Consumption Patterns Over Pay Periods

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CONSUMPTION PATTERNS OVER PAY PERIODS

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This paper establishes a theoretical framework to characterise the optimal behaviour of individuals who receive income periodically but make consumption decisions on a more frequent basis. The model incorporates price uncertainty and imperfect credit markets. The simulated numerical solution to this model shows that weekly consumption functions are ordered such that the functions within the payment period are highest in the first and the last week of the payment cycle for all wealth levels. Using weekly expenditure data from the FES, we estimate the coefficient of relative risk aversion (point estimates are between 1.2 and 7) and the extent of measurement error in the data (which accounts for approximately 60% of the variance in the data).

JEL Classification: D11; D12; D91.

Key Words: Consumption; liquidity constraints; uncertainty; credit cards

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1. INTRODUCTION

The purpose of life-cycle models of consumption is to explain how individuals allocate consumption optimally to different periods of their lifetime given available information about lifetime resources, future uncertainty and the nature of financial markets (see Zeldes (1989a), Deaton (1991), Attanasio et al. (1999), Gourinchas and Parker (2002)). The evidence shows that attitude to risk, the rate of time preference, demographic factors and labour supply are all important determinants of long-run consumption behaviour (see Banks et al. (2001) and Gourinchas and Parker (2002)). The focus of the existing literature is long run consumption behaviour over the life cycle and it is accepted that the frequency of income receipt should be irrelevant for the pattern of expenditure within a payment period (Browning and Collado, 2001). This paper challenges that assertion and argues further that examining such short-run behaviour can help us to understand behaviour in the long-run.

Specifically, in this paper we consider individuals who receive a regular income (known) on a monthly basis while their consumption decisions take place more frequently, say weekly. In the short run income is certain, and uncertainty is assumed to arise from short run variation in prices between weeks. Of course short run randomness in prices faced by individuals may reflect different phenomena, like changes in the local availability of some or all goods, or variation in the opportunity cost of some goods etc. In this paper we assume that uncertainty is gradually "resolved" during a monthly payment period (each week) and we are interested in the optimal consumption decision as that uncertainty is revealed[2]. We expect that the optimal consumption level will not only depend on the level of disposable wealth but also on the specific point in the payment cycle at which the consumption occurs, because of this gradual resolution of uncertainty. In such a world, along a sample path, the individual is relatively wealthier at the time of receipt of her monthly income but faces a somewhat more uncertain future, i.e. behaviour must take into account that there are three more weeks without payment. The last week before payment is due, the individual is relatively poorer but uncertainty may not be so important since the next week brings new disposable wealth. Since in general we may expect individual preferences to exhibit some precautionary motive, consumption behaviour will respond to the changing level of uncertainty.

In the short run the market(s) for unsecured (or even secured) borrowing of a few weeks' duration is/are very thin. This limits the amount of borrowing possible and drives a wedge between the return on very short-term saving and the cost of short-term borrowing on credit cards or bank overdrafts. Thus, this context raises the same fundamental question as life-cycle consumption theory: how is consumption allocated across time periods when both uncertainty and imperfect capital markets prevail. Examining optimal behaviour in this short-run framework will inform us about behaviour under similar circumstances in the long run. Moreover, the short-run nature of the problem allows us to abstract from the influences of demographics and labour supply since we can argue that in the short run these characteristics are essentially fixed.

Our methodology builds on the modelling of long run consumption and follows the work of Zeldes (1989a), Deaton (1991), and draws on the empirical methodology used in modelling inventories and commodity prices in Deaton and Laroque (1992, 1995, 1996) and Chambers and Bailey (1996). Our model of consumption incorporates the features of the short-run environment i.e. periodic receipt of income, imperfect capital markets and uncertainty with respect to consumption. We characterise the optimal solution to the model in terms of first order conditions and then prove the existence of a unique stationary solution. For our empirical work we parameterise the model assuming a felicity function with Constant Relative Risk Aversion. However, since it is not possible to obtain the closed form solutions of the consumption function with this functional form, we use numerical methods to solve the model as in Deaton (1991) and Deaton and Laroque (1995), (for related issues see Judd (1998)). We extend beyond the current consumption literature and use the solution of the structural model to estimate the parameters using the Pseudo Maximum Likelihood Estimator (PMLE) of Gourieroux et al. (1984).

The UK Family Expenditure Survey (FES) is our data source. Each sampled individual in the dataset records item level expenditure in a diary for two consecutive weeks, and provides information on the level and frequency of receipt of regular labour income. For all individuals who are paid monthly we determine the point in their monthly payment cycle at which they are observed and the level of non-durable expenditure for the two observed weeks. Thus, our dataset is a panel of two observations of expenditures in successive weeks of the payment cycle. We exclude from our definition of non-durable expenditure any item that can be purchased but its consumption smoothed within the home e.g. tinned food, shoes, etc…. Prima facia evidence from the FES in Figure 1 shows the average weekly pattern of non-durable expenditure over the payment cycle for nine different income groups. It is clear that for most groups consumption is high in the first week when income is received, then decreases for the second and third and is then relatively higher in the fourth week. This agrees with our intuitive argument above. It is this pattern in weekly expenditure during a month that our structural model is designed to capture.

[2]Adang and Melenberg (1995) have previously examined the issue of intraperiod uncertainty (in relation to income and prices) and find evidence to support uncertainty being gradually resolved during each decision period rather being fully resolved at the beginning of the period.

We use this data from the FES to estimate the structural model of short-run behaviour, and find that the coefficient of relative risk aversion ranges from 1.2 to 7 for different scenarios. This is well within the range in the literature on life-cycle models of consumption for this parameter. Measurement error is also an important issue and we estimate that approximately 50% of the variation in the FES data is due to measurement error. Failing to take account of this issue leads to estimates of risk aversion that are biased and makes if difficult to estimate the model.

Section 2 of this paper presents some illustrative empirical analysis quantifying the evidence presented in Figure 1. We provide more details on the pattern of expenditure decisions over a payment cycle and how this pattern varies with access to credit markets, income and age. Section 3 presents the theoretical model of consumption when income is received periodically, incorporating a limit on borrowing and an interest rate differential between borrowing and saving. We present the numerical solution to this model for assumed values of the parameters. Section 4 describes the estimation procedure. The optimal consumption depends on wealth and prices, while the data contains only expenditure observations. This is the major complication when it comes to the estimation of the parameters of the model. Furthermore, we show that measurement error is a significant issue in the expenditure data available and we extend the estimation procedure to take account of this. We present Monte Carlo results on the performance of the estimator and find that it in all cases the estimated parameters (the coefficient of relative risk aversion and the standard deviation of the distribution of measurement error) are within two standard deviations of the true values. We describe in a similar way the estimation procedure that allows for the misreporting of the point in the payment cycle at which individuals are observed. Finally, we present the estimation results. Section 5 concludes.

2. SHORT-RUN CONSUMPTION DECISIONS: EMPIRICAL EVIDENCE

To assess the importance of short run behaviour in the data we estimate a set of (quasi-)Euler equations which allow for week specific effects, given by

$$\begin{aligned}\Delta \ln c_{it+1} = {} & \alpha \ln y_{it+1} + x_{it+1}\beta + \\ & \gamma_{21}\mathbf{1}_{[\text{week 1 in } t]} + \gamma_{32}\mathbf{1}_{[\text{week 2 in } t]} + \gamma_{43}\mathbf{1}_{[\text{week 3 in } t]} - (\gamma_{21}+\gamma_{32}+\gamma_{43})\mathbf{1}_{[\text{week 4 in } t]} + \varepsilon_{it+1},\end{aligned} \tag{1}$$

where $\Delta \ln c_{it+1}$: individual i's growth in expenditure between two successive weeks,

y_{it+1} : income received at the start of month (i.e. at the beginning of week 1),

x_{it+1} : individual/observation specific controls (like date of survey),

$\mathbf{1}_{[\text{week } k \text{ in } t]}$: dummy, 1 if previous period is k-th in the payment cycle, 0 otherwise,

$\gamma_{k+1,k}$: average expenditure growth between week *k* and *k+1*,,

ε_{it+1}: iid stochastic mean zero error terms.

The terms $\gamma_{k+1,k}$ in this equation capture the change in expenditure between two contiguous weeks in the payment period. We assume here that these changes are invariant to the calendar time and therefore are only week specific. Hence, the sum of all changes between the second and the first week, γ_{21}, the third and the second, γ_{32}, and the fourth and the third, γ_{43}, must add up to the opposite of the change between the first and the fourth week, i.e. $\gamma_{14} = -(\gamma_{21} + \gamma_{32} + \gamma_{43})$. The estimation of this set of equations allows us to assess whether transition through the payment cycle has a significant effect on consumption decisions. We also include income in order to investigate whether liquidity constraints are important in determining short-run behaviour. Significance of income in explaining consumption growth would indicate the importance of liquidity constraints because the individual cannot simply borrow as necessary to achieve their optimal consumption and so the path of consumption cannot be independent of the level of income (see Zeldes 1989b).

Here we outline how we can extend empirical models from the existing literature (Zeldes 1989b) to deal with the problem at hand. One possible simple model to account for the specification in (1) will make the marginal utility of consumption a function of the point in the payment cycle at which the expenditure occurs. The point in the payment cycle acts as a shift factor in the utility function in a similar way to the role of demographic factors in long-run consumption models (Zeldes, 1989b, Deaton, 1992). Loosely speaking, more uncertainty can be thought of as shifting marginal utility down, and therefore less uncertainty will shift marginal utility up. First order conditions from dynamic models imply that marginal utility will be smooth/constant over time and so these shift factors cause differences in the level of optimal consumption over time. By implication consumption must be lower in the weeks with more uncertainty. This is in effect the framework proposed by Zeldes (1989b), where he uses a Constant Relative Risk Aversion (CRRA) felicity function in order to derive an approximation to the Euler equation. We begin with

$$u(c_t, z_t) = \frac{c_t^{1-\rho}}{1-\rho}\exp(z_t),$$

where ρ is the coefficient of risk aversion, c_t is consumption in period *t* and z_t is the realisation of a time specific random shift factor Z_t. Our definition of consumption includes only expenditure on non durable items e.g. perishable goods and food away from home. Hence, we are implicitly assuming separability between non durable consumption and all other consumptions, mostly of durables, of the individual. If the assumption of separability holds and if there is no limit on the amount of borrowing, then in each period the individual maximises the sum of utility in all future periods in her lifetime i.e.

$$\max E_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\rho}}{1-\rho} \exp(Z_t),$$

subject only to a budget constraint governing the evolution of wealth

$$w_{t+1} = (1+r)(w_t + y_t - c_t), \; t = 0...\infty,$$

where β is the discount factor, $y_t = y$ if income is received in time *t* and $y_t = 0$ otherwise (i.e. *y* is regular monthly income), and *r* is the interest rate. In the absence of liquidity constraints and with perfect capital markets, the first order conditions for this problem leads to an Euler equation which implies that discounted marginal utility should be constant on average between time periods, i.e.

$$\frac{c_t^{-\rho} \exp(z_t) \beta (1+r)}{c_{t+1}^{-\rho} \exp(z_{t+1})} = 1 + e_{t+1},$$

where both *r* and β are assumed to be constant over time and across households. The left hand side of this equation should differ from one only by the expectation error e_{t+1}, which has mean zero and is independent of information known at period *t* including monthly income. Taking a log approximation of both sides and rearranging gives the equation for consumption growth as

$$\Delta \ln(c_{t+1}) = \frac{1}{\rho} \left\{ (z_{t+1} - z_t) - \ln(1+r) - \ln(\beta) + \ln(1+e_{t+1}) \right\}.^{3} \qquad (2)$$

Since we can always write $z_{t+1} - z_t = \gamma_{t+1,t} + v_{t+1}$, where $\gamma_{t+1,t} \equiv \mathrm{E}_t [Z_{t+1} - Z_t]$ and $\mathrm{E}_t [v_{t+1}] = 0$, we can express (2) as

$$\Delta \ln(c_{t+1}) = \hat{\gamma}_{t+1,t} + \eta_{t+1},$$

where $\hat{\gamma}_{t+1,t} = \frac{1}{\rho} \left\{ \gamma_{t+1,t} - \ln(1+r) - \ln(\beta) + E\left(\ln(1+e_{t+1})\right) \right\}$ and

$\eta_{t+1} = \frac{1}{\rho} \left\{ \ln(1+e_{t+1}) - E\left(\ln(1+e_{t+1})\right) + v_{t+1} \right\}$. Equation (1) is an empirical specification of (2), where the parameters of the transition dummies are assumed constant over time (more precisely,

[3] Attanasio and Low (2000) suggest that the expectation error will have mean zero when T is sufficiently large. Although we only observe each individual for 2 consecutive time periods, the observations in our data are spread over 52 weeks. Hence we believe that our analysis is not subject to the bias from using cross-section data shown in Carroll (2001) and Ludvigson and Paxson (2001).

transition between weeks with the same position in the payment cycle have the same effect irrespective of the calendar time period of observation).

