27th International Congress on Sound and Vibration
The annual congress of the International Institute of Acoustics and Vibration (IIAV)
11-16 July, 2021
ICSV27
Annual Congress of the International Institute of Acoustics and Vibration (IIAV)

EFFECT OF SPACING BETWEEN A PAIR OF SINGLE DEGREE OF FREEDOM HELMHOLTZ RESONATORS

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This study focuses on the experimental and numerical evaluation of the acoustic performance and underlying physics of sound attenuation from single degree-of-freedom Helmholtz resonators, located in line as an alternative passive sound absorption device for conventional locally resonant liners. Experiments were performed using a grazing flow impedance tube with two highly instrumented Helmholtz resonators, fitted with internal microphones. The relation between the noise attenuation performance of the two resonator system and the spacing between two resonators is characterized in terms of the transmission loss, transmission coefficient and the corresponding frequency bandwidths. The change in sound pressure level (SPL) along the length of the impedance tube for different resonator spacing configurations is also analysed, thereby improving the understanding of the effect of a Helmholtz resonating cavity on the sound field in a duct. Finite element analysis is performed and validated against the experimental data, which has provided interesting results about the pressure and velocity field inside the cavity and in the impedance tube. The results demonstrate that there exists an optimum spacing where the noise attenuation performance of the resulting resonator couple is maximized.

Keywords: Helmholtz resonator, impedance tube, linear acoustics

1. Introduction

Broadband noise attenuation has become a challenging topic in the recent decade for researchers from different backgrounds due to stringent noise emission regulations[1, 2]. The investigations for lightweight, space-efficient , broadband noise attenuators span from the emerging field of acoustic metamaterials to Helmholtz resonators[3]. Although Helmholtz resonators are efficient sound absorbing devices, they come with the penalty of being effective on a narrow band frequency range. The research topic remains arduous due to the unimpressed aspects of the problem, especially the disparity between theoretical and experimental results, not just limited to the resonance frequency but extending to factors affecting the sound absorption or damping characteristics of a resonator. The sound absorption characteristics of a Helmholtz resonator are governed by various geometric factors, which include the resonator cavity volume, the cross-sectional area of the resonator neck and the length of the neck. The resonance frequency of a resonator can be obtained via the classical lumped analysis, given by [4]:

$$f_o = (\frac{c_o}{2\pi})\sqrt{\frac{S_n}{V_c(l_n+\delta_n)}}, \tag{1}$$

where, c_0 is the speed of sound, S_n is the cross-sectional area of neck opening, l_n is the length of the neck and δ_n is the end correction factor which accounts for any discontinuities within the resonator leading to the creation of higher order modes [5, 6].

