Simulation Approach for Assessing the Performance of the γEWMA Control Chart

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Abstract

Purpose –The purpose of this paper is to evaluate the performance of a modified EWMA control chart (γEWMA control chart), which considers data distribution and incorporates its correlation structure, simulating in-control and out-of-control processes. To select an adequate value for smoothing parameter, with these conditions.

Design/methodology/approach –This paper is based on a simulation approach using the methodology for evaluating statistical methods proposed by Morris et al., (2019). Data were generated from a simulation considering two factors that associated with data : i) quality variable distribution skewness (γ) as an indicator of quality variable distribution (f); ii) the autocorrelation structure (φ) for type of relationship between the observations and modeled by AR(1). In addition, one factor associated with the process was considered, i) the shift in the process mean (δ). In the following step, when the chart control is modeled, the fourth factor intervenes. This factor is a smoothing parameter, (λ). Finally, three indicators defined from the Run Length are used to evaluate γEWMA control chart performance with non-normal and non-independent observations, and their interactions.

Findings – Interaction analysis for four factor evidence that the modeling and selection of parameters is different for out-of-control and in-control processes therefore the considerations and parameters selected for each case must be carefully analyzed. For out-of-control processes, it is better to preserve the original features of the distribution (mean and variance) for the calculation of the control limits. It makes sense that highly autocorrelated observations require smaller smoothing parameter since the correlation structure enables the preservation of relevant information in past data.

Originality/value –The γEWMA control chart there has advantages because it gathers, in single chart control: the process and modelling characteristics, and data structure process. Although there are other proposals for modified EWMA, none of them simultaneously analyze the four factors nor their interactions. The proposed γEWMA allows setting the appropriate smoothing parameter when these three factors are considered.

Keywords: Average run length, EWMA control chart, Autoregressive processes, Skewed distributions, Simulation study.

1. Introduction

Manufacturing processes can be monitored by using statistical process control (SPC) charts. Control charts are powerful tools that are used to monitor and analyze a quality variable over a period. Industrial processes perform well if quality variables are maintained within specified limits and under control. If a quality variable exhibits random behavior, the changes are the result of variability that is inherent in the process (common causes), in other cases, non-random variations are due to external factors (special causes). In the event of a special cause, the process is unstable, and it will take time to correct itself before returning to a condition of balance; this time is known as the process time lag (Hori and Skogestad, 2007; Neubauer, 1997).

In practice, control chart limits are often calculated using parameter estimates from an in-control Phase I reference sample. Once an in-control reference sample has been established, the parameters of the process are estimated from this Phase I sample, and the control limits are estimated for use in Phase II (Human et al., 2011; Jensen et al., 2006).

Industrial processes as chemical processes or materials casting, in which preparations or mixtures are made, features such as the statistical distribution of observations and autocorrelation structure data should be analyzed, because if they are ignored could cause wrong conclusion about control process. These features must be analyzed through chart control using the smoothing parameter adequate. Furthermore, the control chart must have ability to identify different levels of shift. Data distribution: The processes that use SPC, usually assume that the distribution of the data is normal, without considering that the data may have other distributions. However, it is important to study the effects of non-normality when individual data is considered.

i) Non-independent data: If the process is influenced by the same causes then autocorrelation can cause non-random behavior. If it is an inherent part of the common-cause variability and cannot be eliminated, then it must be considered in the design of the control chart. The autocorrelation structure is a condition defined by data, and it is modelled by ϕ parameter.
ii) The weight that is assigned for past observations (λ): In industrial processes, often, failures have manifested themselves previously and, therefore, subtle symptoms appear. Hence, the analysis for past observations is an important tool to decide whether the process is going to become out-of-control before this failure occurs. λ is a parameter that is defined by the analyst and used as chart parameters.

iii) Shift in the process mean (δ): The power of a control chart is its performance in identifying various shifts in the process mean. Therefore, knowledge of the size of the changes that the chart can identify becomes a useful tool for process monitoring. δ is a process condition that is unknown, but it is important to analyze chart performance for low, medium, and high shift.

EWMA control charts have optimal properties about control applications because this chart considers the past observations using a model with memory (MacCarthy and Wasusri, 2002). In the literature, Borror et al. (1999), Horng Shiau and Ya-Chen (2005), present several advantages of EWMA control charts: i) EWMA considers the previous and current information of the process; ii) EWMA is robust to nonnormality, iii) EWMA can detect small shifts in the process mean, and iv) EWMA weights samples in geometrically decreasing order so that the most recent samples are weighted most highly while the most distant samples contribute less depending on the smoothing parameter.

The use of EWMA as a tool for monitoring the variability of the process has received attention in the literature. Nevertheless, studies about the simultaneous occurrence of these four features are few. Likewise, there are few studies on chart performance for detecting possible changes in the process, when data do not satisfy the assumptions regarding independence and distribution. According to a review of the literature, at most three problems are analyzed simultaneously. No simultaneous analysis of the four problems was found.

In this article, we study the performance of a modified EWMA control chart (γ EWMA) when three factor about data and process are considered. Finally, the smoothing parameter (λ) is suggested to best perform the EWMA. Three performance measures we used to study the performance of the γEWMA. Different scenarios regarding shift in the process mean are analyzed. The remainder of this paper is organized as follows: In Section 2, the literature review focus on modified EWMA control charts is presented. In Section 3, the basic concept of EWMA is introduced. The simulation methodology and the γEWMA control chart are presented in Section 5. In addition, various cases and the main results are compared in Section 6. The conclusions of this study are presented in the final section.

2. Literature Review

As mentioned above, some industrial processes have four characteristics that influence the conclusions about process control. For this literature review, we considered papers that discussed processes control methods for one or several of these characteristics. In literature, around of 34 papers was found about modified EWMA control chart. However, these modifications analyze one of the four characteristics independently.

Most common EWMA control charts assume normality, which is reflected in publications such as Crowder (1987), Lucas and Saccucci (1990), Jones *et al.* (2001), Koehle *et al.* (2001), Khoo *et al.* (2015), Dawod *et al.* (2017), and Supharakonsakun, *et al.* (2019). A EWMA control charts propriety analyzed the accumulation of previous and current information then the past is hefted, and history is measured in the hope that it is predictive. Among the studies, Lucas and Saccucci (1990) stand out: despite its assumption of independent observations, for this topic, this work plays a pivotal role. The choices of λ and L for the univariate EWMA control chart are discussed in detail, and this work has been cited 1872 times.

However, in our literature review we consider studies that include the distributional assumption. Borror et al. (1999) analyzed the performance of the EWMA control chart while assuming that data are gamma-distributed under six shifts in the process mean and three smoothing parameters. Via the same approach, Horng Shiau and Ya-Chen (2005) discussed the effects of non-normality (t and gamma distributions) and nonindependent (AR(1) process) on EWMA and Shewhart control charts. They show that the EWMA control chart performance is better if the parameters are well selected. Human et al. (2011) used a simulation study of the EWMA control chart behavior for nonnormal distributions, namely, they considered the skewness parameter. They identified a relationship between the distribution and the skewness and studied the effect of the skewness on the performance of the EWMA control chart. Recently, Zheng and Chakraborti, (2016) evaluate robustness to non-normality of Adaptive Exponentially Weighted Moving Average (AEWMA) and they show that AWEMA chart is sensitive to shape assumption and reason why purpose a nonparametric chart called NPAEWMA. Lin *et al.*, (2017) evaluate performance of EWMA median chart under various distributions finding that their proposed is more efficient than the EWMA the EWMA average chart in detecting the shift of the process mean. In addition, Osei-Aning et al (2020) consider normal and non-normal data on bivariate EWMA.

