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Debbie Marie Versoza
Ateneo de Manila University

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University

Maria Alva Q. Aberin
Ateneo de Manila University, maberin@ateneo.edu

Mark Anthony C. Tolentino
Ateneo de Manila University, mtolentino@ateneo.edu

See next page for additional authors

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Authors

Debbie Marie Versoza, Ma. Louise Antonette N. De Las Peñas, Jumela F. Sarmiento, Maria Alva Q. Aberin, Mark Anthony C. Tolentino, and Mark L. Loyola

USING MOBILE TECHNOLOGY TO PROMOTE HIGHER-ORDER THINKING SKILLS IN ELEMENTARY MATHEMATICS

Debbie Marie B. Verzosa

*Department of Mathematics and Statistics
University of Southern Mindanao, Philippines*

Ma. Louise Antonette N. De Las Peñas

*Department of Mathematics
Ateneo de Manila University, Philippines*

Jumela F. Sarmiento

*Department of Mathematics
Ateneo de Manila University, Philippines*

Maria Alva Q. Aberin

*Department of Mathematics
Ateneo de Manila University, Philippines*

Mark Anthony C. Tolentino

*Department of Mathematics
Ateneo de Manila University, Philippines*

Mark L. Loyola

*Department of Mathematics
Ateneo de Manila University, Philippines*

ABSTRACT

The problem of rote-based learning in mathematics is well documented. Mobile technology can provide a potential solution, especially when application (app) design is based on sound pedagogical principles and gamification elements. However, an inventory of available mobile apps for mathematics reveals that many of the available apps are guided by a behaviorist perspective that favors repetition over meaningful learning. This paper reports on the design of mobile mathematics apps that harness gamification techniques to promote higher-order thinking skills (HOTS) even in basic elementary school concepts such as number comparison, and addition and subtraction. The integration of these apps in the classroom is also discussed.

KEYWORDS

mobile technology in teaching mathematics, mobile learning, numeracy apps, higher-order thinking skills, HOTS

1. INTRODUCTION

Despite the long history of reform education in mathematics, rote learning and low-level thinking are still prevalent in mathematics classrooms in many countries (Cox, 2015; Nag et al., 2014). Teachers struggle to teach mathematical concepts meaningfully and instead resort to explicating procedures disconnected from important elementary school concepts such as place value or number magnitude (Verzosa, 2020). Traditional

instruction focuses on teaching rules rather than on promoting critical thinking or problem solving, which are the twin goals of Philippine mathematics education (Department of Education, 2016).

Mobile technology offers a potential solution for mathematical learning, particularly if it is designed well (Calder, 2015). It can provide an additional medium for helping students visualize abstract mathematical concepts and can promote confidence and enjoyment in doing mathematics (Fabian, Topping, & Barron, 2018). A meta-analysis of research on the use of mobile technology demonstrates its potential for developing higher-order thinking skills (HOTS), independent learning, and reflective thinking (Ahmad et al., 2020). Unfortunately, a large proportion of the available apps perform the same function as practice worksheets; and are guided by a behaviorist perspective favoring repetition over meaningful learning (Highfield & Goodwin, 2013; Papadakis, Kalogiannakis, & Zaranis, 2018). This is despite the plethora of research that promotes sense-making and higher-order thinking in mathematics (Burns, 2007; Van de Walle, Karp, & Bay-Williams, 2015).

This paper reports on three numeracy apps that target a wide range of competencies in elementary school mathematics. The design of these apps was guided by the literature on mathematics education and are intended to promote higher-order thinking. The apps are designed in a game like environment. The use of games in mobile devices within lessons in primary mathematics has resulted in improved student engagement and made mathematics more enjoyable (Attard, 2018). Game-based learning is said to provide good exposure and experience in helping the learning process (El-Nasr & Smith, 2006).

These apps are a product of an ongoing government-funded project entitled “Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment”. In this project, mathematical apps also help teachers address the shift in modalities in education brought about by the pandemic. The apps serve as rich resources for both synchronous and asynchronous lessons in blended learning. Because the apps can be installed on mobile devices, these can be used by more school children and teachers who have access to cellphones and tablets, rather than computers and laptops.

