Proof. (a) Decentralization. Also, $\Delta_\omega(m,\{\omega\}) > \Delta_\omega(m,\{\omega,m\})$ because both functions reach their minimum at $\theta = \theta_m$, where both assume the value -K/s_ω but the latter is smoother $\frac{\partial\Delta^2_\omega(m,\{\omega,m\})}{\partial\omega} < \frac{\partial\Delta^2_\omega(m,\{\omega\})}{\partial\omega}$ (see (11.30) and (11.33) in the Appendix). So, $0 > \Delta_\omega(m,\{\omega\}) > \Delta_\omega(m,\{\omega,m\})$. As agents of type ω never want to lobby, it remains to show that majority group m will never want to lobby a policy-maker of type m on their own i.e $\Delta_m(m,\{m\}) < 0$. But it is shown in the Appendix (eqn 11.10) that $\Delta_m(m,\{\omega\}) < 0$. (b) Centralization. First, note that the conditions states rule out an equilibrium where just one minority group lobbies, and we know also that $\Delta_m(m,\{m\}) < 0$. Second, the condition $\Delta_\omega(m,\{\omega,v\}) < 0$ rules out an equilibrium where both minority groups lobby. Third, as $0 > \Delta_\omega(m,\{\omega\}) > \Delta_\omega(m,\{\omega,m\})$, there is no equilibrium where the majority and one minority group lobby. Finally, condition (iii) rules out a lobby of all three types. ♡

So, we see that under decentralization, where only two types of citizens reside in each jurisdiction, the sufficient condition for no-lobby equilibrium is that the minority group will not lobby alone. Also, the majority will never want to lobby a policy maker with their *ex-ante* policy preferences unless there is a "threat" by the minority group. However if the minority group does not want to lobby alone, it does not constitute a threat because, it implies that they no not want to lobby if the majority group does. The intuition for that is very simple, if the minority group does not want to pay the policy maker when there are no other lobbies, it would be inconsistent if they wanted to pay her a bigger amount for a smaller policy deviation if the majority group were induced to lobby. Note that voting for the candidate with their *ex ante* policy preferences it is also the best voting strategy for the majority group if condition (i) holds, because the policy maker will implement their preferred policy choice. Moreover, if the above conditions are satisfied, introducing the lobbying stage to the model will not affect the political equilibrium, i.e. the elected policy makers will be the same as in the benchmark, no

contributions will be paid to them, and no lobbies will form.

Under decentralization the above conditions are necessary and sufficient for an equilibrium without lobbying. The reason for this is that the majority group will never prefer an equilibrium where the candidate of the minority group is elected if no lobby exist; because this equilibrium is always dominate by the one determined by the election of a candidate of the majority group, whatever is the lobbying outcome associated with this voting equilibrium. Under centralization these conditions are only sufficient but not necessary for an equilibrium without lobbying. Suppose one of the conditions of the Proposition is violated: the majority group is one of the extreme groups, let say the one formed by citizens type H, and that the group formed by citizens type L will lobby alone if a candidate type H is elected (i.e. H types vote for her), which formally is $\Delta_L(H,\{L\}) > 0$. However, for the majority group, voting for a candidate type M, who will not lobbied, can be a voting equilibrium preferred to the one determined by voting for the candidate of its type. (i.e. $H_L(M,\{\emptyset\}) > H_L(L,\{L,H\})$ The outcome is that a candidate of the median group is elected and implement her *ex ante* policy preference $t^* = \theta_M$. Note in this case the threat of lobbying has affected the political equilibrium, because the policy choice is now the one preferred by the median group and not by the majority group.

Note that under centralization it is possible to have equilibria where the policy maker has *ex ante* policy preference different from the majority group and no lobbies have formed; this is however possible only when the equilibrium of lobbying subgame for a policy maker from the majority group involves an equilibrium with one or more lobbies. However, even if no lobby exist in equilibrium, the political equilibrium is affected because a different policy maker is elected compared to the benchmark.

In summary, adding a lobby formation stage to the model, will not affect the political equilibrium only if groups have very close preferences over the public good and/ or minority groups are very small.

When lobbying does affect political equilibrium, it is does so in one or both of two ways; either by affecting the policy choice of a given elected policy-maker, or affecting the choice of the policy-maker itself.

Form now on, assume $\Delta_n(m,\{\omega,v,z\}) < 0$, some $n \in \{\omega,v,z\}$, i.e. expression (11.19) in the Appendix is negative for at least one group, and if we substitute the relevant $t(P,\Lambda)s$ we get:

$$\frac{s_\omega + s_v + s_z + 1}{s_\omega}\left(\frac{\theta_v s_v + \theta_z s_z + \theta_P}{s_v + s_z + 1} - \frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{n}\right) + \tag{5.1}$$
$$+\frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{s_\omega}\ln\frac{(s_v + s_z + 1)(\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P)}{n(\theta_v s_v + \theta_z s_z + \theta_P)} - \frac{K}{s_\omega} < 0$$

Similarly we can derive the necessary conditions for a political equilibrium affected by en-

dogenous lobbying, the results are summarized in Proposition 3 and 4.

Proposition 3. *Under decentralization, if* $\Delta_\omega(m,\{\omega\}) \geqslant 0$ *for* $\omega \neq m$, *at least one lobby will form in political equilibrium. Under centralization, if , for* $\omega \neq m$, $\Delta_\omega(m,\{\omega\}) \geqslant 0$, *for* $\omega \neq m$ *for at least one type* ω, *the political equilibrium will be affected by the possibility of endogenous lobbying.*

Proof. *Decentralization.* Assume first a candidate of type m has been elected. There are two possibilities: (i) $\Delta_m(m,\{\omega,m\}) < 0$ and $\Delta_m(m,\{\omega,m\}) \geq 0$. In case (i), clearly an equilibrium with no lobbies is impossible, and $\Lambda = \{\omega\}$ is an equilibrium. In case (ii), $\Lambda = \{m,\omega\}$ is an equilibrium if $\Delta_\omega(m,\{\omega,m\}) \geq 0$. Also: otherwise, there is a one-equilibrium lobby, $\{\omega\}$ if $\Delta_\omega(m,\{\omega,m\}) \geq 0$ and $\Delta_m(m,\{\omega,m\}) < 0$ and there is an equilibrium in mixed strategies otherwise. We can calculate the associate mix strategy equilibrium which is $\left(\frac{\Pi_\omega(m,\{m,\omega\})-\Pi_\omega(m,\emptyset)}{\Pi_\omega(m,\{m,\omega\})-\Pi_\omega(m,\{\omega\})}, -\frac{\Delta_\omega(m,\{\omega\})}{\Delta_\omega(m,\{m,\omega\})}\right)$ for players type m and $\left(\frac{\Delta_m(m,\{m,\omega\})}{\Delta_m(m,\{m,\omega\})+\frac{K}{s_m}}, \frac{\frac{K}{s_m}}{\Delta_m(m,\{m,\omega\})+\frac{K}{s_m}}\right)$for players type ω. Now suppose a candidate of type ω has been elected and no lobby forms: this is impossible, as all citizens of type m would prefer to vote for m, as $H_m(\omega,\emptyset) < H_m(m,\Lambda)$ for all $\Lambda = \emptyset,\{\omega\},\{m\},\{m,\omega\}$. This is because if m is elected, the policy outcome will always be between θ_m and θ_ω, whereas if ω is elected and there is no lobbying, the policy outcome is θ_ω i.e. worst for them. (b) *Centralization:* if $\Delta_\omega(m,\{\omega,v\}) \geqslant 0$ there are four possibilities: (i) $\Delta_v(m,\{v\}) \geqslant 0$ and $\Delta_v(m,\{\omega,v\}) \geqslant 0$. In case (i) $\Lambda = \{\omega,v\}$ is an equilibrium. Case (ii) $\Delta_v(m,\{v\}) \geqslant 0$ and $\Delta_v(m,\{\omega,v\}) < 0$. In case (ii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iii) $\Delta_v(m,\{v\}) < 0$ and $\Delta_v(m,\{\omega,v\}) < 0$. In case (iii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iv) $\Delta_v(m,\{v\}) < 0$ and $\Delta_v(m,\{\omega,v\}) \geqslant 0$. In case (iv) $\Lambda = \{\omega,v\}$ is an equilibrium. if $\Delta_\omega(m,\{\omega,v\}) < 0$ there are four possibilities: (i) $\Delta_v(m,\{v\}) \geqslant 0$ and $\Delta_v(m,\{\omega,v\}) \geqslant 0$. In case (i) $\Lambda = \{v\}$ is an equilibrium. Case (ii) $\Delta_v(m,\{v\}) \geqslant 0$ and $\Delta_v(m,\{\omega,v\}) < 0$. In case (ii), $\Lambda = \{\omega\}$,$\{v\}$ is an equilibrium. Case (iii) $\Delta_v(m,\{v\}) < 0$ and $\Delta_v(m,\{\omega,v\}) < 0$. In case (iii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iv) $\Delta_v(m,\{v\}) < 0$ and $\Delta_v(m,\{\omega,v\}) \geqslant 0$, there is no equilibrium in pure strategies, the equilibrium in mixed strategies is for v $(\frac{\Delta_v(m,\{v,\omega\})}{\Delta_v(m,\{v,\omega\})-\Delta_v(m,\{v\})}$, $\frac{\Delta_v(m,\{v\})}{\Delta_v(m,\{v,\omega\})-\Delta_v(m,\{v\})})$ and for ω $(\frac{\Delta_\omega(m,\{v,\omega\})}{\Delta_\omega(m,\{v,\omega\})-\Delta_\omega(m,\{\omega\})}$, $\frac{\Delta_\omega(m,\{\omega\})}{\Delta_\omega(m,\{v,\omega\})-\Delta_\omega(m,\{\omega\})})$. If voting for $P = m$ at the election stage is not a voting equilibria, the political equilibrium will be anyway affected by lobbying through election. ♡

It is interesting to note that, both under decentralization and under centralization (if $m = M$), the conditions of Proposition 3 implicitly define the conditions for an equilibrium with lobbies. It easy to show that voting for a candidate type $\omega \neq m$ when the associated lobbying outcome is $\Lambda = (\emptyset)$, is always strictly dominated any other equilibria where P is a type $\omega = m$ for any Λ;because citizens type m can always achieve a better outcome when a type m is elected.

Therefore, whoever they vote for, there will always at least one lobby.

If, instead, under centralization $m \neq M$, it is possible that a candidate type M is preferred to the candidate type m, if this voting equilibrium is associated with an equilibrium without lobbies. In this case the political equilibrium is affected by the threat of lobbying, not by the formation of lobbies but by affecting the voting stage. It is useful to state this last results in a more formal way in our next Proposition.