Other feature that we wanted consider is time-dependence. In the event of a special cause, the process is unstable, and it will take time to correct itself and to balance itself; this time is known as the process time lag. The process time lag depends on the ability of a process to continue in a state even after a change has occurred; this ability is known as inertia. During the time lag, the quality variables depend on the same root causes, which leads to autocorrelated data. Nevertheless, independent observations are assumed in most of the procedures that are used in quality control. When this assumption is not satisfied due to biased estimates of parameters, high false-alarm rates and slow detection of process changes can occur (Reynolds and Lu, 1997). In the literature, a few authors have studied modified EWMA charts with autocorrelated data, which they have proposed solutions for analyzing processes that have these characteristics (Crowder, 1987; Wieringa, 1999; Horng Shiau and Ya-Chen, 2005; Patel and Divecha, 2011; Maroš *et al.*, 2011; Morais et al., Asghari Torkamani *et al.*, 2014; Herdiani *et al.*, 2018 and, Okhrin and Schmid, 2019). When process observations are correlated, a methodology used involves fitting an autoregressive model using the time series models. If the observations are independent, ϕ takes the value of zero ($\phi = 0$), and if the observations are non-independent, ϕ takes nonzero values ($\phi \neq 0$). In the literature, the effects of the shift in the process mean and the autocorrelation parameter over ARL have been studied via two approaches: i) the use of standard charts for the residuals of a time-series model that is adjusted over the original data and ii) the use of standard charts with control limits that are adjusted for non-independent observations. Prajapati and Singh (2016) analyzed 110 papers which presented method for monitoring the autocorrelated process parameters. They presented a view broadly and compendious summary of the work on the development of the control charts for variables to monitor processes for autocorrelated data.

Harris and Ross (1991), Koehle *et al.* (2001), and Dawod *et al.* (2017) analyzed the effect of autocorrelation on ARL by considering a shift in the process mean; however, they did not evaluate the effect of the smoothing parameter or the effects of the distribution observations. Ulkhaq and Dewanta (2017) and Supharakonsakun *et al.* (2019) considered the effect of the smoothing parameter but only for normal observations. Borror *et al.* (1999) and Human *et al.* (2011) analyzed the effects of the distribution on ARL under various shifts in the process mean and smoothing parameters; however, they did not evaluate the effect of the autocorrelation observations. Finally, Horng Shiau and Ya-Chen (2005) analyzed the effects of the distribution on ARL under various shifts and under different smoothing parameters in autocorrelated data, they used Wieringa's propose (Wieringa, 1999); however, Horng Shiau and Ya-Chen (2005) did not evaluate the effect of the autocorrelation observations because they presented one value as an autocorrelation parameter.

In referring to control chart performance, in the literature, it has been evaluated using indicators that were derived from the Run-Length (RL). These indicators include the Average-Run-Length (ARL), the Standard-Deviation-Run-Length (SDRL), and the Median-Run-Length (MRL) (B. C. Khoo et al., 2015; Borror et al., 1999; Dawod et al., 2017; Esparza Albarracin et al., 2018; Human et al., 2011; Jones et al., 2001). For instance, it is desirable for the ARL to be large if no assignable cause has occurred, and small if one out-of-control condition has occurred. The effect of parameter estimation on the control chart performance has been studied in the literature. These studies have concluded that the effects of parameter estimation on the control chart properties should not be ignored (Crowder and Wiel, 2014; Dawod et al., 2017; Esparza Albarracin et al., 2018; Jensen et al., 2006; Lucas and Saccucci, 1990; Reynolds and Lu, 1997; Saleh et al., 2013).

In Figure 1, we present a visualization of a network of 34 publications using the methodology proposed by Van Eck and Waltman (2009). This methodology is based on terms that are cited in academic research publications that, for our case, the words are related to the EWMA control chart and the four features for data that are generated in the monitoring of industrial processes (Van Eck and Waltman, 2009). Publications are mainly concentrated between 2005 and 2020; however, in the bibliographic analysis, publications prior to this date were also considered.

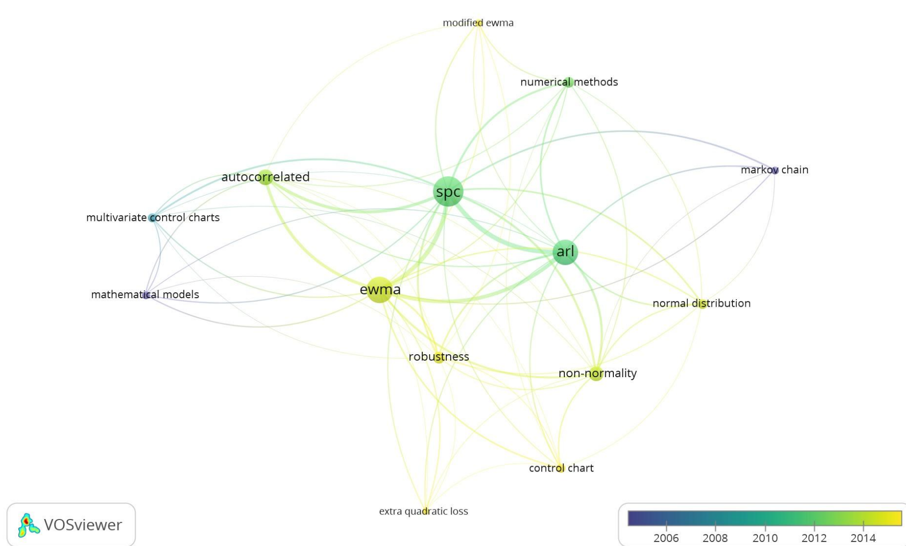

Figure 1. Bibliometric map for EWMA control chart

It is essential to conduct a conjoint analysis on the behaviors of these four features, which are of concern in the industrial field. Therefore, knowledge of the performance of the EWMA control chart that is based on the parameters associated with the four described features enables the identification of possible interactions among these features, thereby enabling the design of more effective control charts.

3. EWMA control chart approach

3.1 EWMA control chart

Assume that $Y_1, Y_2, ..., Y_n$ are independent and identically distributed (i.i.d.) observations with an in-control mean of μ_0 and a standard deviation of σ. The EWMA control chart statistic for individual observations are defined as

$$Z_t = \lambda Y_t + (1 - \lambda)Z_{t-1}, \qquad (1)$$

for $i = 1, 2...., n$, where the constant $0 < \lambda \leq 1$ is the smoothing parameter and Y_t is the current observation. Setting the value of $\lambda = 1$ yields a Shewhart control chart. A value of $\lambda = 1$ implies that only the most recent measurement influences the EWMA. Thus, a large value of $\lambda = 1$ does not give weight for old data; a small value of λ gives more weight for old data. The initial value of Z_t is the target value or the mean in-control value; hence, $z_0 = \mu_0$, and the shift in the mean is $\delta\sigma_Y$, with $\delta \geq 0$. As the observations are independents with variance σ^2, the variance of Z_t is

$$\sigma^2_{Z_t} = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1 - (1 - \lambda)^{2t}\right]. \qquad (2)$$

Therefore, the EWMA control chart is constructed by plotting Z_t against the sample number t. The upper control limit (UCL) and the low control limit (LCL) for the EWMA control chart are as follows:

$$UCL/LCL = \mu_0 \pm L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1 - (1 - \lambda)^{2t}\right]},$$

where $L > 0$ determines the width of the control limits (Crowder, 1987; Lucas and Saccucci, 1990; Roberts, 1959).

3.2 Non-normal EWMA Control Chart

Human *et al.* (2011) assume that $Y_1, Y_2, ..., Y_n$ are i.i.d. observations with an in-control mean μ_{f_0} and standard deviation σ_{f_0}. The observations come from distributions such as uniform, right-triangular, standard normal, T-student, gamma, symmetric bimodal, asymmetric bimodal and contaminated normal distributions. Hence, the values of μ_{f_0} and σ_{f_0} vary according to the distribution. The EWMA control chart statistic is defined by equation (1). The components to Z_t, UCL and LCL for the non-normal EWMA control chart are similar to normal EWMA Control Chart, the difference is that for Z_t, $\mu_0 = \mu_{f_0}$, and the variance to Z_t depende of the variance distribution (σ_{f_0}), and is expressed as shown equation (3).

$$\sigma^2_{Z_t} = \sigma^2_{f_0}\left(\frac{\lambda}{2-\lambda}\right)\left[1 - (1 - \lambda)^{2t}\right] \qquad (3)$$

Authors have used the gamma distribution (Borror et al., 1999; Horng Shiau and Ya-Chen, 2005; Human et al., 2011), the t distribution (Horng Shiau and Ya-Chen, 2005; Human et al., 2011) and the lognormal distribution (Horng Shiau and Ya-Chen, 2005) to compare their performances with control charts that are based on the normal distribution.

