Impact of Dynamical Collapse Models on Inflationary Cosmology

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Inflation solves several cosmological problems at the classical and quantum level, with a strong agreement between the theoretical predictions of well-motivated inflationary models and observations. In this Letter, we study the corrections induced by dynamical collapse models, which phenomenologically solve the quantum measurement problem, to the power spectrum of the comoving curvature perturbation during inflation and the radiation-dominated era. We find that the corrections are strongly negligible for the reference values of the collapse parameters.

Quantum mechanics is very successful, but also problematic. The trouble is that, in its standard formulation, the theory introduces a division between the microscopic quantum world of particles and atoms and the macroscopic world of classical observers [1]. This division should not be part of a fundamental theory of nature, even more so when the theory is applied to the entire universe, where there are no external observers; however, removing it proved to be a difficult task, and no shared solution is yet available.

Models of spontaneous wave function collapse [2,3] attempt to remove the arbitrary quantum-classical divide by modifying the Schrödinger equation. Suitable nonlinear and stochastic terms are added to the standard quantum dynamics, whose effects scale with the mass of the system. The resulting dynamics is such that microscopic systems behave quantum mechanically, but if they interact to form macroscopic objects, they behave the more classically the more massive the objects.

In doing so, collapse models predict a dynamical behavior for matter, which differs from the standard quantum mechanical one: the stochastic terms blur the quantum dynamics, and potentially can be spotted in specific situations. An increasing number of experimental investigations in highly controlled systems [4–21] have set significant bounds on the collapse parameters, at the same time leaving much freedom.

Also cosmology has been used as a playground for collapse models [22–33]. Being the largest and oldest physical system, the Universe can provide relevant information about possible modifications of quantum theory, whose effects would build up during the history of the Universe and would be strongly constrained by the increasing and more and more detailed number of cosmological observations. Moreover, there is still an ongoing discussion on how the inflationary quantum fluctuations evolve into classical stochastic variables [34–41].

A recent paper [32] claims that a straightforward application of the continuous spontaneous localization (CSL) model [42,43], the reference collapse model in the literature, to cosmic inflation, for the most natural choices of the density contrast, leads to results which are incompatible with experimental evidence. Specifically, the CSL correction to the power spectrum of the comoving curvature perturbation is calculated, leading to strongly scale-dependent results, which are disproved by observations. To be quantitative, since during inflation the wavelength of the cosmic microwave background (CMB) modes becomes larger than $r_C = 10^{-7}$ m, which is the reference value of one among the two phenomenological parameters of the CSL model, the authors find $\lambda \ll 5.6 \times 10^{-90}$ s^{-1} as a bound on the second CSL parameter. Therefore, the analysis in [32] rules out a wide class of CSL theories, since in order for the collapse to be effective, $\lambda > 10^{-20}$ s^{-1} [5].

In this letter, we reconsider the application of CSL to standard cosmology, without entering the debate about its foundations [36,38,44–47]. We show that a different, yet a very natural choice of the collapse operator leads to negligible corrections to standard quantum predictions.

Standard inflationary power spectrum.—We briefly overview the standard inflationary dynamics, during which the early Universe underwent an accelerated phase of expansion [48–50], and derive the corresponding power spectrum of the comoving curvature perturbation, for which we will later compute the CSL correction. We refer to [49–51] for a more detailed discussion, which is also summarized in the Supplemental Material [52].