Proposition 4. *Under centralization if (i)* $\Delta_\omega(m,\{\omega\}) \geqslant 0$, $\Delta_\omega(M,\{\omega\}) < 0$, $\Delta_m(M,\{m\}) < 0$, and $\Delta_m(M,\{m,\omega\}) < 0$ or $\Delta_\omega(M,\{m,\omega\}) < 0$ *and (ii)* $H_m(M,\emptyset) > H_m(m,\Lambda(m))$ *for* $\omega \neq m$, $M \neq m$, $\Lambda(m) = \{\omega\}$ *or* $\{m,\omega\}$ *the extreme majority group will elect a policy maker of type M and there will be no lobbies in equilibrium; i.e. the equilibrium is not lobby free but no lobbies will form.*

Proof. For Proposition 3, if $\Delta_\omega(m,\{\omega\}) \geqslant 0$ the equilibrium is not lobby free. $\Delta_\omega(M,\{\omega\}) < 0$ and $\Delta_m(M,\{m\}) < 0$ and $\Delta_m(M,\{m,\omega\}) < 0$ or $\Delta_\omega(M,\{m,\omega\}) < 0$ ensure that $\Lambda^*(M) = \emptyset$ is an equilibrium set of lobbies. Part (ii) says that if $H_m(M,\emptyset) > H_m(m,\Lambda)$ than $M^* = P(m)$.♡

Therefore combining Proposition 3 and 4:

Corollary 1. *If the majority group has extreme policy preferences, introducing the possibility of lobbying ensures that a more "centrist" policy is implemented.*

This follows immediately from Proposition 3, where we have shown that, when lobbying affect political equilibrium, even when no lobby forms, the majority extreme group either vote for a more centrist candidate, or vote for the candidate with their a priori policy preferences who is lobbied by the other extreme group; this is consistent with Felli and Merlo's (2000) finding.

In this section we have defined the conditions under which the political equilibrium is lobby-free, and when it is affected by the introduction of a lobby formation stage. We have shown that a lobby-free equilibrium is possible, under decentralization, when majority and minority groups have quite similar preferences over the public good and, under centralization, when the overall preference heterogeneity in the society is low. Moreover we have show that, when the political equilibrium is not lobby-free, both voting equilibrium and/or lobbying equilibrium can be affected. In particular, in Proposition 4 we have stated the necessary and sufficient conditions under which lobbies will not form in equilibrium but the elected policy maker is of different type from the one selected under the benchmark. In all the other cases, a not lobby-free equilibrium is associated with at least one lobby forming. In the next section we propose three ways of measuring the extent of lobbying and we define the neutrality conditions with respect to them.

6. Measuring the Extent of Lobbying

Lobbying can be characterized in several ways; from the number of lobbies existing, to how much these lobbies have to pay, from how these lobbies can affect policy choices, to how they can influence elections.

In particular in this paper we consider lobbying from three points of view described below. In this section we assume that the equilibrium is not *lobby-free* in the sense of Proposition 2, and we define the *neutrality conditions* with respect to the three ways we characterize lobbying.

6.1. The number of lobbies

The first way considers lobbying as the participation of citizens to the lobbying process, measured by the number of lobbies/lobbyists. From this point of view lobbying will increase or decrease under centralization if the overall number of lobbies and the number of their members are higher. Using the analysis developed in the previous section we can derive the conditions under which no lobby will form in equilibrium (i.e. *lobby-neutrality conditions*).

Proposition 5. *Under decentralization, if* $\Delta_\omega(m,\{\omega\}) \leq 0$ *for* $\omega \neq m$,*there will be a lobby-neutral political equilibrium. Under centralization, if (i)*$\Delta_\omega(m,\{\omega\}) \leq 0$, $\Delta_\omega(m,\{\omega,v\}) \leq 0$,*for* $\omega,\nu \neq m$,*or (ii)* $\Delta_\omega(m,\{\omega\}) \geqslant 0$ *and* $H_m(M,\{\emptyset\}) > H_m(m,\Lambda)$ *for* $\omega \neq m, M \neq m, \Lambda = \{\omega\},\{m,\omega\}$ *there will be a lobby-neutral political equilibrium.*

The first part of Proposition 5 characterizes the conditions for an equilibrium without lobbies under decentralization, (note that this is the same as the first part of Proposition 2). The second and the third parts characterize the (sufficient) conditions for an equilibrium without lobbying under centralization. Part (i) states the conditions for an equilibrium where introducing lobbying does not changes the results of the benchmark, (note that also this is the same as the first part of Proposition 2). However, in addition, we have to take into account conditions (ii); which reflect a situation where the (extreme) majority group prefers to vote for a candidate type M, who is not going to be lobbied, rather than for the candidate of its group, subject to lobbying, this is what already stated in Proposition 4).Note that the conditions set in Proposition 5 are less restrictive than the neutrality conditions set in of Proposition 2 for centralization, because we have to include the case where only the voting equilibrium is affected by endogenous lobbying, while, for decentralization, an equilibrium without lobbies is also lobby-free.

It is useful to remember:

Remark 4. *Under decentralization, an equilibrium without lobbies is lobby-free. Under centralization, an equilibrium lobby-neutral, can be not lobby-free.*

This result is driven by the same logic as the Besley and Coate (2001) paper where it is suggested that citizens vote strategically in order to ensure that lobbies will not have an effect on policy. However if we add a lobby formation stage and we reduce the possible number of candidates this is not always achievable by the majority group.

6.2. The effect of lobbying on policy determination

The second way in which lobbying can be measured consider the difference between the policy choice determined under the benchmark (i.e. $t^*(P^*, \Lambda^*) = t(m, \emptyset)$) and the policy choice determined under the lobbying model. Proposition 6 define the conditions under which in an *not lobby-free* equilibrium the policy choice is the same as that under the benchmark (i.e. *policy-neutrality conditions*).

Proposition 6. *Under decentralization, there is no Nash equilibrium in pure strategy where lobbying does not have an effect on policy in equilibrium. Under centralization, iff* (i) $n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$, $\Delta_\omega(m, \{\Lambda\}) \geqslant 0$, *for,* $m = M$; $\omega, \nu \neq m$ *and* $\Lambda = \{\omega\}, \{\omega, v\}$ *lobbying does not have an effect on policy in equilibrium.*

Proof. *Decentralization.* In order that $t^*(P^*, \Lambda^*) = t(m, \emptyset)$, $\Lambda^* \neq \emptyset$, the only possibility with two types of citizens is that $P^* = m$ and $\Lambda^* \neq \{m\}$. However it easy to show that this can never be an equilibrium in pure strategy. In order that at least a lobby will form, we need that condition for an equilibrium with lobbies of Proposition 3 holds (i.e. $\Delta_\omega(m, \{\omega\}) \geqslant 0$). Moreover, we need that the additional conditions $\Delta_\omega(m, \{m, \omega\}) < 0$ and $\Delta_m(m, \{m, \omega\}) > 0$ hold. Combining all these three conditions together and remembering that $\Delta_m(m, \{m\})$ is always negative, it easy to check that an equilibrium in pure strategy does not exists, see the same case in Proposition3.

Centralization. In order $t^*(P^*, \Lambda^*) = t(m, \{\emptyset\})$, $\Lambda^* \neq \emptyset$,we require either that (i) $P^* = m$ and $\Lambda^* = \{m\}$ or, if $m = M$ and $n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$, that $P^* = m$, and $\Lambda^* = \{v, \omega\}$. Part sub (i): for the same arguments as before the only possible equilibrium is in mixed strategies. Part sub (ii): for $m = M$ and $\omega, \nu \neq m$, if $\Delta_\omega(m, \{\omega\}) \geqslant 0$ and $\Delta_\omega(m, \{\omega, v\}) \geqslant 0$ there is a lobbying equilibrium where $\Lambda = (\omega, v)$ for Proposition 3 part (i) and since $n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$ the policy choice will be $t^*(m^*, \{\omega, v\}^*) = \theta_M$. Note that this is also the most preferred political equilibrium for type m since $H_m(m, \{v, \omega\}) = H_m(m, \{\emptyset\})$, and $H_m(m, \{\emptyset\})$ (the payoffs under the benchmark). $\heartsuit$

Proposition 6 presents the conditions under which in a not lobby-free equilibrium the policy choice is exactly the same that would have occurred under the benchmark. We have proved

that, in a one-lobby equilibrium, this is possible only if the policy maker is a type m and the lobby is the one formed by the majority group. However, we have ruled out that this equilibrium exists in pure strategies, both under centralization and decentralization. In a two-lobby equilibrium, this could occur only under centralization and in very special circumstances, that the majority group has preference type M and that a policy maker type M is lobbied by both extreme groups, which also must have the same power (i.e $n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$,). Finally a three-lobby equilibrium, where lobbies do not affect policy is ruled out, since the necessary condition on preference distribution is the same as the one for two-lobby equilibrium ($n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$), however if this is the case and $\Delta_\omega(m, \{v, \omega, m\}) \geqslant 0$ for both extreme groups, it follows that $\Delta_m(m, \{v, \omega, m\})$ is necessarily negative since $H_m(m, \{m, v, \omega\}) < H_m(m, \{v, \omega\}) = H_m(m, \{\emptyset\})$, because in all three cases $t^*(P^*, \Lambda^*) = \theta_P$ but is the first one group type m will have to pay contributions to the policy maker. In summary, policy choice can be not affected by lobbying only if preference heterogeneity between groups is very high, and the extreme groups are balanced. This is in contrast with the main result of Leaver and Makris (2000), where the only possible equilibrium with lobbies is the one where only the "friendly" lobby will form. In our model, this can only be an equilibrium in mixed strategies and can occur when the majority group is very large relatively to the minority and preference heterogeneity is not very high such that it is too "expensive" for the minority to lobby when the majority does. Note that in this equilibrium the lobby will not pay any contribution because the policy maker implements her *a priori* policy choice, and the lobby do not affect this choice.

Remark 5. *Under decentralization, a not lobby free equilibrium is never policy-neutral. Under centralization, a not lobby free equilibrium is never policy-neutral, also in absence of lobbies, unless* $m = M$, $n_\omega |\theta_\omega - \theta_P| \neq n_v |\theta_v - \theta_P|$ *and* $\Lambda^* = \{\omega, v\}$..

Note if the one of the above conditions in Proposition 6 is not satisfied lobbying will affect policy also in absence of lobbying. As a Corollary of the result.

