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2 ORIGINAL PAPER



A measure of interrater absolute agreement for ordinal categorical data

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8 Abstract

9 A measure of interrater absolute agreement for ordinal scales is proposed capital-1 Aquizing on the dispersion index for ordinal variables proposed by Giuseppe Leti. The procedure allows to overcome the limits affecting traditional measures of interrater 11 12 agreement in different fields of application. An unbiased estimator of the proposed measure is introduced and its sampling properties are investigated. In order to 13 14 construct confidence intervals for interrater absolute agreement both asymptotic results and bootstrapping methods are used and their performance is evaluated. 15 Simulated data are employed to demonstrate the accuracy and practical utility of the 16 17 new procedure for assessing agreement. Finally, an application to a real case is provided. 18

19

20 Keywords Ordinal data · Interrater agreement · Resampling

- 22
- 23

24 **1 Introduction**

- 25 Ordinal rating scales are frequently developed in study designs where several raters
- 26 (or judges) evaluate a group of targets. For instance, in language studies new rating
- 27 scales before their routine application are tested out by a group of raters, who assess
- 28 the language proficiency of a corpus of argumentative (written or oral) texts

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29 produced by a group of writers. Similar situations can be found in organizational, 30 educational, biomedical, social, and behavioural research areas, where raters can be 31 counsellors, teachers, clinicians, evaluators, or consumers and targets can be organization members, students, patients, subjects, or objects. When each rater 32 33 evaluates each target, the raters provide comparable categorizations of the targets. 34 The more the raters categorizations coincide, the more the rating scale can be used 35 with confidence without worrying about which raters produced those categorizations. Hence, the main interest here consists in analysing the extent that raters assign 36 37 the same (or very similar) values on the rating scale (interrater absolute agreement), that is to establish to what extent raters evaluations are close to an equality 38 39 relationship (e.g., in the case of only two raters, if the two sets of ratings are 40 represented by x and y the relation of interest is x = y). Measures of interrater 41 absolute agreement, as Cohen's Kappa [and extensions to take into account three or 42 more raters, e.g., von Eye and Mun (2005)] and intraclass correlations (ICC) [(Shrout and Fleiss 1979; McGraw and Wong 1996)] are usually applied when 43 44 dealing with rating performed by ordinal scales. A first problem of these procedures is that they are not originally defined for ordinal scales, and so they have to be 45 46 adapted. For instance, the application of indices based on Cohen's Kappa need to assign numerical values to the ordinal level of the scale; intraclass correlation 47 48 indices are based on ANOVA for repeated measures approach for interval data. 49 Another limitation of the above mentioned measures is that they are affected by the 50 restriction of variance problem [e.g., LeBreton et al. (2003)], that consists in an attenuation of estimates of rating similarity caused by an artefact reduction of the 51 52 between-subjects variance in ratings. For instance, this happens in language studies 53 when the same task is defined for native (L1) and non-native (L2) writers, and the 54 analysis compare rater agreement in the two groups separately. Even in the presence of a very good absolute agreement, Cohen's Kappa coefficient and intraclass 55 56 correlations can take low values, especially for L1 group, because the range of 57 ratings provided by the raters are concentrated on one or two very high levels of the 58 scale (a range restriction that determines a between-target variance restriction). 59 In order to overcome the restriction of variance problem, measure for absolute

60 agreement (or consensus) have been proposed, see (LeBreton and Senter 2008) for a 61 review. The main underlying idea is to measure the within-target variance of ratings 62 (i.e., the between-rater variance) separately for each target, and summarize the 63 results in a final average index (usually normalized in the interval [0, 1]). In this 64 approach, the influence of the low level of the between-target variance is removed 65 by separate analysis of the ratings of each target. One of the most popular index in 66 this group, denoted by r_{WG} , was proposed by James et al. (1984), (1993). Let X be 67 an ordinal categorical variable with K categories (e.g. a Likert scale), the index r_{WG} 68 can be expressed as

$$r_{WG} = 1 - \frac{s_X^2}{\sigma_E^2} \tag{1}$$

70 where s_X^2 is the observed between-rater variance of the ratings and σ_E^2 is the 71 between-rater variance obtained from a theoretical null distribution representing a

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complete lack of agreement among raters. Roughly speaking, the null distribution conceptually represents no agreement, which means that to calculate r_{WG} , one makes a direct comparison between the observed variance in raters' ratings with the variance one would expect if there was no agreement among raters. Higher numbers indicate a greater agreement.

For raters in perfect agreement we have $s_X^2 = 0$, with a corresponding value $r_{WG} = 1$. In applications, r_{WG} values greater than 0.7 (possibly 0.8) are considered associated with high level of interrater absolute agreement [see (LeBreton and Senter 2008), p. 836 Table 2]. Often researchers define the no agreement, or the null distribution, in terms of a uniform distribution. When the null distribution is assumed as uniform, the equation for the corresponding variance is

$$\sigma_E^2 = \frac{K^2 - 1}{12}$$
(2)

84 where K refers to the total number of levels of the scale X.