We estimate this Euler equation using data from the FES in the UK. As described in the introduction, the dataset allows us to create a panel of expenditure observations for two consecutive weeks for each monthly paid individual in the survey. The FES for 1996/7, 1997/98 and 1998/99 are used but the sample is restricted to individuals who are in full time employment, are usually paid monthly and the reported income is their usual income. The estimations exclude "outliers" which are defined as persons whose non-durable consumption in any one week is greater than their monthly income. These observations are less than 3% of the sample and are distributed almost equally between credit card holders and non-holders. Their inclusion leads to income being insignificant in each sample, but the results for the transition dummies are qualitatively similar.

We carry out the estimation for different groups within the sample and we examine how the significance of the payment cycle differs between these groups. Firstly, we separate the sample into individuals who may be constrained with respect to short-term credit based on whether they are recorded as holding a credit card in the dataset. This depends on the individual paying an annual charge for her card and we supplement this indicator by also recording whether the individual made any purchases on credit during her diary period. While this is not a perfect indicator of liquidity, close to 55% of our sample is classified as unconstrained which seems realistic although conservative. With this information we mis-classify as constrained individuals who either did not use their credit card during the fortnight, or did not pay or recall paying an annual charge during the previous year although they may have a card. However, following the findings of Japelli et al. (1998), this classification may still be more accurate than a wealth to income ratio. While the effect of uncertainty may still be significant for those with access to credit, the importance should be reduced and so we do not expect the $\gamma_{k+1,k}$ coefficients to be as large or as significant as for those without access to credit. In addition, we do not expect α, the income coefficient, to be significant for the group with access to credit since we believe they are not constrained. We also split the sample at median income to examine whether the uncertainty and liquidity constraints are important only for those with relatively low income. Finally, we separate the sample based on age. Gourinchas and Parker (2002) present evidence that the buffer stock model of accumulation of assets for precautionary reasons applies only to those below middle age, following which accumulation of assets for retirement and bequests becomes important. Thus we consider a sample split at age 45.

[Insert Table 1 here]

[Insert Table 2 here]

The results of the estimation of (1) are presented in Tables 1 and 2. Monthly dummies were included to account for calendar time specific effects, although the coefficients are not reported. We first consider the effect of the changing level of uncertainty from moving through the payment cycle for the entire sample (Table 1). This is measured relative to the effect of going from week four when uncertainty is least to week one when it is greatest. The importance of uncertainty in the sample as a whole for every year is clear. γ_{43}, the parameter that measures the transition between weeks three and four (at which point uncertainty is resolved) is positive and significant, while we find that γ_{21}, the coefficient which measures the transition between weeks one and two (at which point uncertainty is at its highest) is negative and significant. We carry out F-tests where the null hypothesis is the joint insignificance of the transition dummies and find that we reject the null hypothesis for all years of data. Income is also significant for the pooled samples of constrained and unconstrained individuals in both 1997/8 and 1998/9 but not in 1996/7. This indicates the possibility of liquidity constraints since consumption growth should be independent of any information known at the time of the decision, in particular regular certain income.

The differential effects of uncertainty are also clear when the sample is split into those with and without a credit card (Table 1). For the constrained group, the resolution of uncertainty remains significant while the week dummy variables are generally insignificant for the unconstrained group. The null hypothesis of joint insignificance is clearly rejected for the non credit card holders in all years of data. However, we also reject the null for the unconstrained sample in 1998/9, but in line with expectations, fail to reject the null hypothesis in 1996/7 and 1997/8. The results from splitting the sample based on income and age (Table 2) indicates clearly the importance of the changing level of uncertainty for lower income and younger individuals. The week dummies are significant in all years for both of these groups. They are significant in the above median income group in 1996/7, however the F-tests for all other older and relatively wealthier groups strengthen the conclusion that their behaviour is not systematically influenced by uncertainty.

Considering each sub sample in turn gives ambiguous results as far as the significance of income is concerned. For example, contrary to our expectations, we find that income is insignificant for the non credit card holders in 1996/7 and 1997/8 and that it is significant for the credit card holders in 1997/8 and 1998/9. The misclassification of unconstrained individuals into the constrained subsample may explain these results. Furthermore, income is generally the greatest determinant of access to credit, hence it also possible that we are capturing some effect of the selection.[4] Splitting the sample at median income and at age 45, does not strengthen further the evidence on the presence of liquidity constraints. Indeed, income is significant for the below median income group only in 1996/7 and is insignificant for the below age 45 group in 1997/8.

An alternative explanation for the observed pattern in expenditure could be a positive correlation between the timing of the payment of regular bills, and other regular expenditure some of which we define as short term consumption. If so, such a pattern should be observed in the expenditure of all individuals, irrespective of the frequency of income receipt. In particular, evidence of a cyclical pattern in the expenditure of weekly paid individuals would provide evidence in favour of this explanation. Unfortunately, we cannot carry out the analysis above for weekly paid individuals because the data does not allow us to identify when they pay regular bills. However, a cyclical pattern in expenditure implies the same variance in the growth of expenditure between weeks for all payment frequencies. A smaller variance of expenditure growth for weekly paid individuals implies a smoother consumption path than the cyclical pattern evident in expenditure of monthly paids. We show in Table 3 that the mean growth rate is smaller in absolute terms while the standard deviation of the growth rate is larger for monthly paid individuals. We test the null hypothesis of equality of the variances across the two groups against the alternative that the weekly paid have a smaller variance using a conventional F-test. The results show that the null is rejected at all levels of significance. We conclude therefore that a cyclical pattern is more pronounced among those who are paid monthly. We interpret this finding consistent with optimal behaviour by monthly paid individuals as they make the transition through the payment cycle.

[Insert Table 3 here]

3. MULTI CONSUMPTION PERIODS WITHIN EACH PAYMENT PERIOD

3.1 *Model of Consumption*

The previous section established that the position in the payment cycle and credit constraints are important determinants of consumption in the short-run and this section develops a model of consumption behaviour which captures these features. There are three possible situations for the individual: payment occurs in this period and not in the next period, payment does not occur either in this period or next period, and payment does not occur this period but

[4] In results not reported here, we correct for selectivity in credit card ownership and include the selectivity term in Eq. (1) using a Heckman selection estimator. As expected, income is very significant in the selection stage. The level of income and the payment cycle are both insignificant in the quasi-Euler equation for the credit card owners. Hence, it is very likely that the significance of income in the results in Tables 1 and 2 may be the consequence of the selectivity in or out of the constrained group.

does occur next period. Therefore a three period model is sufficient to capture the characteristics of behaviour and simplifies the presentation of the model[5]. In this context, we define wealth in week one as the amount of assets (positive or negative) *before* income is received and, in all other weeks as the amount of assets (positive or negative) at the start of the week after the relevant interest rate has been applied. Here "cash in hand" refers to the sum of wealth and income in the first week and the level of wealth in subsequent weeks. We present the optimal consumption function as the relationship between consumption and wealth, given price. Income enters the budget constraint only in the first week of the payment cycle and the maximum borrowing in each time period is defined as the limit on "monthly" borrowing $\bar{d}$, discounted by the cost of borrowing, δ, which is incurred "weekly". The weekly borrowing limits are given by $\bar{d}_1 \equiv \bar{d}/(1+\delta)^2$, $\bar{d}_2 \equiv \bar{d}/(1+\delta)$, and $\bar{d}_3 \equiv \bar{d}$, and therefore the minimum wealth at the start of each week is given by $\underline{w}_{t+1} \equiv -(1+\delta)\bar{d}_t$. We also assume that the interest rate for borrowing is greater than the return on saving. The budget constraint in any time period t is $p_t c_t \leq w_t + y_t + d_t$, and the evolution of wealth follows the process

$$w_{t+1} = (w_t + y_t - p_t c_t)(1+r) + d_t(r-\delta),$$

where p_t: price draw in the current period,

c_t: consumption in the current period

w_t: wealth at the start of the current period,

y_t: regular income such that $y_t = y$ if $t\,\text{mod}(3) = 0$, and $y_t = 0$ otherwise.

d_t: debt in period t, such that $0 \leq d_t \leq \bar{d}_t$,

r: return on savings,

δ: cost of borrowing.

We assume that y_t, $\bar{d}$, r and δ are exogenous and known with certainty and that the differential between borrowing and saving rates is constant. Total expenditure in period t is given by $p_t c_t$.

We assume that individuals are infinitely lived, and consumption is chosen to maximise the discounted sum of utility flows

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the budget constraint (given above) in each period *t*. Standard dynamic programming arguments suggest that we consider the value of the optimal consumption plan at time t given

[5] When we deal with data from the FES (Sections 2 and 4) we assume four weeks in the payment cycle

wealth *w* and price *p*. However, the value function differs between weeks in the payment cycle because the function reflects the proximity to receipt of income. The week 1 value function takes account of the fact that cash on hand is at its highest in that week but that any debt will incur an interest cost in each week until the next payment. The value function in week 3 reflects the receipt of income in the following period, the reduction in uncertainty and the immediate repayment of debt incurred. Thus we write a set of period specific Bellman equations, which define the optimal consumption path as a function of wealth and price in each week, i.e. w and p.