3.3 Modified EWMA control chart with non-independent observations

In the literature is proposed a modified EWMA control chart for non-independent observations (Crowder, 1987; Horng Shiau and Ya-Chen, 2005; Wieringa, 1999). Let Y_t be the quality variable that is observed at time t of the $AR(1)$ process that is described by:

$$Y_t - \mu = \phi\left(Y_{t-1} - \mu\right) + \varepsilon_t, \quad for\ t \in T \qquad (4)$$

Y_t is the observed value at time t, ϕ is a constant that satisfies $\phi \in (-1, 1)$ and $\{\varepsilon_t\}$ is a sequence of i.i.d. disturbances with mean zero $\left(\mu_\varepsilon = 0\right)$ and constant variance σ_ε^2.

The corresponding EWMA statistic at time t is defined by equation (1). The initial value z_0 for the statistic is chosen to be the in-control or target the process mean ($z_0 = \mu_0 = 0$). For determining the control limits of the modified EWMA control chart for $AR(1)$ data, the variance of Z_t for large t is approximated as

$$\sigma_{Z_t}^2 \approx \left(\frac{\sigma_\varepsilon^2}{1-\phi^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}\right). \qquad (5)$$

The UCL and LCL for the modified EWMA control chart are as follows:

$$\frac{UCL}{LCL} = \mu_\varepsilon \pm L\sigma_\varepsilon\sqrt{\left(\frac{1}{1-\phi^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}\right)}$$

Authors such as Harris and Ross (1991), Koehle et al. (2001), and Dawod *et al.*, (2017) considered the non-independent observations in their simulation studies.

3.4 Modified EWMA with distributional consideration - γEWMA control chart

Let Y_t be the quality variable that is observed with the gamma, lognormal and standard normal distributions at time t of the $AR(1)$ process that is described by equation (4), where $\{\varepsilon_t\}$ is a sequence of i.i.d. innovations with mean μ_ε and constant variance σ_ε^2. The innovations come from the gamma, lognormal, and standard normal distributions. Hence, the values of μ_ε and σ_ε vary according to the distribution.

The corresponding EWMA statistic at time t is defined by equation (1). The initial value z_0 for the statistic is chosen to be the in-control or target the process mean ($z_0 = \mu_0 = \frac{\mu_\varepsilon}{(1-\phi)}$). Note, the values of μ_ε and σ_ε vary according to the distribution. For determining the control limits of the γEWMA control chart for $AR(1)$ data, the approximate variance of Z_t for large t is expressed as equation (5), then we propose UCL and LCL for γEWMA control chart:

$$UCL/LCL = \mu_0 \pm L\sigma_\varepsilon\sqrt{\left(\frac{1}{1-\phi^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}\right)}$$

4. Performance measures

The performance of a control chart technique is typically evaluated in terms of the Run-Length (RL), which is defined as the number of observations that are plotted before a signal is observed. Three performance measures that are based on RL are as follows: i) the Average Run Length (ARL), which measures the number of points that, on average, will be plotted on a control chart, before an out-of-control condition is indicated. If in-control process, the ARL indicated a false alarm and if out-of-control process, ARL indicated a real condition; ii) the Average Relative Distance (ARD), which compares the ARL performances between skewed and normal distributions (Horng Shiau and Ya-Chen, 2005), and iii) the Average Reference Ratio (ARR), which measures the control chart relative capability to identify a condition process (in-control and out-of-control) (Dawod et al., 2017).

One of the most common measures of control-chart performance is the ARL, which is the expected number of consecutive samples to be taken until the sample statistic falls outside the control limits. For an in-control process, a chart should have a large ARL. In contrast, for an out-of-control process, a chart should have a small ARL, namely, it should detect the out-of-control condition quickly (Jones et al., 2001). The statistical design of control charts considers the in-control and out-of-control ARLs that result from the sample size and the control limits that are chosen by the user (Human et al., 2011; Roberts, 1959).

When the process is in-control and there is no change in the quality variable ($\delta = 0$), the ARL is denoted as ARL_0. A large ARL_0 is desired. Using the normal distribution with independent observations as reference, the range of reference values are $ARL_0 \geq 370.4$. In contrast, for an out-of-control process with changes in the quality variable ($\delta > 0$), the ARL is denoted as ARL_1. For out-of-control processes, with normal distribution and independent observations, small ARL_1 is desired (close to 1).

The second performance measure is the ARD. Given the values of ϕ, λ, γ, δ and f, as in Horng Shiau and Ya-Chen (2005), we define the ARD between the ARL value of the chart for the distribution under study and that of the standard normal distribution by

$$ARD(\delta|\phi, \lambda, \gamma, f) = \frac{ARL_{NonNormal}(\delta|\phi,\lambda,\gamma,f) - ARL_{Normal}(\delta|\phi,\lambda,\gamma,f)}{ARL_{Normal}(\delta|\phi,\lambda,\gamma,f)}$$

If the process is under control, namely, $\delta = 0$, we denote the ARD as ARD_0. If $-1 < ARD_0 < 0$, the normal distribution outperforms the skewed distributions (gamma and lognormal) because its ARL_0 exceeds those of the other studied distributions. If $ARD_0 \rightarrow 0$, the skewed and the normal distributions have similar ARL_0 values. Then, if $ARD_0 > 1$, the skewed distributions outperform the normal distribution because the ARL_0 is larger than the ARL_0 of the normal distribution. If the process is out-of-control, namely, $\delta > 0$, we denote the ARD as ARD_1. Since for out-of-control processes a small ARL_1 is desirable, ARD_1 exhibits opposite behavior to ARD_0. If $-1 < ARD_1 < 0$, the skewed distributions outperform the normal distribution. If $ARD_1 \rightarrow 0$, the skewed and the normal distributions have similar ARL_1 values. Then, if $ARD_1 > 0$, the normal distribution outperforms the skewed distributions.

The third measure is the ARR. Given the values of ϕ, λ, γ, δ and f, we modified the ratio that is defined in Dawod et al. (2017) by considering the relative distance

$$ARR = \frac{ARL(\delta|\lambda,\gamma,\phi)}{ARL(\phi=0, \delta, \gamma=0|\lambda)}$$

Note that the reference case is the normal distribution with uncorrelated observations. If the process is in-control, namely, $\delta = 0$, we denote the ARR as ARR_0. If $ARR_0 < 1$ and the observation have known features (λ, γ, ϕ), then the chart control will have lower capacity to identify in-control condition than the reference case. If the process is out-of-control, namely, $\delta > 0$, we denote the ARR as ARR_1. If $ARR_1 > 1$, and the observation have known features (λ, γ, ϕ), then the chart control will have low capacity to identify out-of-control condition than the reference case. Table 1 lists the performance measures and their features.

	Performance measure	Support	Reference case	Acceptable values for reference case
In-control	ARL_0	$0 < ARL_0 < \infty$	$ARL_{Normal}(\phi = 0, \delta = 0)$	$ARL_0 \geq 370.4$
process	ARD_0	$-1 < ARD_0 < \infty$	$ARL_{Normal}(\delta = 0)$	$-1 < ARD_0 < 0$
$(\delta = 0)$	ARR_0	$0 < ARR_0 < \infty$	$ARL_{Normal}(\phi = 0, \delta = 0)$	$0 < ARR_0 < 1$
Out-of-control	ARL_1	$0 < ARL_1 < \infty$	$ARL_{Normal}(\phi = 0, \delta > 0)$	$ARL_1 = 1$
process	ARD_1	$-1 < ARD_1 < \infty$	$ARL_{Normal}(\delta > 0)$	$ARD_1 \geq 0$
$(\delta > 0)$	ARR_1	$0 < ARR_1 < \infty$	$ARL_{Normal}(\phi = 0, \delta > 0)$	$ARR_1 > 1$

Table 1. Performance measures, supports, reference cases and acceptable values

5. Methodology

The simulation study that we will describe is based on the methodology for evaluating statistical methods (Morris et al., 2019). The objective of this study is to evaluate the performance of the γEWMA control chart with non-normal and non-independent observations and their interactions through the RL.