85 The index r_{WG} and other indices reviewed in LeBreton and Senter (2008) (e.g., standard and average deviation indices) allow to avoid the problem of variance 86 87 restriction, but as traditional measures of interrater agreement they are defined only for interval data. Besides, the accuracy of r_{WG} depends strongly on the specification 88 89 of the null distribution. One disadvantage of r_{WG} is the ambiguity in choosing the reference distribution. Although (James et al. 1984) recommended using the 90 uniform distribution, Lindell and Brandt (1997) recommended using maximum 91 92 dissensus. Burke et al. (1999), however, cautioned against the use of maximum dissensus because its use may lead to the overestimation of interrater agreement. 93 94 Finally, depending on the choice of the null distribution, negative values could be 95 obtained for r_{WG} . For these reasons, in this contribution we propose a new procedure to measure absolute agreement for ordinal rating scales by using the dispersion 96 index proposed by Leti (1983) (pp. 290-297) for ordinal variables. In this way, we 97 98 take into consideration the ordinal level of the measurement scales. The new 99 measure is not affected by restriction of variance problems and does not depend on the choice of a particular null distribution. In this paper we assume a two-way 100 101 random sampling design, where the sampling design involves a sample of raters as 102 well as a sample of targets, all of which are rated by each sampled rater.

The paper is organized as follows. In Sect. 2 the dispersion index proposed by 103 Leti (1983) (pp. 290–297) for ordinal variables is introduced and its sampling 104 105 properties are analyzed in Sect. 3. Such results allow to construct confidence interval without resorting to bootstrap method, as generally happened for inference 106 on measure of interrater absolute agreement, see (Cohen et al. 2001) and reference 107 108 therein. Section 4 contains results of a simulation experiment used to illustrate both 109 the performance of the proposed interrater agreement index and to compare it with 110 the bootstrap method in constructing confidence intervals. Finally, in Sect. 5 an 111 application to real data is performed.

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112 2 Leti index as a measure of interrater absolute agreement113 for ordinal scales

The dispersion of an ordinal categorical variable can be measured by the index proposed in Leti (1983) (pp. 290–297), which is given by

$$D = 2\sum_{k=1}^{K-1} F_k (1 - F_k)$$
(3)

where K is the number of categories of the variable X and F_k is the cumulative 117 proportion associated to category k, for k = 1, ..., K. It is interesting to notice that D 118 119 has properties of within and between dispersion decomposition analogous to the 120 well-known variance decomposition (Grilli and Rampichini 2002). Index (3) is 121 nonnegative and it is easy to prove that D = 0 if and only if all observed categories are equal (absence of dispersion). The maximum value of the index (D_{max}) is 122 obtained when all observations are concentrated in the two extreme categories of the 123 124 variable (maximum dispersion), and it is

$$D_{max} = \frac{K-1}{2} \tag{4}$$

126 as N is even,

$$D_{max} = \frac{K - 1}{2} \left(1 - \frac{1}{N^2} \right)$$
(5)

128 as *N* is odd, *N* being the total number of observations. For *N* moderately large, the 129 maximum of the index can be assumed equal to (K - 1)/2. Hence, it is possible to 130 define a measure of dispersion normalized in the interval [0, 1] given by

$$d = \frac{D}{D_{max}} = \frac{2}{K - 1}D.$$
(6)

- 132 The lower the value of d the higher the raters agreement. Note that, when d = 0
- 133 (maximum agreement between raters) $r_{WG} = 1$. When d = 1

$$r_{WG} = 1 - \frac{(K-1)^2}{4} \frac{1}{\sigma_E^2}$$
(7)

135 and if the uniform distribution is assumed as null distribution, (7) becomes

$$r_{WG} = \frac{4 - 2K}{K + 1} \tag{8}$$

137 taking value lower than zero when K > 2. In accordance with (LeBreton et al.

138 2005) out-of-bounds values ($r_{WG} < 0$ or $r_{WG} > 1$) are generally setted to zero. 139 Unlike r_{WG} , *d* can never be out of the range [0, 1].

Advantages of our proposal respect to measures of absolute agreement like r_{WG} are:

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- *d* takes into consideration the ordinal level of the measurement scales: (i)
 - (ii) d allows to avoid the problem of restriction of variance;
 - (iii) d does not depend by the formulation of a null distribution for normalization:
- the sampling proprieties of d are known, as showed in Sect. 3. (iv)

148 **Remark 1** In order to homogenize the values assumed by d and r_{WG} , the index 149 1 - d can be considered.

Suggestions for interpreting the value of 1 - d appropriately are in Table 1, 150 where a comparison between r_{WG} and d(1-d) is reported. More specifically, 151 datasets with different level of raters agreement have been generated and the indices 152 153 r_{WG} , d and 1 - d have been computed.

As reported in LeBreton et al. (2003) values of r_{WG} greater than 0.7 (possibly 154 0.8) are considered associated with high level of interrater absolute agreement. As 155 shown in Table 1 the same consideration holds for the 1 - d index. 156

157 Finally, in this paper a single item on Likert scale with K categories has been considered. For J items, the index r_{WG} (denoted by $r_{WG(J)}$) can been defined as 158 shown in Cohen et al. (2001). Analogoulsy to $r_{WG(J)}$, extensions to J items for d 159 160 index based on the average of J values d_i , each computed for each single item, can 161 be considered.