We assume that price does not exhibit any persistence. This simplifies our analysis, in particular expectations are not conditional on the information contained in previous price levels and/or the history of price observed so far. The realisation of future price is denoted by π and we assume both p and π are drawn from the distribution $\boldsymbol{F}$ with support $\mathcal{P}$ over which expectations are taken. We assume that the support is closed interval $\left[\underline{p}, \overline{p}\right]$ of $\mathbb{R}_*^+$, with $\underline{p} > 0$ and $1 < \overline{p} < \infty$. The support therefore includes 1. Furthermore we assume that $\mathrm{E_F}\left[p\right] = 1$.The week specific Bellman equations are given by

$$V_1\left(w;p\right) = \max_{\substack{c,d \\ \text{subject to} \\ pc \leq w+y+d \\ 0\leq d\leq \frac{\bar{d}}{(1+\delta)^2}}} \left\{u\left(c\right) + \beta\, \mathrm{E}_{\mathcal{P}}\left[V_2\left((1+r)(w+y-pc)+(r-\delta)d;\pi\right)\right]\right\},$$

$$V_2\left(w;p\right) = \max_{\substack{c,d \\ \text{subject to} \\ pc \leq w+d \\ 0\leq d\leq \frac{\bar{d}}{(1+\delta)}}} \left\{u\left(c\right) + \beta\, \mathrm{E}_{\mathcal{P}}\left[V_3\left((1+r)(w-pc)+(r-\delta)d;\pi\right)\right]\right\}, \tag{3}$$

$$V_3\left(w;p\right) = \max_{\substack{c,d \\ \text{subject to} \\ pc \leq w+d \\ 0\leq d\leq \bar{d}}} \left\{u\left(c\right) + \beta\, \mathrm{E}_{\mathcal{P}}\left[V_1\left((1+r)(w-pc)+(r-\delta)d;\pi\right)\right]\right\},$$

where the felicity function $u(c)$ is assumed to be increasing, monotonic and concave. We assume that the discount factor β is such that $\beta(1+\delta)$ <1 i.e. the consumer is "impatient" and has a preference for consumption in the present. For a given wealth and price realisation, the control variable consumption is chosen to maximise the right hand side, subject to a lower limit on negative wealth. The problem fulfils the conditions for continuity and differentiability of the value function as given by Benveniste and Scheinkman (1979). Indexing each specific week in the payment cycle by *k*, the set of Lagrangian equations which define the constrained maximum are given by

which more closely resembles weekly decision making and monthly income receipt.

$$L_k\left(c,d;\gamma,\mu_1,\mu_2\right)=u\left(c\right)+\beta E_{\wp}\left[V_{k+1}\left((1+r)(w+y_k-pc)+d(r-\delta);\pi\right)\right]$$
$$-\psi_k\left(pc-(w+y+d)\right)+\mu_{k1}\left(d\right)+\mu_{k2}\left(\bar{d}-d\right),$$

for $k = 1,2,3$ and where $k+1 = 1$ if $k=3$. This gives rise to the first order conditions (where * denotes the optimal consumption level)

$$\begin{aligned}
\frac{\partial L_k\left(c^*,d^*;\gamma_k,\mu_{k1},\mu_{k2}\right)}{\partial c}&=u'\left(c^*\right)-\beta(1+r)p\frac{\partial EV_{k+1}}{\partial w}\left(.^*\right)-\psi_k p=0,\\
\frac{\partial L_k\left(c^*,d^*;\gamma_k,\mu_{k1},\mu_{k2}\right)}{\partial d}&=\beta(r-\delta)\frac{\partial EV_{k+1}}{\partial w}\left(.^*\right)+\psi_k+\mu_{k1}-\mu_{k2}=0,\\
\frac{\partial L_k\left(c^*,d^*;\gamma_k,\mu_{k1},\mu_{k2}\right)}{\partial \psi_k}&=pc^*-(w-y+d^*)\le 0,\\
\frac{\partial L_k\left(c^*,d^*;\gamma,\mu_{k1},\mu_{k2}\right)}{\partial \mu_{k1}}&=d^*\ge 0,\\
\frac{\partial L_k\left(c^*,d^*;\gamma_k,\mu_{k1},\mu_{k2}\right)}{\partial \mu_{k2}}&=\bar{d}-d^*\ge 0.
\end{aligned} \tag{4}$$

The solution to this problem can be characterised by examining different values of the multipliers. Only four combinations of the multipliers are possible. We now describe each possible "regime" of the solution. In what follows $\lambda(c)$ denote the marginal utility of consumption.

Regime 1: $pc^* < w+y_k$, $d^* = 0$, ψ_k =0, μ_{k1} >0, μ_{k2} =0, $\frac{1}{p}\lambda\left(c_k^*\right)=\beta(1+r)\mathrm{E}\left[\frac{\lambda\left(c_{k+1}^*\right)}{\pi}\right]$.

In this regime the agent does not consume all cash in hand. Hence borrowing is zero, μ_{k1} >0, μ_{k2} =0, and the positive wealth carried over to the next period earns return *r*. The Euler equation for this regime (the final equality above) implies that the capital market imperfections do not constrain behaviour. The real marginal utility of current optimal consumption is given by the value of marginal utility of optimal consumption in the next period, discounted at $\beta\left(1+r\right)$.

It is never optimal to borrow and not consume that extra cash when the cost of borrowing is greater than the return on savings. This rules out regimes where ψ_k = 0, $\mu_{k1}=\mu_{k2}$ = 0, and ψ_k = 0, $\mu_{k1}>0$, $\mu_{k2}=0$. In all other cases ψ_k >0, i.e. consumption is greater than or equal to cash in hand and assets are run down or at least not accumulated.

Regime 2: $pc^* = w+y_k$, $d^* = 0$, ψ_k >0, μ_{k1} >0, μ_{k2} = 0, $\frac{1}{p}\lambda\left(c_k^*\right)=\frac{1}{p}\lambda\left(\frac{w+y_k}{p}\right)$.

The individual consumes the entirety of her cash in hand, ψ_k >0, and does not borrow, μ_{k1} >0. She is neither patient enough to save at *r* nor impatient enough to borrow at δ. The current marginal utility is simply the marginal utility of available cash in hand. No assets are carried over to the next period.

Regime 3: $pc^* = w + y_k + d^*$, $0 < d^* < \bar{d}$, $\psi_k > 0$, $\mu_{k1} = \mu_{k2} = 0$, $\frac{1}{p}\lambda\left(c_k^*\right) = \beta(1+\delta)\mathrm{E}\left[\frac{\lambda\left(c_{k+1}^*\right)}{\pi}\right]$.

In this third regime, the level of debt d^* is positive but below the limit, $\psi_k > 0$ and $\mu_{k1} = \mu_{k2} = 0$, and consumption is equal to the sum of cash in hand and d^*. Negative wealth of $-(1+\delta)d^*$ is carried into the next period. The Euler equation is fulfilled in an unconstrained way because optimal borrowing is below the limit. The real marginal utility of optimal consumption in the current period is equal to the value of optimal consumption in the next period discounted at $\beta(1+\delta)$.

Regime 4: $\overline{pc} = w + y_k + \bar{d}_k$, $d^* = \bar{d}$, $\psi_k > 0, \mu_{k1} = 0$, $\mu_{k2} > 0$, $\frac{1}{p}\lambda\left(c_k^*\right) = \frac{1}{p}\lambda\left(\frac{w + y_k + \bar{d}_k}{p}\right)$.

In this last regime the individual borrows up to her limit, $\psi_k > 0$, $\mu_{k2} > 0$ and $\mu_{k1} = 0$. Even at the maximum level of borrowing, consumption is still below that which would be chosen if the borrowing limit did not exist. In this case, the solution is to consume the maximum amount possible (cash in hand plus maximum borrowing $\bar{d}_k$), and carry maximum debt $-(1+\delta)\bar{d}_k$ into the next period. The Euler equation implies the solution for current marginal utility is equal to the marginal utility of the sum of all available cash in hand and maximum borrowing.

The optimality conditions for these four regimes can be summarised in an augmented Euler equation that relates consumption decisions in any two periods, *k* and *k+1*

$$\frac{1}{p}\lambda\left(c_k^*\right) = \max\left\{\begin{array}{l} \beta(1+r)\mathrm{E}\left[\frac{\lambda\left(c_{k+1}^*\right)}{\pi}\right], \\ \min\left\{\frac{1}{p}\lambda\left(\frac{w+y_k}{p}\right), \beta(1+\delta)\mathrm{E}\left[\frac{\lambda\left(c_{k+1}^*\right)}{\pi}\right]\right\}, \\ \frac{1}{p}\lambda\left(\frac{w+y_k+\bar{d}_k}{p}\right) \end{array}\right\}, \qquad (5)$$

The minimum in this expression arises because capital market imperfections prevent an individual from receiving a return equal to δ on saving. If this is optimal but saving at r is not, the individual will not save and so will have a higher level of consumption (and therefore a lower marginal utility) than if saving at δ were possible. The minimum condition captures this limitation on behaviour. Hence this expression is a generalisation of the first order conditions in Deaton and Laroque (1995).

We follow Deaton and Laroque (1992, 1996) and assume that a stationary solution to (5) exists. This solution defines the optimal relationship between expenditure, wealth and prices, and so the consumption function in each week of the payment cycle, *k*, described by nominal value of wealth and prices, is given by $c_k^* = g_k(w;p)/p$.

We assume $\frac{\partial g_k(w;p)}{\partial w} > 0$, and $\frac{\partial g_k(w;p)}{\partial p} < 0$. The marginal utility of money/wealth $q_k[w;p]$ is given by the real value of marginal utility of optimal consumption $\lambda(c_k^*)/p$ and therefore $q_k[w;p] = \frac{1}{p}\lambda\left(\frac{1}{p}g_k(w;p)\right)$. Writing the Euler equation (5) in terms of the marginal utility of money, gives the following set of functional equations to which there are unique solutions $q_k[w;p]$ given by

$$q_k[w;p] = \max\left\{\begin{array}{l} \beta(1+r)\int_{\wp} q_{k+1}\left[(1+r)(w+y_k - p\lambda^{-1}(pq_k[w;p]);\pi\right]dF(\pi), \\ \min\left\{\frac{1}{p}\lambda\left(\frac{w+y_k}{p}\right), \beta(1+\delta)\int_{\wp} q_{k+1}\left[(1+\delta)(w+y_k - p\lambda^{-1}(pq_k[w;p]);\pi\right]dF(\pi)\right\}, \\ \frac{1}{p}\lambda\left(\frac{w+y_k+\bar{d}_k}{p}\right) \end{array}\right\}. \quad (6)$$

For ease of notation define the functions

$$H_1(q_k[w;p],w,p) = \beta(1+r)\int_{\wp} q_{k+1}\left[(1+r)(w+y_k - p\lambda^{-1}(pq_k[w;p]);\pi\right]dF(\pi),$$

$$\underline{\lambda} = \frac{1}{p}\lambda\left(\frac{w+y_k}{p}\right),$$

$$H_2(q_k[w;p],w,p) = \beta(1+\delta)\int_{\wp} q_{k+1}\left[(1+\delta)(w+y_k - p\lambda^{-1}(pq_k[w;p]),\pi\right]dF(\pi),$$

$$\bar{\lambda} = \frac{1}{p}\lambda\left(\frac{w+y_k+\bar{d}_k}{p}\right),$$

where $H_1(q_k[w;p],w,p)$ and $H_2(q_k[w;p],w,p)$ are the expected discounted marginal utilities of saving, respectively, borrowing, for current wealth and price, *w*, *p*, and marginal utility of money $q_k[w;p]$ in week *k*. Following the arguments in Deaton and Laroque (1992) and in Bailey and Chambers (1996), we show the following theorem

Theorem 1. *There is a unique set of functions* $q_k[w,p]$, *the stationary solutions to the functional equations* (6) *above, which are continuous in wealth and price and non-increasing in wealth. In addition the solutions can be characterised as follows*

$$q_k[w;p] = H_1\left(q_{k+1}[w;p], w, p\right), \qquad \text{when } \beta(1+r)\int_{\wp} q_{k+1}[0;\pi]dF(\pi) \geq \underline{\lambda};$$

$$q_k[w;p] = \underline{\lambda}, \qquad \text{when } \begin{cases} \beta(1+r)\int_{\wp} q_{k+1}[0;\pi]dF(\pi) \leq \underline{\lambda}, \text{ and} \\ \beta(1+\delta)\int_{\wp} q_{k+1}[0;\pi]dF(\pi) \geq \underline{\lambda}; \end{cases}$$

$$q_k[w;p] = H_2\left(q_{k+1}[w;p], w, p\right), \qquad \text{when } \begin{cases} \beta(1+\delta)\int_{\wp} q_{k+1}\left[-(1+\delta)\overline{d}_k;\pi\right]dF(\pi) \geq \overline{\lambda}, \text{ and} \\ \beta(1+\delta)\int_{\wp} q_{k+1}[0;\pi]dF(\pi) \leq \underline{\lambda}; \end{cases}$$

$$q_k[w;p] = \overline{\lambda} \qquad \text{when } \beta(1+\delta)\int_{\wp} q_{k+1}\left[-(1+\delta)\overline{d}_k;\pi\right]dF(\pi) \leq \overline{\lambda}.$$

Proof*:* Available upon request.