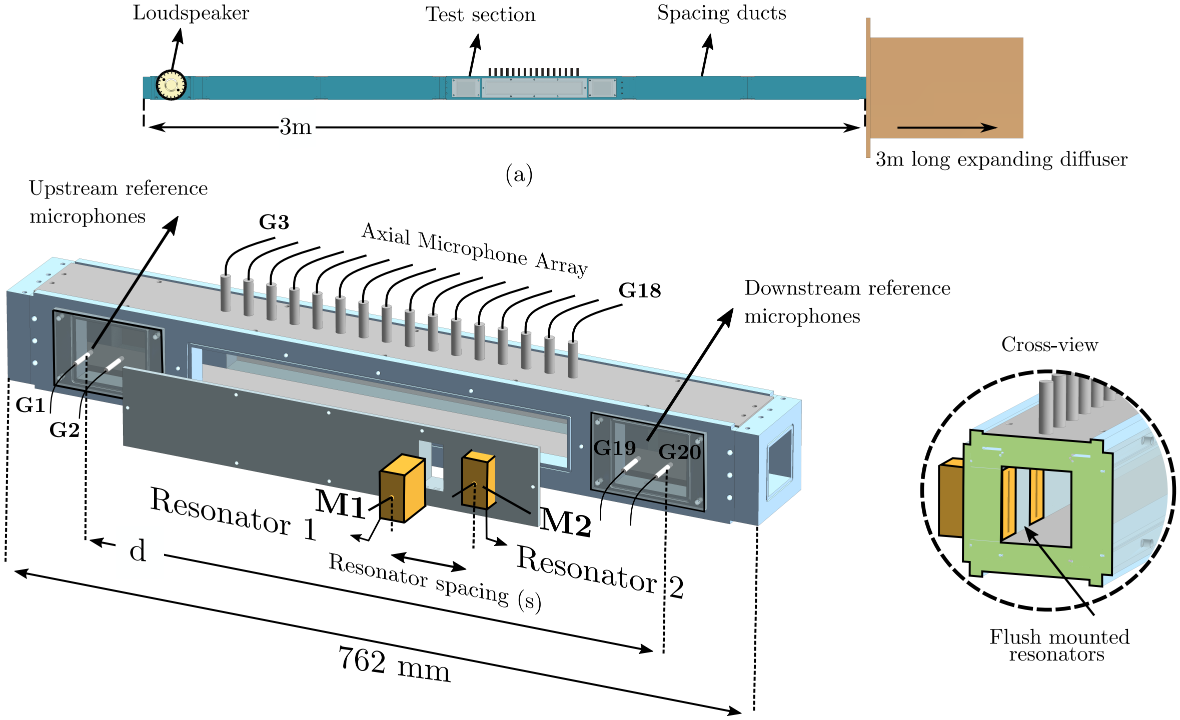
Extensive research has been carried out on understanding the effect of changing various geometric parameters, discussed in Equation 1, on the sound absorption characteristics of a Helmholtz resonator. The effect of changing the neck geometry such as the neck opening cross-sectional area, neck location and neck length for a resonator with circular and rectangular cavity were studied by Ingard [5]. The study illustrated a decrease in resonance frequency with increasing neck length whereas an increase was seen when the neck cross sectional area was increased. The study also developed end corrections to consider the excitation of higher order modes at the junction between the cavity and resonator neck. The effect of changing cavity depth and width for both a fixed and variable volume resonator was studied by Chanaud [6]. In the case where the resonator volume and dimensions of the neck were fixed, the study showed a change in resonance frequency with changing orifice position whereas the shape of the orifice had no significant effect on the same. The effect of cavity volume and orifice position has also been studied by Selamet et.al [7, 8, 9]. These studies were focussed on analysing the effect of changing the resonator length as a function of its diameter, on the transmission loss and resonance frequency.

Although a Helmholtz resonator is very effective at attenuating sound at its resonance frequency, there exists a need to find novel ways to increase the bandwidth of frequencies that can be attenuated. Helmholtz resonator cavities are spaced uniformly within an acoustic liner, when used for commercial applications. While there is a diverse range of literature on how changing the parameters of a single resonator affects its sound absorption capability, the effect of spacing between consecutive resonators remains to be investigated. The present work focuses on the effect of changing the spacing between two resonators, to investigate its effect on the transmission loss characteristics and change in the sound pressure levels between the inlet and the outlet of the duct to which the resonators are mounted on. An optimised resonator spacing, keeping the dimensions of the resonator fixed, might be beneficial in a range of applications [10, 11, 12].

2. Experimental Setup

2.1 Testing facility

Experiments were carried out at the University of Bristol Grazing Flow Impedance Tube Facility. The schematic of the facility is presented in Fig. 1. The Impedance Tube is a 50.4mm x 50.4mm square cross-section tube, 3000 mm in length with a 3000 mm long diffusing section to reduce air velocity and minimise acoustic reflections back into the test section. Flow speeds of up to Mach 0.3 can be achieved within the test section using a 15kW centrifugal fan. Sound pressure levels of up to 130dB can be achieved in the test section via two BMS 4592ND compression drivers with the acoustic pressure data obtained via GRAS 40PL microphones. Data acquisition was achieved with a NI PXIe-4499 modules mounted in a NI PXIe-1082 chassis. The loudspeakers were excited at white noise at an amplitude of 10Vpp via a Tektronix function generator. MatlabR2016a was used to interface between the data acquisition system and the signal generator to run the data acquisition code. The tests were performed for 16s at a sampling frequency of 2^{15} Hz.