2. HIGHER-ORDER THINKING SKILLS

A classic framework for analyzing student tasks is provided by Bloom’s Revised Taxonomy (Anderson & Krathwohl, 2001). Remembering and understanding comprise the lowest level of thinking. These consist of tasks that require students to copy, listen, memorize, or compute. By contrast, verbs associated with the higher levels of mathematics require students to investigate, represent, predict, construct or create (Van de Walle et al., 2015). Students who demonstrate higher-order thinking do not just know how to manipulate numeric symbols. They can form their own strategies and apply their knowledge to solve non-routine tasks, or tasks that cannot be solved by a specific algorithm.

2.1 Foundational Ideas in Elementary Mathematics

Higher-order thinking requires knowledge of foundational mathematics concepts. In the elementary grades, some of the most important ideas involve number sense (Yang, Li, & Lin, 2008), which includes various components such as number magnitude (Siegler & Lortie-Forgues, 2014), the effect of operations, and conceptual place value (Ellemor-Collins & Wright, 2011). These concepts provided the pedagogical basis for the apps designed through our project.

3. NUMERACY APPS

3.1 Ordering Game

The *Ordering Game* is based on a game described by a recognized expert in mathematics education, Marilyn Burns (cited by Silva, n.d.). Burns’ game involves whole numbers, but our team’s mobile app extends the game to include fractions and integers (Figure 1). In this game, one or two numbers are generated at random (Figure 1a). The user must choose one number and drag it to a row. As the process repeats, the user places more and

more numbers on the row with the condition that the numbers appear in ascending order. The goal of the game is to fill-up the entire row within the allowed number of chances, indicated at the bottom part of the screen. The game becomes more difficult as it progresses because it becomes more likely that a randomly selected number cannot be correctly placed in any of the remaining slots.

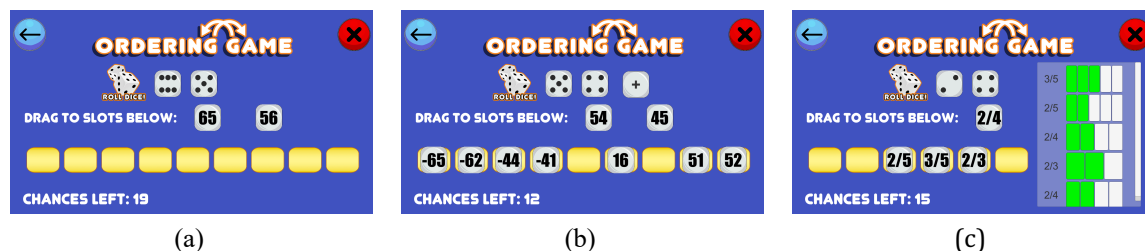


Figure 1. Screenshots from the *Ordering Game* for (a) Whole Numbers, (b) Integers, and (c) Fractions

The game offers opportunities to develop higher-order thinking skills. First, it pushes the user to reason with numerical magnitude, which is one of the key learning competencies in elementary mathematics (Siegler & Lortie-Forgues, 2014). Numerical magnitude understanding is necessary for estimating and comparing the sizes of numbers. Second, the game provides a natural environment to learn probability concepts. For example, if the randomly generated numbers are within the range 11 to 66 and the randomly generated number is 60, then the user must consider the likelihood that the next generated numbers will be higher or less than 60. Since there is a relatively low probability that a succeeding number will be greater than 60, then it is wise to place 60 somewhere near the right end of the row.

The app also presents appropriate representational media that enable users to reason mathematically. The game presents a variant of a number line estimation task, which is one of the tasks often used to measure symbolic numerical magnitude understanding (Fazio et al., 2014). In the case of fractions (Figure 1c), a regional model of each fraction is also shown, to support the visualization and comparison of fraction magnitudes.

Additional options are also available for each number type. For whole numbers (Figure 1a), users may choose among varying number range choices from 0 to 99,999. For integers (Figure 1b), users may choose to play within a number range between from -99,999 to 99,999. In the case of fractions (Figure 1c), users may choose to work with fractions with denominators from 1 to 6 or denominators from 1 to 9. In addition, users may also choose among Easy, Medium, or Challenging levels, which determine the number of chances available in each game.