Corollary 2. *If only one lobby will form in equilibrium this will never be the "friendly" one, i.e. the one formed by citizens sharing the same preferences as the policy maker.*

This is consistent with Austen-Smith and Wright (1992), where if a lobby forms, it will be the one which disagrees with the policy maker *ex-ante* policy preferences.

6.3. The size of lobbies

This way of measuring lobbying takes into account the overall amount of contributions paid to the policy-makers in equilibrium; it focuses on the resources spent by the lobbyists to organize

their activity and to pay the policy maker. In the next Proposition we define the conditions under which lobbies do not pay contribution in equilibrium(i.e. *contribution-neutrality conditions*) given that lobbies will form, i.e. given that lobby-neutrality conditions are not satisfied.

Proposition 7. *If the equilibrium is not lobby-neutral is never contribution-neutral.*

Proof. Given the truthful contribution functions derived in the Appendix (11.6), $C_{\omega}^{T}(t(P,\Lambda)) = 0$ iff $t(P,\Lambda) = t(\omega,\{\omega\})$, for $\omega = L, M, H$. Is it proved in Proposition 3 that this can never be an equilibrium in pure strategies of the lobbying subgame because for any ω $\Delta_{\omega}(\omega,\{\omega\}) < 0$. A full characterization of this equilibrium is in Proposition 3. ♡

Again, this results, consistently with Proposition 6, is in contrast with Leaver and Makris' (2000) findings, where lobbies never pay contributions in equilibrium, because if lobbies will form it can only be the friendly one, which, in absence of lobbies' competition, does not need to pay. In our model, however, the reverse is true: if a lobbies will form in equilibrium it will never be the one sharing the policy maker's same *ex-ante* policy preferences.

Remark 6. *Money always changes hands.*

Each of these three ways of measuring lobbying will be discussed more in details in the next sections, where we describe some scenario and possible political equilibrium arising under centralization and decentralization.

7. Centralization and the Number of Lobbies

In this section we compare centralization and decentralization outcomes using our first definition of lobbying: the number of lobbies formed under the two settings. We will develop three scenarios. The first two scenarios define the sufficient conditions under which centralization increases the number of lobbies in the economy: the first one describes an equilibrium with no lobby under decentralization and at least one lobby under centralization, the second one an equilibrium with one lobby under decentralization and two lobbies under centralization. The third scenario shows that also the reverse can be true: it is possible to move from an equilibrium with four lobbies under decentralization to an equilibrium with two lobbies under centralization.

Scenario 1, summarized in Proposition 8, describes a political equilibrium where the number of lobbies increases under centralization; in particular we move from an equilibrium with no lobby under decentralization to an equilibrium with at least one lobby under centralization. This can occur when one of the extreme groups is the majority group, let us say L, both under decentralization and under centralization. Regarding preference distribution, each group

has preferences quite similar to the neighbor's group but the overall heterogeneity is quite high. Under these conditions we can show that it is possible to have no lobbying under decentralization, since for the minority group lobbying a policy maker with quite similar preferences is worthless, and at least one lobby under centralization.

Proposition 8: *Suppose that* $L = m(A)$ *and* $H = m(B)$ *with decentralization and,* $L = m$ *with centralization . Under decentralization if (i)* $\Delta_M(A)(L, \{M\}) < 0$ *and, (ii)* $\Delta_M(B)(H, \{M\}) < 0$. *Under centralization if (iii)* $\Delta_H(L, \{H\}) \geqslant 0$, *and (iv)* $H_m(M, \emptyset) < H_m(m, \Lambda^*))$, *for* Λ^* *there will be an equilibrium lobby-free under decentralization and an equilibrium not lobby neutral under centralization.*

Proof. *Decentralization*: if $\Delta_M(A)(L, \{M\}) < 0$ and $\Delta_M(B)(H, \{M\}) < 0$, for Proposition 2, the equilibrium is lobby-free. *Centralization:* if $\Delta_H(L, \{H\}) \geqslant 0$, the equilibrium is not-lobby free, (i.e. the equilibrium is affected by lobbying, either through lobby formation or/and voting) for Proposition 2. Part (iii): now we have exclude that voting for a candidate type M if $\Lambda^*(M) = \emptyset$ is not a voting equilibrium (i.e. $H_m(M, \emptyset) < H_m(m, \Lambda)$) if $\Lambda(M) = \emptyset$ is a lobbying equilibrium for $P = M$; so condition (ii) in Proposition 4 is violated. Moreover $\Lambda(H) = \emptyset$ cannot be a political equilibrium, because it is always dominated by any other strategies for types M.♡

Proposition 8 depicts the sufficient conditions under which the number of lobbies increases under centralization. If conditions (i) and (ii) are satisfied for the minority groups in both jurisdictions there will be an equilibrium lobby-free under decentralization. Condition (iii) defines lobbying decision taken by the minority groups under centralization given that the candidate with *ex-ante* policy preference of the majority group is elected. It says that H types will lobby under centralization if the other groups do not. This imply that any Nash equilibria in pure strategies will have at least one lobby if the L type candidate is elected. If $\Delta_L(L; \{L, H\}) \geqslant 0$, and $\Delta_H(L; \{H, L\}) \geqslant 0$, then $\Lambda(L) = \{L, H\}$ .If $\Delta_L(L; \{L, H\}) < 0$, and $\Delta_H(L; \{H, L\}) < 0$, then $\Lambda(L) = \{H\}$. In order that $\Lambda^*(\omega) \neq \emptyset$, we must rule out the possibility that type m will never vote for a candidate $\omega \neq m$ if the associate lobby set is $\Lambda(\omega) = \emptyset, (\text{i.e.} H_m(M, \emptyset) < H_m(m, \Lambda))$. Moreover, this implies that there will be at least one lobby if the candidate M type is elects. We do not need any restriction about the outcome of the lobbying subgame if the candidate H is elected because the outcome without lobbying will be dominated by any other possible voting equilibrium for citizens type L.

The next example, summarized in the following Proposition, shows the sufficient conditions for an equilibrium with one lobby under decentralization and two lobbies under centralization.

Scenario 2 is summarized in Proposition 9. In one jurisdiction preference heterogeneity among groups is quite high, such that there is an equilibrium with one lobby under decentral-

ization, and, in the other jurisdiction, the two groups have quite similar preferences, such that there is no lobbying when policy is decentralized. Moving to the case when policy is centralized, the number of lobbies in the economy will increase from one to two, as stated in Proposition 9.

Proposition 9: *Suppose that* $M = m(A), m(B), m$. *Under decentralization if (i)* $\Delta_L(A)(M, \{L\}) \geqslant 0$, $\Delta_M(A)(M, \{M, L\}) < 0$, *and* $H_M(A)(M, \{L\}) \geqslant H_M(A)(\omega, \Lambda(\omega))$ *in A and (ii)* $\Delta_H(B)(M, \{H\}) < 0$, *in B and, under centralization, (iii)* $\Delta_\omega(M, \{\omega, v\}) \geqslant 0$ *for* $\omega, v \neq m$, *there will be an equilibrium with one lobby in jurisdiction A and no lobbies in jurisdiction B under decentralization and an equilibrium with two lobbies under centralization.*

Proof. *Decentralization.* If $\Delta_L(A)(M, \{L\}) \geqslant 0$, $\Delta_M(A)(M, \{M, L\}) < 0$ the unique Nash equilibrium of the lobbying subgame is $\Lambda = \{L\}$, and this is also a political equilibrium since $H_M(A)(M, \{L\}) \geqslant H_M(A)(\omega, \Lambda(\omega))$, in A. If $\Delta_H(B)(M, \{H\}) < 0$ the unique political equilibrium is $P^*(M) = M$, $\Lambda^* = \emptyset$, from Proposition 2. *Centralization.* If $\Delta_\omega(M, \{\omega, v\}) \geqslant 0$ the unique Nash equilibrium of the lobbying subgame is $\Lambda = \{L, H\}$, . since $\Delta_L(A)(M, \{L\}) = \Delta_L(M, \{L\}) \geqslant 0$ and $\Delta_H(B)(M, \{H\}) = \Delta_H(M, \{H\}) < 0$. This is also a political equilibrium since for type M, this is the best policy choice they can achieve: i.e. $|t(M, \{L, H\}) - t(M, \emptyset)| < |t(\omega, \Lambda) - t(M, \emptyset)|$.♡

The above expressions define the necessary and sufficient conditions for the minority group L to form a lobby in jurisdiction A when the policy maker is an M type, while condition (ii) is the sufficient and necessary conditions for no-lobby equilibrium in jurisdiction B. In both cases, in the two jurisdictions the representatives of the majority groups are selected. If we move to centralization and condition (iii) is satisfied, we have a completely different scenario: the candidate type M is elected and lobbied by the two extreme groups (L and H). This can happen because, the group L will lobby no matter what the group H does, if a candidate type M is elected; the group H will lobby only if the group L does. So, under centralization, the group H, which was not lobbying under decentralization, is induced to lobby by the group L under centralization. This can happen when preferences are asymmetrically distributed around the majority group. Finally, given that both the extreme group will lobby, the median group does not need to lobby, since the policy choice will be anyway close to its ideal policy choice.

However it is possible that the same scenario under centralization depicted in Scenario 2 can be achieved from a very different decentralized equilibrium as illustrated in our next Proposition.

Scenario 3 presents a case where preference heterogeneity among groups is very high such that under decentralization, both citizen types will form a lobby in A and in B. Moving to centralization will reduce the number of lobbies because the majority group will not lobby, since

the lobbies formed by the extreme groups offset each other' influence.