Data were generated from a simulation in which five factors were considered the quality variable distribution (f), quality variable distribution skewness (γ), the shift in the process mean (δ), the autocorrelation structure (φ) for type of relationship between the observations and modeled by AR(1), and the weight for past observations which modeled through smoothing parameter, (λ). The parameters γ *and* ϕ are defined by data, λ is a parameter that is defined by the analyst and used as chart parameter, and δ is a process condition that is unknown and we analyze chart performance for shift low, medium, and high.

In the simulation, the quality variable distribution and autocorrelation structure were considered. In each replication, we generated a random sample of the quality variable Y from an $AR(1)$ process that was described by equation (4), with the gamma, lognormal and normal distributions. The probability density functions (p.d.f.s) $f(x; \theta)$ that we used and expressions for their means and variances are presented in Table 2.

The distributions that are used in our study are described as follows:

1. The gamma distribution is positively skewed and is bounded below zero. This distribution may occur when monitoring a quality variable such as the time until failure (Mahmoodian, 2016), the waiting times for life events (Aksoy, 2000) and waiting times in call centers (Avramidis *et al.*, 2004).
2. The lognormal distribution is a continuous probability distribution of a random variable of which the logarithm is normally distributed. The distribution is positively skewed and can be used as a distribution of quality variables in semiconductor processes (Ross (2018)), cutting tool wear processes (Ghosh, 2018) and accelerated life tests (Zhang *et al.*, 2015).
3. The normal distribution is a continuous probability distribution that is used in the pharmaceutical industry (Gershon, 1991), chemical industry (Morud, 1996), health care industry (Tsacle and Aly, 1996), software industry (Mahanti and Evans, 2012), and food industry (Lim *et al.*, 2014).

	ϕ					
λ	0.0	0.1	0.3	0.5	0.7	0.9
0.1	2.703	2.678	2.623	2.554	2.450	2.211
0.2	2.860	2.836	2.784	2.714	2.609	2.369
0.3	2.925	2.906	2.860	2.796	2.695	2.455
0.4	2.958	2.944	2.906	2.848	2.753	2.516
1.0	3.000	2.999	2.994	2.979	2.927	2.712

Table 2. Control limit L for attaining approximately the incontrol ARL of 370.4 under various values of f and λ for γEWMA control chart

Other factor that was considered in the simulation was the quality variable distribution skewness (γ). We considered positive skewness due to the features of quality variables such as the time until failure, the waiting times for life events and waiting times in call centers. Like Horng Shiau and Ya-Chen (2005), we used three values of skewness to analyze the performance of the modified chart in terms of the symmetry degree. Table 3 presents the values of skewness of the distributions under study. For the gamma distribution, the skewness increases as the value of μ decreases, namely, as the scale parameter increases. For the lognormal distribution, the skewness increases as the value of σ increases, e.g., as the standard deviation of a random variable's natural logarithm increases.

Distribution	Probability density function (p.d.f.)	Mean $E(Y)$	Variance $V(Y)$
Gamma	$f(y;\mu,\sigma) = \frac{1}{\mu^{\sigma}\Gamma(\sigma)} y^{\sigma-1} e^{-\frac{y}{\mu}}; y \geq 0, \mu > 0, \sigma > 0$	$\sigma\mu$	$\sigma\mu^2$
Lognormal	$f(y;\mu,\sigma) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left[-\frac{(ln(y)-\mu)^2}{2\sigma^2}\right]; y > 0, \mu, \sigma > 0$	$e^{\frac{\mu+\sigma^2}{2}}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Normal	$f(y;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]; -\infty < y < \infty, \ -\infty < \mu < \infty, \sigma > 0$	μ	σ^2

Table 3. Probability density functions of random variables.

The fourth factor that was considered in the simulation study was (δ), the autocorrelation structure (φ) for the type of relationship between the observations and modeled by AR(1). We consider only positive values, namely, ϕ = 0.0, 0.1, 0.3, 0.5, 0.7 and 0.9, due to the inertial effect process.

The first factor considered was the weight that was given for past data (λ). Therefore, in this study, we consider a wide range of values of $\lambda = 0.1,\ 0.2,\ 0.3,\ 0.4$ and 1.0 aiming select adequate value of λ that present the best performance according to the performance measures. Additionally, the value of L was selected for various combinations of two parameters (λ and ϕ), as presented in Table 4. The code of the algorithm for calculating the combinations of λ and L is in a github repository (https://github.com/ousuga/gEWMA-Paper). It was developed in this study. The Table 4 presents the control limit L for attaining approximately the in-control ARL of 370.4 for the γEWMA control chart, with the specified values of λ and ϕ. A special case of the combination of λ and ϕ is $\lambda = 1.0$ and ϕ = 0.0, which yields a value of L=3.00, which is a typical value in the Shewhart control chart parameters.

Skewness	Gamma	Lognormal	Normal
0.0	–	–	$\mu = 0.00, \sigma = 1.00$
1.0	$\mu = 4.00, \sigma = 1.00$	$\mu = 0.00, \sigma = 0.30$	–
1.5	$\mu = 1.75, \sigma = 1.00$	$\mu = 0.00, \sigma = 0.45$	–
2.0	$\mu = 1.00, \sigma = 1.00$	$\mu = 0.00, \sigma = 0.55$	–

Table 4. Values of skewness of gamma, lognormal and normal distributions.

The last factor that was considered was the shift in the process mean (δ). If the process is in an in-control state, the shift takes the value of zero ($\delta = 0$), and if the process is in an out-of-control state, the shift takes non-zero values ($\delta > 0$). In this study, we consider a wide range of shift values: 0, 0.10, 0.15, 0.20, 0.25, 0.30, 0.50, 0.75, 1.00 and 2.00.

The study was conducted based on Monte Carlo simulations in R Core Team (2019). The codes are in the github repository (https://github.com/ousuga/gEWMA-Paper). All simulations were based on N=1000000 replications and the number of subgroups that were considered on each simulated control chart was n=5000 to avoid truncation of the RL at a value of the number of subgroups. The following algorithm was used in the simulation study to estimate the RL values:

1. Input the values of f, ϕ, (λ, L), and δ and the parameters of distribution f.
2. Calculate $\mu_Z = \frac{\mu_f}{(1-\phi)}$.
3. Calculate $\sigma^2 = \sigma_f^2$, $\sigma_Y = \sqrt{\sigma^2/\left(1-\phi^2\right)}$ and $\sigma_Z = \sqrt{\left(\frac{\sigma_f^2}{1-\phi^2}\right)\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}\right)}$.
4. Calculate $CL = \mu_Z + L * \sigma_Z$.
5. Calculate $\delta_S = \delta * \sigma_Y$.
6. Set the number of samples to $n = 5000$.
7. Generate Y_t, $t = 1, ..., n$, from the $AR(1)$ model according to the distribution f, the parameters of the distribution f and the values of ϕ and n.
8. Calculate $Y_t + \delta_S$.
9. Set $z_0 = \mu_Z$, $i = 0$ and $t = i + 1$.
10. Calculate the EWMA statistic $Z_t = \lambda Y_t + (1-\lambda)Z_{t-1}$.
11. If $\left|Z_t\right| > CL$, return a run length of $RL = t$; otherwise, set $t = i + 1$ and return to Step 9.
12. Repeat Steps 9-11 N times to obtain N RLs.
13. Estimate the summary measures of the N RLs, such as the minimum, the maximum, the average, the standard deviation, and percentiles 1, 25, 50, 75, and 99.

6. Results

Three performance measures ARL, ARD and ARR were used to assess the performance of the EWMA control chart according to the parameters λ, f, γ, ϕ and δ. The definitions and interpretations were presented in Section 3. To carry out the objective of the simulation study, the results were analyzed through plots. In the figures, the behavior of studied factors can be seen simultaneously, which facilitates the analysis of the relationships between them. The tables representing the figures are in the supplementary material (https://github.com/ousuga/gEWMA-Paper). We used plots of each factor vs. the performance measure (univariate effects plot) and bivariate interaction plots vs. the performance measure. The univariate effect plots were produced by averaging the performance measure at each level of each simulation factor and plotting these averages along a vertical line. Then, the larger the distance between the factor levels, the larger the effect of the factor on the average value of the performance measure. The bivariate interaction plots were produced by averaging the performance measure for each level of one of the two factors on each level of the second factor, plotting these averages versus the second factor, and obtaining curves by joining the points for each level of the first factor. Then, the less parallel the curves are, the more interaction occurs between the two factors. Additionally, we used boxplots to explain the relationships among data distribution (f), the skewness (γ), and the redefinition of the shift in the process mean (δ) levels. The analysis of the figures are discussed according to the simulation results and the acceptable values for the reference case are shown in Table 1.