3.2 Target Number Game

In the *Target Number Game*, users can choose one of the following targets: Greatest Sum, Least Sum, Greatest Difference or Least Difference (Figure 2a). Two sets of three-digit numbers are given and the goal is to rearrange the six digits, by swapping two at a time, so that the answer, either the sum or difference, shows the target. The game provides feedback on whether the user is able to get the target answer. It also gives hints so that the user can eventually reach the optimal solution.

To reach the target, users must have the knowledge of numerical magnitudes and the effect of operations. Higher-order thinking emerges from the number of considerations required to solve the problem. How do you form two three-digit numbers, from the given six digits, to form the largest or smallest possible sum? For example, to get the greatest sum in Figure 2b, the user should place the two largest digits in the hundreds place of each of the three-digit numbers. Then the next two largest digits should be in the tens places. The user can also discover that although the maximum sum is unique the two sets of three-digit numbers are not unique. In Figure 2c, another possible choice for the addends are 621 and 851. To get the least sum, the user should use the opposite strategy, that is, to place the two largest digits in the ones place, and so on.

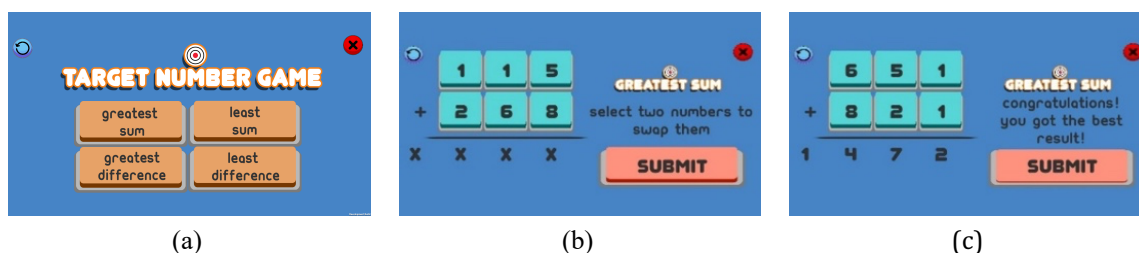


Figure 2. Screenshots from the *Target Number Game* (a) Main Menu, (b) Example of Greatest Sum, and (c) An Optimal Solution to an Example of Greatest Sum

Finding the greatest or least difference requires even more thinking. To find the greatest difference of two three-digit numbers formed using six digits, the user should realize that the first three-digit number must be as large as possible and the second three-digit number as small as possible. The most challenging target is finding the least difference. Unlike the other problems, there is no simple strategy such as placing the largest digits in the hundreds place or the ones place. The reason for the difficulty is that the difference must be a positive number. Thus, it is not only a matter of making the first three-digit number as small as possible and making the second three-digit number as large as possible. For example, in Figure 3a, the user must rearrange the digits to get the least difference. The user will try out different possibilities such as in Figure 3b, which will not give the target answer. Thus, the user will try out other arrangements until the target is achieved (Figure 3c). By playing the game a number of times, the user will be able to find a strategy to find the least difference.

Pilot testing revealed that even adults need to think to solve problems presented by the *Target Number Game*. But since the app involves only basic addition and subtraction, it can present the same thinking opportunities for elementary school children.

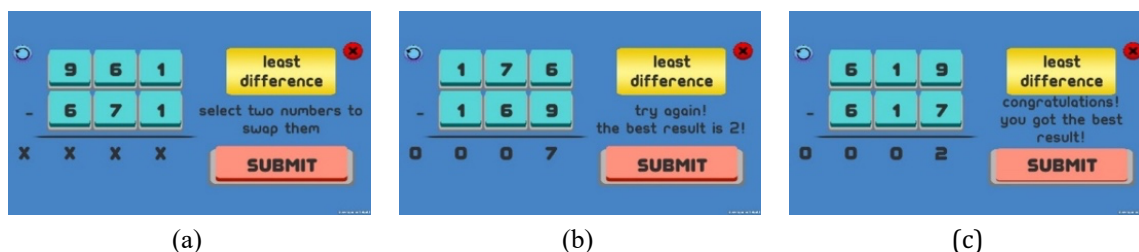


Figure 3. Screenshots of finding the least difference from the *Target Number Game*