Proposition 10: *Suppose that* $M(k) = m(k)$, $k = A, B$. *If, under decentralization: (i)* $\Delta_\omega(k)(M, \{\omega\}) \geqslant 0$, $\Delta_\omega(k)(M, \{\omega, M\}) \geqslant 0$, $\Delta_M(k)(M, \{\omega, M\}) \geqslant 0$, $H_M(k)(M, \{\omega, M\}) \geqslant H_M(k)(\omega, \{\Lambda(k)^*(\omega)\})$*with* $k = A, B$, and $\omega = L$ *in* A *and* $\omega = H$ *in* B, *and, under centralization, (ii)* $\Delta_\omega(M, \{L, M\}) \geqslant 0$, *with* $\omega = L, H$ *and there will be an equilibrium with two lobbies in jurisdiction A and B under decentralization and an equilibrium with two lobbies under centralization.*

Proof. *Decentralization.* If $\Delta_\omega(k)(M, \{\omega\}) \geqslant 0$, $\Delta_\omega(k)(M, \{\omega, M\}) \geqslant 0$, $\Delta_M(k)(M, \{\omega, M\}) \geqslant 0$ the unique Nash equilibrium of the lobbying subgame is $\Lambda(A) = \{L, M\}$, and $\Lambda(B) = \{M, H\}$. Moreover $H_M(k)(M, \{\omega, M\}) \geqslant H_M(k)(\omega, \{\Lambda^*(\omega)\})$ ensures that voting $P(m) = M$ is also a political equilibrium. *Centralization.* the unique Nash equilibrium of the lobbying subgame is clearly $\{L, H\}$, and this is also the equilibrium set of lobby in the political equilibrium, see Proposition 8.♡

This example shows that the number of lobbies can decrease under decentralization. We start from a situation where both groups form a lobby under decentralization to a situation where only the extreme groups will lobby under centralization. Condition i) represents the sufficient condition for two-lobby equilibrium under decentralization and condition ii) ensures that the majority group will not lobby when the other two groups do under centralization, which is also the best outcome that can be achieved by the majority group. This is possible when preference heterogeneity is already very high under decentralization, such that both group lobbies. If we move to centralization, and the median group is the majority group, and the two extreme groups offset each others influence by lobbying, the majority group does not need to form a lobby to preserve its interest. In this situation, the majority group will be strictly better off under centralization, since its able to achieve its favorite policy choice without lobbying.

We have presented some examples where both the number of lobbies can increase or decrease under centralization. Since the model is very general and allows both for different preference distribution between and within jurisdictions and preference heterogeneity across groups, we cannot establish a unique relationship between centralization and lobbies. However we can make some considerations: i) in general if centralization increases preference heterogeneity, (for sure preference heterogeneity will not decrease!), *ceteris paribus*, lobbying is more likely to occur; ii) if centralization increases the number of groups in the economy, lobbying is less likely to occur because citizens have more options at the election stage and prevent lobbying by voting for alternative candidates; finally iii) if centralization affects groups size lobbying increases,

given the assumption that the cost of lobbying constitution is fixed and equally shared among members.

8. Centralization and Lobbying Influence on Policy

In this section lobbying is measured by its influence on policy decision. We first present three scenarios where policy choice can be affected in a different way depending on the centralization /decentralization decision , finally, we add some general observations.

In the first scenario we show that, despite the fact that no-lobby will form in equilibrium both under decentralization and centralization, the political equilibrium can be not lobby-free in the latter case. The second case analyzed correspond to Scenario 3 presented in the previous section, where, under decentralization, all groups lobby, and under centralization, only the extreme ones: in this case, compared to the benchmark, lobbying affects policy more under decentralization than centralization.

Scenario 4 presents elections, lobbying and policy choice in two jurisdictions A and B. The majority group in A are L type citizens, and, in B, H types, and $n_L > n_H, n_M$. Preference heterogeneity between the extreme groups and the median group is relatively low, such that under decentralization no lobby will form in equilibrium and the candidate of the majority is elected. Under centralization, however, the majority group L, is not able to achieve its optimal policy choice by voting for the candidate with its ex -ante policy preferences, since she will be lobbied by type H. In this situation the group type $m = L$, may prefer the equilibrium generated by the election of the candidate with M preferences if it is associate with an equilibrium with no lobby. This scenario is described in more formal way in our next Proposition.

Proposition 11: *Suppose that* $L = m(A)$, $H = m(B)$ *with decentralization and, and* $L = m$ *with centralization. Under decentralization if (i)* $\Delta_M(A)(L, \{M\}) < 0$ *and, (ii)* $\Delta_M(B)(H, \{M\}) < 0$. *Under centralization if (i)* $\Delta_H(L, \{H\}) \geqslant 0$, $\Delta_H(M, \{H\}) < 0$, $\Delta_L(M, \{M\}) < 0$, *and* $\Delta_L(M, \{L, H\}) < 0$ or $\Delta_H(M, \{L, H\}) < 0$ *and (ii)* $H_L(M, \emptyset) > H_L(L, \Lambda^*(L))$ *for* $\omega = H$, $L = m$, *no lobby will form in equilibrium but lobbying affect policy under centralization.*

Proof. *Decentralization.*If *(i)* $\Delta_M(A)(L, \{M\}) < 0$ *and, (ii)* $\Delta_M(B)(H, \{M\}) < 0$ for Proposition 2 the equilibrium is lobby-free. *Centralization.* The proof is in Lemma 4.♡

This scenario shows that the threat of lobbying can change the majority group voting strategy such that the political equilibrium is *lobby-neutral* but *not lobby-free*. In other words lobbies do not have to exist to affect policy, because voters anticipate the political equilibrium associated with the election of every candidate and vote for the one who in equilibrium select the policy choice closest to their ideal one and it is the cheapest to buy. Note that when the majority group

has extreme policy preferences lobbying ensures that a more centrist policy is implemented either via the election of a "moderate" candidate or by the formation of opposing lobbies. However, note that the possibility that an equilibrium lobby-neutral but not lobby-free is only possible under centralization, where three groups of citizens exist. Under centralization every equilibrium without lobby is always lobby-free, since the majority group will never vote for the candidate of the minority if the equilibrium is lobby-neutral, since this equilibrium is dominated by any others determined by voting for the candidate of the majority.

Scenario 5. The next scenario presents a case where decentralization affects policy more than centralization. Compared to Scenario 4, the majority groups are now M type in both jurisdiction, and preference heterogeneity between extreme and median groups is higher and or minorities are bigger. Here, under decentralization, the majority groups in both jurisdictions vote for the candidate with their preferences, who is lobbied by the minorities. Under centralization the candidate of the median group is elected and lobbied by both extreme groups. This scenario is described in more formal way in our next Proposition.

Proposition 12: *Suppose that* $, M = m(A), m(B)$ *under decentralization and* $M = m$ *under centralization. Under decentralization, if (i)* $\Delta_\omega(k)(M, \{\omega\}) \geqslant 0$, $\Delta_M(k)(M, \{M, \omega\}) < 0$, *and (ii)* $H_M(k)(M, \{\omega\}) \geqslant H_M(k)(\omega, \Lambda(\omega))$ *and, under centralization, (iii)* $\Delta_\omega(M, \{\omega\}) < 0$, $\Delta_\omega(M, \{\omega, v\}) \geqslant 0$ *for* $\omega, v \neq m$, *the effect of lobbying is higher under decentralization.*

Proof. The proof is in proposition 9.♡

Conditions (i) says that if the majority group votes for its candidate the minority group will lobby the policy maker and the majority group will not. Moreover, according to condition (ii) this outcome is the one preferred by the majority group compared to any other feasible outcomes. The second part of the Proposition is the same as the previous one. The overall policy deviation under decentralization is the difference between the policy outcome with one lobby and the benchmark, which is:

$$d^d = \sum_k \left| t^*(k)\left(M, \{\omega\}\right) - t^B(k)\left(M, \{\emptyset\}\right) \right| = \left| \frac{n_L\left(\theta_L - \theta_M\right)}{n_L + 1} \right| + \left| \frac{n_H\left(\theta_H - \theta_M\right)}{n_H + 1} \right| \tag{8.1}$$

where $t^*(k)\left(M, \{\omega\}\right)$ is the policy choice under lobbying and $t^B(k)\left(M, \{\emptyset\}\right)$ under the benchmark. Note that if $\left| \frac{n_L(\theta_L - \theta_M)}{n_L + 1} \right| = \left| \frac{n_H(\theta_H - \theta_M)}{n_H + 1} \right|$ there is not policy deviation in equilibrium since the two groups offset each other influence.

The deviation under centralization is:

$$d^c = |t^*(M, \{\omega, v\}) - t^*(M, \{\emptyset\})| = |\frac{n_L(\theta_L - \theta_M) + n_H(\theta_M - \theta_H)}{n_L + n_H + 1}| \tag{8.2}$$

Taking the difference $d^c - d^d$ after some manipulation, we have that the effect of lobbying under centralization is higher if:

$$\frac{n_L(\theta_L - \theta_M)(2n_L + n_H + 2)}{(n_L + n_H + 1)(n_L + 1)} > \frac{n_H n_L(\theta_H - \theta_M)}{(n_L + n_H + 1)(n_H + 1)} \tag{8.3}$$

which is never satisfied since the LHS is negative and the RHS positive.

We have presented two scenarios where the effect of centralization on lobbying is analyzed. Even with respect to this way of measuring lobbying we have shown that the effect of centralization on lobbying and its direction are not unique. We have shown that, under decentralization, is not possible to have an equilibrium with lobbies and no effect on policy. The reason for that is if a lobby exists in equilibrium it will be the one opposing the policy maker, and if two lobbies form there will be preference dilution, since the public good is continuous. On the contrary, under centralization, lobbies may be counteractive, in the sense that they form only to contrast the effect of an opposing lobby. In this situation, in the (very unusual!) event that they have the same power, their overall effect on policy will be zero. Of course, if coordination between groups were possible, both groups will be better off if they would not form a lobby. However in Scenario 4, we have shown that, under centralization, lobbies do not have to exist to affect policy, since the only possibility that they may form will affect the voting behavior of the majority group.

The question whether or not lobbying affects policy in equilibrium has been debated by several papers. On one hand, Besley and Coate (1999) and Leaver and Makris (2000) found that lobbying never affects policy. In the first case, Besley and Coate in their citizen-candidate model with exogenous lobbies, suggest that the reason is that citizens can predict lobbying activity and offset their influence by strategically voting for a candidate of a different type; of course this is possible when the set of possible candidate is wide enough for this choice. In the second case, the authors suggest that free-riding prevents lobbies to form, unless no contributions will be due in equilibrium, and this situation corresponds to the one where only the friendly lobby will form. On the other hand, Felli and Merlo (1999) argue with this result and demonstrate that lobbying always matter. They show that an "extremist" candidate is elected and implements a "centrist" policy, which differs from the median voter preferred outcome. Our results confirm Felli and Merlo finding, in the case that the majority group have extreme policy preferences.

In general, in our setting, we have shown that, when lobbies form, what determines their overall effect on policy are the different alternatives available to citizens. More in details, in the benchmark we have shown that the optimal outcome for each group of citizens is the one

achieved by the policy maker belonging to their group. However when lobbying is introduced, the policy maker does not necessarily have the same objective function as the citizens of her group anymore, because she can increase her utility by "selling" her policy choices to the highest bidders. Knowing this, the citizens belonging to the majority group can try to offset lobbying by voting for another candidate or by lobbying themselves, whatever they do, their objective is to minimize the policy deviation generated by lobbying. Their success depends on several factors:(i) the existence of many voting alternatives which generate better outcomes, (ii) their relative size compared to the other groups, (iii) the cost of forming a lobby.