3 Sampling properties of *d* index 162

A sample of n_R raters and a sample of n_T targets are drawn by simple random 163 sampling without replacement from a finite population of targets and raters, 164 respectively. Let us denote with X_{ii} the score given by the *j*th rater to the *i*th target 165 on a K-point scale, for $i = 1, ..., n_T$ and $j = 1, ..., n_R$. Formally, X_{ij} s are 166 random variables having K categories categorical independent 167 with $p_k^{(ij)} = P(X_{ij} = k)$, for $i = 1, ..., n_T$, $j = 1, ..., n_R$ and k = 1, ..., K. In the sequel 168 we assume that both the targets and the raters are homogeneus (targets-raters 169 homogeneity assumption), which implies that the probability $p_k^{(ij)} = p_k$ does not 170 depend on rater j or target i, for $i = 1, ..., n_T$, $j = 1, ..., n_R$, k = 1, ..., K. As a 171 consequence of *homogeneity assumptions*, the variables X_{ij} are independent and 172 173 identically distributed (i.i.d.).

| Table 1 Comparison between r_{WG} and $d(1-d)$ | r _{WG} | d | 1 - d |
|---|-----------------|------|-------|
| | 0.07 | 0.81 | 0.19 |
| | 0.34 | 0.61 | 0.39 |
| | 0.49 | 0.53 | 0.47 |
| | 0.74 | 0.32 | 0.68 |
| | 0.83 | 0.14 | 0.86 |
| | - | | |

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| Proof | 144 145 146 147 |
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174 **Remark 2** With regard to the raters homogeneity, variability in scores provided by 175 raters may depend on a number of raters characteristics such as their expertise, 176 familiarity with the assessment process, or amount of training raters received prior 177 to the rating task, etc. Cumming et al. (2002) showed that rating was positively 178 influenced by earlier rating experience and by experience as an EFL/ESL or English 179 L1 teacher. Thompson (1991) indicated that training in linguistics and knowledge of 180 other languages may lead to higher degrees of interrater reliability. Roughly 181 speaking, assuming raters homogeneity means to eliminate the effect of such 182 characteristics on raters score.

183 Evaluations of interrater agreement can be applied to a number of different
184 contexts and are frequently encountered in social, medicine, psychology and
185 education. An application in medicine and in education are illustrated in Examples 1
186 and 2, respectively.

187 **Example 1** Gleason grading is a used grading system for prostatic carcinoma. The Gleason Score is the grading system used to determine the aggressiveness of 188 prostate cancer. This grading system can be used to choose appropriate treatment 189 190 options. The Gleason Score ranges from 1 to 5 and describes how much the cancer from a biopsy looks like healthy tissue (lower score) or abnormal tissue (higher 191 192 score). In Allsbrook et al. (2001) 46 needle biopsies containing prostatic carcinoma were assigned Gleason scores by 10 urologic pathologists. Clearly the urologic 193 pathologists do not necessarily give the same grading for each patient. However, we 194 195 would expect that they tend to agree with each other. The hypothesis that X_{ii} are *i.i.d* comes from the assumption of targets and raters homogeneity. With regard to 196 197 Allsbrook et al. (2001) study: (i) the 10 urologic pathologists are homogeneous since they have the same background knowledge and familiarity with grading 198 199 system; (ii) the 46 patients are homogeneous because affected by the same kind of 200 prostatic carcinoma.

201 **Example 2** A study of agreement among raters in educational research is in Kuiken 202 and Vedder (2014), where raters' judgements of writing performance in L2 and L1 has been analyzed. More specifically, all texts in L2 and L1 were rated by expert 203 raters on both communicative adequacy and linguistic complexity on a six-point 204 205 Likert scale. All raters were experienced L2-teachers and native speakers of the 206 target language. Furthermore, they are homogeneous with respect to the familiarity 207 with the assessment process and the amount of training raters received prior to the 208 rating task.

As previously stressed, the dispersion of an ordinal categorical variable can be measured by the index (3).

211 With regard to *i*th target, let us denote with $\widehat{F}_k^{(i)}$ the empirical cumulative 212 distribution function defined as

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$$\widehat{F}_{k}^{(i)} = \frac{1}{n_{R}} \sum_{j=1}^{n_{R}} I_{(X_{ij} \le k)}$$
(9)

where the numerator represents the number of raters giving score less than or equal to k to the *i*th target. It is known that $E(\widehat{F}_k^{(i)}) = F_k^{(i)} = F_k$, where the last equality comes from the *targets homogeneity assumptions*. Furthermore, $V(\widehat{F}_k^{(i)}) = F_k(1 - F_k)$ and $Cov(\widehat{F}_k^{(i)}, \widehat{F}_l^{(i)}) = min(F_k, F_l) - F_kF_l$. In order to estimate (3), for each target *i* the following estimator can be defined

$$\widehat{D}_{i} = 2 \sum_{k=1}^{K-1} \widehat{F}_{k}^{(i)} (1 - \widehat{F}_{k}^{(i)}).$$
(10)

220 As stressed in Piccarreta (2001), (10) can be alternatively expressed as

$$\widehat{D}_{i} = \sum_{k=1}^{K} \sum_{l=1}^{K} |k - l| \widehat{p}_{k}^{(i)} \widehat{p}_{l}^{(i)}$$

$$= \frac{1}{n_{R}^{2}} \sum_{j=1}^{n_{R}} \sum_{j'=1}^{n_{R}} |X_{ij} - X_{ij'}|$$
(11)

222 where

$$\hat{p}_{k}^{(i)} = \frac{1}{n_{R}} \sum_{j=1}^{n_{R}} I_{(X_{ij}=k)}$$
(12)

224 is an unbiased estimator of p_k .

Proposition 1 The random variable (r, v) $n_R(\hat{p}_1, \ldots, \hat{p}_K)'$, with $\hat{p}_k = \sum_{i=1}^{n_T} \hat{p}_k^{(i)}/n_T$ for $k = 1, \ldots, K$, follows a multinomial distribution with parameters n_R and (p_1, \ldots, p_K) .

The expression (11) allows to compute easily the expectation and the variance of estimator (10) as shown in Proposition 2, see Lomnicki (1952) for details.