The results are organized as follows. First, we justify the use of the skewness parameter (γ) instead of the distribution (f). Later, we analyze the results for the in-control state and both the univariate effects and the bivariate interaction effects on the ARL_0performance measure (the results are similar if the ARD_0 and ARR_0 are analyzed, we use the ARL_0 case as illustration). In this case, we only analyze the parameters λ, γ and ϕ since the mean shift is zero ($\delta = 0$). Finally, we present the results for the out-of-control state for the three performance measures, where we follow the same structure as in the previous case but including the shift in the process mean (δ) factor. As discussed in Section 4, λ is a parameter that is defined by the analyst, whereas γ and ϕ are defined by the data, and δ is a process condition that is unknown. Therefore, the bivariate interaction analysis will be addressed by this last factor.

In the following analysis, we used the skewness factor (γ) instead of the distributions (f). As presented in Table 3, the parameters of the distributions were chosen such that a skewness value (γ) was obtained. Figure 2 shows that distributions with equal skewness behave similarly for both the in-control and out-of-control states. The colors in the top row match the colors in the bottom row. For the in-control state, distributions with larger skewness (Gamma(1,1) and Lognormal(0,0.55), for which $\gamma = 2$) present low ARL_0, namely, the false-alarm rate increases as the skewness decreases. A similar pattern is observed for the out-of-control state. In addition, the results for the normal distribution, for which the skewness is 0, demonstrate that the coefficient of variance for the ARL_0 is almost 0.0% and that for the ARL_1 is approximately 72.2%. As we have discussed, the effect of the skewness seems to be more important than the distribution. From here onwards, we will analyze scenarios with the skewness parameter (γ). As the results are similar among all three performance measures, we consider the ARL case for illustration.

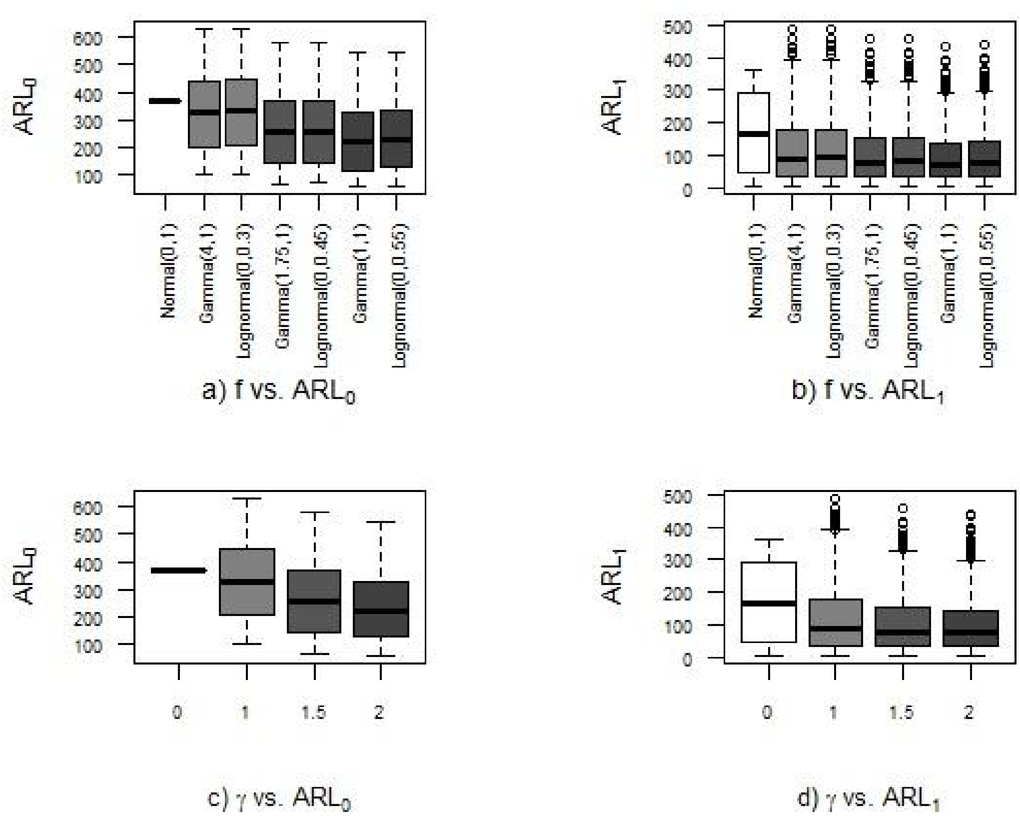

Figure 2. Effects of the distribution and skewness on the ARL performance: (a) distribution on the ARL_0; (b) distribution ARL_1; (c) skewness ARL_0 and (d) skewness on the ARL_1.

6.1 Results for in-control processess ($\delta = 0$)

Univariate effects for the in-control state are presented in Figure 3. Since the magnitude of the effect for λ is almost the same as that for ϕ, this figure shows that the ARL_0 is equally affected by the autocorrelation among observations (ϕ) and by the selection of the value of the parameter λ. The ARL_0 is more influenced by these two factors than by the skewness of the distribution. When the analysis was conducted for each simulation factor separately, we found that the smaller the skewness and λ, and the larger the autocorrelation among observations, the higher the ARL_0.

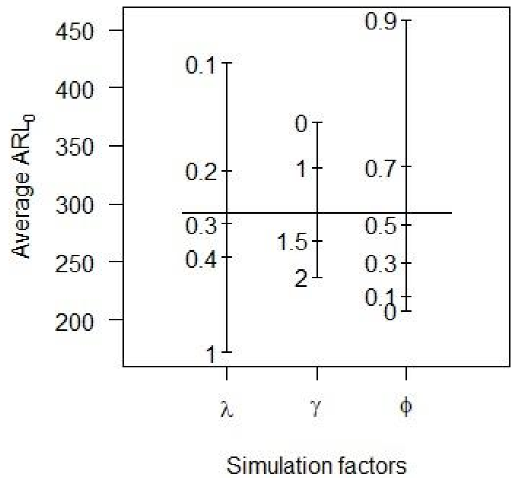

Figure 3. Univariate effects for the in-control process.

Figure 4 presents the results for the bivariate interaction effects of ϕ vs. λ for the ARL_0conditioned on the γ parameter: i) there is no important interaction between ϕ and λ while conditioning on the skewness of the distribution, ii) the

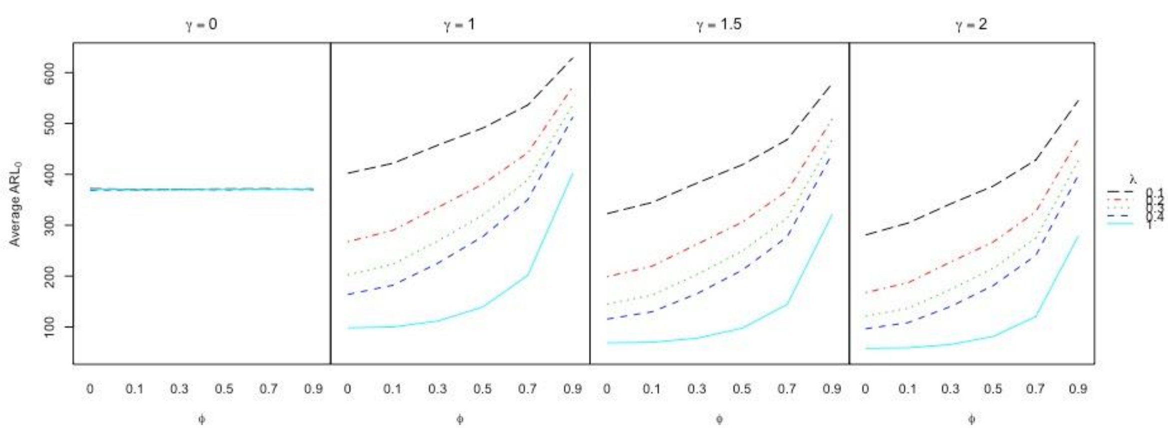

Figure 4. Bivariate interaction effects of ϕ vs. λ for the in-control process. (a) γ = 0; (b) γ = 1; (c) γ = 1.5; (d) γ = 2.