A final consideration must be added; note that allowing for lobbying in the model, there can be can affect on political equilibrium, without any lobby formed. The reason for that is that citizens take into account lobbying generated by the election of each candidate, when they make their voting decision. In this respect the "threat" of lobbying can itself affect the political equilibrium, without observing any lobby formation.

9. Centralization and the Size of Lobbies

The last way we measure lobbying is the amount of contributions paid to the policy maker(s) and the costs of lobby formation. This way of measuring lobbying is strictly related to the effect of lobbying on policy choice but also to the number of lobbies in the economy. Because, on one hand, the more the policy maker moves away from her ideal point without lobbying, and gets closer to a lobby preferred policy choice, the more she receives; but on the other hand, when more than one lobby exists, the threat for a lobby that the policy maker will deviate toward lobbying preferences will make the lobby pay. In this section we provide an example which show the different amount of contributions paid.

We present a simple case, depicted in Scenario 5 to illustrate our point. In this scenario a policy maker type M is elected and the two extreme groups will lobby both under centralization and decentralization. However, in this section we show that the amount of contributions payed in equilibrium are higher under centralization for a smaller policy deviation.

Proposition 12: *If conditions of Proposition 11 hold, the contributions paid by the lobbyists to the policy maker under centralization are higher than under decentralization for a smaller policy deviation.*

Note that under these conditions policy makers type M are always elected and the extreme groups will form a lobby both under centralization and decentralization.

With respect to the policy deviation, note that under decentralization the policy makers will

effectively move their policy choice toward lobbies preferences, while under centralization, since the two lobbies have opposite interests, they will just offset each other and they will not be able to move the policy from the *ex ante* policy maker's ideal policy choice if lobbies have the same power.

With respect to the amount of contributions, if the policy is decentralized, each lobby offers the policy maker of its jurisdiction an amount that exactly compensates her loss of utility, determined by a policy choice closer lobby's preference; which are for each lobby formed by ω types:

$$C_{\omega}(k)(M,\{\omega\}) = \frac{n_{\omega}\left(\theta_{\omega}-\theta_{M}\right)}{\left(n_{\omega}+1\right)} + \theta_{M}\ln\left(\frac{\theta_{M}}{\left(n_{\omega}\theta_{\omega}+\theta_{M}\right)}\left(n_{\omega}+1\right)\right), \omega = L,H; k = A,B \quad (9.1)$$

After substituting $\theta_H = \theta_M + d$ and $\theta_L = \theta_M - d$, and remembering that $n_H = n_L = n$, the overall amount of contributions in the economy is:

$$C_L(A)(M,\{L\}) + C_H(B)(M,\{H\}) = \theta_M \ln \frac{\left[\theta_M\left(n+1\right)\right]^2}{\left(\left[\theta_M\left(n+1\right)\right]^2 - n^2d^2\right)} \quad (9.2)$$

And under centralization each lobby pays to the same policy maker the following amount:

$$\begin{aligned} C_{\omega}(M,\{L,H\}) &= \frac{n_{\omega}\left(\left(n_v+1\right)\left(\theta_{\omega}-\theta_v\right)+\left(\theta_{\omega}-\theta_M\right)\right)}{\left(n_{\omega}+n_v+1\right)} + \\ &\quad \left(n_v\theta_v+\theta_M\right)\ln\left(\frac{\left(n_v\theta^j+\theta_M\right)}{\left(n_{\omega}\theta_{\omega}+n_v\theta_v+\theta_M\right)}\frac{\left(n_{\omega}+n_v+1\right)}{\left(n_v+1\right)}\right) \end{aligned} \quad (9.3)$$

The overall amount of contributions in the economy is:

$$\begin{aligned} C_L(M,\{L,H\} + C_H(M,\{L,H\} &= \theta_M\left(n+1\right)\ln\frac{\left[\theta_M\left(n+1\right)\right]^2}{\left(\left[\theta_M\left(n+1\right)\right]^2 - n^2d^2\right)} + \\ &\quad + nd\ln\frac{\left(\theta_M\left(n+1\right)+nd\right)}{\left(\theta_M\left(n+1\right)-nd\right)} \end{aligned} \quad (9.4)$$

Note that (9.4) is greater than (9.2), since (9.4) is equal to (9.2) plus a positive number.

In Proposition 8, we have shown how the same lobbies and the same type policy maker have different gain from centralization: the lobbies will have to pay overall more for a smaller policy deviation.

A general consideration is: when the same groups form a lobby under centralization and decentralization, the policy makers will prefer policy centralization (because they can fully exploit lobbies' rent) to decentralization, where it is more likely that only one lobby will form, and therefore they will not have any extra gain from it.

10. Discussion and Conclusion

This paper has analyzed the relationship between the level of centralization of policy decision and lobby formation. We have developed a formal framework where we combine the citizen-candidate model with the menu-auction model of lobbying, extended to endogenous lobbying. We have discussed the relationship between our results and existing results in the lobbying literature, and their implications for the analysis of endogenous lobby and policy centralization.

First, in our model there is no equilibrium in pure strategies where lobbying can be counteractive in Leaver and Makris (2000) way, in the sense that only the "friendly" lobby[11] will form in order to counteract the possibility that other groups will organize; in this case no contributions will be offered to the policy maker. However, in our setting, lobbying can be counteractive in the sense of Austen-Smith and Wright (1992), where two lobbies form in order to offset each others influence, in this case positive contributions are offered to the policy maker. This form of lobby competition can occur when preference heterogeneity between groups is high and groups are large, and, in the special case where lobbies have the same "power" (i.e.$n_\omega |\theta_\omega - \theta_P| = n_v |\theta_v - \theta_P|$),there will not effect of lobbying on policy.

Second, a part from the special case described above, lobbying generally induces the policy maker to change her policy; this can happen both when only one lobby exists or when lobbies compete. In the first case the lobby will be the one formed by citizens with preference opposing the policy maker's, and it requires that preferences heterogeneity between groups is not very high. When more than one lobby exists in equilibrium, since the policy choice is an average between lobbies and policy maker preferences, the overall effect will increase the less lobbies are able to counteract each others. Consistently with Felli and Merlo (2002), lobbying may influence policy and move the policy choice toward the center, when the majority group has extreme preferences; this can happen either by the election of a more centrist candidate compare to the candidate with the preference of the majority group, or by the formation of antagonist lobbies. Finally, we have presented the case where an equilibrium not lobby free may be lobby neutral. In other words, adding a lobbying stage to the model, affects the political equilibrium even when no lobby exists in equilibrium, In this situation the policy choice will be less extreme than the optimal

[11] The one by thegroup sharing the same preferences of the policy maker.

choice for the majority group.

In order to compare the political equilibrium under policy decentralization and policy centralization we have proposed three ways in which the extent of lobbying can be measured: i) the number of lobbies, ii) their impact on policy iii) the amount of resources transferred to the policy maker from the lobbyists. We have shown that lobbying is affected by the level of government who decides the policy. However, contrary to the *preference dilution* effect, which defines an inverse relationship between the level of centralization and lobbying; we have shown that the direction of this relationship is actually *ambiguous*, with respect to all three ways of measuring lobbying. We have presented scenarios where lobbying is higher under centralization and scenarios where the opposite is true under all three ways of measuring lobbying.

Other general results are; groups are more likely to form a lobby when the minorities have preference quite dissimilar to majority and when they are large (i.e. uniform distribution of preferences). Both results seem to support the idea that lobbying is higher under centralization where we expect both higher preference heterogeneity and groups of bigger size. However we have also shown that lobbying is less likely to form when citizens are divided into many different groups. We have found that lobbies are less likely to form in small jurisdiction, or in jurisdictions where the majority group is very large compare to minority groups, or when where residents are fragmented in many groups.

In particular, in Scenario 1, we have illustrated an example where the political equilibrium is not affected by lobbying under decentralization but it is under centralization. In this situation lobbying ensures that the policy choice is more centrist that the one would have occurred in the benchmark. The welfare consequences of lobbying are therefore an increase in the overall welfare for the society, if the fix costs of lobby constitution are less than the welfare gain, since the policy choice is closer to the average of residents' ideal points, which is the efficient outcome under the Samuelson rule.[12]

Scenarios 2, and 5 present similar situations. In the first case, there will be one lobby in one jurisdiction formed by the minority group, under decentralization, and two lobbies formed by the extreme group under centralization. In the second case, both the extreme groups lobby under centralization and decentralization. Under decentralization, in the case where the lobby forms, the policy choice will be biased toward the minority, and therefore the majority of residents will

[12] The notion of efficiency used here is the one determined by the Samuelson rule, where the sum of marginal rate of substitution is equal to the marginal rate of transformation, which in our case is:$G^* = \sum \theta^i$,therefore $t^* = \frac{G^*}{N} = \frac{\sum \theta^i}{N}$
(i.e. the optimal policy choice that the economy can achieve is determined by an average of its members preferences.) However under plurality voting, if citizens vote sincerely over the set of candidates, this outcome is not achieved, because the majority group is able to elect its preferred candidate.

be worse off from lobbying. Under centralization, if the lobbies have the same power, the welfare consequences for the society are negative, there will be no influence on policy in either direction but there will be a waist of resources associated with the cost of lobby formation (which in reality can be thought as the costs of renting offices, organizing people, collecting information etc.).[13] Under centralization, if lobbies do not have the same power, the policy choice will move toward the ideal point of the "strongest" lobby, but the welfare consequences are ambiguous: first, we have to take into account the waist of resources associated with lobby constitution, second, we have to consider whether or not the policy choice gets close to the optimal policy choice under the Samuelson rule.

Remark 7. *If lobbying does not have any effect on policy it is always harmful, and limiting the possibilities of lobbying by increasing the cost of entrance will improve welfare.*

In these situations the society will not have any benefits from lobbying and any measure preventing groups to organize will increase the welfare of the society by K amount for each lobby. For instance it is possible to make lobbying more difficult by restricting the regulations.

Moreover, in Scenario 5, where the same extreme lobbies will form under centralization and decentralization and policy makers type M being elected, the consequences on policy and the amount of resources spent by the lobbyists are different. In this situation the lobbyists will have to pay a smaller amount to the policy maker for a bigger policy deviation under decentralization.