The theorem sets out the conditions for each regime: positive saving is optimal when the wealth level is sufficiently high so that the real marginal utility from consuming all cash on hand, $\underline{\lambda}$, is below the discounted value of the marginal utility of money from zero savings. In this case, consumption is substituted towards the future so that current marginal utility increases and future marginal utility of money decreases until the unique level of saving that results in equality between the two terms is reached. When the wealth level is such that the real marginal utility from consuming cash in hand is above the discounted marginal utility of zero savings, the individual has an incentive to increase consumption and carry negative wealth into the next week. However, the cost of borrowing δ may bring the discounted future marginal utility of zero borrowing above the marginal utility of cash in hand, in which case the preference for higher consumption is not strong enough for the individual to borrow at δ. In this case, neither borrowing nor saving occurs. If the wealth level is sufficiently low, it is optimal for the individual to have a positive level of borrowing (below the limit) which decreases current marginal utility of consumption while increasing the future marginal utility of wealth. Borrowing remains below the limit while the marginal utility from maximum negative wealth in the next period is above the marginal utility of maximum possible consumption today. If however, the individual would choose a level of consumption so high relative to the real value of wealth that borrowing greater than the maximum possible would be optimal, then at maximum debt $\overline{d}$ the individual would further decrease current marginal utility of consumption by borrowing and increasing consumption, and so the optimal outcome is to borrow the maximum available.

3.2 *Numerical Solution to the model*

The functional equation (6) is analytically intractable and so we solve the model using the standard numerical methods of dynamic programming (see Deaton (1991, 1992) for a clear exposition and Judd, (1998)). The calculations are simplified because the optimal consumption function is homogenous of degree 1 in y (See Imrohoroglu, (1989), and Gourinchas and Parker, (2002)). This allows us to calculate the relationship between consumption as a proportion of y and wealth as a proportion of y. This reduces significantly the computation burden. We set the upper bound on wealth to a multiple of income, while the lower bound is defined by the credit limit. We discretise the wealth space into W grid points where $\omega =1,.,.,.,W,$ and the price distribution with M grid points where $m=1,.,.,.M$ so that the solution is calculated at each point in the $W \times M$ matrix of grid points. Linear interpolation (or non linear methods like splines) allows us to calculate the optimal consumption for any combination of wealth and price within the grid. In calculating the solution, we assume CRRA preferences with felicity function of the form

$$u(c) = \frac{1}{1-\rho} c^{1-\rho} \quad for \;\; \rho > 1$$

where ρ is the coefficient of relative risk aversion. This numerical solution is shown in Figure 2 where we assume β = 0.985, and ρ = 2.5, values generally accepted in the literature for these parameters. We exaggerate the cost of borrowing in this case so that the regimes of the solution are clearly evident. Consumption and wealth are shown as proportions of income. We assume that the limit on borrowing is set at two-thirds of income and that prices are distributed uniformly with mean of 1 and support from 0.75 to 1.25. The solution is shown for the same price draw in each week. We show the solution for four weeks within each payment cycle which most closely resembles the short-run problem faced by individuals who are paid monthly and consume weekly.

[Insert Figure 2 here]

This solution means that for any given wealth level and draw of price, we can calculate the consumption behaviour of the individual in any week. The liquidity constraint only binds in week four so that even when there is already some borrowing in week two or three, the remainder is never exhausted in those weeks because of the infinite cost of zero consumption. Hence, the fourth regime is evident only in week four. The other regimes, and the kinks at which the solutions switch between these regimes, are evident in all weeks. For all levels of wealth, consumption is highest in the week when income is received, lowest in the next week and the increasing over the remaining weeks. However, this ranking of the weekly consumption functions is not unique to the CRRA utility function: the same pattern holds for other utility functions within the class of HARA functions because these preferences imply precautionary motives. It is also evident from the solution that a minimum consumption of approximately one quarter of income is always attainable in the first week of the payment cycle even when debt is at its maximum. This is a critical feature of the model and it has important implications when we fit the model to the data.

4. ESTIMATION OF MODEL

The previous section described a model of short-run consumption which incorporates the characteristics of the environment faced by many individuals: periodic receipt of income, a differential between the cost of borrowing and the return on saving, and an upper bound on the amount of borrowing possible. Income is certain and uncertainty arises because the price of weekly consumption is random. In this section we show how the model can be estimated with repeated expenditure data alone and how we can extend the estimation procedures to take account of some measurement error in the data. We carry out a Monte Carlo study of the performance of the estimator both before and after we include measurement error in the parameter vector. We then describe the data and the results of the estimation on a subsample of the FES in the UK.

4.1 *Estimation without Measurement Error*

The simulated policy function shown in Figure 2 allows us to relate any level of wealth and price to the optimal consumption by linear interpolation between grid points. In what follows we drop the index for the week (*k*) and assume that for each individual observation the index t is informative about both calendar time and position in the individual payment cycle. Therefore the weekly consumption function, $g_t\left(w; p, \theta\right)$, is a function of wealth and price given the vector of parameters θ, and implicitly depends on the individual's position in the payment cycle. If wealth and price were observed, estimating the model from the data available would then be relatively straightforward. However, neither of these variables is observed. Instead, we exploit the fact that we observe two successive weekly expenditures for each individual. We use the policy functions to give the relationship between expenditure observations in adjacent time periods rather than between wealth and prices in the same time period. We achieve this by taking expenditure in week t and inverting the policy function for that week for a given price. That gives the level of wealth consistent with the expenditure observation, i.e. observed expenditure gives an implied level of wealth

$$w_t = g_t^{-1}\left(p_t c_t; p_t, \theta\right).$$

The success of this strategy relies upon the monotonicity of the function to give a unique association between an expenditure observation and wealth level. Once the implied wealth level

w_t is calculated, the cash on hand carried into the *next* period *t+1* can be calculated using the equation for the evolution of wealth

$$w_{t+1} = (1+\zeta)(g_t^{-1}(p_t c_t; p_t, \theta) + y_t - p_t c_t),$$

where *y=y* if income is received in period *t*, and *y=0* otherwise, $\zeta = \delta$ if $(w_t - p_t c_t) <$ 0, and $\zeta = r$ otherwise. Conditional on expenditure in *t* and price outcomes in *t* and *t+1*, (p_t, p_{t+1} respectively), expenditure in *t+1* can therefore be calculated as

$$p_{t+1}c_{t+1} = g_{t+1}\left((1+\zeta)\left(g_t^{-1}(p_t c_t; p_t, \theta) + y - p_t c_t\right); p_{t+1}, \theta\right).$$

Taking the expectation over the price distribution in the second period, we can obtain the expectation of expenditure for week *t+1* conditional upon the observed level of expenditure in the week *t*, the price outcome in the week *t*, and the parameter vector θ. This expectation is given by

$$E_{p_{t+1}}[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta] = E_{p_{t+1}}\left[g_{t+1}\left((1+\zeta)\left(g_t^{-1}(p_t c_t; p_t, \theta) + y - p_t c_t\right); p_{t+1}, \theta\right) \mid p_t c_t, p_t, \theta\right].$$

However the implied level of cash on hand $w_t = g_t^{-1}(p_t c_t; p_t, \theta)$ must be calculated for all possible first period prices p_t. Therefore taking an expectation again over the price distribution in the first period gives the conditional expectation of expenditure in *t+1* given expenditure in *t* as

$$\begin{aligned} e(p_{t+1}c_{t+1};\theta) &= E_{p_t}\left[E_{p_{t+1}}[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta] \mid p_t c_t, \theta\right] \\ &= E_{p_t}\left[E_{p_{t+1}}\left[g_{t+1}\left((1+\zeta)\left(g_t^{-1}(p_t c_t; p_t, \theta) + y - p_t c_t\right); p_{t+1}, \theta\right) \mid p_t c_t, p_t, \theta\right] \mid p_t c_t, \theta\right]. \end{aligned}$$

Applying the Law of Iterated Expectations to the calculation of a variance, we obtain the conditional variance of expenditure in *t+1* given expenditure in *t* as,

$$\begin{aligned} v(p_{t+1}c_{t+1};\theta) &= E_{p_t}\left[E_{p_{t+1}}\left[\left(p_{t+1}c_{t+1} - e(p_{t+1}c_{t+1};\theta)\right)^2 \mid p_t c_t, p_t, \theta\right] \mid p_t c_t, \theta\right] \\ &= E_{p_t}\left[E_{p_{\tau+1}}\left[\left(p_{t+1}c_{t+1} - E_{p_{t+1}}[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta]\right)^2 \mid p_t c_t, p_t, \theta\right] \mid p_t c_t, \theta\right] \\ &\quad + E_{p_t}\left[\left(E_{p_{t+1}}[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta] - e(p_{t+1}c_{t+1};\theta)\right)^2 \mid p_t c_t, \theta\right]. \end{aligned}$$

The first term arises from the uncertainty of prices in the second period and is the variance of each possible future value of expenditure $p_{t+1}c_{t+1}$ around each conditional expectation of future expenditure $E_{p_{t+1}}[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta]$. The expectation of this variance is taken over the distribution of current prices p_t. The second term is the variance of each expectation of future expenditure $E_{p_{t+1}}\left[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta\right]$ (given current price p_t), around the expectation unconditional on price, $E_{p_t}\left[E_{p_{t+1}}\left[p_{t+1}c_{t+1} \mid p_t c_t, p_t, \theta\right] \mid p_t c_t, \theta\right]$. This term arises because current price is not observed by the econometrician and so the conditional expectation of future expenditure must be calculated over all possible prices today. Deaton and Laroque (1995) refer to these terms as the *within* and *between* variance respectively. The *within* variance captures the variance of future expenditure conditional on each current price draw and is common to the individual and the econometrician. The *between* variance captures the variance across possible current price draws and exists only for the econometrician because current price is not observed. The model is estimated using the Pseudo Maximum Likelihood Estimator (PMLE) of Gourieroux et al., (1984). Following Deaton and Laroque (1995, 1996), we use a second order PMLE i.e. we use the first two moments of the conditional distribution of the expenditure in *t+1* given expenditure in *t*, and use the p.d.f. of the normal distribution to calculate the pseudo likelihood function. The non-differentiability of the policy function at the points where the solution switches between the four regimes carries through to the conditional moments of the distribution of expenditure. Therefore the pseudo likelihood function will also be non-differentiable with respect to the parameters at these points. However, Laroque and Salanié (1994) show that PMLE gives consistent estimates of the parameters despite this non-differentiability and also show that a second order PMLE is almost as efficient in finite samples as Full Information Maximum Likelihood, when estimating parameters from "badly-behaved" likelihood functions arising from non-differentiable models. Michaelides and Ng (2000) in addition prove the superiority of PMLE over simulation methods in estimating parameters in models where the likelihood is non-differentiable.

The mean of the pseudo log-likelihood function can be written as

$$\ln L = \frac{1}{N}\sum_{j=1}^{N}\ln l_j(\theta) = -\frac{1}{N}\sum_{j=1}^{N}\frac{\left(p_{jt+1}c_{jt+1} - e\left(p_{jt+1}c_{jt+1};\theta\right)\right)^2}{v\left(p_{jt+1}c_{jt+1};\theta\right)} - \frac{1}{N}\sum_{j=1}^{N}\ln\left(v\left(p_{jt+1}c_{jt+1};\theta\right)\right). \quad (7)$$

The asymptotic variance covariance matrix of $\sqrt{N}\left(\hat{\theta}-\theta_0\right)$ for the PMLE is calculated as

$$V = J^{-1}\left(G'G\right)J^{-1}, \quad (8)$$

where G is a $N\times rows(\theta)$ matrix with generic element $G_{ji} = \frac{\partial \ln l_j}{\partial \theta_i}$ and J is a $rows(\theta)\times rows(\theta)$ matrix with generic element $J_{ih} = \frac{\partial^2 \ln L}{\partial\theta_i \partial\theta_h}$.