We reconsider the dynamics of the perturbation of the scalar field ϕ on a flat Friedmann-Lemaître-Robertson-Walker metric $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$ in the presence of a scalar potential $V(\phi)$, where $\eta_{\mu\nu}$ is the Minkowski metric, $a(\eta)$ the

scale factor, and η is the conformal time. In terms of the gauge-invariant Mukhanov-Sasaki variable $\delta\phi_G$ [53,54], the action of the scalar perturbations is given by [53–55]

$$S = \frac{1}{2} \int d\eta \int d\mathbf{x} \left[\dot{u}^2 - \delta^{ij}\partial_i u \partial_j u + \frac{\ddot{z}}{z}u^2 \right], \quad (1)$$

where we have introduced the rescaled field $u(\eta,\mathbf{x}) = a\delta\phi_G$. Further, $\mathbf{x}$ are the comoving coordinates, $\dot{u} = du/d\eta$, and $z(\eta) = aM_P\sqrt{2\epsilon}/c_s$. Here $\epsilon = -H^{-2}dH/dt$ is the slow-roll parameter, $H = a^{-1}da/dt$ is the Hubble parameter, t is the cosmic time, and c_s stands for the speed of sound ($c_s = 1$ during inflation, and $c_s = 1/\sqrt{3}$ during the radiation-dominated era). During inflation, we will work under the slow-roll approximation assuming $\epsilon \ll 1$ and $d\epsilon/dt \approx 0$. Moreover throughout this Letter, we will work in reduced Planck units ($\hbar = 1$, $c = 1$, and $M_P^2 = 1/8\pi G$).

Upon quantization, $\hat{u}(\eta,\mathbf{x})$ can be expressed in terms of the creation and annihilation operators as

$$\hat{u}(\eta,\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} [v_k(\eta)\hat{a}_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{HC}], \quad (2)$$

where HC denotes the Hermitian conjugate, the creation and annihilation operators satisfy $[\hat{a}_\mathbf{k}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$, and the modes $v_k(\eta)$ are determined by

$$v_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right), \quad (3)$$

under a perfect de Sitter approximation [49–51]. Given the above definitions, one can compute quantities of interest such as the variance of the comoving curvature perturbation $\hat{\mathcal{R}} = \hat{u}/z$. In the comoving gauge, where the comoving observers measure zero energy flux ($T_{0i} = 0$), $\mathcal{R}$ determines the spatial curvature $^{(3)}R_{\text{com}}$ on the hypersurface of constant η through $^{(3)}R_{\text{com}} = (4/a^2)\nabla^2\mathcal{R}$ [51].

The mean squared quantum expectation value

$$\langle 0|\hat{\mathcal{R}}^2(\mathbf{x},\eta)|0\rangle = \int d\ln k \mathcal{P}_\mathcal{R}(k,\eta), \quad (4)$$

defines the corresponding power spectrum $\mathcal{P}_\mathcal{R}$. The latter reads

$$\mathcal{P}_\mathcal{R}(k,\eta) = \frac{c_s^2}{2\epsilon M_P^2} \frac{k^3}{2\pi^2} \frac{|v_k(\eta)|^2}{a^2(\eta)}. \quad (5)$$

The modes probed by the CMB exit the horizon well before the end of inflation. For these modes, the expectation values are indistinguishable from classical stochastic averages [34,35]. This allows us to use the expression in Eq. (4) and relate it to observations [34,35,37]. In general, the

power spectrum can be parametrized as $\mathcal{P}_\mathcal{R} = A_\mathcal{R}^*(k/k_*)^{n_\mathcal{R}^*-1}$ where the values of $A_\mathcal{R}^*$ and $n_\mathcal{R}^*$ are determined by Planck data [56] at the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ to be $A_\mathcal{R}^* = (2.099 \pm 0.014) \times 10^{-9}$ and $n_\mathcal{R}^* = 0.9649 \pm 0.0042$ at the 68% confidence level. We remark that the expression for $\mathcal{P}_\mathcal{R}$ in Eq. (5) is valid not only for inflation, but also for the radiation-dominated era. Both stages will be of interest in this Letter.

Cosmological application of collapse models.—The application of collapse models to cosmology has been previously considered, with motivations ranging from explaining the origin of the cosmic structure [36,65–69] and constructing chronogenesis and cosmogenesis models [70], to implementing an effective cosmological constant [71]. In particular, the phenomenological parameters of the CSL model have been previously constrained through a consideration of the heating of the intergalactic medium [22–24] and spectral distortions of the CMB radiation [26]. Moreover, previous works have studied the modifications due to CSL to the spectra of primordial perturbations at a scalar and tensorial level [27–30,32,33].