230 **Proposition 2** The estimator \hat{D}_i has expectation

$$E(\widehat{D}_i) = \left(1 - \frac{1}{n_R}\right)D\tag{13}$$

232 and variance given by

$$Var(\widehat{D}_i) = \left(\frac{1}{n_R^2} - \frac{1}{n_R^3}\right) (4\sigma^2 + 4(n_R - 2)J - 2(2n_R - 3)D^2) = V$$
(14)

234 where

| $\langle \!\!\! \Delta \!\!\!$ | Sp | rin | gei |
|--------------------------------|----|-----|-----|
| | | | |

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$$\sigma^{2} = Var(X_{ij}) = \sum_{k=1}^{K} k^{2} p_{k} - \left(\sum_{k=1}^{K} k p_{k}\right)^{2}$$
(15)

236

$$J = \sum_{k=1}^{K} \sum_{h=1}^{K} \sum_{l=1}^{K} |k-h| |k-l| p_k p_h p_l.$$
(16)

237

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238 *Proof* Both (13) and (14) come from the results in Lomnicki (1952). With regard to239 (13), we have

$$E(\widehat{D}_{i}) = E\left(\frac{1}{n_{R}^{2}}\sum_{j=1}^{n_{R}}\sum_{j'=1}^{n_{R}}|X_{ij} - X_{ij'}|\right)$$

$$= \frac{n_{R}(n_{R}-1)}{n_{R}^{2}}E\left(\frac{1}{n_{R}(n_{R}-1)}\sum_{j=1}^{n_{R}}\sum_{j'=1}^{n_{R}}|X_{ij} - X_{ij'}|\right)$$

$$= \frac{n_{R}(n_{R}-1)}{n_{R}^{2}}2\sum_{k=1}^{K-1}F_{k}(1 - F_{k})$$

$$= \left(\frac{n_{R}-1}{n_{R}}\right)D.$$
(17)

241 For the variance (14) we obtain

$$Var(\widehat{D}_{i}) = Var\left(\frac{1}{n_{R}^{2}}\sum_{j=1}^{n_{R}}\sum_{j'=1}^{n_{R}}|X_{ij} - X_{ij'}|\right)$$

$$= \left(\frac{n_{R}-1}{n_{R}}\right)^{2} Var\left(\frac{1}{n_{R}(n_{R}-1)}\sum_{j=1}^{n_{R}}\sum_{j'=1}^{n_{R}}|X_{ij} - X_{ij'}|\right)$$

$$= \left(\frac{n_{R}-1}{n_{R}}\right)^{2} \frac{1}{n_{R}(n_{R}-1)} (4\sigma^{2} + 4(n_{R}-2)J - 2(2n_{R}-3)D^{2})$$
(18)

$$= \left(\frac{1}{n_R^2} - \frac{1}{n_R^3}\right) (4\sigma^2 + 4(n_R - 2)J - 2(2n_R - 3)D^2).$$

243

244 **Remark 3** For n_R sufficiently large, we have

$$Var(\widehat{D}_i) \approx \frac{4(J-D^2)}{n_R}.$$
 (19)

- As an estimator of d index (6) we consider
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$$\widehat{d} = \frac{\widehat{\overline{D}}}{D_{max}} = \frac{1}{D_{max}} \left(\frac{1}{n_T} \sum_{i=1}^{n_T} \widehat{D}_i \right).$$
(20)

where $\widehat{\overline{D}}$ is an estimator of *D* obtained averaging the n_T estimates $\widehat{D}_1, \ldots, \widehat{D}_{n_T}$. In Proposition 3 both the sampling properties and the asymptotic distribution of \widehat{d} are analyzed for large n_T (*e.g.*, $n_T > 30$) and moderate n_R (*e.g.*, $n_R = 7 - 10$).

251 **Proposition 3** The estimator \hat{d} has expectation

$$E(\hat{d}) = \left(\frac{n_R - 1}{n_R}\right)d\tag{21}$$

253 and variance

$$V_d = \left(\frac{1}{D_{max}}\right)^2 \frac{V}{n_T} \tag{22}$$

- 255 where V is given in (14). Furthermore, since $\hat{D}_1, \ldots, \hat{D}_{n_T}$ are i.i.d., for the central
- 256 limit theorem, as n_T goes to infinity the random variable \hat{d} tends to a standard
- 257 normal distribution with mean and variance given by (21) and (22), respectively.

Remark 4 If the homogeneity assumption is violated then the X_{ij} random variables are independent but not identically distributed. The main result in this area is the Liaponouv's Theorem, (see Billingsley 1995). The theorem strengthens the requirement of finite variance requiring that the X_{ij} have finite moments of order $(2 + \delta)$, for some $\delta > 0$. Clearly, the convergence to normal distribution could be slower.

In Proposition 4 an unbiased estimator of d is proposed and its asymptotic distribution is evaluated.

266 Proposition 4 From (21), an unbiased estimator of d can be defined as follows

$$\widehat{d^*} = \frac{n_R}{n_R - 1} \widehat{d}.$$
(23)

As a consequence of Proposition(3), the distribution of \hat{d}^* is approximately normal with mean d and variance

$$V_{d^*} = \left(\frac{n_R}{n_R - 1}\right)^2 \left(\frac{1}{D_{max}}\right)^2 \frac{V}{n_T}.$$
(24)

270

The proof of Proposition 4 follows from Proposition 3. The above results are useful to construct point and interval estimates of *d*. They are also useful for testing both the statistical significance of the index (that is the null hypotheses $H_0: d = 0$) and null hypothesis such as $H_0: d \le d_0$, where d_0 be a real number in [0, 1]. Consider the hypothesis problem

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$$\begin{array}{ll}
\left(\begin{array}{ccc}
H_0: & d \le d_0 \\
A_1: & d > d_0
\end{array}$$
(25)

As a consequence of Proposition 4, a test with an asymptotic significance level α consists in accepting H_0 whenever

$$\widehat{d}^* \le d_0 + z_\alpha \sqrt{\widehat{V}_{d^*}} \tag{26}$$

where z_{α} is the α -th quantile of the standard normal distribution and \hat{V}_{d^*} is an estimate of variance (24).