Remark 8. *In an equilibrium where lobbyists and policy maker are the same under decentralization and centralization, the minority groups will prefer decentralization and the majority centralization.*

This can be relevant if the choice to centralize or decentralize is endogenous. Consider the proposal of moving from centralization to decentralization by mean of a *referendum*, if votes are aggregated at central level and the number of citizens belonging to the extreme groups is bigger than the majority group, decentralization will pass; it will not otherwise. Suppose that, instead, citizens are asked their opinion about moving from decentralization to centralization, if voted are aggregated at the decentralized level, centralization will passed, because it is preferred by the majority in the two jurisdictions. Note that under the benchmark, where the

[13] Note that the potential waist of resources generated by lobbying and stressed by the rent-seeking literature arises from the cost of forming a lobby rather than by the contributions offered to the policy maker (among others Tullock (1967)) or by the additional incentive to run for an office that citizens have in the citizen-candidate model with lobbying (Besley and Coate (2001)).

possibility of lobbying is not taken into account, citizens are indifferent between centralization and decentralization, if you abstract from economies of scale arguments.

Scenario 3 shows how it is possible that the number of lobbies decrease under centralization, by moving from an equilibrium with overall four lobbies in the first case to an equilibrium with only two lobbies in the second one. This can occur when preferences heterogeneity between groups is already very high under decentralization, such that it is optimal for both groups to form a lobby and try to "bribe" the policy maker with preferences of the majority (the group with M preferences in both jurisdictions). Under centralization, instead, the majority "centrist" group will not lobby, since both extreme groups will lobby and in doing that they already offset each other influence. In this situation the majority group will benefit from centralization since, if the extreme group have the same power, they can achieve their ideal policy choice without lobbying.[14] The overall welfare effect for the society is however ambiguous: on one hand, under decentralization the costs associated with lobby formation are higher because more lobbies will form, on the other hand, compare to the benchmark, the policy choice is the efficient one.[15] However, when more than a lobby exists the policy maker is able to capture most of the surplus,[16] and lobbyists may be worse off, instead.

Remark 9. *Lobbying may be welfare improving for the economy, but when more than one lobby exists most of the benefits go to the policy maker.*

Finally, Scenario 4 shows that, under centralization, a lobby-neutral equilibrium can be not lobby-free. In this situation, similarly to Scenario 1, the policy choice is biased toward the center compared to the benchmark, and the policy choice is closer to the efficient one , remember the costs associated with lobby formation are zero, since no lobby will form in equilibrium. Therefore:

Remark 10. *A lobby-neutral but not lobby-free equilibrium always welfare improving for the society compare to the benchmark.*

There is still a lot to be done to develop a more complicated picture of the effect of policy centralization on lobbying. First, in this model we only have accounted for preference heterogeneity among consumers, a more sophisticated representation of the reality could improve the

[14] Note that if we look at the result of the benchmak, centralization and decentralization are indifferent for the majority group since they can always achive their ideal policy choice.

[15] Given the notion of truthful equilibrium develop by Bernheim and Whinston (1986) the policy maker maximizes jointly her and lobbyists' utilities. It follows immediately that in a world where every individual lobbies the optimal provision of public is achieved and the society on the whole achieves the maximum level of utility feasible.

[16] See DGH for a discussion about that.

analysis: like different factor owners, different income distribution. Second, we assume that citizens vote sincerely over the set of candidate, it would be interested to explore strategic voting and endogenous lobbying.

Finally, centralization and decentralization are here depicted in a very stylized way, it would probably be worth to take into account different concepts of centralization like the existence of different levels of government at the same time.

References

[1] Austen-Smith, D., 1987, Interest Groups, Campaign Contributions, and Probabilistic Voting, *Public Choice*, 54, 123-39.

[2] Austen-Smith, D., J.R. Wright, 1994, Counteractive Lobbying, *American Journal of Political Science*, 38, 25-44.

[3] Bernheim, B.D. and M.D., Whinston, 1986, "Menu auctions, resource allocation, and economic influence", *Quaterly Journal of Economics* 101, 1-131.

[4] Besley,T., T. Coate, 1997, "An economic model of representative democracy", *Quaterly Journal of Economics,* 112, 85-114.

[5] Besley,T., T., Coate, 2001, "Lobbying and welfare in a representative democracy", *Review of Economic Studies,* Vol 68 (1), 67-82.

[6] Besley T., T., Coate, 1999, "Centralized versus Decentralized Provision of Local Public Goods: A Political Economy Analysis", NBER working paper W7084.

[7] de Melo, J., A. Panagariya, D. Rodrik ,1993, "The new regionalism: a country perspective", in J. de Melo and A. Panagariya (eds), *New Dimensions in Regional Integration*, CUP.

[8] Dixit, A., G., Grossman and E. Helpman, 1997, "Common agency and coordination: general theory and application to government policy making", *Journal of Political Economy*, 107, 1-39.

[9] Felli, L. and A., Merlo, 2000, "Endogenous Lobbying", Working Paper, New York University.

[10] Fishman, R., and R., Gatti, 2000, "Decentralization and Corruption: Evidence Across Countries", World Bank Policy Research Working Paper 2290.

[11] Hamilton, A., J. Madison and J. Jay, 1981 [1787], The Federalist papers, 10, Roy P. Fairfield, 2nd edn, Baltimore and London: JHUP.

[12] Leaver, C. and M.,Makris, 2000, "Passive industry interest in a large polity: a large Poisson Game approach to endogenous lobby formation", Working Paper, University of Bristol, CMPO 00/21.

[13] Lockwood, B., 2002, "Distributive Politics and the Benefits of Decentralization, *Review of Economic Studies, 69(2), 1-23.*

[14] Mitra, D., 1999, "Endogenous lobby formation and endogenous protection: a long run model of trade determination", *American Economic Review*, 89, 1116-1134.

[15] Oates, W., 1972, *Fiscal Federalism*, Harcourt-Brace, NY.

[16] Olson, M., 1965, *The logic of collective action*, Harward University Press, Cambridge, MA.

[17] Root, H., 1999, "The importance of Being Small", mimeo.

[18] Treismam, D., 1999, "Decentralization and Corruption: Why are Federal States perceived to be More Corrupt?", UCLA Department of Political Science, mimeo.

[19] Tullock, G., 1967, "The Welfare Costs of Tariffs, Monopolies and Theft, Western" Economic Journal, 5, 224-32.

[20] Wright, J.R., 1996, *Interest Groups and Congress*, Allyin & Bacon, Boston.

11. Appendix

11.1. Derivation of the truthful contributions

Definition 11.1. *Truthful contributions relative to level of utility* u_i *are defined as follows:*

$$\begin{aligned} C_l^T(t, u_i^\circ) &= \max\left[0, H^l(t) - u^\circ\right] \\ &\quad \max\left[0, \sum_{i\in l}(1 - t + \theta^i \ln tn - u_i^\circ)\right] \end{aligned} \tag{11.1}$$

Definition 11.2. *Let* $\left\{C_i^T\right\}_{i\in l}$ *be a truthful equilibrium relative to* u°*, for all* $i\in\ l$*. Then* $(\{u_i{}^\circ\}_{i\in l}, t^\circ)$ *is characterized by:*
a)

$$t^\circ = \arg\max_t H^P\left[t, \left\{C^T\left(t^\circ, u_i{}^\circ\right)\right\}_{i\in l}\right];$$

b) for every $i \in l$*,*

$$H^P\left[t^\circ, \left\{C^T\left(t^\circ, u_i{}^\circ\right)\right\}_{i\in l}\right] = \max_{t\in T} H^p\left[t, \left\{C_j^T\left(t^\circ, u_i{}^\circ\right)\right\}_{j\neq i}, 0\right]$$

The explanation is as follows. Condition a) simply says that the policy maker maximizes her utility, taking into account lobbies' contribution payments; condition b) says truthful contribution are offered such that the policy maker receives the same utility he would have got without lobbies' offer.

Using condition (b) of Definition 2, we calculate the corresponding member of lobby and policy maker welfare levels, for one lobby:

$$1 - t(P, \{\omega\}) + \theta_P \ln t(P, \{\omega\})n + C^T(t(P, \{\omega\})) = 1 - t(P, \{\emptyset\}) + \theta_P \ln t(P, \{\emptyset\})n \tag{11.2}$$

Condition (11.2) means that the policy maker gets exactly the same utility with one or zero lobbies, and the contributions she receives compensate the loss of welfare due to moving the policy choice away from her ideal point. The payment offered by the lobby is:

$$C^T(t(P, \{\omega\}) = \max\left[0, \sum(t(P, \{\omega\}) - t(P, \{\emptyset\}) + \theta_P \ln \frac{t(P, \{\emptyset\})}{t(P, \{\omega\})})\right] \tag{11.3}$$

For two lobbies, the following condition must hold for every lobbyist and policy maker:

$$1 - t(P,\{\omega,v\}) + \theta_P \ln t(P,\{\omega,v\})n + C_\omega^T(t(P,\{\omega,v\})) + C_v^T(t(P,\{\omega,v\}))$$
$$= 1 - t(P,\{v\}) + \theta_P \ln t(P,\{v\})n + C_\omega^T(t(P,\{v\}))$$
$$1 - t(P,\{\omega,v\}) + \theta_P \ln t(P,\{\omega,v\})n + C_\omega^T(t(P,\{\omega,v\})) + C_v^T(t(P,\{\omega,v\}))$$
$$= 1 - t(P,\{\omega\}) + \theta_P \ln t(P,\{\omega\})n + C_v^T(t(P,\{\omega\})) \quad (11.4a)$$

And the truthful contributions are:

$$C_\omega^T(t(P,\{\omega,v\})) = \max \begin{bmatrix} 0, \sum[(t(P,\{\omega,v\}) - t(P,\{v\}) + \theta_P \ln \frac{t(P,\{v\})}{t(P,\{\omega,v\})} + \\ s_v(t(P,\{\omega,v\}) - t(P,\{v\}) + \theta_v \ln \frac{t(P,\{v\})}{t(P,\{\omega,v\})})] \end{bmatrix}$$
$$C_v^T(t(P,\{\omega,v\})) = \max \begin{bmatrix} 0, \sum[(t(P,\{\omega,v\}) - t(P,\{\omega\}) + \theta_P \ln \frac{t(P,\{\omega\})}{t(P,\{\omega,v\})} + \\ s_\omega(t(P,\{\omega,v\}) - t(P,\{\omega\}) + \theta_\omega \ln \frac{t(P,\{\omega\})}{t(P,\{\omega,v\})})] \end{bmatrix} \quad (11.5)$$

In general, contributions offered to the policy maker type θ^p can be expressed by:

$$C_\omega^T(t(P,\Lambda)) = (t(P,\Lambda) - t(P,\Lambda/\{\omega\}) + \theta_P \ln \frac{t(P,\Lambda/\{\omega\})}{t(P,\Lambda)} +$$
$$\sum\nolimits_{l \in \Lambda/\{\omega\}} s_l(t(P,\Lambda) - t(P,\Lambda/\{\omega\}) + \theta_l \ln \frac{t(P,\Lambda/\{\omega\})}{t(P,\Lambda)}) \quad (11.6)$$

where $C_\omega^T(t(P,\Lambda))$ are the contributions paid by the lobby formed by ω types if they form a lobby when groups $l \in \Lambda$ lobby, $t(P,\Lambda)$ is the policy choice with Λ lobbies and $t(P,\Lambda/\{\omega\})$ is the policy choice with $\Lambda/\{\omega\}$ lobbies. These contributions compensate the policy maker for the loss of utility due to i) moving away from her ideal point, and/or ii) the loss of the contributions paid by the other lobbyists.