3.3 Grid Game

In *Grid Game*, a 10 by 10 grid is shown on the screen, together with a beginning number and a target number. The user has to click the buttons (e.g., +1, -1, +10, -10) in order to reach the target number. For example, if the beginning number is 35 and the target is 49 (Figure 4a), then a possible solution is to click +10 once and +1 four times. The learning objective is to support children’s structuring of multi-digit numbers by training them to increment and decrement by tens starting any number, such as knowing that 56 and 10 more is 66 (Ellemor-Collins & Wright, 2011).

There are several levels available. In the *Explore* level, any solution is accepted, as long as the target number is reached. At the more advanced levels, the user has to complete the task in the shortest or most efficient way possible, as indicated by the number of allowed moves (“Moves Left” in Figures 4b and 4c). These levels promote higher-order thinking skills by forcing the user to move from counting by ones or just pressing +1 or -1 multiple times to reach the target. An efficient strategy requires knowledge of incrementing and decrementing by 10 and then pressing +1 or -1 a few times.

There are also variations in the type of visual support offered by the app. In the *Advanced* level, only a blank grid is shown (Figure 4b); in the *Challenging* level, no grid is shown (Figure 4c).

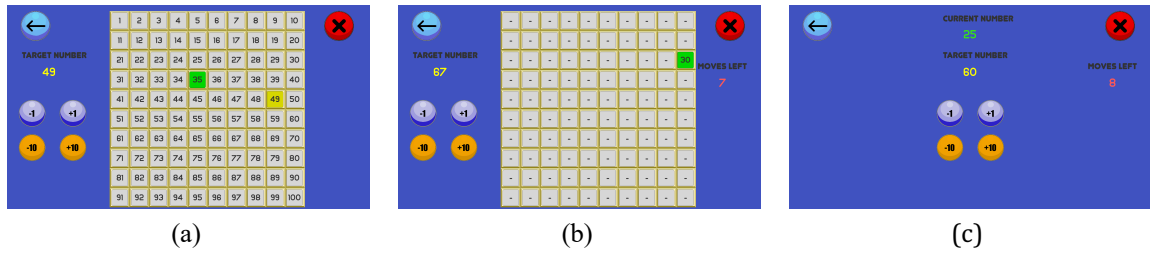


Figure 4. Screenshots from the *Grid Game* for Whole Numbers (a) Explore, (b) Advanced, and (c) Challenging levels

Because of the base-10 structure of the number system, the app can also be extended to larger whole numbers (in the range of 1-1000; Figure 5a), decimals (Figure 5b), and integers (Figure 5c). As in the previous app, the *Grid Game* involves only addition and subtraction and yet offers opportunities for higher-order thinking. For example, to get from 577 to the target 979 (Figure 5a) in 6 moves requires a correct combination of buttons +1, -1, +10, -10, +100, and -100. If the user fails to solve the problem, the app presents the same problem again so that the user always has chances to think of the most effective strategy.

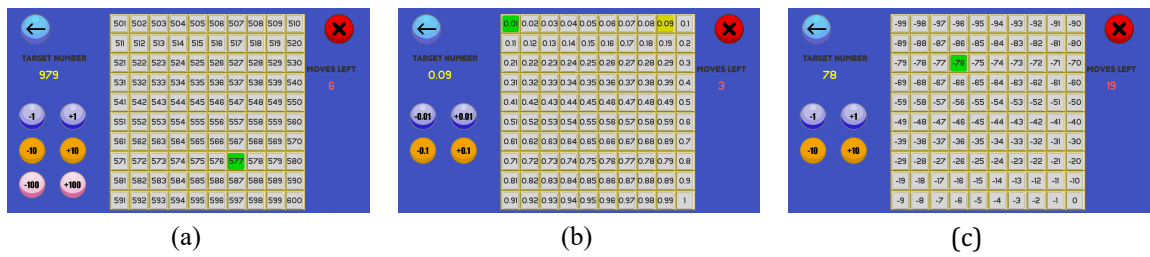


Figure 5. Screenshots from the *Grid Game*: (a) Whole Numbers (range 1-1000), (b) Decimals, (c) Integers