282 The performance \hat{d}^* has been evaluated in Sect. 4 by a simulation study and it 283 has been compared with the bootstrap method. With regard to the size of d, the 284 judgment depends on the application context. Researchers should gain experience 285 using the proposed index to understand which values might be expected to be 286 obtained for d in various situations and how to interpret these values. For instance, 287 one of the main questions in multilevel data analysis is whether it is appropriate to 288 aggregate data and to use the aggregated measures to make inferences about higher 289 level units. A necessary precondition for aggregation is that there is an agreement 290 among the individuals who form the group with regard to the aggregated construct. 291 In this context, the problem is to evaluate if the degree of agreement justifies data 292 aggregation. From this perspective, the hypothesis test (25) assumes a fundamental 293 importance.

294 **Remark 5** If the index 1 - d introduced in Remark 1 is considered, as a 295 consequence of Proposition 4, the distribution of $1 - \hat{d}^*$ is approximately normal 296 with mean 1 - d and variance given by (24).

297 4 Simulation study

In this section a simulation study has been performed. The aim is: (i) to evaluate the performance of \hat{d}^* ; (ii) to compare the normal approximation for the distribution of \hat{d}^* with the bootstrap method. Such a method is generally used in constructing confidence intervals of interrater agreement measures but its use is recommended when n_R is sufficiently large (e.g., $n_R > 20$), see Cohen et al. (2001). Alternative methods based on bootstrap to construct confidence intervals are compared in the simulation.

We focus on confidence intervals for the index d because confidence intervals indicate the range within which the population parameter d (the intervater agreement in the population) is likely to fall, as well as precision of this estimate (i.e., the size of the range).

A finite population of size $N_T = 150$ targets and $N_R = 28$ raters was generated from a multinomial model with parameters $N_R = 28$ and probabilities $(p_1, p_2, p_3, p_4, p_5) = (0.1, 0.2, 0.35, 0.25, 0.1)$. Then, the finite population consists in a matrix *P* of size $N_T \times N_R$. The value of *d* index (6) is 0.61.

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From the population, S = 1000 samples were drawn according to a simple 313 314 random sampling without replacement on the basis of the following two-step procedure. First of all, a simple random sample of size $n_R = 7$ from the $N_R = 28$ 315 316 raters has been selected. This is equivalent to select a simple random sampling 317 without replacement of columns in the finite population matrix P, the result is a 318 matrix P_R of size $N_T \times n_R$. Secondly, a simple random sampling of size $n_T = 50$ 319 from $N_T = 150$ targets has been drawn. This means to draw a simple random 320 sampling of $n_T = 50$ rows from P_R .

In order to construct confidence intervals for the index *d*, both the asymptotic result in Proposition 4 and bootstrapping procedures are used. The bootstrap methods are described in points (2)–(4) below, where we assume that B = 1000bootstrap samples are drawn from each initial sample *s*. Formally, confidence intervals for *d* of level $1 - \alpha = 0.95$ have been constructed using the following methods:

(1) Normal approximation For the initial sample s (for s = 1, ..., S), the confidence interval $[L^s_{Norm}, U^s_{Norm}]$ based on the asymptotic normal approximation is given by

$$L_{Norm}^{s} = \hat{d}^{*} - z_{1-\alpha/2} \sqrt{\hat{V}_{d^{*}}}; \quad U_{Norm}^{s} = \hat{d}^{*} - z_{\alpha/2} \sqrt{\hat{V}_{d^{*}}}$$
(27)

- 332 where \hat{d}^* and \hat{V}_{d^*} are the estimates of d and V_{d^*} , respectively.
- 333 (2) *Percentile method* For the initial sample *s* (for s = 1, ..., S), the confidence 334 interval $[L_{Perc}^s, U_{Perc}^s]$ is obtained by taking $\alpha/2$ and $1 - \alpha/2$ quantiles of the *B* 335 bootstrap samples. Formally

$$L_{Perc}^{s} = \mathcal{Q}_{\alpha/2}; \quad U_{Perc}^{s} = \mathcal{Q}_{1-\alpha/2}$$
(28)

338

339 (3) Bootstrap-t interval For the initial sample s (for s = 1, ..., S), the confidence 340 interval $[L_T^s, U_T^s]$ is computed as follows

$$L_{T-int}^{s} = \widehat{d^{*}} - t_{1-\alpha/2}\sqrt{\widehat{V}_{d^{*}}}; \quad U_{T-int}^{s} = \widehat{d^{*}} - t_{\alpha/2}\sqrt{\widehat{V}_{d^{*}}}$$
(29)

342 where t_{α} is the α th percentile of the distribution of z_b^* (for b = 1, ..., B) with

$$z_b^* = \frac{\widehat{d_b^*} - \widehat{d^*}}{\widehat{se}_b^*}.$$
(30)

345 In (30) \hat{d}_b^* is the estimate of d^* based on the *b*th bootstrap sample and \hat{se}_b^* is 347 the standard error based on the data in the *b*th bootstrap sample.