6.2 Results for out-of-control processess (δ ≠ 0)

The univariate effects for the out-of-control process are presented in Figure 5. Figure 5(a) presents the results for the ARL_1(for which lower values are better): i) the shift in the process mean (δ) is the simulation factor with the strongest effect on the ARL_1 since the difference among its levels is the largest; for this measure performance, it is important to consider the value of the shift in the process mean since the value of the shift significantly affects the speed at which the chart control detects an out-of-control state; the second highest effect for this performance measure is the autocorrelation among observations (ϕ), followed by the skewness of the distributions (γ), and the λ is the factor with the lowest effect, and ii) when the analysis is done for each factor separately, we found that the larger the γ, λ and δ, and the smaller the ϕ, the lower the ARL_1. Figure 5(b) presents the results for the ARD_1, where the reference case involves departures from the normal distribution assumption and changes in the shift: i) the shift in the process mean (δ) and λ are the factors that have the strongest effects on the ARD_1, followed by γ and ϕ. ii) when the analysis is done for each factor separately, we found that the larger the γ, and λ, and the smaller the ϕ and δ, the lower theARD_1, and iii) although the smallest ARL_1 values are achieved when $\delta = 2$, they fail to exceed the ARL_1 of the reference case by far; improvements in the detection have a greater impact at lower shift values. Figure 5(c) shows the results for the ARR_1, where the reference case involves departures from the normal distribution and independence assumptions, and changes in the shift: i) the autocorrelation between the observations is the factor that has the strongest effect on the ARR_1, followed by δ and λ, and ending with γ; therefore, when data have an autocorrelation structure, it is important to model that structure, and ii) the direction of the effects is the same than for the ARD_1, but the shift presents a nonlinearity effect for the three last levels.

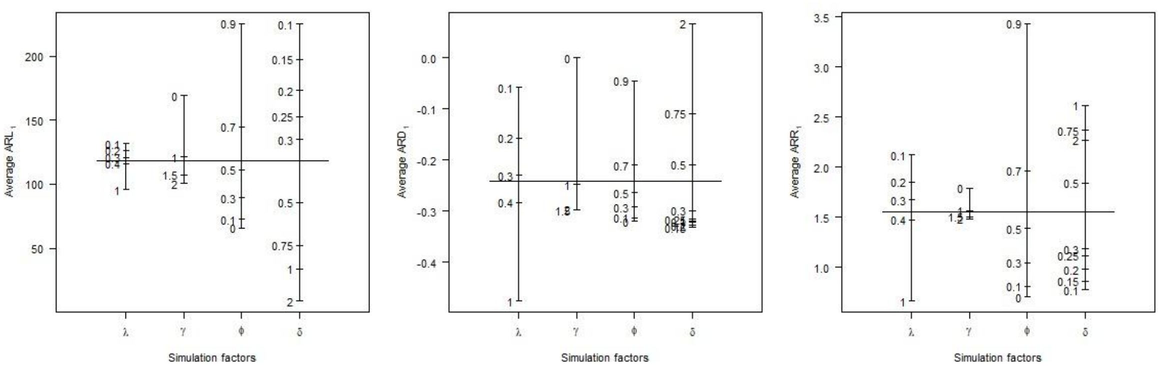

Figure 5. Univariate effects for the out-of-control process. (a) Average ARL_1. (b) Average ARD_1. (c) Average ARR_1.

The shift in mean is a process condition that has been frequently reported in the literature. Analyzing its impact on the performance of the chart is very important. In order to keep the analysis easy to read, we decided to redefine its levels. As is observed in the three plots of Figure 6, this parameter can be clustered in three groups: δ_{Low}, where $\delta \leq 0.3$, δ_{Middle}, where $\delta = 0.5$, and δ_{High}, where $\delta \geq 0.75$. A justification of this redefinition can be seen in Figure 6 as well.

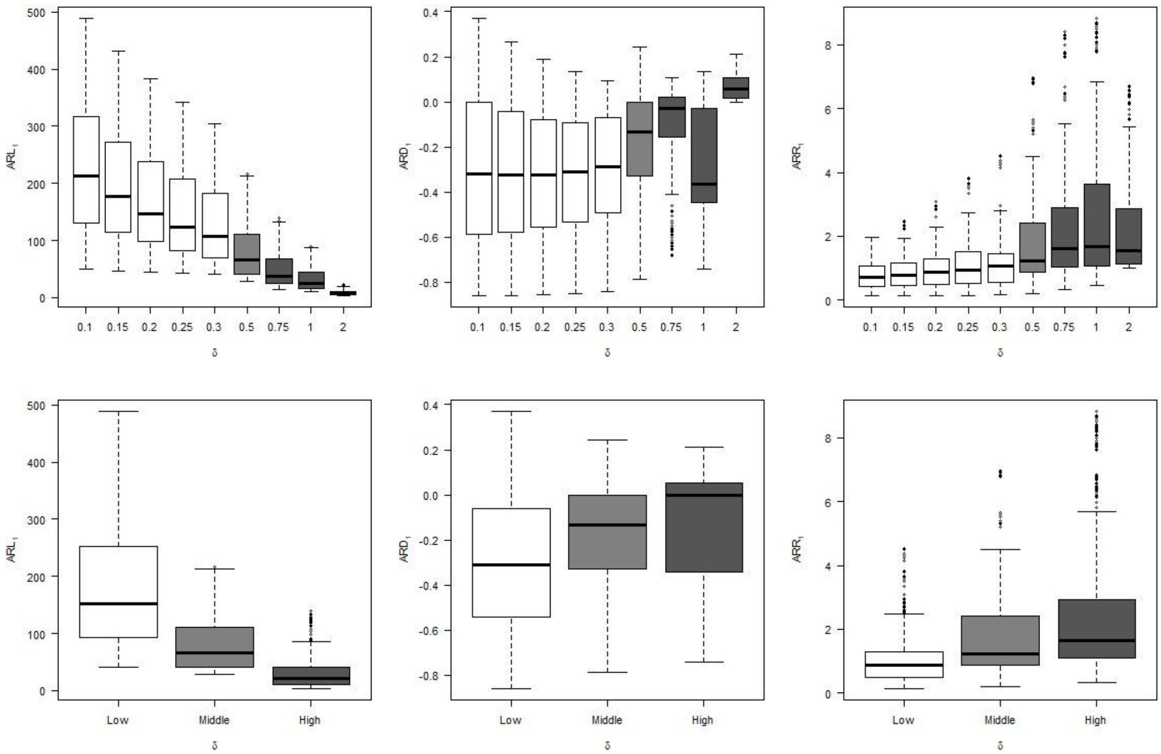

Figure 6. Redefinition of the δ levels. (a) Effects of the original δ levels on the ARL_1, ARD_1 and ARR_1. (b) Effects of the new δ levels on the ARL_1, ARD_1 and ARR_1.

Figure 7, Figure 8, and Figure 9 present the results for the peer interaction (λ,ϕ) for the average ARL_1, ARD_1 and ARR_1 at the three levels of δ(δ_{Low},δ_{Middle},δ_{High}). If the $ARL_1$of these three plots are compared, the average ARL_1improves at higher levels of δ.This means that the speed of out-of-control status detection increases when the shifts in the mean that the user wants to detect become larger.