In this Letter, we study the CSL correction to the power spectrum of the scalar perturbations during inflation and the radiation-dominated era. As discussed in detail in the Supplemental Material [52], the CSL dynamics can be mimicked by adding a stochastic Hamiltonian $\hat{H}_{\text{CSL}}$ to the standard quantum Hamiltonian $\hat{H}_0$. The former is given by

$$\hat{H}_{\text{CSL}}(\eta) = \frac{\sqrt{\lambda}(4\pi r_C^2)^{3/4}}{m_0} \int d\mathbf{x}\xi_\eta(\mathbf{x})\hat{L}_{\text{CSL}}(\eta,\mathbf{x}), \quad (6)$$

where m_0 is a reference mass set equal to that of a nucleon, $\hat{L}_{\text{CSL}}(\eta,\mathbf{x})$ is the CSL collapse operator yet to be chosen, and $\xi_\eta(\mathbf{x})$ is a white Gaussian noise characterized by zero average $\mathbb{E}[\xi_\eta(\mathbf{x})] = 0$ ($\mathbb{E}$ denotes the stochastic average over the noise) and the correlation function

$$\mathbb{E}[\xi_\eta(\mathbf{x})\xi_{\eta'}(\mathbf{y})] = \frac{\delta(\eta - \eta')}{a(\eta)} \frac{e^{-a^2(\eta)(\mathbf{x}-\mathbf{y})^2/(4r_C^2)}}{(4\pi r_C^2)^{3/2}}. \quad (7)$$

Note that the model is defined in terms of two parameters λ and r_C, which are the collapse rate and the space correlator of the noise, respectively. The numerical values of these parameters are constrained by experimental evidence. We will come back to this later.

By considering $\hat{H}_{\text{CSL}}$ as a small perturbation to the full dynamics, one can exploit the standard perturbative approach in the interaction picture and compute the time evolution of a general operator $\hat{O}(\eta)$ to the leading order. By following standard calculations [57], one can express the expectation value of $\hat{O}(\eta)$ as

$$\bar{O} \equiv \mathbb{E}\langle\hat{O}(\eta)\rangle = \langle\hat{O}(\eta)\rangle_0 + \delta\bar{O}(\eta)_{\text{CSL}}, \quad (8)$$

where we account also for the stochastic average $\mathbb{E}$. Here, $\langle \hat{O}(\eta) \rangle_0$ is the expectation value given by standard cosmology, and $\delta \bar{O}(\eta)_{\text{CSL}}$ stands for the CSL correction, which reads

$$\delta \bar{O}(\eta)_{\text{CSL}} = -\frac{\lambda}{2m_0^2} \int_{\eta_0}^{\eta} \frac{d\eta'}{a(\eta')} \int d\mathbf{x}' d\mathbf{x}'' e^{-\frac{a^2(\eta')(\mathbf{x}''-\mathbf{x}')^2}{4r_C^2}}$$
$$\times \langle \psi | [\hat{L}_{\text{CSL}}^I(\eta', \mathbf{x}''), [\hat{L}_{\text{CSL}}^I(\eta', \mathbf{x}'), \hat{O}^I(\eta)]] | \psi \rangle, \quad (9)$$

where the superscript "I" indicates that the operators are evaluated in the interaction picture and $|\psi\rangle$ is the initial state of the system, which we will later set equal to the Bunch-Davies vacuum state $|0\rangle$ [49–51].