4. GAMIFICATION FEATURES

Employing gamification, or the application of game design elements, techniques, and principles, in education across different subjects and different learning activities, has been prevalent in the past years. Primarily, gamification is used to motivate and engage students better, with the hope of increasing the effectiveness of learning materials, as well as to hone skills such as creative thinking and independent learning (Caponetto et al., 2014). Moreover, video and mobile games enjoy widespread popularity among the youth; for instance, in the Philippines, it is reported that there are approximately 40 million gamers with around 75% of them playing games on mobile devices (Elliot, 2020). Thus, it is opportune to use gamified environments in education to take advantage of students' familiarity with and disposition towards games.

The design of the *Ordering Game*, *Target Number Game*, and *Grid Game*, as described in the previous section, was guided by gamification principles and took advantage of game-like elements and settings to maximize their effectiveness and encourage student participation. More specifically, these three apps adhere to the Educational Games Design Model (Ibrahim & Jaafar, 2009), which consists of Game Design, Pedagogy, and Learning Content Modelling. Table 1 provides the details of the apps' Game Design.

Table 1. Game Design of *Ordering Game*, *Target Number Game*, and *Grid Game*

	Element	Implementation
Usability	Satisfaction	The vibrant color theme, large font styles, and visual designs in the three apps are catered to its target users, who are primarily grade school students. The apps feature different modes and difficulty levels for more diverse experiences.
	Efficiency	The three apps feature simple menus and interfaces. Users can immediately go into a game level after a few menu selections. Transitions between levels are quick and the controls are simple and responsive.
	Effectiveness	The three apps simulate learning activities that can help students develop their numeracy skills. Each app allows the users, with the guidance of an adult, to customize the app's topic and level to match the students' level.
Multimodal	Multimedia	The apps make use of texts, icons, figures, simple animations, and sound prompts.
	Interaction	The apps are interactive with users being able to tap, click, or drag objects on the screen directly using touch gestures, or indirectly using an input device.
Fun	Challenge	The difficulty level of the games can be adjusted so that users can start with easy levels and progress to more challenging levels on their own or as instructed by their teachers. The topics covered as well as game elements can be adjusted.
	Clear goals	Each app has a single clear goal. The user is prompted once the game is done or the goal is achieved.
	Uncertain outcome	The problem in each game is randomly generated but strategically planned. Moreover, solving each problem requires higher-order thinking. For example, in <i>Ordering Game</i> , there is some uncertainty in the sequence of numbers that will be generated, so users are potentially confronted with a new set of challenges each time.
	Self-esteem	The customization of the difficulty levels allows the user to build their self-esteem as they successfully win more difficult levels.

On Pedagogy, the previous section details how the strategy for each game requires knowledge and skills on numeracy. To summarize, the strategy for *Ordering Game* requires estimation skills and knowledge of number magnitude; for *Target Number Game*, the strategy requires knowledge of number magnitude and the effect of operations; and for *Grid Game*, the strategy requires knowledge of incrementing/decrementing by ones and tens. With the tiered difficulty levels implemented in the apps, users can develop knowledge and skills as they encounter the need to formulate new strategies for the increasingly difficult levels. Moreover, the three apps can be used independently, requiring little guidance from adults or teachers.

On Learning Content Modelling, the three apps were designed so that the math topics involved therein are aligned with some of the most essential learning competencies for Grades 1 to 6 (Department of Education, 2020) prescribed by the Philippine Department of Education. These apps can also be used as remediation or enrichment activities for students in higher grade levels. As previously described, the contents and game structure of each app were designed to promote higher-order thinking skills so that even older children or adults can be challenged by the apps.