According to Figure 7 (λ and ϕ interaction for δ_{Low}), if the average $ARL_1$is analyzed, the normal distribution and skewed distribution behave in opposite ways; while for the normal distribution (γ = 0), a value of $\lambda = 0.1$is recommended at any value of ϕ (although for $\phi = 0.9$, $\lambda = 0.1,..., 1$seem to produce similar values of ARL_1), for the skewed distributions (γ = 1, 1.5, 2), a value of $\lambda = 1$ is more appropriate at any value of ϕ. When the average $ARD_1$is analyzed, the skewed distributions outperform the normal distribution (i.e., $ARD_1 > 0$) for all cases except when autocorrelation is high ($\phi > 0.7$) and the value of the weight for past observations, λ, is low. Finally, when the average $ARR_1$is analyzed (remember that the reference case is that in which the distribution is normal and the observations are independent), the normal distribution (γ = 0) behaves less well when the value of the autocorrelation is higher and the value of λis smaller; the skewed distributions outperform the normal distribution with independent observations (i.e., $ARR_1 < 1$) when the value of λis higher.

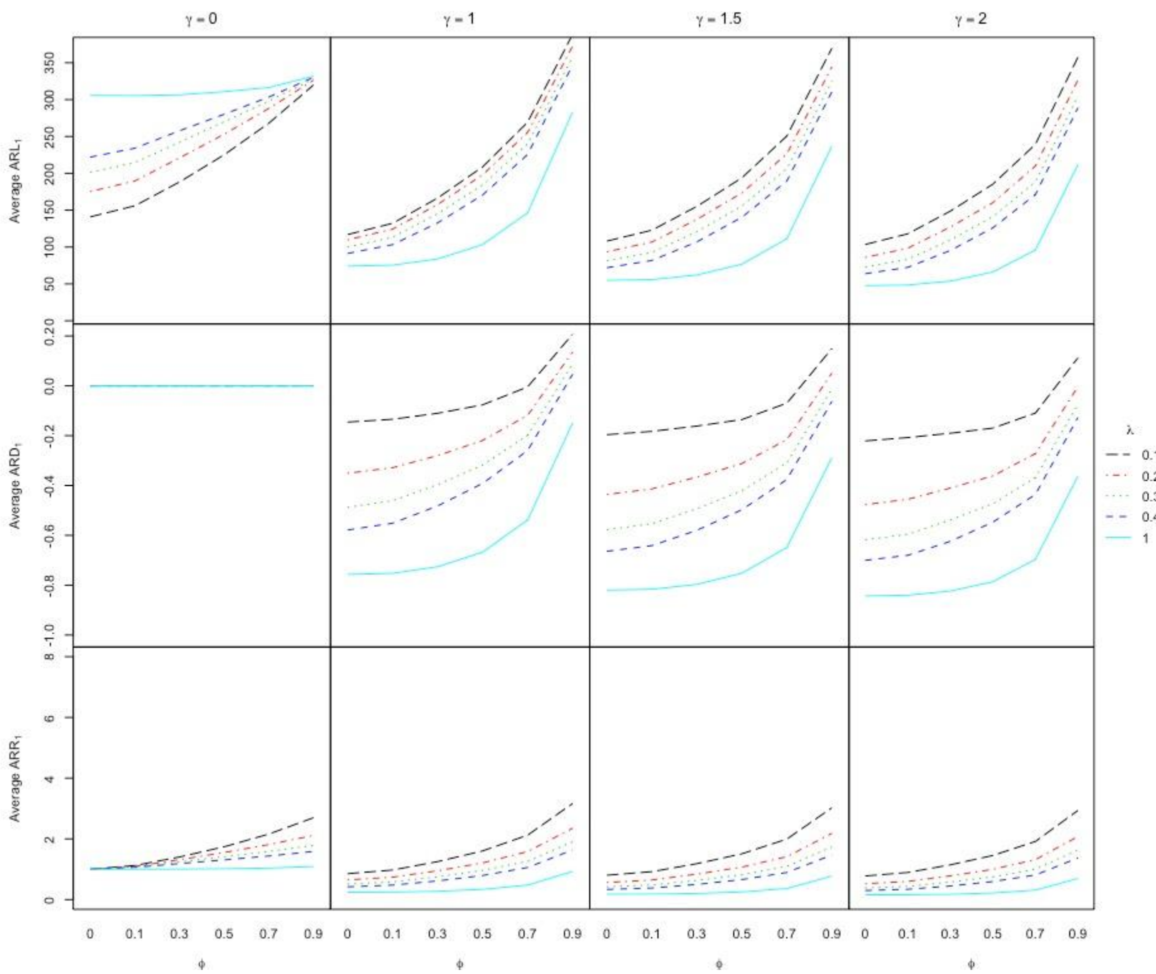

Figure 7. λ and ϕ interaction for the average ARL_1, ARD_1 and ARR_1 for δ_{Low}.

Figure 8 depicts the λ and ϕ interaction for δ_{Middle}. In contrast with the results of δ_{Low}(Figure 7) for the average ARL_1, the interaction between λ, ϕ and γ is important for the analysis of the data. The recommendations for the normal case remain the same as for δ_{Low}. Nevertheless, for the skewed distributions (γ >0) the conclusions change: i) when $\gamma = 1$, a value of $\lambda = 0.1$ is more appropriate if $\phi < 0.5$, for observations with higher autocorrelation, a value of $\lambda = 1$is preferable, ii) when γ = 1.5, a value of $\lambda = 0.1$ is more appropriate if $\phi < 0.1$, for observations with higher autocorrelation, a value of $\lambda = 1$is preferable, and iii) when γ = 2 and $\phi < 0.1$ any value of λ do not produce significant differences in the averageARL_1, for observations with higher autocorrelation, a value of $\lambda = 1$is preferable. When the average $ARD_1$is analyzed, the skewed distributions outperform the normal distribution (i.e., $ARD_1 > 0$) for all cases except when autocorrelation and the value of the weight for past observations are low. Finally, when the average $ARR_1$is analyzed the skewed distributions outperform the normal distribution with independent observations (i.e., $ARR_1 < 1$) when $\lambda = 1$; observe that the performance of the γEWMA control chart improves at lower values of ϕ.