11.2. Lobbying Stage

11.2.1. *Centralization: three types of citizens*

One lobby and tree types of citizens

Then substituting $C(\omega,P,\Lambda)$ into (2.6), we get the payoff to a type ω if lobbies Λ form:

$$H(\omega,P,\Lambda) = 1 - t(P,\Lambda) + \theta_\omega \ln(t(P,\Lambda)n) - \frac{1}{s_\omega}[C(\omega,P,\Lambda) + K]$$

If lobby ω does not form, a type ω gets

$$H(\omega,P,\Lambda/\{\omega\}) = 1 - t(P,\Lambda/\{\omega\}) + \theta_\omega \ln(t(P,\Lambda/\{\omega\})n)$$

We denote $H_\omega(P,\Lambda)$ the payoffs of citizens type θ_ω, when his lobbying decision is $\lambda_\omega = v$ where v=$\{0,1\}$. The gain to forming a lobby is defined as $\Delta_\omega(P,\Lambda) = H(\omega,P,\Lambda) - H(\omega,P,\Lambda/\{\omega\})$. If no other lobby has formed therefore the set of possible lobbies lobbies is be $\Lambda = \{\emptyset\},\{\omega\}$.

$$\begin{aligned} H_\omega(P,\{\omega\}) &= 1 - t(P,\{\omega\}) + \theta_\omega \ln t(P,\{\omega\})n + \\ &\quad \frac{1}{s_\omega}\left[t(P,\{\emptyset\}) - t(P,\{\omega\}) + \theta^p \ln \frac{t(P,\{\omega\})}{t(P,\{\emptyset\})}\right] - \frac{K}{s_\omega} \qquad (11.7) \\ H_\omega(P,\{\emptyset\}) &= 1 - t(P,\{\emptyset\}) + \theta_\omega \ln t(P,\{\emptyset\})n \qquad (11.8) \end{aligned}$$

So if $\Delta_\omega(P,\{\omega\}) \geqslant 0$ citizens type ω will form a lobby in absence of any other lobbies. After some manipulations

$$\Delta_\omega(P,\{\omega\}) = \frac{s_\omega + 1}{s_\omega} t(P,\{\emptyset\}) - t(P,\{\omega\}) + \frac{\theta_\omega s_\omega + \theta_P}{s_\omega} \ln \frac{t(P,\{\omega\})}{t(P,\{\emptyset\})} - \frac{K}{s_\omega} \qquad (11.9)$$

A subgame perfect equilibrium of the lobbying stage with one lobby requires that for one group the above expression is positive and for the other two groups is negative.

Substituting $t(P,\{\emptyset\}) = \theta_P$ and $t(P,\{\omega\}) = \frac{\theta_\omega s_\omega + \theta_P}{s_\omega + 1}$ if $\omega \neq P$, $t(P,\{\omega\}) = \theta_P$ and $t(P,\{\emptyset\}) = \theta_P$ if $\omega = P$ (11.9) becomes:

$$\begin{aligned} \Delta_{1\equiv}\Delta_\omega(\omega,\{\omega\}) &= -\frac{K}{s_\omega - 1} < 0 \qquad \text{if } \omega = P \qquad (11.10) \\ \Delta_{2\equiv}\Delta_\omega(v,\{\omega\}) &= \theta_P - \theta_\omega + \frac{\theta_P + \theta_\omega s_\omega}{s_\omega} \ln \frac{\theta_P + \theta_\omega s_\omega}{(s_\omega + 1)\theta_P} - \frac{K}{\theta_\omega} \qquad \text{if } \omega \neq P, v = P \qquad (11.11) \end{aligned}$$

Two lobbies and tree types of citizens

If citizens type v form a lobby, citizens type ω lobbying decision becomes:

$$\begin{aligned} H_\omega(P,\{\omega,v\}) &= 1 - t(P,\{\omega,v\}) + \theta_\omega \ln t(P,\{\omega,v\})n + \\ &\quad \frac{1}{s_\omega}\left[t(P,\{v\}) - t(P,\{\omega,v\}) + \theta_P \ln \frac{t(P,\{\omega,v\})}{t(P,\{v\})}\right] + \qquad (11.12) \\ &\quad + \frac{s_v}{s_\omega}\left[t(P,\{v\}) - t(P,\{\omega,v\}) + \theta_v \ln \frac{t(P,\{\omega,v\})}{t(P,\{v\})}\right] - \frac{K}{s_\omega} \\ H_\omega(P,\{v\}) &= 1 - t(P,\{v\}) + \theta_\omega \ln t(P,\{v\})n \qquad (11.13) \end{aligned}$$

As before, we define $\Delta_\omega(P,\{\omega,v\}) \geqslant 0 = H_\omega(P,\{\omega,v\}) - H_\omega(P,\{v\})$ the difference in the payoffs for of the lobby- not lobby decision for a citizens type θ_ω,which, as before, after some manipulations becomes:

$$\begin{aligned}\Delta_\omega(P,\{\omega,v\}) &= \frac{s_\omega+s_v+1}{s_\omega}\left(t(P,\{v\})-t(P,\{\omega,v\})\right)+\\ &\quad+\frac{\theta_v s_v+\theta_\omega s_\omega+\theta_P}{s_\omega}\ln\frac{t(P,\{\omega,v\})}{t(P,\{v\})}-\frac{K}{s_\omega}\end{aligned}$$

A subgame perfect equilibrium of the lobbying stage with two lobbies exists if for two groups the above expression is positive and for the other is negative.

Substituting $t(P,\{v\})=\frac{\theta_v s_v+\theta_P}{s_v+1}$ and $t(P,\{\omega,v\})=\frac{\theta_v s_v+\theta_\omega s_\omega+\theta_P}{s_\omega+s_v+1}$ if $\omega\neq P$ and $v\neq P$, $t(P,\{v\})=\frac{\theta_v s_v+\theta_P}{s_v+1}$ and $t(P,\{\omega,v\})=\frac{\theta_v s_v+\theta_\omega s_\omega}{s_\omega+s_v}$ if $\omega=P$ and $v\neq P$ and $t(P,\{v\})=\theta_P$ and $t(P,\{\omega,v\})=\frac{\theta_v s_v+\theta_\omega s_\omega}{s_\omega+s_v}$ if $\omega\neq P$ and $v=P$,we obtain the following expressions.

If the policy maker is not a citizen type ω, and the existing lobby is formed by citizens v with $v\neq P$,,lobbying decision for type ω citizens is:

$$\begin{aligned}\Delta_{3\equiv}\Delta_\omega(P,\{\omega,v\}) &= -\frac{K}{n_\omega}+\frac{n_v\left(\theta_v-\theta_\omega\right)+\left(\theta_P-\theta_\omega\right)}{(n_v+1)}+\\ &\quad+\frac{n_v\theta_v+n_\omega\theta_\omega+\theta_P}{n_\omega}\ln\frac{(n_v+1)\left(n_v\theta_v+n_\omega\theta_\omega+\theta_P\right)}{(n_v+n_\omega+1)(\theta_P+n_v\theta_v)} \qquad (11.14)\\ \text{for } n_\omega &= s_\omega,\, n_v{=}\, s_v,\ \omega\neq P\text{and } v\neq P\end{aligned}$$

For citizens sharing the same preferences as the policy maker, the decision of whether or not to lobby, if another group does is:

$$\begin{aligned}\Delta_{4\equiv}\Delta_\omega(\omega,\{\omega,v\}) &= -\frac{k}{n_\omega-1}+\frac{n_v\left(\theta_v-\theta_\omega\right)}{(n_v+1)}+\\ &\quad+\frac{s_v\theta_v+(n_\omega-1)\theta_\omega+\theta_P}{n_\omega-1}\ln\frac{(s_v+1)\left(s_v\theta_v+(n_\omega-1)\theta_\omega+\theta_P\right)}{(n_v+n_\omega)(\theta_P+n_v\theta_v)} \qquad (11.15)\\ \text{for } n_\omega &= s_\omega-1,\, n_v{=}\, s_v,\ \omega=P\text{and } v\neq P\end{aligned}$$

If the existing lobby is formed by citizens sharing the same preferences of the policy maker, lobbying decision for citizens type ω is expressed by:

$$\begin{aligned}\Delta_{5\equiv}\Delta_\omega(v,\{\omega,v\}) &= -\frac{K}{n_\omega}+(\theta_v-\theta_\omega)+\frac{s_v\theta_v+n_\omega\theta_\omega}{n_\omega}\ln\frac{s_v\theta_v+n_\omega\theta_\omega}{(n_v+n_\omega)\theta_v}\\ \text{for } n_\omega &= s_\omega,\, n_v{=}\, s_v-1,\ \omega\neq P\text{and } v=P \qquad (11.16)\end{aligned}$$

Three lobbies and tree types of citizens

Similarly, we can proceede to calcolate $\Delta_\omega(P,\{\omega,v,z\})$.

$$\begin{aligned} H_\omega(P,\{\omega,v,z\}) &= 1-t(P,\{\omega,v,z\})+\theta_{_\omega}\ln t(P,\{\omega,v,z\})n+ \\ &\quad \frac{1}{s_\omega}\left[t(P,\{\omega,v\})-t(P,\{\omega,v,z\})+\theta_P\ln\frac{t(P,\{\omega,v,z\})}{t(P,\{\omega,v\})}\right]+ \\ &\quad +\frac{s_v}{s_\omega}\left[t(P,\{\omega,v\})-t(P,\{\omega,v,z\})+\theta_v\ln\frac{t(P,\{\omega,v,z\})}{t(P,\{\omega,v\})}\right]+ \\ &\quad +\frac{s_z}{s_\omega}\left[t(P,\{\omega,v\})-t(P,\{\omega,v,z\})+\theta_z\ln\frac{t(P,\{\omega,v,z\})}{t(P,\{\omega,v\})}\right]-\frac{K}{s_\omega} \quad (11.17) \\ H_\omega(P,\{v,z\}) &= 1-t(P,\{\omega,v\})+\theta_{_\omega}\ln t(P,\{\omega,v\})n \quad (11.18) \end{aligned}$$

As before, we define $\Delta_\omega(P,\{\omega,v,z\})$=$H_\omega(P,\{\omega,v,z\})-H_\omega(P,\{v,z\})$ as the difference in the payoffs for of the lobby- not lobby decision for a citizens type θ^i.The general expression for an equilibrium with two lobbies is the following

$$\begin{aligned} \Delta_\omega(P,\{\omega,v,z\}) &= \frac{s_\omega+s_v+s_z+1}{s_\omega}\left(t(P,\{v,z\})-t(P,\{\omega,v,z\})\right)+ \quad (11.19) \\ &\quad +\frac{\theta_\omega s_\omega+\theta_v s_v+\theta_z s_z+\theta_P}{s_\omega}\ln\frac{t(P,\{\omega,v,z\})}{t(P,\{v,z\})}-\frac{K}{s_\omega} \end{aligned}$$

An subgame perfect equilibrium of the lobbying stage with two lobbies exists if for all groups the above expression.

