ON THE ROLE OF THE INCOMPRESSIBILITY CONSTRAINT IN SOFT DIELECTRIC COMPOSITES

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Summary In this work we demonstrate that the incompressibility constraint customarily adopted in literature to model soft dielectric composites may lead to incorrect predictions. In fact, although in these composites each phase may individually be assumed to be incompressible, for high-phase contrast in terms of elastic moduli the volumetric deformation of the softest phase can provide a non-negligible contribution to the effective behaviour. To reach our goal, we determine the effective electric response of a two-phase Dielectric Laminated Composite (DLC) actuator, whose softest phase is described by a constitutive law admitting volumetric deformation. Our results, discussed in the light of the limit case in which the softest phase consists of void, are expected to aid the design of high-performance hierarchical DLCs.

PROBLEM STATEMENT

We investigate the two-phase DLC actuator illustrated in Figure 1, which is built by repeating a unit cell constituted by an incompressible phase a and a considerably softer phase b, the latter being compressible. We describe the DLC effective behaviour in terms of the macroscopic deformation gradient $\mathbf{F}$ and the nominal electric field $\mathbf{E}$, along with their work-conjugate first Piola-Kirchhoff stress $\mathbf{S}$ and nominal electric displacement $\mathbf{D}$. All the microscopic fields and the material constants (such as the shear modulus μ and the electric permittivity ϵ) are indicated with the subscripts a and b, depending on the phase.

We assume the following free-energy density for the phase a, based on neo-Hookean hyperelasticity [1, 2]:

$$W_a = \frac{\mu_a}{2}\left[\mathrm{tr}(\mathbf{F}_a^{\mathrm{T}}\mathbf{F}_a) - 3\right] - \frac{\epsilon_a}{2}|\mathbf{F}_a^{-\mathrm{T}}\mathbf{E}_a|^2 \tag{1}$$

where tr and $^{\mathrm{T}}$ are the trace and transpose operators and $|\mathbf{v}|$ is the modulus of $\mathbf{v}$. For the phase b, we adopt the following *decoupled* free-energy density:

$$W_b = \frac{\mu_b}{2}\left(\frac{\mathrm{tr}(\mathbf{F}_b^{\mathrm{T}}\mathbf{F}_b)}{J_b^{2/3}} - 3\right) + \frac{K_b}{2}\left(\frac{J_b^2 - 1}{2} - \ln J_b\right) - \frac{\epsilon_b J_b}{2}|\mathbf{F}_b^{-\mathrm{T}}\mathbf{E}_b|^2 \tag{2}$$

in which $J_b = \det \mathbf{F}_b$ and K_b is the bulk modulus.

We consider a voltage-driven actuator subject to the following macroscopic boundary conditions. We apply increasing transverse nominal electric field $E_2 = \Delta\phi/h_0$ (see Figure 1), whereas we disregard edge effects, such that $E_1 = 0$. For what concerns the mechanics, we assume plane-strain and hamper macroscopic shear deformations, such that the sole non-vanishing components of $\mathbf{F}$ are the longitudinal stretch $F_{11} \equiv \lambda_1$, the transverse stretch F_{22}, and $F_{33} = 1$. Correspondingly, the effective direct stress components vanish, that is $S_{11} = 0$ and $S_{22} = 0$, while, in general, $S_{12} \neq 0$, $D_1 \neq 0$, $D_2 \neq 0$. We solve the problem by the homogenisation approach for hierarchical DLCs based on the continuity relationships holding at the interfaces [3, 1, 2, 4], assuming that the microscopic fields are spatially uniform in each phase.

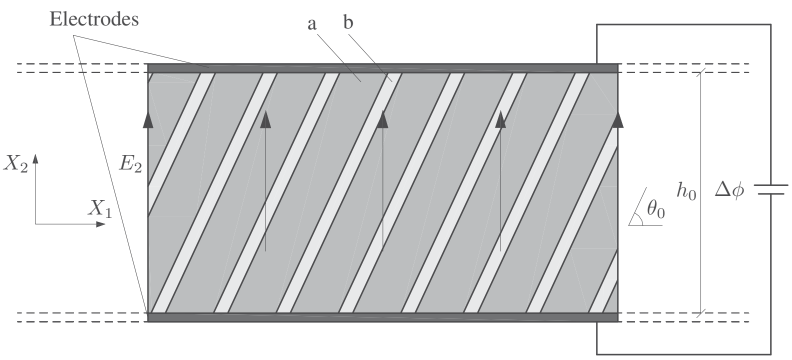

Figure 1: Geometric parameters of the DLC actuator consisting of incompressible phase a and compressible phase b. $\Delta\phi$ is the applied voltage drop across the electrodes, h_0 is the actuator thickness in the undeformed configuration, and θ_0 is the lamination angle.