We now turn to the specification of the collapse operator $\hat{L}_{\text{CSL}}(\eta, \mathbf{x})$. In standard nonrelativistic CSL, $\hat{L}_{\text{CSL}}(\eta, \mathbf{x})$ is defined as the mass density operator $m\hat{a}^\dagger \hat{a}$ [42,43]. Although to this date there is no satisfactory generalization of collapse models to the relativistic regime [2,72–82], different choices for the collapse operator have been proposed in the cosmological setting. Nevertheless, to our knowledge, all such choices are either linear or, to leading order, linearized in the field perturbation $\hat{u}$ and its conjugate momentum [83]. Some authors have chosen the collapse operator to be the rescaled variable $\hat{u}$ itself [33,46], while others have chosen the perturbed matter-energy density $\hat{\delta \rho}$ [32], which to leading order, is linear in $\hat{u}$ and $\hat{\dot{u}}$ in standard cosmological perturbation theory. With these choices of the collapse operator, when one describes the cosmological perturbations in the Fourier space, the corresponding modes evolve independently, exactly as in standard cosmology [50,51,55]. However, when generalizing a model, one should retain its characteristic traits. In the case of a generalization of the CSL model, one would like the collapse operator to couple different Fourier modes as in the standard case [17], which is not possible when the collapse operator is linear in the fields.

Here, we take the collapse operator to be $\hat{L}_{\text{CSL}}(\eta, \mathbf{x}) = \hat{\mathcal{H}}_0(\eta, \mathbf{x})$, the Hamiltonian density operator of scalar cosmological perturbations, which is identified by $\hat{H}_0(\eta) = \int d\mathbf{x} \hat{\mathcal{H}}_0(\eta, \mathbf{x})$. This choice is a natural, though not unique, relativistic generalization of the nonrelativistic mass density. Indeed, in flat spacetime, there is no distinction between the Hamiltonian density of the system and the matter-energy density ρ which, in turn, was considered as a possible generalization of CSL even in Friedmann–Lemaître–Robertson–Walker cosmology [32]. The role played by gravitational degrees of freedom in the reduction of the quantum mechanical wave function is still a subject of active debate [84–88]. In this light, and given that the unitary part of the time evolution is governed by the full Hamiltonian of the system, we find it more natural for the collapse operator to be given by $\hat{\mathcal{H}}_0(\eta, \mathbf{x})$. This choice contains contributions from the perturbations of both the standard Einstein-Hilbert term and the matter sector of the full action, while the perturbed matter-energy density $\delta \rho$ is obtained only from the latter [50]. In addition, even in standard perturbation theory, $\hat{\mathcal{H}}_0(\eta, \mathbf{x})$ is quadratic in the field variable $\hat{u}$ and its conjugate momentum, and therefore is also quadratic in the creation and annihilation operators, analogous to the mass density of the standard CSL model.

CSL and inflation.—During inflation, the Hamiltonian density operator reads [51] $\hat{\mathcal{H}}_0^I(\eta, \mathbf{x}) = \frac{1}{2} \{ \hat{\dot{u}}^2(\eta, \mathbf{x}) + [\nabla \hat{u}(\eta, \mathbf{x})]^2 - (2/\eta^2) \hat{u}^2(\eta, \mathbf{x}) \}$, which is the Hamiltonian density of the scalar perturbations in the Heisenberg picture in standard cosmology, where one does not have additional contributions coming from collapse dynamics. Taking into account Eq. (2), we have

$$\hat{\mathcal{H}}_0^I(\eta, \mathbf{x}) = \int d\mathbf{q} d\mathbf{p} \frac{e^{i(\mathbf{p}+\mathbf{q})\cdot\mathbf{x}}}{2(2\pi)^3} [b_\eta^{\mathbf{p},\mathbf{q}} \hat{a}_\mathbf{p} \hat{a}_\mathbf{q} + d_\eta^{\mathbf{p},\mathbf{q}} \hat{a}_{-\mathbf{q}}^\dagger \hat{a}_\mathbf{p}$$
$$+ b_\eta^{\mathbf{p},\mathbf{q}*} \hat{a}_{-\mathbf{p}}^\dagger \hat{a}_{-\mathbf{q}}^\dagger + d_\eta^{\mathbf{p},\mathbf{q}*} \hat{a}_{-\mathbf{p}}^\dagger \hat{a}_\mathbf{q}],$$