Throughout the analysis we assume that the price distribution is the same in all periods and that draws from that distribution are *iid*. To simplify the calculations we consider M discrete values from $\wp$, the support of ***F***. The expectations are therefore calculated simply as weighted averages of the outcome for each price draw where the weights are the probability of each price outcome in both periods, denoted by pr(p_m) and pr(p_h)

$$e\left(p_{t+1}c_{t+1};\theta\right)=E_{p_t}\left[E_{p_{t+1}}\left[p_{t+1}c_{t+1}\mid p_tc_t,p_t,\theta\right]\mid p_tc_t,\theta\right]$$
$$=\sum_{h=1}^{M}\sum_{m=1}^{M}\mathrm{pr}\left(p_m\right)\mathrm{pr}\left(p_h\right)g_{t+1}\left(\left(1+\zeta\right)\left(g_t^{-1}\left(p_tc_t;p_m,\theta\right)-p_tc_t\right);p_h,\theta\right), \quad (9)$$

$$v\left(p_{t+1}c_{t+1};\theta\right)=E_{p_t}\left[E_{p_{t+1}}\left[\left(p_{t+1}c_{t+1}-e\left(p_tc_t;\theta\right)\right)^2\mid p_tc_t,p_t,\theta\right]\mid p_tc_t,\theta\right]$$
$$=\sum_{h=1}^{M}\sum_{m=1}^{M}\left(\mathrm{pr}\left(p_m\right)\mathrm{pr}\left(p_h\right)\left(g_{t+1}\left(\left(1+\zeta\right)\left(g_t^{-1}\left(p_tc_t;p_m,\theta\right)-p_tc_t\right);p_h,\theta\right)-e\left(p_tc_t;\theta\right)\right)^2\right) \quad (10)$$

These are calculated for each observation and are then substituted into the pseudo likelihood function in (7) which is then maximised with respect to θ.

Monte Carlo Study

We carry out a Monte Carlo study to investigate the small sample properties of the PMLE in this framework using β = 0.95, δ = 0.4% and r = 0. The parameter vector in the estimation procedure could include $\beta,\rho,\delta,r,\bar{d}$ and possibly the parameters of the price distribution. However we do not try to identify all those parameters from the data and instead restrict the estimation to the coefficient of risk aversion ρ. The replications are carried out for three values of ρ (1.5, 2.5, 5), and for two different price distributions (normal and uniform). In the case of the uniform distribution, prices can take on any of six equally spaced discrete values between 0.75 and 1.25, for which the expectation is one. We use a normal distribution $N(1,0.17^2)$ so that it has the same support as the uniform distribution but twice the variance. This is discretised by six points, each point being the mid-points of the interval within which 1/6 of the density lies. (See Deaton and Laroque, 1996). For each of these set-ups we draw 100 datasets of either 500 or 1000 observations. When estimating just one parameter, we use a simple golden-section procedure to find the maximum and find this very fast and reliable. The results are presented in Table 4. The empirical distribution of the estimator is very close to the asymptotic variance given by (8) and in all cases the estimated parameter is within two standard deviations of the true value. All the maxima are robust to changing the starting values and the results suggest that there are no obvious problems within the estimation procedure.

[Insert Table 4]

[Insert Figure 3]

4.2 *Estimation with Measurement Error*

Figure 3 shows the conditional expectation in (9) and it is obvious that over a large part of the expenditure values the conditional expectation is close to a 45^o line in all weeks. Therefore we do not expect to see very large changes in expenditure between adjacent weeks. However this is not what we find in the data. Figure 4(a) shows the kernel estimates of the density of the proportional change in expenditures between weeks two and three from a simulated dataset where prices are drawn according to the distribution $N(1,0.2^2)$, $\beta = 0.95$, $\rho = 2.5$, $\bar{d} = 0.66$. Figure 4(b) shows the same for the dataset from the FES. The mean change between expenditure in weeks 2 and 3 in the simulated data is –1.5% with a standard deviation of 22%, while in the FES data the mean is –13% and the standard deviation is 90%. Clearly the variation between weeks in the FES is enormous relative to the simulated data. Even if we double the variance in the price distribution from 0.04 to 0.08, the standard deviation in the simulated data only increases to 35%. It does not seem reasonable that these differences arise from greater variance in prices faced by actual individuals than we allow for in the simulation (a variance of 0.08 implies the standard deviation of the price distribution is 0.28 which seems unrealistic).

In addition, a simple linear regression of expenditure in *t+1* on *t* gives a coefficient close to 0.75 in the simulated dataset while in the actual data the coefficient is closer to 0.35. This is symptomatic of the attenuation bias resulting from measurement error. We now describe how we extend the estimation procedure to account for the presence of classical measurement error.

[Insert Figure 4 here]

If classical measurement error exists in the data, then the observed expenditure is the sum of true expenditure $p_t c_t^*$ plus noise ε_t, i.e. $\widetilde{p_t c_t} = p_t c_t^* + \varepsilon_t$. The conditional expectation of interest is now

$$e\left(\widetilde{p_{t+1}c_{t+1}};\theta\right) = E_{p_t}\left[E_{p_{t+1}}\left[\widetilde{p_{t+1}c_{t+1}} \mid \widetilde{p_t c_t}, p_t, \theta\right] \mid \widetilde{p_t c_t}, \theta\right]$$

and the variance becomes

$$v\left(\widetilde{p_{t+1}c_{t+1}};\theta\right) = E_{p_t}\left[E_{p_{t+1}}\left[\left(\widetilde{p_{t+1}c_{t+1}} - e\left(\widetilde{p_t c_t};\theta\right)\right)^2 \mid \widetilde{p_t c_t}, p_t, \theta\right] \mid \widetilde{p_t c_t}, \theta\right].$$

In this case, it is the expectation and variance of noisy expenditure in *t+1* that are used in the PMLE in (7) and these moments are conditional on the observation of the noisy expenditure in *t*. However, the methodology outlined in the previous section to estimate the parameters of the model in Section 3 is based on the relationship between true expenditure data in adjacent time periods. Therefore we need a way to recover the true expenditure observation from the contaminated data, in order to calculate the conditional moments of the distribution of expenditure in the following time period. We make the following (relatively) strong parametric assumptions: we assume i) $\mathrm{E}[\varepsilon_t]=0$, $\mathrm{E}\left[p_t c_t^*\right]=\mu$, ii) $\mathrm{E}\left[p_t c_t^*,\varepsilon_t\right]=0$, i.e. the true value of expenditure and the error are uncorrelated, and iii) the noisy observation and the error itself are jointly normally distributed. This joint distribution is given by

$$\begin{pmatrix}\varepsilon_t \\ \widetilde{p_t c_t}\end{pmatrix} \sim N\left(\begin{pmatrix}0 \\ \mu\end{pmatrix},\begin{pmatrix}\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2+\sigma^2_{p_t c_t}\end{pmatrix}\right),$$

and the expectation of the error conditional upon a noisy observation of expenditure is then

$$E\left[\varepsilon \mid \widetilde{p_t c_t}\right]=\frac{\sigma^2}{\sigma^2+\sigma^2_{p_t c_t}}\left(\widetilde{p_t c_t}-\mu\right).$$

Given an observation and a corresponding draw from the distribution of $\varepsilon_t \mid p_t c_t$, $\widetilde{p_t c_t}-\varepsilon \mid \widetilde{p_t c_t}$ gives the "true" observation. By using this in the procedure above to find the implied wealth, we can calculate the conditional expectation of the noisy data, given a value for the error and the parameters. The expectation of observed expenditure in week *t+1* given a draw from the conditional distribution of the error, is given by

$$\tilde{e}\left(\widetilde{p_{t+1}c_{t+1}};\widetilde{p_t c_t},\varepsilon \mid \widetilde{p_t c_t},\theta\right)=E_{p_t,p_{t+1}}\left[\widetilde{p_{t+1}c_{t+1}} \mid \widetilde{p_t c_t}-\varepsilon \mid \widetilde{p_t c_t},\theta\right].$$

This expression can be integrated over the entire conditional distribution of $\varepsilon_t \mid p_t c_t$ and the conditional expectation of expenditure in *t+1* allowing for the measurement error becomes

$$e\left(\widetilde{p_{t+1}c_{t+1}};\theta\right) = E_{\varepsilon\mid\widetilde{p_t c_t}}\left[\tilde{e}\left(\widetilde{p_{t+1}c_{t+1}};\widetilde{p_t c_t},\varepsilon \mid \widetilde{p_t c_t},\theta\right)\mid \widetilde{p_t c_t},\theta\right] \qquad (11)$$

Similarly the variance can be written as

$$v\left(\widetilde{p_{t+1}c_{t+1}};\theta\right)=E_{\varepsilon\mid\widetilde{p_t c_t}}\left[E_{p_t,p_{t+1}}\left[\left(\widetilde{p_{t+1}c_{t+1}}-e\left(\widetilde{p_{t+1}c_{t+1}};\theta\right)\right)^2 \mid \widetilde{p_t c_t},\varepsilon \mid \widetilde{p_t c_t},\theta\right]\mid \widetilde{p_t c_t},\theta\right]$$

$$=E_{\varepsilon\mid\widetilde{p_t c_t}}\left[E_{p_t,p_{t+1}}\left[\widetilde{p_{t+1}c_{t+1}}^2 \mid \widetilde{p_t c_t},\varepsilon \mid \widetilde{p_t c_t},\theta\right]\widetilde{p_t c_t},\theta\right]+\sigma^2-e\left(\widetilde{p_t c_t};\theta\right)^2. \qquad (12)$$

In order to integrate with respect to the conditional distribution of ε_t given the expenditure in *t*, we use Gauss-Hermite quadrature, (see Judd, (1998)).

The difference between the conditional expectation with and without measurement error is shown in Figure 5 where the variance of the measurement error is assumed equal to 0.1. The effect of measurement error is clearly significant. Note that if $\sigma=0$, the model with measurement error "reduces" to the model without measurement error.

[Insert Figure 5 here]

We now include the standard deviation of measurement error in the parameter vector to be estimated. We change the parameterisation of the parameters of the model. First σ^2 must be restricted to be between zero and the variance in the data i.e. measurement error cannot account for more than 100% of the variation in the data. In addition we find that the reparameterisation of ρ makes it easier to find the global maximum of the pseudo-likelihood. The re-parameterisation is such that the estimated parameter vector θ is given by

$$\rho = \underline{\rho} + \left(\left(\bar{\rho} - \underline{\rho}\right)/1 + \exp\left(-\theta_1\right)\right),$$

$$\sigma = \underline{\sigma} + \left(\left(\bar{\sigma} - \underline{\sigma}\right)/1 + \exp\left(-\theta_2\right)\right),$$

where $\left[\underline{\rho}, \bar{\rho}\right] = [1,11]$ and $= [\underline{\sigma}, \bar{\sigma}] = \left[10^{-5}, \sigma_{\widetilde{p_t c_t}}\right]$.

Monte Carlo Study and Results

We again carry out a Monte Carlo study using the true parameters as the starting values and the results are given in Table 5. The mean of the estimates are all within one standard error of the true value for both ρ and σ. There are significant differences however between the empirical variation in the estimates and the mean of the asymptotic errors in Table 5. Results from the asymptotic variance-covariance are systematically above those from the empirical values and the number of observations does not affect this, although the relative difference is greater for ρ than σ. However, this is similar to the results for the PMLE from the Monte Carlo studies carried out in Laroque and Salanié (1994). The results also show that doubling the number of observations reduces the standard error by approximately $1/\sqrt{2}$ as we would expect from first order asymptotic theory.