5. INTEGRATION OF THE APPS INTO THE CLASSROOM

The nature and gamification features of the apps described in the previous sections ensure opportunities for students to engage in non-routine problems which require analysis, estimation, making predictions and conclusions, and reasoning. Such engagement promotes active learning and the prospect of developing higher-order thinking skills. There are several approaches in which the app can be integrated in the classroom to support student learning and develop higher-order thinking skills among students.

All the three apps (*Ordering Game*, *Target Game* and *Grid Game*) can be used to provide input for meaningful mathematical activities. When these are utilized, it should be directed towards (1) reasoning; (2) inference making; (3) creation of a strategy and (4) collaborative interaction with classmates. To illustrate, in

the instance of *Ordering Game* shown in Figure 1a, the teacher can ask questions such as: *Which of the numbers 56 and 65 should be used? In which position should it be placed? Why?* These questions engage students and can develop reasoning skills, form inferences and eventually devise a winning strategy.

Similarly, in the *Target Game*, the teacher can pose questions as a scaffolding technique towards stronger understanding and reasoning. In the problem shown in Figure 2b, teachers can ask: *Which two numbers can be placed in the hundreds place to get a four-digit sum? What numbers should be placed in the tens digit to get the greatest sum? How many possible answers are there?* After several examples, the teacher can lead the students to make a generalization on how to create the greatest sum from the given digits.

In the Grid game task shown in Figure 5a, teachers can ask: *Which of the buttons should be used to go from 577 to 979 within 6 moves? Is it possible to do it if the +1 button is used? Why? How about if the +10 button is used? How many solutions are possible to get the target number?*

The team has also conducted a series of webinars and training for teachers to introduce them to the apps and highlight its potential for developing higher-order thinking skills. Initial feedback is promising. Some comments indicate that the apps are “very enjoyable and motivating” and “will help students ignite their interest” through “its game like features”, and that learners “will be able to enjoy, think and [be] challenged a lot”. A more robust implementation will form the next phase of this ongoing project (although initial implementation has started for the apps in October 2020 in selected Department of Education School Divisions). It is anticipated that the apps, which are designed on the basis of research in mathematics education (Ellemor-Collins & Wright, 2011; Siegler & Lortie-Forgues, 2014; Van de Walle et al., 2015) and mobile technologies (Fabian, 2018; Ibrahim & Jaafar, 2009) can facilitate mathematical thinking.

6. CONCLUSION

The numeracy apps presented here address an urgent need, especially because many of the available apps focus on rote learning rather than on developing deep conceptual understanding and higher-order thinking (Papadakis, Kalogiannakis, & Zaranis, 2018). The three apps described in this paper were designed on the basis of mathematical and pedagogical frameworks, and on gamification strategies. Further, the apps are aligned with official learning competencies, while also promoting higher-order thinking skills. The apps were designed to run on mobile technologies to encourage and enhance the student learning experience. Another intention was to maximize the benefits of mobile technologies such as mobility and portability, to overcome time and space constraints in the learning environment. Beyond pedagogical and gamification designs, the role of the teacher and the educational context are crucial elements in the success of integrating the mathematical apps in learning.

The development of the mathematical apps described in this paper is funded by a government agency, and partnerships with various school divisions had been forged. The next step in the project is to study the effectiveness of the apps for student learning, and possibly how these affect the students’ attitude toward mathematics, which will be conducted with partner Department of Education School divisions.