where we have defined

$$\begin{pmatrix} b_\eta^{\mathbf{p},\mathbf{q}} \\ d_\eta^{\mathbf{p},\mathbf{q}} \end{pmatrix} = \begin{pmatrix} j_\eta^{p,q} \\ l_\eta^{p,q} \end{pmatrix} - \left[(\mathbf{p} \cdot \mathbf{q}) + \frac{2}{\eta^2} \right] \begin{pmatrix} f_\eta^{p,q} \\ g_\eta^{p,q} \end{pmatrix}, \quad (10)$$

and

$$f_\eta^{p,q} = v_p v_q, \quad g_\eta^{p,q} = v_p v_q^*, \quad j_\eta^{p,q} = \dot{v}_p \dot{v}_q, \quad l_\eta^{p,q} = \dot{v}_p \dot{v}_q^*. \quad (11)$$

We can now compute the corrections $\delta \mathcal{P}_\mathcal{R}$ to the power spectrum of the curvature perturbation $\hat{\mathcal{R}}$ at the end of inflation. The first step of the procedure, which is fully reported in the Supplemental Material [52], is to compute the correction to the evolution of $\hat{\mathcal{R}}^2 = (\hat{u}/z)^2$ due to CSL, by evaluating $\delta \bar{\mathcal{R}}^2(\eta)_{\text{CSL}}$ according to Eq. (9), for the given choice of the collapse operator, starting from the Bunch-Davies vacuum state $|0\rangle$. We find

$$\delta \bar{\mathcal{R}}^2(\eta_e)_{\text{CSL}} = C_{\eta_e} \int d\mathbf{q} d\mathbf{p} \int_{\eta_0}^{\eta_e} d\eta e^{-\frac{r_C^2(\mathbf{p}+\mathbf{q})^2}{4a^2(\eta)}} \mathcal{F}_\eta^{\mathbf{p},\mathbf{q}}, \quad (12)$$

where η_e is the conformal time at the end of inflation, $C_{\eta_e} = -\lambda r_C^3 / (8\epsilon_{\text{inf}} M_P^2 a^2(\eta_e) m_0^2 \pi^{9/2})$, and

$$\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}} = \mathfrak{Re}[b_\eta^{\mathbf{p},\mathbf{q}} d_\eta^{\mathbf{q},\mathbf{p}} (f_{\eta_e}^{q,q})^* - b_\eta^{\mathbf{p},\mathbf{q}} (b_\eta^{\mathbf{q},\mathbf{p}})^* g_{\eta_e}^{p,p}]. \quad (13)$$

To provide an estimate of Eq. (12), we first notice that, during inflation, the scale factor is inversely proportional to the conformal time $a(\eta) \simeq -(H_{\text{inf}}\eta)^{-1}$, with the Hubble parameter H_{inf} that can be approximated to a constant. Thus, the argument of the Gaussian in Eq. (12) becomes $-r_C^2 H_{\text{inf}}^2 \eta^2 (\mathbf{p} + \mathbf{q})^2$. To get a feeling of the orders of

magnitudes involved, the typical value of $r_c = 10^{-7}$ m $\sim 10^{27} M_P^{-1}$ is much bigger than $H_{\text{inf}} \sim 10^{-5} M_P$, so one has $r_c H_{\text{inf}} \gg 1$. This implies that the Gaussian in Eq. (12) will suppress the integrand for all values of $\mathbf{p}$ and $\mathbf{q}$ except for those where $p\eta \ll 1$ and $q\eta \ll 1$. Under such conditions, we can safely expand $\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}}$ for small $p\eta$ and $q\eta$ and determine the leading contribution to Eq. (12). Thus, we obtain

$$\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}} \simeq -\frac{1}{8 p^3 q^4 \eta^8} \left(\frac{4 p^3 q \eta^6}{\eta_e^2} + \frac{32 q^4 \eta^6}{9\eta_e^2} \right). \qquad (14)$$