[Insert Table 5 here]

4.3 *Estimation over Four Weeks*

The two-week period over which individuals in the FES record their expenditure data is randomly assigned and we can safely assume that individuals have no control either over their pay date. Therefore we expect that the weeks of expenditure observations should be evenly distributed across the four weeks of the payment cycle if the sampling is random. Table 6 shows the frequency of the first observation week across the payment cycle. Clearly there is a significant under recording of week 4[6]. This may be due to mis-reporting in the data or due to our allocation of expenditure weeks to payment cycle weeks. To overcome this, we estimate the model as if the week of observation was not recorded in the data. (While this could be interpreted as throwing away information, the results from this analysis lend strength to our results).

[6] We allocate individuals to week four if it is between 24 and 31 days since the last receipt of income.

[Insert Table 6 here]

When the week within the payment cycle is unknown, the conditional expectation over all weeks can easily be calculated using the Law of Iterated Expectations. Taking the term for the conditional expectation allowing for measurement error in (11), we can calculate this over all four weeks (indexed by *k*) as

$$e\left(\widetilde{pc};\theta\right)=E_k\left[e\left(\widetilde{p_k c_k};\theta\right)\right] \tag{13}$$

The probability of being observed in any one week is exactly 0.25 (under random sampling), and so this conditional expectation simply involves calculating the expectation for each week and then taking an average. Calculation of the variance is slightly more complex and again using the Law of Iterated Expectations for the variance, (12) becomes

$$v\left(\widetilde{pc};\theta\right)=E_k\left[v\left(\widetilde{p_k c_k};\theta\right)|\,k\right]+E_k\left[e\left(\widetilde{p_k c_k};\theta\right)^2|\,k\right]-\left(E_k\left[e\left(\widetilde{p_k c_k};\theta\right)|\,k\right]\right)^2. \tag{14}$$

The first term is the expectation of the week specific variances, the second term is the expectation of the square of each week specific conditional expectation and the final term is the expectation of the week specific conditional expectations squared. Again these moments are substituted into (7) to estimate θ.

4.4 *Data Description*

As noted in Section 2, the FES provides expenditure patterns for two consecutive weeks for each individual in the survey and allows us to calculate exactly at what point during the payment cycle each individual is observed. The sample of interest for the estimation of our model is those individuals who are in paid employment and receive income on a monthly basis. The model however, is one of non-durable consumption and so income should be interpreted as a measure of disposable income, i.e. after regular payments such as housing and heating costs, credit repayments etc. are made. The FES gives very detailed information regarding loans and higher purchase schemes outstanding for all individuals but records housing and heating costs at a household level only. Therefore we use a sub-sample of the data used in Section 2. We only include single individuals who are heads of households. For these individuals we can deduct regular costs from monthly labour income to give a measure of disposable income available for non-durable consumption. Using single heads of households also abstracts from the issue of resource pooling in households who receive income at different points in time. The data on these individuals are pooled for three years of the FES 1996-99 giving a total of 727 observations.

[Insert Table 7 here]

The first panel in Table 7 provides some summary statistics with the sample split by median income. As expected, those with higher income also have greater housing costs and credit repayments and are slightly older. The second panel of Table 7 shows the mean of weekly expenditure as a proportion of monthly disposable income for each week in the cycle. There is a large variation in the expenditure share in each week and for plausible values of the parameters, our model would not predict such a wide variation without allowing for measurement error. Notably, the standard deviation of expenditure is greater for those with income below the median. In addition, there are 208 observations of expenditure less than 0.2 of disposable income in the first week which is inconsistent with the minimum expected in week 1 as shown in Figure 2 for acceptable values of the parameters. There is also greater variation in the proportion of expenditure in each week among those with income below the median.

4.5 *Results of estimation of model with Measurement Error*

We report the results of the estimation of the model in Table 8 for different assumptions about the values of the exogenous parameters. The basic specification assumes $\beta = 0.985$, $\delta = 0.4\%$ giving an annualised rate of 25%, $r=0$, $\overline{d} = 0.9$ and a normal distribution for prices[7]. We assume two different values of the discount factor and four different variances for the price distribution, while keeping the mean of prices equal to one. A credit limit of 90% of disposable income is consistent with the debt to income ratio of 25% - 30% reported in Ludvigson (1999), given that the ratio of disposable income to regular income is close to 60% in Table 7. The values chosen for β maintain the assumption from the model in Section 3 that $\beta(1+\delta)<1$. These results are presented in the first two panels of Table 8 for the two values of the discount factor. We also carry out the estimation for interim values β (reported in the last panel of the table) holding the price distribution constant at $N(1,0.2^2)$ (given that this value for the price uncertainty gives the highest pseudo-log likelihood in the first two panels of the table). This implies that 95% of price draws will lie between 0.6 and 1.4, so an individual faces quite a large amount of uncertainty in each week.

[Insert Table 8 here]

Focussing on the upper two panels of the table shows that the estimate of ρ decreases as the variance of the price distribution is reduced. Interestingly, the value of the pseudo likelihood

[7] We examine the level of macro price uncertainty using data from the Retail Price Index (RPI) and the Shop Price Index (SPI), from Dec. 1997 to Aug. 2000. The standard deviation of each of these series normalised by their respective means (so that the series has mean equal to 1) is 0.08% and 0.5% respectively, while the standard deviations of the detrended series are 0.55% and 0.59%. In addition, data from the Bank of England on the variation in expected inflation implies a standard deviation of annual inflation between 0.25% and 1%. We consider these measures as lower bounds of the true variation in prices faced by the individual and thus, we assume values for the standard deviation of prices between 0.5% and 20% in the estimation.

function is also increasing in price uncertainty and this points towards a higher level of uncertainty being a more realistic assumption. In our basic specification the estimates of ρ lie between 1.7 and 2.9. There is a significant difference between these estimates and those for the lower value of the discount factor in the second panel were the estimates are between 6.7 and 6. Table 8 shows that the estimate of ρ decreases monotonically with the value of the discount factor. This arises because both parameters have a similar effect on the conditional expectation of expenditure. A higher degree of impatience implies a lower conditional expectation of future expenditure as expenditure is brought forward to the present, cet. par. A lower risk aversion also decreases the conditional expectation of future expenditure as less precautionary saving occurs and current consumption increases. This correlation is also evident in Gourinchas and Parker (2002) who estimate both of these parameters using data from the PSID and CEX. Their model is one of life-cycle consumption with income uncertainty, but the results also show that lower estimated values of β are consistent with higher estimates of ρ. The mean pseudo log likelihood is highest in Table 8 for the rate of time preference equal to 5% and the standard deviation of prices = 0.2, i.e. individuals are relatively impatient and face substantial uncertainty. A rate of time preference between weeks higher than 5% does not seem to be realistic and therefore the estimation is not carried out for discount factor β less than 0.95.

The point estimates for the standard deviation of the measurement error are very robust to different assumptions about the rate of time preference, price uncertainty, etc. The estimated values all lie between 0.1 and 0.12 giving the variance of measurement error between 0.01 and 0.0144 and are significantly different from zero. Given the variance of the data is 0.0225, the estimates imply that measurement error accounts for between 44% and 64% of the total variation in the data. Hence measurement error is a significant issue in the data and any results which ignore reporting errors may be considerably biased. However, the estimates increase as the assumed variance for prices decrease. This makes intuitive sense because the less of the variance in the data that is attributed to the model, the more of that variation that will be explained by measurement error. The highest value of the mean log likelihood function occurs at the highest assumed value of the variance of the price distribution and the lowest value of the discount factor. Using this criteria, the proportion of variation explained by actual measurement error lies close to 50%. Figure 6 shows the conditional expectation using the estimated values in column one of the first panel of Table 8. The model (allowing for measurement error) fits the data reasonably well when conditioning on expenditure in weeks two and three. However for expenditure in weeks four and one, the model does not perform as well. Firstly, the consumption function in week four is steep for low values of wealth, (see Figure 2). Hence, there is a range of expenditure in week four that implies maximum debt at the end of that week. The model predicts that at minimum wealth, expenditure in week 1 should be at least 0.2 even at the highest price, and therefore a range of week 4 expenditure observations imply the same minimum expected expenditure in week 1. Secondly, all actual expenditure observations below 0.2 in week 1 can only be consistent with minimum wealth and hence all of these observations have the same expectation of expenditure in week 2.

4.6 Further Estimation Results over four weeks

The results from estimating the model without using the observed week in the payment cycle, (described in Section 4.3) are given in Table 9. The most noticeable difference between the results in Tables 9 and 8 is the drop in the values for ρ for each specification. The parameter estimates now range between 1.1 and 4.5. As before, the estimated coefficient of risk aversion increases when the variance of the price distribution is increased. The rate of time preference has a strong effect on these results also with a higher time preference (and lower discount factor) correlated with a higher estimate of relative risk aversion. There is only a small reduction in the estimates of σ, relative to the results in Table 8. However, these lower values of σ reduce the range of the variation in the data that can attributed to measurement error to between 40% and 54%. As in the previous results, the mean pseudo likelihood is highest when we assume $p \sim N(1,0.2^2)$, $\beta = 0.95$.

4.7 Discussion

A coefficient of relative risk aversion of 6 implies that the certainty equivalent income, for a gamble with 2 payoffs of €50,000 and €100,000, each with probability 0.5, is €57,082; for a coefficient of 4, the value is €60,570 and for 2 the value is €66,667. The behavioural consequences of these two measurements are clearly distinct. However, a range of this size is comparable to the range of estimates in the life-cycle literature which ranges between 1 and 5. Gourinchas and Parker (2002) estimate a structural model of consumption from entry into the labour market through to retirement and find estimates between 0.5 and 2. Estimates in Weber (2002) based on a life-cycle consumption model with preferences which are non-separable over time, lie between 0 and 3.8. Other estimates of relative risk aversion have been based on asset pricing models relating consumption volatility to the returns from optimal portfolios. These models require very large levels of risk aversion to explain the risk premium on equities and so these estimates have generally been much greater than those based on the life-cycle models. Blake (1996) estimates the coefficient to be between 7 and 47 and Mankiw and Zeldes (1991) estimate it to be 26.3.

Measurement error is a problem that many studies have highlighted: Hall and Mishkin (1982) assume that classical measurement error exists in data on the level of consumption but not in income data, while most recently Gourinchas and Parker (2002) allow for classical measurement error in the log of consumption. Carroll (1994) criticised Dynan (1993) on the basis that the consumption data used in that study was composed mostly of noise and this drove the results that precautionary motives were not significant. Altonji and Siow (1987) show that measurement error could be responsible for the conflicting conclusions about the validity of the life-cycle model. Lusardi (1996) deals with measurement error issues by using data from two different datasets and concludes that "*measurement error is a severe problem and it should be considered seriously when estimating the prediction of the permanent-income model in microdata*". However, even these last two papers arrive at different conclusions: Altonji and Siow fail to find sufficient evidence against the permanent income model while Lusardi rejects the model finding a significant effect of income growth on consumption growth.