348 (4) *Pivotal method* For the initial sample *s* (for s = 1, ..., S), the confidence 349 interval $[L_{Pivot}^s, U_{Pivot}^s]$ is computed as follows

$$L_{Pivot}^{s} = 2\widehat{d^*} - Q_{1-\alpha/2}; \quad U_{Pivot}^{s} = 2\widehat{d^*} - Q_{\alpha/2}$$
(31)

352 where $Q_{\alpha/2}$ and $Q_{1-\alpha/2}$ are the $\alpha/2$ and $1 - \alpha/2$ quantiles of the *B* bootstrap 353 estimates \widehat{d}_{b}^{*} , for b = 1, ..., B.

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354 As far as the methods described in steps (2)-(4) are concerned, from each of the 355 S = 1000 initial samples, the B = 1000 bootstrap samples were selected according 356 to the following methods:

357 1 Nonparametric bootstrap From each initial sample s, the bth bootstrap sample is 358 selected as follows: (i) a simple random sample with replacement of r = 7 raters 359 has been selected from the original sample of raters; (ii) a simple random sampling with replacement of n = 50 writers has been drawn from the original 360 sample of writers. Then, bootstrap is applied to the raters sample as well as the 362 targets sample in order to take into account the variability in \hat{d}^* due to the two-363 way random sampling design (where the sampling design involves a sample of raters and a sample of targets). Clearly, when the sampling design involves only 364 365 the raters the proposed methodology resembles that used in literature, see Cohen 366 et al. (2001) and reference therein.

Parametric bootstrap From each initial sample s, the bth bootstrap sample is 367 2 generated according the multinomial model specified in Proposition 1. 368

- 369 3 Pseudo-Nonparametric bootstrap The nonparametric bootstrap described in 370 point (1), is based on the assumption that the data are *i.i.d.*, see Efron (1979). 371 Since survey data are not necessarily *i.i.d.*, many bootstrap resampling methods 372 have been proposed in the context of survey sampling. These methods are 373 obtained after making some modifications to the classical *i.i.d.* bootstrap in 374 order to adapt it for survey data. For a review of bootstrap methods in the 375 context of survey data, see Mashreghi et al. (2016). The class of pseudopopulation bootstrap methods consists in creating a pseudo-population by 376 377 repeating the units of the initial sample and drawing from such a pseudo-378 population bootstrap samples with the same design as the initial one. In order to 379 illustrate how a pseudo-population is constructed, let us assume that a simple 380 random sample without replacement has been selected from a finite population of size N. A pseudo-population of size N can be created by repeating the 381 selected sample, N/n times. This method, was first introduced by Gross (1980). 382 383 In practice N/n is rarely an integer, in this case a method to build a pseudo-384 population of size N was proposed by Booth et al. (1994). In this method, a 385 pseudo-population is first constructed by replicating k = |N/n| times each unit 386 of the original sample s. Then, the pseudo-population is completed by taking a simple random sample of size N - nk without replacement from s. Taking into 387 388 account the two-way sampling design of both targets and raters, the pseudopopulation has been generated according the following two step procedure: 389
- 390

| 391 | Step 1 | the ratings of $N_R = 28$ raters have been reconstruted replicating the |
|-----|--------|--|
| 392 | | columns of the original sample s, $k_R = N_R/n_R = 28/7 = 4$ times. As a |
| 393 | | consequence, this first step generates a sample s_R of size $n_T = 50$ and |
| 394 | | $n_R = N_R = 28;$ |
| | | |

395 Step 2 the points of $N_T = 150$ targets have been reconstruted replicating the rows 396 of the sample s_R obtained in Step 1, $k_T = N_T/n_T = 150/50 = 3$ time.

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397 The accuracy of confidence intervals has been evaluated by the following indicators.

398 (1) Estimated coverage probability, in per cent, for the interval

$$ECP = \frac{100}{S} \sum_{s=1}^{S} I(L_t^s \le d \le U_t^s).$$
(32)

(2) Estimated left-tail and right-tail errors (lower and upper error rates) in per cent

$$LE = \frac{100}{S} \sum_{s=1}^{S} I(L_t^s > d),$$
(33)

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$$RE = \frac{100}{S} \sum_{s=1}^{S} I(U_t^s < d).$$
(34)

409 (3) Estimated average length (AL) of all 1000 simulated intervals given by

$$AL = \sum_{s=1}^{S} \frac{U_t^s - L_t^s}{S}$$
(35)

412

413 where I(a) = 1 if a is true and I(a) = 0 elsewhere, and t = Norm, 414 T - int, Perc, Pivot.

415 **4.1 Simulation results**

416 Tables 2 presents the outcomes achieved in the simulation study. More specifically, 417 the estimated coverage probabilities of 95% confidence intervals (CP), the estimated 418 left-tail (LE) and right-tail (RE) errors (nominal values is 2.5% for both) and the 419 average length (AL) for the index *d*, when ($n_R = 7, n_T = 50$), are reported. The 420 *d* value is equal to 0.61.

421 As reported in Table 2, the confidence intervals obtained with the normal 422 approximation perform very well. Coverage probabilities are larger than 95%423 nominal value (99.4%) with an average length of 0.16. Furthermore, the normal 424 confidence intervals construction is simple, as it does not require resampling from 425 the initial sample. Figure 1 shows the kernel density of the *d* index estimated from 426 the 1000 original samples. The bandwidth selection rule is as proposed by Sheather 427 and Jones (1991).