By substituting Eq. (14) in Eq. (12), we find the leading order correction due the CSL to the mean squared value of the comoving curvature perturbation:

$$\delta \bar{\mathcal{R}}^2(\eta_e)_{\text{CSL}} = \int d \ln k \, \delta \mathcal{P}_\mathcal{R}(k, \eta_e), \qquad (15)$$

with

$$\delta \mathcal{P}_\mathcal{R}(k, \eta_e) \simeq -\frac{17}{36} \frac{\lambda H_{\text{inf}}^3}{\epsilon_{\text{inf}} \pi^2 M_P^2 m_0^2} \ln\left(\frac{\eta_e}{\eta_0} \right). \qquad (16)$$

This is the CSL correction to the power spectrum $\mathcal{P}_\mathcal{R}$ computed during inflation. We notice that $\delta \mathcal{P}_\mathcal{R}(k, \eta_e)$ is independent from k and r_c. This is just an artifact of the leading order expansion in $k\eta$. Indeed, by looking at the exact expression for $\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}}$ presented in the Supplemental Material [52], it is clear that the exact expression for $\delta \mathcal{P}_\mathcal{R}(k, \eta_e)$ depends both on r_c and the modes k. Moreover, the leading order expansion was justified by noticing that $r_c H_{\text{inf}} \gg 1$, and therefore indirectly relies on the largeness of r_c compared with the length scale H_{inf}^{-1}.

Equation (16) can be used to set upper bounds on λ. To obtain the numerical value of $\delta \mathcal{P}_\mathcal{R}$, we set $\eta_0 \approx -k_*^{-1} \approx -10^{60} M_P^{-1}$, where $k_* = 5 \times 10^{-60} M_P$ is the pivot scale, which first crosses the horizon at the e-folding number N_* satisfying $a(N_*) = k_*/H(N_*)$. In this way the dynamics is restricted up to the time at which the largest scales of interest 2×10^{-4} Mpc$^{-1} \lesssim k_{\text{CMB}} \lesssim 2$ Mpc^{-1} exit the horizon during inflation. The e-folding number N_* satisfies $50 \le N_* \le 60$ [56]. We fix $N_* = 60$. The scale factor at the end of inflation $a(\eta_e)$ can then be determined from the relation $a(\eta_e) = a(N_*) \exp(N_*)$. By setting $\epsilon_{\text{inf}} = 0.005$ [32], we find

$$\delta \mathcal{P}_\mathcal{R}(k, \eta_e) \sim \lambda/\lambda_{\text{GRW}} \times 10^{-34}, \qquad (17)$$

where $\lambda_{\text{GRW}} = 10^{-16}$ s^{-1} [2]. By comparing $\delta \mathcal{P}_\mathcal{R}(k, \eta_e)$ with the observational error of $\mathcal{P}_\mathcal{R}$, which is of order $\approx 10^{-11}$ [56], one obtains an upper bound $\lambda \lesssim 10^7$ s^{-1}, which is 17 orders of magnitude weaker than the state-of-art result $\lambda \lesssim 10^{-10}$ s^{-1} [21].

CSL and the radiation-dominated era.—In standard cosmology, the power spectrum is frozen at the end of

inflation for large scales [51]. However, as pointed out in [32,89], this may not be the case when the collapse dynamics is also taken into account. We now calculate the CSL contribution to the evolution of $\hat{\mathcal{R}}^2$ during the radiation-dominated era, which lasts from time η_e to $\eta_r = 3 \times 10^{60} M_P^{-1}$ which is estimated by using the fact that $a(\eta_r)/a(\eta_e) \approx 3 \times 10^{26}$ [90]. Notice that, as a first approximation, we are not including effects due to the reheating stage [90], and directly consider the radiation-dominated era as following the inflationary one.