The FES has been used often to analyse many different issues e.g. in Banks et al, (1998) for the analysis of saving and retirement behaviour, in Blundell et al (1993) in the estimation of demand systems, in Browning et. al (1985) to analyse household consumption and labour supply decisions, Harmon and Walker (1995) to estimate the return to schooling, to mention just a few. However dealing systematically with the issue of measurement error is a relatively new practice (Bound et al, 2001). Lewbel (1996) considers the effect of measurement error in the FES in the particular case of the estimation of demand for fuel. He concludes that measurement error has a statistically significant effect and finds that estimates change by more than 15% once the estimation takes measurement error into account. To our knowledge results presented here are the first estimates of the extent of measurement error in the expenditure data in the FES data. Our results suggest that any further analysis using either consumption or income data from the FES takes into account the fact that there is substantial measurement errors (which may distort any empirical results based on the data if ignored)

5. SUMMARY AND CONCLUDING REMARKS

In this paper, we examine short run consumption behaviour. So far, the short run has been overlooked in the literature on consumption. However understanding consumption decisions in the short-run involves understanding the effect of uncertainty and imperfect capital markets on the allocation of consumption over time. Our analysis of the short-run is made more difficult because of the periodic receipt of income. In particular, this leads to the optimal level of consumption depending, in a non-trivial way, on the particular point in the payment cycle at which the consumption decision is made. However, in the short run some aspects of behaviour can be assumed constant (e.g. demographics, labour supply) and this is clearly advantageous when it comes to the empirical analysis. Unsurprisingly, the optimal consumption rule/policy is still based upon smoothing the marginal utility of consumption over time just as in the long run. Hence, the same process achieves the optimal outcome in the short and long run. Consequently, our analysis of behaviour in the short-run is informative about the dependence of the optimal consumption path on uncertainty and imperfect capital markets, for example in a life-cycle setting. Exploiting this similarity to extract valuable information about preferences and behaviour in the long run from the behaviour that we observe in the short-run is, in our mind, the main contribution of this paper. Indeed, it is striking that our empirical findings are broadly consistent with findings found elsewhere in the consumption literature concerned with the life cycle.

In particular, we find that our estimates of the coefficient of relative risk aversion lie generally between 1.2 and 7 depending on the specific scenario chosen. This range of values is well inside the range of acceptable values provided by the literature. Accounting for measurement error proves to be important. Measurement error accounts for a significant part of the observed variance of our data (about 60%) and its omission leads to estimates of the parameter of risk aversion that are substantial biased. In fact ignoring the measurement error makes it difficult to estimate the model since a significant proportion of the data observations are extreme "outliers".

Our work raises some questions about the joint modelling of short run and long run behaviour. In particular it would be interesting to understand how behaviour in the short run (i.e. periods where income receipt, labour supply and expenditure on durables are fixed) interacts with behaviour in the long run (i.e. when all of the above can change) in a world where markets are imperfect and individuals are liquidity constrained, and whether the data at hand would allow us to measure/uncover this interaction.

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TABLE 1

Estimation of Euler Equation on Monthly paid individuals:

Sample split by Liquidity Constraint

	All observations			Non Credit Card holders			Credit Card holders		
	98/99	**97/98**	**96/97**	**98/99**	**97/98**	**96/97**	**98/99**	**97/98**	**96/97**
N	3225	3169	2688	1475	1407	1346	1750	1762	1342
llastpay	0.060*	0.060**	0.044	0.180*	0.026	0.073	0.082*	0.099**	0.024
	0.024	*0.024*	*0.029*	*0.034*	*0.035*	*0.041*	*0.036*	*0.034*	*0.046*
γ_{21}	-0.076**	-0.010	-0.069**	-0.084*	0.023	-0.055	-0.068	-0.038	-0.081
	0.027	*0.027*	*0.029*	*0.035*	*0.038*	*0.041*	*0.040*	*0.039*	*0.044*
γ_{32}	-0.043	-0.049	-0.032	0.007	-0.075*	-0.063	-0.088*	-0.029	-0.0003
	0.028	*0.025*	*0.030*	*0.035*	*0.038*	*0.041*	*0.042*	*0.035*	*0.044*
γ_{43}	0.067*	0.081**	0.125**	0.084*	0.112**	0.191*	0.050	0.053	0.052
	0.026	*0.027*	*0.031*	*0.037*	*0.040*	*0.043*	*0.037*	*0.036*	*0.045*
constant	-0.518*	-0.416**	-0.331	-0.202	-0.186	-0.529*	-0.708*	-0.678**	-0.168
	0.173	*0.170*	*0.201*	*0.234*	*0.238*	*0.270*	*0.263*	*0.250*	*0.331*
F-test	5.16	3.37	5.75	2.79	3.77	7.02	3.75	0.96	1.30
p-value	*0.002*	*0.018*	*0.001*	*0.039*	*0.01*	*0.000*	*0.011*	*0.412*	*0.272*
Mean Income	1138	1081	969	917	888	820	1324	1235	1120

*denotes significance at the 95% level, ** at the 99% level, standard errors are given in italics except for F-test

TABLE 2

Estimation of Euler Equation for Monthly paid individuals:
Sample split by Median Income and Age

	Below Median Income			Above Median Income			Age ≤45			Age≥45		
	98/99	**97/98**	**96/97**	**98/99**	**97/98**	**96/97**	**98/99**	**97/98**	**96/97**	**98/99**	**97/98**	**96/97**
							2240	2209	1935	985	966	753
llastpay	0.067	0.022	0.096*	0.106	0.037	-0.174	0.062*	0.053	0.097**	0.043	0.076	-0.060
	0.042	*0.038*	*0.047*	*0.068*	*0.073*	*0.092*	*0.031*	*0.030*	*0.036*	*0.040*	*0.039*	*0.049*
γ_{21}	-0.129**	-0.040	-0.047	-0.016	0.015	-0.083	-0.120**	-0.040	-0.112**	0.019	0.042	0.029
	0.035	*0.036*	*0.040*	*0.042*	*0.041*	*0.045*	*0.032*	*0.033*	*0.036*	*0.046*	*0.051*	*0.053*
γ_{32}	-0.047	-0.034	-0.042	-0.047	-0.060	-0.028	-0.029	-0.045	-0.051	-0.070	-0.056	0.017
	0.036	*0.035*	*0.040*	*0.041*	*0.037*	*0.045*	*0.034*	*0.031*	*0.035*	*0.048*	*0.046*	*0.058*
γ_{43}	0.118**	0.105**	0.132**	0.016	0.060	0.113**	0.086**	0.085**	0.200**	0.026	0.074	-0.078
	0.035	*0.037*	*0.04*	*0.040*	*0.039*	*0.049*	*0.032*	*0.032*	*0.036*	*0.047*	*0.049*	*0.062*
constant	-0.507	-0.257	-0.702	-0.961	-0.160	1.300*	-0.584	-0.380	-0.715	-0.291	-0.487	0.475
	0.279	*0.246*	*0.294*	*0.501*	*0.540*	*0.662*	*0.220*	*0.213*	*0.245*	*0.271*	*0.278*	*0.344*
F-test	7.81	2.71	3.67	0.67	1.39	2.25	6.45	2.72	11.04	0.73	1.41	0.55
p-value	*0.000*	*0.044*	*0.012*	*0.57*	*0.245*	*0.081*	*0.000*	*0.043*	*0.000*	*0.537*	*0.238*	*0.652*
Median Income	1000	963	880									
Mean Income	623	608	574	1674	1567	1366	1110	1081	972	1195	1123	963

*denotes significance at the 95% level, ** at the 99% level, standard errors are given in italics except for F-test

TABLE 3

Test for equality of variance of growth rates between monthly and weekly paid individuals

	96/97		97/98		98/99	
	Weekly	Monthly	Weekly	Monthly	Weekly	Monthly
N	1604	2688	1676	3169	1645	3225
Mean growth rate of expenditure	-0.049	-0.042	-0.072	-0.062	-0.084	-0.062
Standard deviation of growth rate	0.770	0.934	0.776	0.891	0.841	0.904
F-Test of equality of variances	0.680		0.759		0.867	
p-value	*0.000*		*0.000*		*0.001*	

TABLE 4

Monte Carlo results without Measurement Error

$\beta = 0.95, \overline{d} = 0.66$

Uniform Distribution						
ρ	1.5		2.5		5	
Sample Size	500	1000	500	1000	500	1000
Mean Estimate	1.5025	1.5030	2.5088	2.5053	4.9637	5.1430
Sample SE	0.0199	0.0138	0.1022	0.0733	0.2170	0.1552
Asymptotic SE	0.0189	0.0134	0.0984	0.0692	0.2473	0.1359

Normal Distribution of Prices						
ρ	1.5		2.5		5	
Sample Size	500	1000	500	1000	500	1000
Mean Estimate	1.4746	1.5032	2.4956	2.5108	5.0861	4.985
Sample SE	0.0141	0.0150	0.0876	0.0758	0.1477	0.1981
Asymptotic SE	0.0134	0.0144	0.0734	0.0756	0.1253	0.1756

TABLE 5

Monte Carlo results with Measurement Error

$\sigma = 0.01, \quad \widetilde{p_t c_t} \sim N\left(0.25, 0.05^2\right) \; \sigma^2_{p_t c_t} + \sigma^2 = 0.0026$

ρ	2.5		5	
Sample Size	500	1000	500	1000
Mean Estimate ρ	2.3860	2.4740	5.1343	4.9797
Sample SE	0.22311	0.1317	0.5075	0.2836
Asymptotic SE	0.24352	0.2157	0.8303	0.5095
Mean Estimate σ	0.0099	0.0101	0.00975	0.0102
Sample SE	0.0004	0.0003	0.0005	0.0003
Asymptotic SE	0.0006	0.0004	0.0007	0.0004

TABLE 6

Frequency of Week of Observation

	Week 1	Week 2	Week 3	Week 4	Total
N	189	185	198	155	727
Frequency %	26	25.45	27.24	21.32	100

TABLE 7

*Descriptive Statistics**

	Below Median Income		Above Median Income	
	Mean	Std Dev	Mean	Std Dev
Income	781.65	*200.85*	1556.79	*608.05*
Housing Cost	250.58	*135.63*	450.60	*281.91*
Credit Repayments	42.77	*73.30*	71.07	*103.22*
Disposable Income	437.13	*192.79*	969.88	*569.60*
Age	32.95	*10.06*	35.68	*8.62*
Weekly expenditure as a share of monthly disposable income				
Week 1	0.28	*0.30*	0.19	*0.17*
Week 2	0.28	*0.30*	0.20	*0.25*
Week 3	0.27	*0.28*	0.21	*0.25*
Week 4	0.30	*0.31*	0.21	*0.20*

*All monetary amounts are monthly or monthly equivalents

TABLE 8

Estimates of Coefficient of Relative Risk Aversion ρ *and Measurement Error* σ

$\bar{d}$ = 0.9, β = 0.985

Price Distribution	$N(1,0.2^2)$	$N(1,0.1^2)$	$N(1,0.05^2)$	$N(1,0.01^2)$	$N(1,0.005^2)$
Mean loglikelihood	2.774	2.741	2.734	2.732	2.732
ρ	2.927	2.434	1.869	1.794	1.753
se	*6.e-05*	*4e-06*	*3e-04*	*2e-04*	*9.e-05*
σ	0.116	0.120	0.120	0.120	0.120
se	*2e-06*	*2e-06*	*3e-06*	*2e-06*	*2e-06*

$\bar{d}$ = 0.9, β = 0.95

Price Distribution	$N(1,0.2^2)$	$N(1,0.1^2)$	$N(1,0.05^2)$	$N(1,0.01^2)$	$N(1,0.005^2)$
Mean loglikelihood	2.816	2.756	2.736	2.729	2.729
ρ	6.684	6.697	6.072	6.386	6.386
se	*0.009*	*6e-04*	*2e-05*	*1e-06*	*1e-05*
σ	0.109	0.118	0.119	0.120	0.120
se	*4e-06*	*2e-06*	*3e-06*	*2e-06*	*1e-06*

$\bar{d}$ = 0.9, $p \sim N(1,0.2^2)$

β	0.985	0.975	0.965	0.95
Mean loglikelihood	2.774	2.792	2.804	2.816
ρ	2.927	4.010	5.10	6.684
se	*6e-05*	*3e-04*	*0.001*	*0.009*
σ	0.116	0.112	0.111	0.109
se	*2e-06*	*2e-06*	*3e-06*	*4e-06*

TABLE 9

Estimates of Coefficient of Relative Risk Aversion ρ and Measurement Error σ when week of observation is assumed unknown