Substituting $t(P,\{v,z\})=\frac{\theta_v s_v+\theta_z s_z+\theta_P}{s_v+s_z+1}$ and $t(P,\{\omega,v,z\})=\frac{\theta_\omega s_\omega+\theta_v s_v+\theta_z s_z+\theta_P}{n}$ into (11.19) we obtain the lobbying decision by citizens type ω when the other two groups have formed a lobby. The procedure is the same as before so we omit it.

Proof of Proposition 1.

The first part of the Proposition analyses the relationship between preference heterogeneity, calculates as the distance between different groups preference parameter θ.

In what follow we analitically study the different Δ functions which represents, the lobbying non-lobbying decisions faced by each group, once the identity of the policy maker is known, and anticipating other groups' behaviour.

i) and ii) **Preference Heterogeneity**

First Derivatives

$$
\begin{aligned}
\frac{\partial \Delta_1}{\partial \theta_\omega} &= 0 \quad \text{for any } \theta_\omega \ \text{ if } \omega = P \\
\frac{\partial \Delta_2}{\partial \theta_\omega} &= \ln \frac{\theta_P + n_\omega \theta_\omega}{(n_\omega + 1)\,\theta_P} = 0 \qquad \text{if } i \neq P \qquad (11.20) \\
\frac{\partial \Delta_3}{\partial \theta_\omega} &= \ln \frac{(n_v + 1)}{(n_j + n_\omega + 1)} \frac{n_v \theta_v + n_\omega \theta_\omega + \theta^p}{(\theta_P + n_v \theta_v)} \ \text{ if } \omega \neq P \text{ and } \ \text{if } v \neq P \qquad (11.21) \\
\frac{\partial \Delta_4}{\partial \theta_\omega} &= \ln \frac{(n_v + 1)}{(n_v + n_\omega)} \frac{(n_v \theta_v + n_\omega \theta_\omega)}{(\theta_\omega + n_v \theta_v)}, \qquad \text{if} \quad \omega = P; v \neq P \qquad (11.22) \\
\frac{\partial \Delta_5}{\partial \theta_\omega} &= \ln \frac{n_v \theta_v + n_\omega \theta_\omega}{(n_\omega + n_v)\,\theta_v} \qquad \text{if } \ \omega \neq P;\ v = P \qquad (11.23)
\end{aligned}
$$

which gives the following solutions:

$$
\begin{aligned}
\frac{\partial \Delta_1}{\partial \theta^i} &= 0 \qquad \text{for any } \theta \qquad (11.24) \\
\frac{\partial \Delta_2}{\partial \theta^i} &= 0; \qquad \text{for } \theta_\omega = \theta_P \qquad (11.25) \\
\frac{\partial \Delta_3}{\partial \theta^i} &= 0 \qquad \text{for} \quad \text{for } \theta_\omega = \frac{\theta_P + n_v \theta_v}{n_\omega + 1} \qquad (11.26) \\
\frac{\partial \Delta_4}{\partial \theta^i} &= 0 \qquad \text{for } \theta^i = \theta_v \qquad (11.27) \\
\frac{\partial \Delta_5}{\partial \theta^i} &= 0 \qquad \text{for } \theta^i = \theta_P \qquad (11.28)
\end{aligned}
$$

Second Derivatives

$$
\begin{aligned}
\frac{\partial^2 \Delta_1}{\partial \theta^i} &= 0 \qquad \text{if } \omega = P \qquad (11.29) \\
A &\equiv \frac{\partial^2 \Delta_2}{\partial \theta_\omega} = \frac{n_\omega}{\theta_P + n_\omega \theta_\omega} > 0 \qquad \text{if } \omega \neq P \qquad (11.30) \\
B &\equiv \frac{\partial^2 \Delta_3}{\partial \theta_\omega} = \frac{n_\omega}{n_v \theta_v + \theta^p + n_\omega \theta_\omega} \equiv A > 0 \qquad \text{if } \ \omega \neq P \text{ and } \ \text{if } v \neq P \qquad (11.31) \\
C &\equiv \frac{\partial^2 \Delta_4}{\partial \theta_\omega} = \frac{n_\omega - 1}{n_v \theta_v + n_\omega \theta_\omega} \equiv B > 0 \quad \text{if} \quad \omega = P;\ v \neq P \qquad (11.32) \\
D &\equiv \frac{\partial^2 \Delta_5}{\partial \theta_\omega} = \frac{n_\omega}{n_v \theta_v + n_\omega \theta_\omega} \equiv C > 0 \quad \text{if } \ \omega \neq P;\ v = P \qquad (11.33)
\end{aligned}
$$

Conditions (11.11), (11.14), (11.15), (11.16) and (11.19) are monotonically decreasing until they reach their minimum at the value defined in (11.25), (11.26), (11.27) and (11.28), and then they become monotonically increasing. Let us implicitly define $\underline{\theta}(\omega, \Lambda)$ and $\overline{\theta}(\omega, \Lambda)$ the lower and upper values that make (11.11), (11.14), (11.15), (11.16) and (11.19) equal to 0. At these value citizens type ω are indifferent between lobbying and not lobbying. Note that as θ_ω move away from θ_P the functions increases, and therefore the likelihood for lobbying increases.

iii) Group Size

Moreover if we take the derivative of (11.30), (11.32) and (11.33) with respect to group size, we get:

$$\frac{\partial B}{\partial n_\omega} = \frac{n_v\theta_v + \theta_P}{(n_v\theta_v + \theta_P + n_\omega\theta_\omega)^2} > 0 \tag{11.34}$$

$$\frac{\partial B}{\partial n_v} = -\frac{n_\omega}{(n_v\theta_v + \theta_P + n_\omega\theta_\omega)^2} < 0 \tag{11.35}$$

$$\frac{\partial C}{\partial n_\omega} = \frac{1}{\theta_v}\frac{n_v + 1}{(n_v + n_\omega)^2} > 0 \tag{11.36}$$

$$\frac{\partial C}{\partial n_v} = -\frac{1}{\theta_v}\frac{n_v - 1}{(n_v + n_\omega)^2} < 0 \tag{11.37}$$

$$\frac{\partial C}{\partial n_\omega} = \frac{n_v\theta_v}{(n_v\theta_v + n_\omega\theta_\omega)^2} > 0 \tag{11.38}$$

$$\frac{\partial C}{\partial n_v} = -\frac{n_\omega\theta_\omega}{(n_v\theta_v + n_\omega\theta_\omega)^2} < 0 \tag{11.39}$$

This implies that the (11.21), (11.22) and (11.23), become steeper for bigger its own group size and smoother for the other group size. Note that as n_ω increases the minimum value of the above functions increases.

Proof of Lemma 3.

Calculation of a mixed strategy equilibrium when $\Delta_m(m,\{m\}) < 0$, $\Delta_\omega(m,\{\omega\}) \geqslant 0$, $\Delta_\omega(m,\{\omega,m\}) < 0$, and $\Delta_m(m,\{\omega,m\}) \geqslant 0$.

The expected payoff for a type m given that types ω do not lobby is $E\Pi_m(N)$ and types ω do lobby is $E\Pi_m(L)$.

$$\begin{aligned} E\Pi_m(N) &= q * \Pi_m(m,\{\emptyset\}) + (1-q)\Pi_m(m,\{\omega\}) \\ E\Pi_m(L) &= q * \Pi_m(m,\{m\}) + (1-q)\Pi_m(m,\{m,\omega\}) \end{aligned}$$

$$\begin{aligned} q * \Pi_m(m,\{\emptyset\}) - q * \Pi_m(m,\{m\}) &> (1-q)\Pi_m(m,\{m,\omega\}) - (1-q)\Pi_m(m,\{\omega\}) \\ q\frac{K}{s_m} &> (1-q)\Delta_m(m,\{m,\omega\}) \\ q &> \frac{\Delta_m(m,\{m,\omega\})}{\Delta_m(m,\{m,\omega\}) + \frac{K}{s_m}} = q^* \end{aligned}$$

Similarly, the expected payoff for a type ω given that types m do not lobby is $E\Pi_\omega(N)$ and types ω do lobby is $E\Pi_\omega(L)$.

$$\begin{aligned} E\Pi_\omega(N) &= p * \Pi_\omega(m, \emptyset) + (1-p)\Pi_\omega(m, \{m\}) \\ E\Pi_\omega(L) &= p * \Pi_\omega(m, \{\omega\}) + (1-p)\Pi_\omega(m, \{m, \omega\}) \end{aligned}$$

$$\begin{aligned} -p\Delta_\omega(m, \{\omega\}) &> (1-p)\Delta_\omega(m, \{m, \omega\}) \\ p &> \frac{\Pi_\omega(m, \{m, \omega\}) - \Pi_\omega(m, \emptyset)}{\Pi_\omega(m, \{m, \omega\}) - \Pi_\omega(m, \{\omega\})} \equiv p^* \end{aligned}$$

note that $\frac{\Pi_\omega(m,\{m,\omega\})-\Pi_\omega(m,\emptyset)}{\Pi_\omega(m,\{m,\omega\})-\Pi_\omega(m,\{\omega\})} < 1$ since $\Pi_\omega(m, \{\omega\}) - \Pi_\omega(m, \emptyset) < 0$

Mixed strategy for type m is $(q^*, 1-q^*)$,

Mixed strategy for type ω is $(p^*, 1-p^*)$