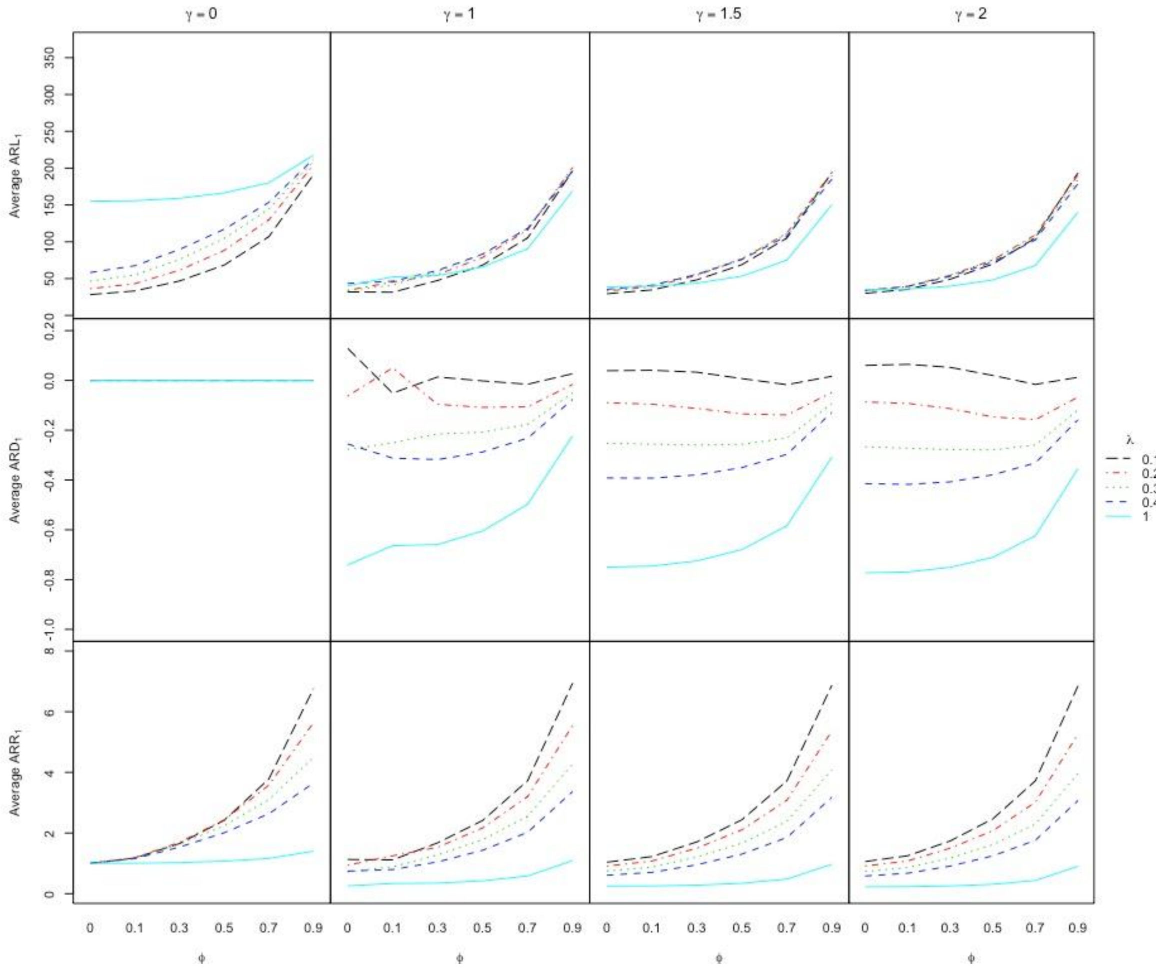

Figure 8. λ and ϕ interaction for the average ARL_1, ARD_1 and ARR_1 for δ_{Middle}.

Figure 9 depicts the λ and ϕ interaction for δ_{High}. In this case, the selection of the λparameter seems to be less important (due to small significant differences in the average ARL_1at different values of λ) when the the average $ARL_1$is analyzed. Nevertheless, the average ARD_1shows that a value of λ=1 would be a good choice if the normal distribution needs to be outperformed. In a similar way the analysis of the ARR_1shows that a value of λ=1 for skewed distributions outperforms the normal distribution with independent observations, although when autocorrelation increases the performance could be improved.

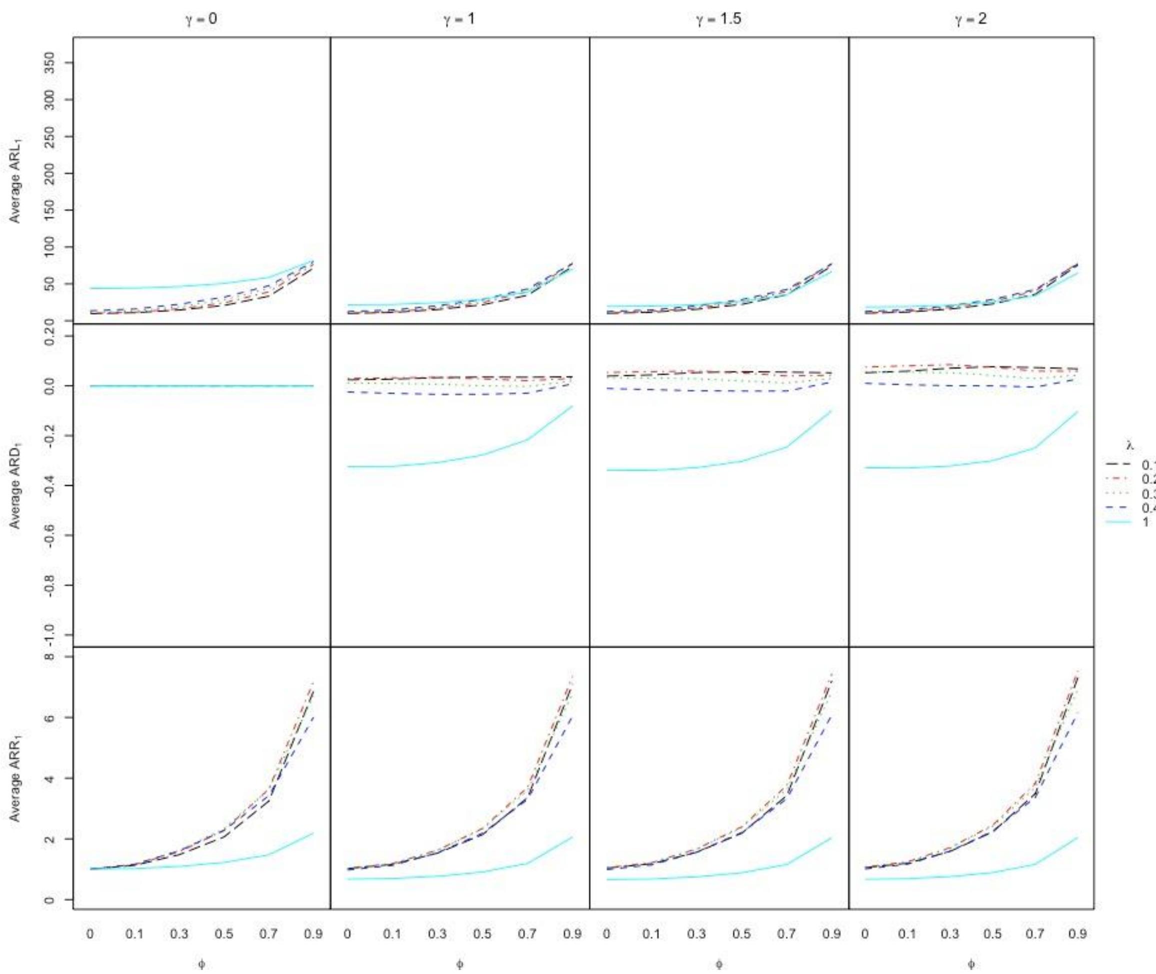

Figure 9. λ and ϕ interaction for the average ARL_1, ARD_1 and ARR_1 for δ_{High}.

7. Conclusions

We used three performance measures based on Run Length to evaluate the detecting ability a point outside the control limits of a control chart when the statistical distribution of the observations, the autocorrelation structure, and the shift in the process mean is considered in the modeling because we proposed γEWMA control chart. Our findings are in line with the results in (i) Human et al. (2011), who studied the non-normality assumption (they used sixteen different distributions), and (ii) Horng Shiau and Ya-Chen (2005), who studied the effects of non-normality (*t* and Gamma with different parameters) and non-independent data ($\phi = 0.6$) under different shifts (δ between 0 and 3, each 0.25) and smoothing parameters (λ = 0.05, 0.1, 1). We show that the impact of the simulation factors are different for each performance measure and shift in the process mean is very important to analyze its impact on the sensitivity of the chart. Besides, the analysis of the interaction between the simulation factors provides more insights for the analysis of the control chart.

For the in-control process, three performance measures showed similar results, it concludes that, there is no important interaction between ϕ and λ, and the normal distribution ($\gamma = 0$) behaves differently than the skewed distributions. For the out-of-control process (δ≠0), the performance of the γEWMA control chart is measured based on the smoothing parameter selection and jointly analyzes three process or data conditions. This paper shows that when observations consider process characteristics the γEWMA control chart is far more useful than the EWMA control chart in detecting small, middle and high shifts when the data distribution is symmetric; for middle and high shifts and non-symmetric data distribution, the γEWMA control chart behaves better as the autocorrelation diminishes. The γEWMA control chart requires less time to detect small shifts in the process mean as the level of autocorrelation decreases.

The analysis of λ and ϕ interactions for the three performance measures in the γEWMA control showed it makes sense that highly autocorrelated observations require smaller λ for in-control processes; this is not true for the out-of-control scenario. The correlation structure enables the preservation of relevant information in past observations. Also, the analysis of γ, ϕ and λ interaction, for the in-control scenario, showed a good performance of the γEWMA control chart when data follow a skewed distribution and the λ parameter is properly chosen, under this scenario.

For out-of-control processes, it is better to preserve the original features of the distributions, mean and variance, for the calculation of the control limits. The analysis of γ, ϕ and λ interaction showed that the performance of the γEWMA control chart for skewed data with $\gamma = 1.0,\ 1.5$ and 2.0 outperforms in most cases the performance of the γEWMA control chart for the symmetric data with $\gamma = 0.0$. The larger the skewness and the shift in mean, the better its performance. The selection of the λ parameter, conditioned in the data and process characteristics, is determinant for the performance of the chart.

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