The percentile method has a good performance with coverage probability larger than 91%. The worst methods are the *Pivot* and *T*-int methods. The lower and upper error rates, giving us an idea of how skewed the distribution of the *d* estimator is, are not well balanced. With regard to the methods used to generate the bootstrat samples, the *parametric* approach performance is strictly related to the estimation of the multinomial probabilities. As previously stressed, each row in the initial sample *s* provides an estimate of $(p_1, p_2, p_3, p_4, p_5)$ and the mean of such estimates defines the

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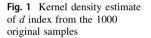
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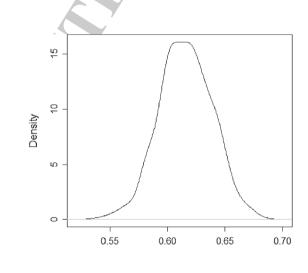
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| | | $n_R = 7$ | | |
|--------|------------|---------------|------------|----------------------|
| Method | Indicators | Nonparametric | Parametric | Pseudo-Nonparametric |
| Normal | СР | 99.4 | 99.4 | 99.4 |
| | LE | 0.6 | 0.6 | 0.6 |
| | RE | 0 | 0 | 0 |
| | AL | 0.16 | 0.16 | 0.16 |
| T-int | CP | 26.2 | 72.4 | 28.8 |
| | LE | 73.8 | 26.2 | 71.2 |
| | RE | 0 | 1.4 | 0 |
| | AL | 0.18 | 0.08 | 0.15 |
| Perc | СР | 92.8 | 91.2 | 92.8 |
| | LE | 0 | 8.8 | 0 |
| | RE | 7.2 | 0 | 7.2 |
| | AL | 0.23 | 0.10 | 0.18 |
| Pivot | СР | 27 | 79.2 | 30 |
| | LE | 73 | 19.6 | 70 |
| | RE | 0 | 1.2 | 0 |
| | AL | 0.23 | 0.10 | 0.18 |

Table 2 Performance of different confidence intervals for d when $n_R = 7$, d = 0.61





estimated probabilities $(\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4, \hat{p}_5)$ of the multinomial distribution used to generate the bootstrap samples as specified in Proposition 1. In Table 3, the minimum, the maximum, the mean and the standard deviation of the distribution of \hat{p}_k (for k = 1, 2, 3, 4, 5) estimated from the original 1000 samples are reported.

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| Table 3 Descriptive statistics of \hat{p}_k distribution, for $k =$ | Parameter | True value | Min | Max | Mean | Sd |
|--|-----------|------------|------|------|------|------|
| 1, 2, 3, 4, 5 and $d = 0.61$ | p_1 | 0.10 | 0.05 | 0.15 | 0.10 | 0.01 |
| | p_2 | 0.20 | 0.14 | 0.25 | 0.19 | 0.02 |
| | p_3 | 0.35 | 0.29 | 0.41 | 0.35 | 0.02 |
| | p_4 | 0.25 | 0.20 | 0.33 | 0.26 | 0.02 |
| | p_5 | 0.10 | 0.06 | 0.16 | 0.10 | 0.02 |

439 As Table 2 shows, the *pseudo-nonparametric* approach taking into account the 440 sample selection effects has a slightly better performance than the *nonparametric* 441 approach both in terms of coverage probabilities and average lengths for all methods 442 (T - int, Perc, Pivot).

443 Finally, note that in the *nonparametric* approach the resampling with replace-444 ment from $n_R = 7$ raters generates a replication of columns of the bootstrap sample introducing a false agreement between raters and as a consequence an underesti-445 446 mation of d. This fact is showed in Table 4 where the mean of the d estimates over 447 both the 1000 original samples s and over the bootstrap replications b are reported.

448 Such means have been computed both for the original population with d = 0.61449 and for a population with d = 0.41, showing as the magnitude of bias depends also on the original agreement degree between raters. That is, the higher the raters 450 451 agreement (low values of d), the smaller the bias in the d estimator introduced by the resampling with replacement. Clearly, such a bias is also present in the *pseudo*-452 453 *nonparametric* approach but with a smaller magnitude, thank to the construction of 454 the pseudo-population that mitigates such a phenomenon. As Table 4 shows, the parametric approach produces null bias estimates. 455

456 The simulation in Table 2 has been repeated for a population with d = 0.41. The 457 results are reported in Table 5.

458 In conclusion, the most competitive method in terms of performance and 459 computational time seem to be the normal. Finally, among the alternative methods based on bootstrap the percentile method in the *parametric* approach seems to 460 461 perform better.

| Approach | Mean of \hat{d}^* (d=0.61) | Mean of \hat{d}^* (d=0.41) |
|------------------------------------|------------------------------|------------------------------|
| Nonparametric | 0.53 | 0.36 |
| Parametric Pseudo-nonparametric | 0.61 0.55 | 0.41 0.37 |

Table 4 The mean of d over the initial samples s and over the bootstrap replications b

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| | | $n_R = 7$ | | |
|--------|------------|---------------|------------|----------------------|
| Method | Indicators | Nonparametric | Parametric | Pseudo-nonparametric |
| Normal | СР | 98.2 | 98.2 | 98.2 |
| | LE | 1.8 | 1.8 | 1.8 |
| | RE | 0 | 0 | 0 |
| | AL | 0.13 | 0.13 | 0.13 |
| T-int | CP | 60.2 | 83.2 | 61.2 |
| | LE | 39.8 | 14.8 | 38.8 |
| | RE | 0 | 2 | 0 |
| | AL | 0.18 | 0.10 | 0.14 |
| Perc | СР | 93.2 | 93.8 | 93.2 |
| | LE | 0 | 5.8 | 0 |
| | RE | 6.8 | 0.4 | 6.3 |
| | AL | 0.19 | 0.10 | 0.15 |
| Pivot | СР | 64.8 | 84.6 | 65.4 |
| | LE | 35.2 | 12.6 | 34.6 |
| | RE | 0 | 2.8 | 0 |
| | AL | 0.19 | 0.10 | 0.15 |