During this era, the Hamiltonian density reads $\hat{\mathcal{H}}_0^I(\eta, \mathbf{x}) = \frac{1}{2}\{\dot{\hat{u}}^2(\eta, \mathbf{x}) + \frac{1}{3}[\nabla \hat{u}(\eta, \mathbf{x})]^2\}$, where the quantized field $\hat{u}(\eta, \mathbf{x})$ can still be expressed as in Eq. (2), and related to $\hat{\mathcal{R}}$ via $\hat{u} = z\hat{\mathcal{R}}$, but now the modes $v_k(\eta)$ are determined as the solutions of the equation $\ddot{v}_k(\eta) + \frac{1}{3} k^2 v_k(\eta) = 0$. By solving this equation and matching the curvature perturbation and its derivative with those at the end of inflation [32], one can obtain the explicit form for $v_k(\eta)$, which we report in the Supplemental Material [52]. By following the same choice as for inflation, we fix the collapse operator as $\hat{L}_{\text{CSL}} = \hat{\mathcal{H}}_0$. Therefore, in the interaction picture, the collapse operator can be rewritten as in Eq. (10), where $b_\eta^{\mathbf{p},\mathbf{q}}$ and $d_\eta^{\mathbf{p},\mathbf{q}}$ now follow

$$\begin{pmatrix} b_\eta^{\mathbf{p},\mathbf{q}} \\ d_\eta^{\mathbf{p},\mathbf{q}} \end{pmatrix} = \begin{pmatrix} j_\eta^{p,q} \\ l_\eta^{p,q} \end{pmatrix} - \frac{1}{3} (\mathbf{q} \cdot \mathbf{p}) \begin{pmatrix} f_\eta^{p,q} \\ g_\eta^{p,q} \end{pmatrix}, \qquad (18)$$

where $j_\eta^{p,q}$, $l_\eta^{p,q}$, $f_\eta^{p,q}$, and $g_\eta^{p,q}$ are defined in terms of the radiation-dominated era mode $v_k(\eta)$ as described in Eq. (11). It follows that the CSL correction $\langle 0 | \delta \hat{\mathcal{R}}_{\text{CSL}}^2(\eta_r) | 0 \rangle$ to the mean squared value of the comoving curvature perturbation generated during the radiation-dominated era has the same structure as in Eq. (12), with (η_e, η_r) substituting (η_0, η_e) and $C(\eta_r) = -\lambda r_c^3/(48 M_P^2 a^2(\eta_r) m_0^2 \pi^{9/2})$ replacing $C(\eta_e)$.

To quantify the effect, we first notice that during this era the scale factor is proportional to conformal time: $a(\eta) = (\eta - 2\eta_e)/(H_{\text{inf}} \eta_e^2)$. This expression for the scale factor neglects possible effects during reheating, as it is obtained by matching the well-known expressions for $a(\eta)$ and its derivative during inflation and the radiation-dominated era, as it was also derived in [32]. For times η close to η_e, all the modes of cosmological interest are outside the horizon and satisfy the condition $p\eta_e \ll 1$ and $q\eta_e \ll 1$.

As was the case in the inflationary era, the leading order contribution to $\delta \mathcal{P}_\mathcal{R}$ is now obtained by expanding in $p\eta_e \ll 1$, $p\eta \ll 1$ and $p\eta_r \ll 1$. This justification comes from looking at the exact functional form of $\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}}$ during the radiation-dominated era where the $p\eta$ and $p\eta_r$ terms lead to rapid oscillations of the integrand in the limit $p\eta \gg 1$ and $p\eta_r \gg 1$. This is in contrast to the inflationary era, where the subhorizon contribution $p\eta \gg 1$ is instead suppressed by the exponential term. For more details we refer to the discussion in the Supplemental Material [52]. Within this

approximation, $\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}}$ is given to leading order by

$$\mathcal{F}_\eta^{\mathbf{p},\mathbf{q}} \simeq -\frac{54}{\epsilon_{\text{inf}}^3 \eta_e^4 q^3}. \tag{19}$$