$\bar{d}$ = 0.9, β = 0.985

Price Distribution	N(1,0.2^2)	N(1,0.1^2)	N(1,0.05^2)	N(1,0.01^2)	N(1,0.005^2)
Mean loglikelihood	2.852	2.826	2.824	2.823	2.823
ρ	2.322	1.509	1.280	1.187	1.172
se	*6e-05*	*4e-05*	*1e-04*	*9e-05*	*8e-05*
σ	0.105	0.109	0.109	0.109	0.109
se	*4e-06*	*3e-06*	*3e-06*	*3e-06*	*3e-06*

$\bar{d}$ = 0.9, β = 0.95

Price Distribution	N(1,0.2^2)	N(1,0.1^2)	N(1,0.05^2)	N(1,0.01^2)	N(1,0.005^2)
Mean loglikelihood	2.891	2.846	2.828	2.822	2.821
ρ	4.531	4.542	4.138	3.954	4.092
se	*4e-05*	*1e-04*	*1e-04*	*3e-04*	*9e-05*
σ	0.109	0.107	0.109	0.109	0.109
se	*4e-06*	*4e-06*	*3e-06*	*4e-06*	*3e-06*

$\bar{d}$ = 0.9, p~ N(1,0.2^2),

β	0.985	0.975	0.965	0.95
Mean loglikelihood	2.852	2.877	2.894	2.891
ρ	2.322	3.599	3.800	4.531
se	*6e-05*	*0.019*	*0.001*	*4e-05*
σ	0.105	0.101	0.099	0.109
se	*4e-06*	*4e-06*	*5e-06*	*4e-06*

FIGURE 1

Average Non-Durable Expenditure per Income Group over Payment Cycle[8]

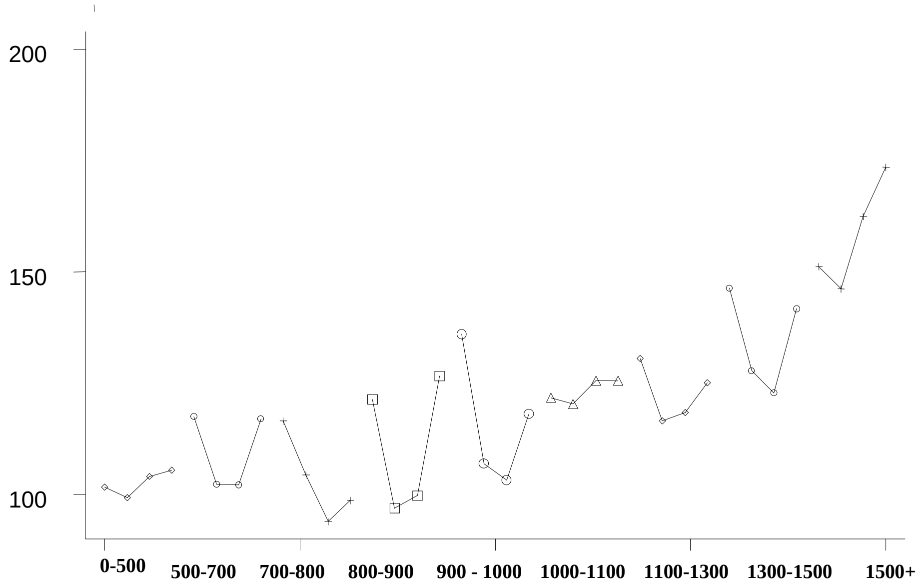


FIGURE 2

Numerical Solution to Model with Four Weeks

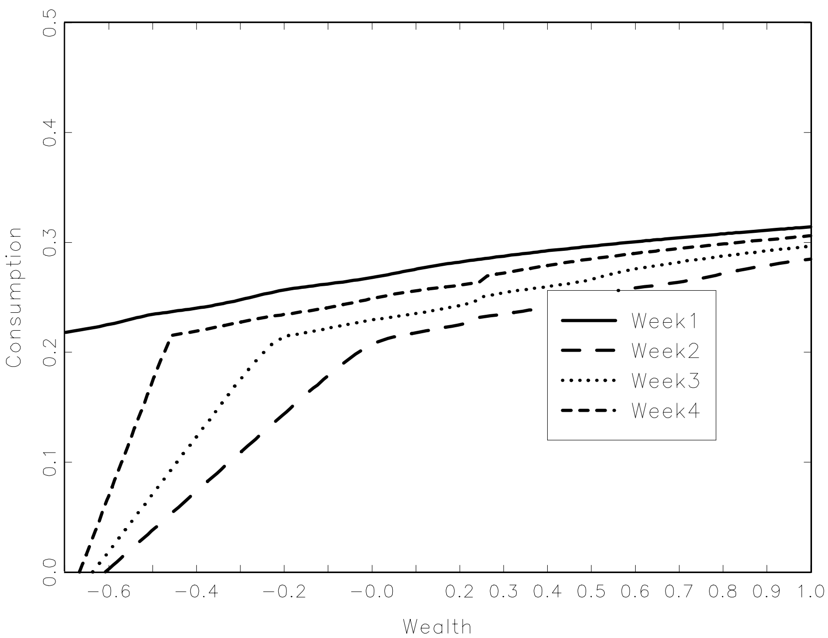


FIGURE 3

Conditional Expectation

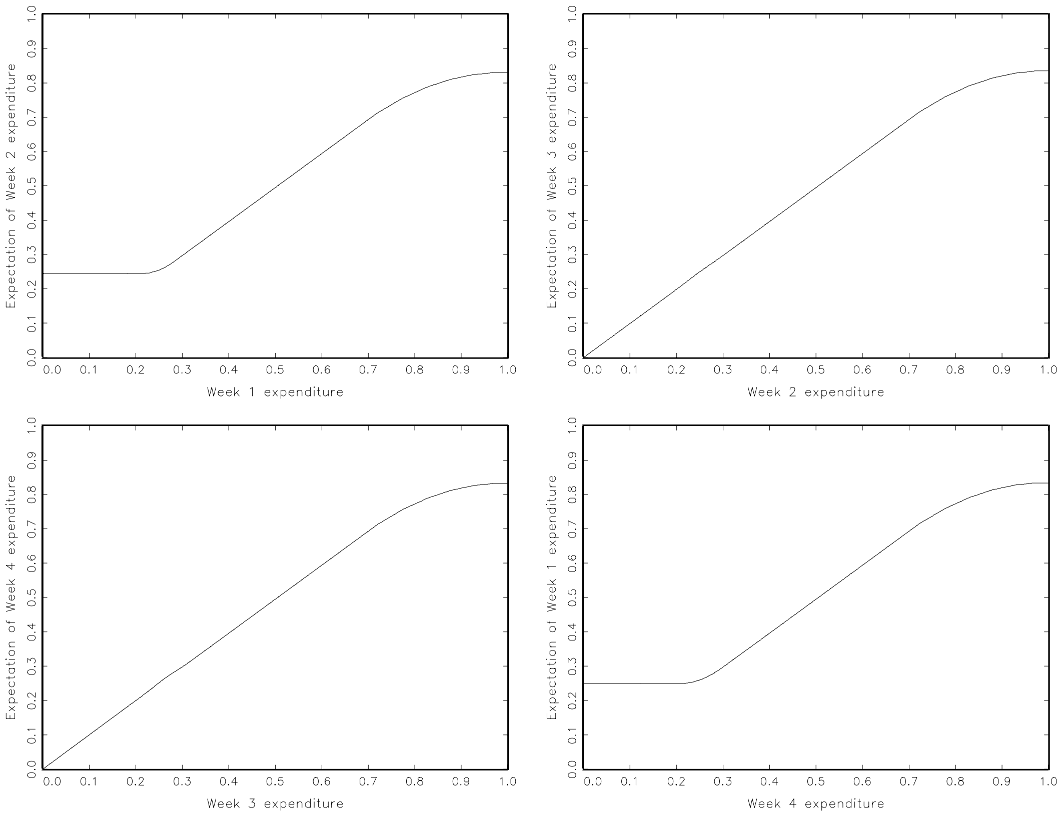
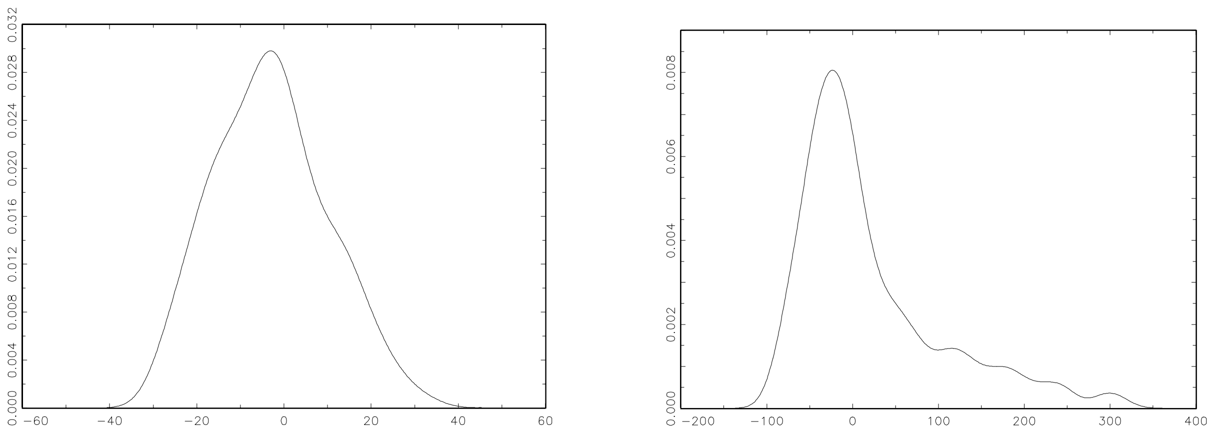


FIGURE 4

% change in expenditure between second and third week



(a) Simulated data **(b) FES data**

[8] The income groups are defined as: 0-499; 500-699; 700-799; 800-999; 1000-1099; 1100-1299; 1300-1499; greater than 1500.

FIGURE 5

Conditional Expectation with Measurement Error

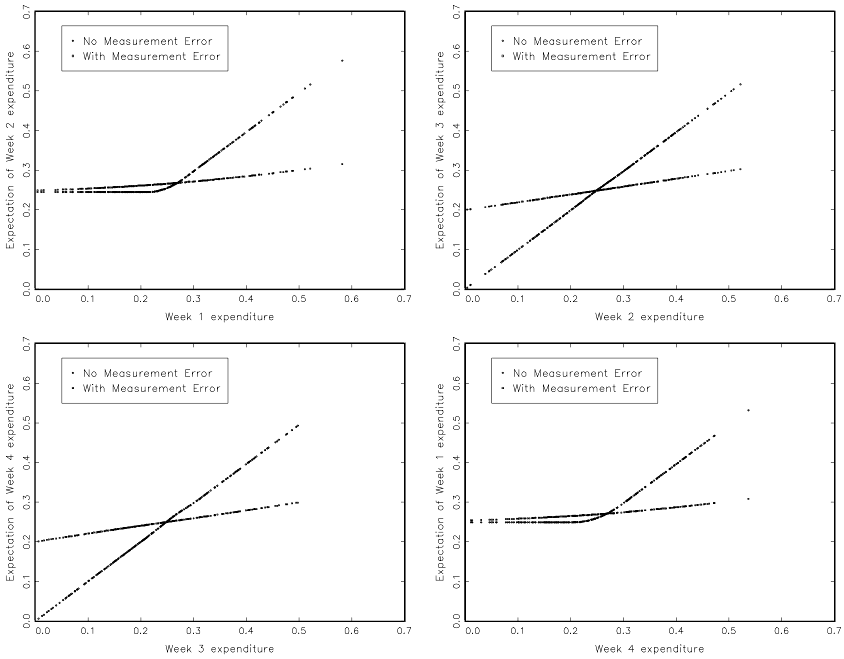


FIGURE 6

Conditional Expectation using FES data: $\bar{d}$ *= 0.9,* ρ *= 2.9,* σ *= 0.12,* β*=0.985*