Table 5 Performance of different confidence intervals for d when $n_R = 7$, d = 0.41

462 5 An application on real data: the assessment of language 463 proficiency

The aim of this section is to apply the methodology illustrated in the previous 464 465 sections on an empirical data set, we have analysed ratings obtained in a research conducted at Roma Tre University [see (Nuzzo and Bove 2020), for a detailed 466 467 description]. The main aim of the study was to investigate the applicability of a six-468 point Likert scale for functional adequacy (an aspect of language proficiency) developed by Kuiken and Vedder (2017) to texts produced by native and non-native 469 470 writers, and to different task types (narrative, instruction, and decision-making 471 tasks). The scale comprises four subscales, corresponding to the four dimensions of functional adequacy identified by the authors of the scale: content, task require-472 473 ments, comprehensibility, coherence and cohesion [the reader is referred to Kuiken 474 and Vedder (2017) for a detailed presentation of scales and descriptors]. 20 native 475 speakers of Italian (L1) and 20 non-native speakers of Italian (L2) participated in 476 the study as writers. All the texts produced by L1 and L2 writers (120 texts in total for the three tasks) were assessed by 7 native speakers of Italian on the Kuiken and 477 478 Vedder six-point Likert scale. The raters did not have any specific experience in 479 judging written texts, and can therefore be categorized as being non-expert. For our 480 purposes, we have selected ratings concerning only the narrative task and the 481 subscale comprehensibility. Just to give a general idea of the subscale, definitions of 482 levels 1 and 6 are reported in the following:

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| 483 | Level 1: | The te | xt is 1 | not a | t all con | npre | hens | ible. Ide | eas | and purposes | are | uncle | arly |
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| 484 | | stated | and | the | efforts | of | the | reader | to | understand | the | text | are |
| 485 | | ineffec | ctive. | | | | | | | | | | |

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The text is very easily comprehensible and highly readable. The ideas Level 6: and the purpose are clearly stated.

The results of the interrater agreement analysis for the subscale are summarized in 488 Table 6, where the intraclass correlation ICC(A, 1), as defined in McGraw and 489 Wong (1996), and the average values of r_{WG} , as defined in LeBreton and Senter 490 (2008), the coefficient of variation CV, \hat{d} and \hat{d}^* are shown for L1, L2 and total 491 groups. The intraclass correlation ICC(A, 1) provides a low-moderate level of 492 493 agreement for the total group (0.67). The results for the average values of CV (12.16%) seems in accord with *ICC*(A, 1), while the average value of $r_{WG} = 0.87$, 494 $\hat{d} = 0.22 (1 - \hat{d} = 0.78)$ and $\hat{d}^* = 0.25 (1 - \hat{d}^* = 0.75)$ highlight a higher level of 495 agreement. As it was observed in Bove et al. (2018), when the analysis focuses 496 separately on the two subgroups of L1 and L2 students, results regarding the L1 497 group deserve particular attention. Interrater agreement measured by intraclass 498 499 correlation is very low in the L1 group (ICC(A, 1) = 0.14). Analysing the 500 dispersion of the ratings given to this subgroup, it comes out that most of the 501 raters used almost exclusively levels 5 and 6 of the scale. Such a range restriction 502 caused the very low value of the intraclass correlation, despite the substantial 503 agreement among the raters that scored all the L1 texts in the same high levels. This problem does not regard the results for the other indices of Table 6: $r_{WG} = 0.90$; 504 CV = 8.12%; $\hat{d} = 0.17$ $(1 - \hat{d} = 0.83)$; $\hat{d}^* = 0.19$ $(1 - \hat{d}^* = 0.81)$. that show a 505 very good level of absolute agreement. Finally, the standard deviation of \hat{d}^* 506 computed on the basis of formula (24) is equal to 0.05. As a consequence, the 507 508 $(1 - \alpha) = 0.95$ confidence interval using the normal approximation for the total group is [0.15, 0.35] and the error is at most 0.10. 509

510 6 Conclusions

In this paper a measure of interrater absolute agreement for ordinal scales is 511 proposed. Such a measure is not affected by restriction of variance problems and 512 does not depend on the choice of a particular null distribution. An unbiased 513 estimator of the proposed measure is introduced and its sampling properties are 514 515 investigated. In the simulation study confidence intervals for the proposed interrater 516 agreement index are constructed using the normal approximation, the parametric

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| Table 6 <i>ICC</i> (<i>A</i> , 1) and average of r_{WG} , <i>CV</i> , \hat{d} and \hat{d}^* for the | Group | Ν | ICC(A, 1) | r _{WG} | CV% | â | \widehat{d}^* |
|---|-------|----|-----------|-----------------|-------|------|-----------------|
| comprehensibility subscale in the L1, L2 and the total groups | L1 | 20 | 0.14 | 0.90 | 8.12 | 0.17 | 0.19 |
| the D1, D2 and the total groups | L2 | 20 | 0.63 | 0.84 | 16.20 | 0.28 | 0.32 |
| | Total | 40 | 0.67 | 0.87 | 12.16 | 0.22 | 0.25 |

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 \square CP 517 and nonparametric bootstrap. Furthermore, a pseudo-nonparametric bootstrap taking 518 into account the sampling design is also implemented. As previously stressed, the 519 resampling involves both raters and targets sample. Confidence intervals obtained 520 with the normal approximation seem to perform very well both in terms of coverage 521 probability and computational cost.

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