By following the procedure delineated above for inflation and reported in detail in the Supplemental Material [52], we derive the CSL correction to the power spectrum of the curvature perturbation at the end of the radiation-dominated era:

$$\delta\mathcal{P}_\mathcal{R}(k,\eta_r) = \frac{9\lambda H_{\text{inf}}^3 \eta_e^2}{2\epsilon_{\text{inf}}^3 M_P^2 (\eta_r - 2\eta_e)^2 \pi^2 m_0^2} \ln\left(\frac{2\eta_e - \eta_r}{\eta_e}\right). \tag{20}$$

As for the CSL contribution during inflation, we notice that $\delta\mathcal{P}_\mathcal{R}(k,\eta_r)$ is independent from k and r_C. This occurs for the same reasons as during the inflationary stage. In terms of λ_{GRW}, the correction in Eq. (20) reads

$$\delta\mathcal{P}_\mathcal{R}(k,\eta_r) \sim \lambda/\lambda_{\text{GRW}} \times 10^{-81}, \tag{21}$$

which can be safely considered as negligible with respect to the contribution obtained during inflation reported in Eq. (17).

Discussion.—Although there is no general consensus on how to generalize the CSL model to a relativistic scenario as required in a cosmological setting, some requirements have already been pointed out [32,33,72,73]. We propose a different generalization of the CSL model and study its effects on the scalar curvature perturbations and corresponding power spectrum. We find that the corrections, when compared with observations [56], provide upper bounds which by the end of inflation are already 17 orders of magnitude weaker than those from state-of-art ground based experiments [21]. A detailed study concerning possible modifications of other features of the CMB pattern, such as the presence of acoustic peaks, clearly goes beyond the scope of this letter. However, in light of the negligible corrections obtained in Eqs. (17) and (21) to the standard quantum mechanical power spectrum, we expect our choice of the collapse operator to be fully compatible with observations. Furthermore, the negligible corrections obtained in our work are in strong contrast to those obtained in Ref. [32]. As our calculations show, this difference is a consequence of the fact that the Hamiltonian density of the perturbations is several orders of magnitude smaller than the perturbed matter-energy density in standard cosmology. This difference in results for the two choices of the collapse operator is also confirmed in our analysis performed using the perturbative approach within the interaction picture framework [52].

Moreover, we find that the stringent constraints set on the collapse parameters in Ref. [32] are not fully self-consistent for the following reason. The measure of cosmological perturbations is quantified by the power spectrum which is defined in Eq. (5). In addition to the standard dynamics, collapse also contributes to the value of the power spectrum

with $\delta\mathcal{P}_\mathcal{R}$ proportional to λ. Working within perturbation theory limits the magnitude of the possible collapse induced corrections that can be trusted and hence the range of λ that can be observationally constrained. Indeed, if $\delta\mathcal{P}_\mathcal{R}$ is much greater than the classical value ϕ^2/M_P^2 then the assumption of $\delta\phi$ being much smaller than ϕ, which is fundamental for the application of linear cosmological perturbation theory, breaks down. We suspect that the application of linear perturbation theory in Ref. [32] is not valid for the entire range of λ values that the authors have excluded. For example, for $\lambda = 10^{-16}$ s^{-1} they find $\delta\mathcal{P}_\mathcal{R} = 10^{\sim 50}$ that should be compared to the classical value of $\phi^2/M_P^2 \sim 1$ which is typically the case during inflation.

We briefly comment on the claims made by the authors of Ref. [32] in their recent work of Ref. [58]. There, it is claimed that the power spectrum vanishes for our choice of the collapse operator. However, as our results show, this is not the case. Moreover, the proof provided in Ref. [58] relies on the assumption that the collapse operator leads to a fully localized wave function, which does not hold in general, and in fact needs not be applied to calculate the variance [52]. We have considered a physically consistent definition of the power spectrum, and with a well-motivated choice of collapse operator obtained theoretical corrections that are consistent with observations.

Finally, our work stresses that any eventual validation or discard of the CSL model cannot be made without addressing the issue of its generalization to the relativistic regime, which—without question—is becoming the subject of present and future research.

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