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REFERENCES

- Ahmad, M. et al., 2020. Mobile Technology in Enhancing Students' Higher Order Thinking Skill. *Journal of Physics: Conference Series*, Vol. 1529, 042057.
- Anderson, L.W. & Krathwohl, D. R. (Eds.), 2001. *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives: Complete Edition*. Addison Wesley Longman, New York, USA.
- Attard, C., 2018. Mobile Technologies in the Primary Mathematics Classroom: Engaging or Not? In N. Calder, K. Larkin, & N. Sinclair (Eds.), *Using Mobile Technologies in the Teaching and Learning of Mathematics*, Springer, pp. 51-65.
- Burns, M., 2007. *About Teaching Mathematics: A K-8 resource* (3rd ed.). Math Solutions Publications, Sausalito, CA.
- Calder, N., 2015. Apps: Appropriate, Applicable, and Appealing? In T. Lowrie, R. Jorgensen (Zevenbergen) (Eds.), *Digital Games and Mathematics Learning*. Springer, Dordrecht, the Netherlands, pp. 233-250.
- Caponetto, I. et al., 2014. Gamification and Education: a Literature Review. *Proceedings of the 8th European Conference on Games-Based Learning - ECGBL 2014*, pp 50-57.
- Cox, R. D., 2015. "You've Got to Learn the Rules:" A Classroom-level Look at Low Pass Rates in Developmental Math. *Community College Review*, Vol. 43, No. 3, pp. 264-286.
- Department of Education (DepEd), 2016. K to 12 Curriculum Guide: Mathematics. [Online]. Available: https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf
- Department of Education (DepEd), 2020. K to 12 Most Essential Learning Competencies with Corresponding CG Codes. [Online]. Available: <https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf>
- El-Nasr, M. S. & Smith, B. K., 2006. Learning through Game Modding. *Computer Entertainment*, Vol. 4, No. 1, Art. 3B, pp. 1-20.
- Ellemor-Collins, D. & Wright, R., 2011. Developing Conceptual Place Value: Instructional Design for Intensive Intervention. *Australian Journal of Learning Difficulties*, Vol. 16, No. 1, pp. 41-63.
- Elliott, R., 2020. The Philippines' Games Market: Data and Insights. [Online]. Available: <https://newzoo.com/insights/articles/data-and-insights-on-the-philippines-games-market/#:~:text=Share%3A,Southeast%20Asia's%20overall%20games%20market>.
- Fabian, K., Topping, K.J, & Barron, I.G, 2018. Using Mobile Technologies for Mathematics: Effects on Student Attitudes and Achievement. *Education Tech Research Dev*, Vol. 66, pp.1119-1139.
- Fazio, L.K. et al., 2014. Relations of Different Types of Numerical Magnitude Representations to Each Other and to Mathematics Achievement. *Journal of Experimental Child Psychology*, Vol. 123, pp. 53-72.
- Highfield, K. & Goodwin, K., 2013. Apps for Mathematics Learning: A Review of 'Educational' Apps from the iTunes App Store. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics Education: Yesterday, Today and Tomorrow (Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia)*. Melbourne, Australia, pp. 378-385.
- Ibrahim, R. & Jaafar, A, 2009. Educational Games (EG) Design Framework: Combination of Game Design, Pedagogy and Content Modeling. *Proceedings of the 2009 International Conference on Electrical Engineering and Informatics, ICEEI 2009*, pp. 293-298.
- Nag, S. et al., 2014. Literacy, Foundation Learning and Assessment in Developing Countries: Final Report. *Education Rigorous Literature Review*. Department for International Development, United Kingdom.
- Papadakis, S., Kalogiannakis, M., & Zaranis, N., 2018. Educational Apps from the Android Google Play for Greek Preschoolers: A Systematic Review. *Computers and Education*, Vol. 116, pp. 139-160.
- Siegler, R. S. & Lortie-Forgues, H., 2014. An Integrative Theory of Numerical Development. *Child Development Perspectives*, Vol. 8 No. 3, pp. 144-150.
- Silva, M. (n.d.) Place Value Activity Package. [Online]. Available: <http://www.mathematicshed.com/uploads/1/2/5/7/12572836/place-value-activity-pack.pdf>
- Van de Walle, J.A., Karp, K S., & Bay-Williams, J. M., 2015. *Elementary and Middle School Mathematics: Teaching Developmentally* (9th ed.). Pearson, Essex, UK.
- Verzosa, D. M. B., 2020. Pre-service Elementary Teachers' Reasoning in Introducing Multidigit Subtraction. *International Journal of Mathematical Education in Science and Technology*, Vol. 51, pp. 876-892.
- Yang, D. C., Li, M. N., & Lin, C. I., 2008. A Study of the Performance of 5th Graders in Number Sense and its Relationship to Achievement in Mathematics. *International Journal of Science and Mathematics Education*, Vol. 6, pp. 789-807.