Using [2] we found the Nash's position optimal control

$$k^* = \frac{1}{2}x^4, \quad m^* = \frac{1}{2}x^3, \quad a^* = \frac{1}{2}x^2, \quad w^* = \frac{1}{2}x, \quad p^* = 0$$

for our forest ecosystem model with
$Q_1 = \alpha x^6 + \frac{1}{2}\left(\frac{1}{2}x^8 + x^6 + x^4 + x^2\right), Q_2 = \alpha x^6 + \frac{1}{2}\left(x^8 + \frac{1}{2}x^6 + x^4 + x^2\right),$
$Q_3 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + \frac{1}{2}x^4 + x^2\right), Q_4 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + x^4 + \frac{1}{2}x^2\right),$
$Q_5 = \alpha x^6 + \frac{1}{2}\left(x^8 + x^6 + x^4 + x^2\right).$
For this optimal control productivity x asymptotically tends to zero with $t \to +\infty$, i.e. to characteristic observed value of productivity in region, but dynamics of forest ecosystem is not asymptotically stable. Since $u_i^* > 0$ for $x > 0$, then we have a slowly degrading forest.

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DIFFERENTIAL EQUATIONS OF MOTIONS OF MULTI-AXIS SYSTEMS

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The problem of program motion synthesis is generally solved without uniqueness and control functions realizing the motion and minimizing a functional must be obtained.

Differential equations of motions of multi-axis systems based on linear spepping motors [1, 2] can be represented as

$$\dot{x}_i = p_i(\mathbf{x}) + u_i(\mathbf{x})b_i(\mathbf{x}), \quad i = 1, \dots, n. \tag{1}$$

where $\mathbf{x} = (x_1, \dots, x_n)$ are generalized device coordinates, $\mathbf{u} = (u_1, \dots, u_n)$ is the control vector.

The problem consists in forming controls $u = (t, \mathbf{x})$ such that $u \in \mathbf{R}^r$ and corresponding solution of the system (1) satisfies the additional conditions

$$\omega_k(t, \mathbf{x}) = 0, \quad k = 1, \dots, r. \tag{2}$$

However, if $\mathbf{x} = \mathbf{x}(t)$ is a solution satisfied the program (2) then $\omega_k(t, \mathbf{x}(t)) \equiv 0$, $k = 1, \dots, r$.

Whence

$$\frac{d}{dt}\omega_k(t, \mathbf{x}(t)) \equiv 0, \quad k = 1, \dots, r$$

or

$$\sum_{i=1}^{n}\left(\frac{\partial\omega_k(t,\mathbf{x})}{\partial x_i}(p_i(\mathbf{x}) + u_i b_i(\mathbf{x})) + \frac{\partial\omega_k(t,\mathbf{x})}{\partial t}\right) \equiv 0,$$

when $\mathbf{x}$ satisfies (2).

The last expression is equivalent to the condition

$$\sum_{i=1}^{n}\left(\frac{\partial\omega_k(t,\mathbf{x})}{\partial x_i}(p_i(\mathbf{x}) + u_i b_i(\mathbf{x})) + \frac{\partial\omega_k(t,\mathbf{x})}{\partial t}\right) = R_k(t, \mathbf{x}, \omega_k), \quad k = 1, \dots, r, \tag{3}$$

where R_k is the arbitrary functions such that $R_k(t, \mathbf{x}, 0) \equiv 0$.

Therefore, the condition (3) is neccessary and sufficient for implementing the program (2) along solution $\mathbf{x} = \mathbf{x}(t)$ of system (1). It can be used for calculating the neccessary controls $u_i(t, \mathbf{x})$, $i = 1, \dots, r$.

As $r < n$, the system (3) defines the controls ambiguously, and the functional must be minimized on free controls additionally. E.g. the control optimization problem with constraints

$$u(t,\mathbf{x}) \in \tilde{U}, \quad \omega_k(t,\mathbf{x}) = 0, \quad k = 1,\dots,r$$

can be considered for each time moment.

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SOME GEOMETRICAL ASPECTS OF INFINITE-DIMENSIONAL DYNAMICAL SYSTEMS

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Denote $U = C^2([t_0,t_1],U_1), V = C([t_0,t_1],V_1)$, where U_1, V_1 are normed linear spaces over the field of real numbers $\mathbb{R}$.

Let the position of infinite-dimensional dynamical system be described by the function $u \in U$, satisfying the conditions $u|_{t=t_0} = u_0,\ u|_{t=t_1} = u_1$, where u_0, u_1 are elements from U_1. In what it follows we will use the notations and the terminology of papers [1-3]. Let us consider the bilinear form $\langle\cdot,\cdot\rangle : U_1 \times V_1 \to \mathbb{R}$ and the kinetic energy $T[u,u_t] = \frac{1}{2}\langle u_t, A_u u_t\rangle$, where A_u is a linear Gateaux differentiable operator, in general depending on u in a nonlinear way.

Let us denote $A'_u(h;g) = \frac{d}{d\varepsilon}A_{u+\varepsilon g}h|_{\varepsilon=0}$, $F[u] = \int\limits_{t_0}^{t_1} T[u,u_t]dt$ and let $f(t,u,u_t)$ be a density of acting forces.

Theorem 1. *There is an equality* $gradF[u] = \frac{1}{2}(A_u + A_u^*)u_{tt} + \frac{1}{2}[A'_u(u_t;u_t) + A_u^{*\prime}(u_t;u_t) - A_u^{\prime *}(u_t;\cdot)u_t]$, *where* $(\cdots)^*$ *is a conjugate operator.*

Theorem 2. *The operator* k_{1u} *of the kind*

$$k_{1u}[u_t] = A'_u(u_t;u_t) + A_u^{*\prime}(u_t;u_t) - A_u^{\prime *}(u_t;\cdot)u_t$$

is an analog of the Christoffel symbols of the first kind.

Let us note that the equations of motion of the considered dynamical system can be presented in the form

$$\frac{1}{2}(A_u + A_u^*)u_{tt} + \frac{1}{2}k_{1u}[u_t] - f(t,u,u_t) = 0. \tag{1}$$

Theorem 3. *If there exists the inverse operator* $(A_u + A_u^*)^{-1}$, *then the operator* k_{2u} *of the kind*

$$k_{2u}[u_t] = (A_u + A_u^*)^{-1}k_{1u}[u_t]$$

defines an analog of the Christoffel symbols of the second kind.

Theorem 4. *Operator* $\frac{D}{Dt}$ *defined by*

$$\frac{Du_t}{Dt} = u_{tt} + (A_u + A_u^*)^{-1}k_{1u}[u_t]$$

is an analog of the covariant derivative of u_t *with respect to* t.

Consequence. *Evolutionary equation (1) can be written in the form*

$$\frac{1}{2}(A_u + A_u^*)\frac{Du_t}{Dt} - f(t,u,u_t) = 0.$$

This form of equation has a special interest in connection with Riemannian geometry.

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