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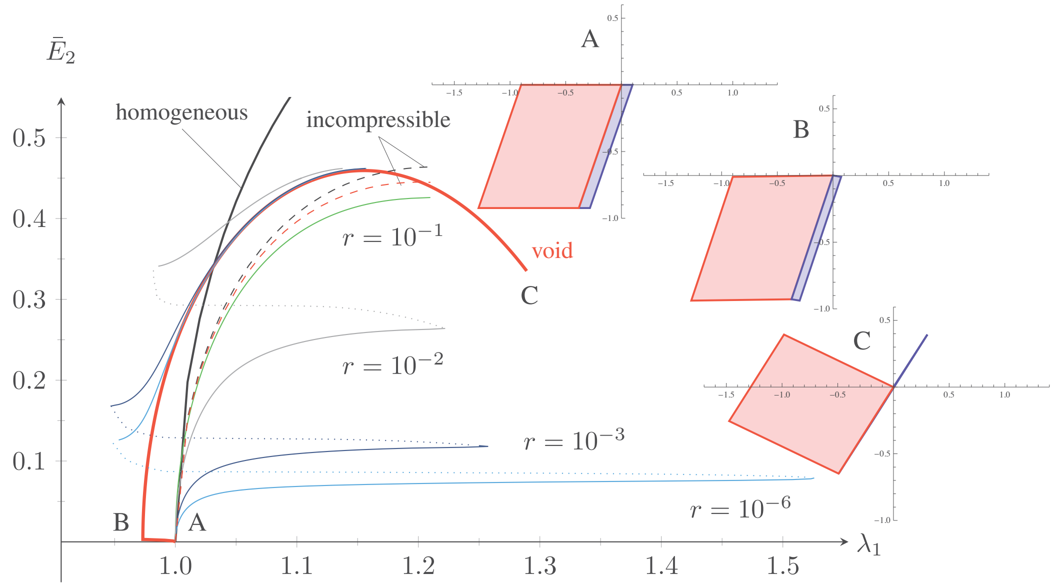

Figure 2: Performance of the compressible DLC actuator for varying $r = K_b/\mu_a$. Other parameters are $\theta_0 = 3\pi/8$, initial volume fraction of the compressible phase equal to 0.1, $\mu_a/\mu_b = 3 \times 10^3$, and $\epsilon_a/\epsilon_b = 10$. For the case in which phase b is void (red curves) we refer to $\mu_a = 10$ MPa and provide the configurations assumed by the two-phase unit cell at stages A (rest), B, and C.

RESULTS, DISCUSSION, AND PERSPECTIVES

Figure 2 illustrates the actuator response in terms of λ_1 as a function of the nondimensional electric field $\bar{E}_2 = E_2\sqrt{\epsilon_a/\mu_a}$, by varying the bulk modulus K_b through the ratio $r = K_b/\mu_a$. To discuss the results, we consider the case, referred to as the "limit case", in which the compressible phase b is void, for which the potential (2) has the electrostatic contribution only. In this case, we observe a behaviour characterised by three branches. The first branch (from A to B) is an initial *transient*, in which phase b suddenly shrinks whereas the continuity condition of stress at the interfaces cannot be properly satisfied. After this transient, a state of microscopically uniform *Cauchy* shear stress is reached all over the DLC. In this situation, holding up to the final stage C, all the electromechanical conditions are satisfied, as we can analytically prove. Our analysis also sets $\theta_0 = 3\pi/8$ as the lamination angle maximising J_b after the initial transient. After stage B, a second branch with increasing $\bar{E}_2$ follows, in which the DLC elongates as the result of local shear deformations and rotations. This branch ends with a maximum that we ascribe to the onset of a pull-in electromechanical instability. In the final branch $\bar{E}_2$ must decrease and the analysis ends when the phase b completely vanishes.

For a solid compressible phase b, the curves in Figure 2 share important features with the limit case, after an interesting high-performance branch. Then, in a transient response (dotted part of the curves), analogous to the first branch in the limit case, phase b largely shrinks while violating interface stress continuity. After this transient, the response approaches the limit case (from the left in Figure 2) for suitably small r. About the actuation response, for relative small r we observe an enhancement with respect to both the homogeneous DLC consisting of the sole phase a (black curve) and the incompressible DLC (black dashed curve), whose response is predicted through the effective potential obtained in [2]. Interestingly, the latter behaviour is very close to that of the incompressible DLC whose phase b is void (red dashed curve), which, in turn, is significantly different from the limit case, or any case with small r. This gives a clear indication that the incompressibility constraint in dielectric composites with large phase contrast should be cautiously adopted.

The foregoing results summarise those under publication in [5]. In this contribution we aim at further explaining the behaviour above by resorting to two strategies, towards relaxing the problem at hand and modelling actual hierarchical DLCs. First, we will consider imperfect interfaces. Second, we will allow the DLC to experience a non-vanishing macroscopic shear deformation F_{12}, to be constitutively related to the shear stress S_{12} through a linear law. Such a relation may, in fact, be particularised to both the case above and the case where one sets $S_{12} = 0$ to leave F_{12} free, for which, for any solid phase b, the obtained actuation responses continuously satisfy all the electromechanical conditions, being free from transients such as those reported above [5].

References

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