(b)

Figure 1: Schematics of (**a**) the grazing flow impedance tube, (**b**) the test section of the impedance tube with microphone naming convention and the cross section-view of the test section.

2.2 Test Samples and Experimental program

For experimental analysis, two one degree of freedom resonators tuned to 1360 Hz (f_o) were manufactured by laser cutting walls of the resonators from 3 mm Perspex, with finger edge joints to facilitate the resonator sealing process. Knowles Omni-Directional (FG) 2.56 DIA electret condenser microphones (M1 and M2) were flush mounted to the top internal wall of the two resonators. These microphones are permanently glued to the resonator walls to obtain a perfect air seal to ensure accurate measurements. The resonators have an internal cavity dimension of 36.4 mm x 35 mm x 14 mm. The neck length is 3 mm and the slit opening dimensions are 36.4 mm x 3 mm. Following calibration of microphones M1 and M2, the walls were sealed together with acrylic sealing liquid and the sealing checked by comparing the transmission loss obtained from the sealed resonator to that of the numerical model from COMSOL Multiphysics.

The two single-degree-of-freedom resonators were flush mounted to the internal wall of the impedance tube working section. Data was acquired via 22 surface pressure transducers in total. Microphones G1,G2,G19 and G20 mounted upstream and downstream of the test section, respectively, to evaluate transmission loss and coefficient, followed by microphones G3-G18, covering the length of the liner section to measure the sound pressure level (SPL) decay along the length of the test section. Microphones M1 and M2 measure the acoustic signal inside the two test resonators. In order to have a clear understanding of the effect of spacing, on the sound attenuation, thirteen different spacing configurations between

$0.13 < s/\lambda < 1$ were investigated experimentally, where 's' is the spacing between the two resonators normalised by the wavelength (λ) of the resonance frequency (f_o).

3. Numerical Setup

Finite Element Analysis simulations were carried out on COMSOL Multiphysics to obtain the transmission loss data for the different Helmholtz resonator spacing configurations, as well as the resonator cavity pressure and velocity information. The Pressure Acoustics interface was coupled with the Thermoviscous Acoustics interface to model thermal and viscous losses in all simulations. A free triangular mesh was used for the impedance tube domain and a boundary layer mesh type was used for the domain consisting of the two resonators. A port boundary condition was used to setup an incident wave at one boundary to model a source and prescribe a non-reflecting condition at the end of the waveguide i.e. a termination. The maximum element size for the mesh was chosen to be 2mm, which is smaller than 1/6th of the wavelength of the highest frequency of interest i.e. 3000 Hz. The minimum element size chosen was 0.12 mm. The maximum element growth rate was 1.2 with a curvature factor of 0.3 and resolution of narrow regions of 3.

4. Results and discussion

Figures 2a and 2b compare the experimental transmission coefficient and transmission loss results respectively, for a single resonator compared to two resonators mounted on to the impedance tube test section. Including a second resonator has considerable effects on both the transmission loss and coefficient. At a transmission coefficient of 0.5, the bandwidth (Δf_0) increases by 114 Hz or 104% when a second resonator is introduced, at a spacing s/λ = 0.5, as compared to the single resonator case. Introduction of a second resonator causes a substantial increase in the transmission loss by 42 dB or 302% compared to a single resonator case, at a spacing s/λ = 0.25. A study was therefore carried out to characterise the effect of spacing between two resonators on the sound pressure level in the duct, the transmission loss and transmission coefficient.

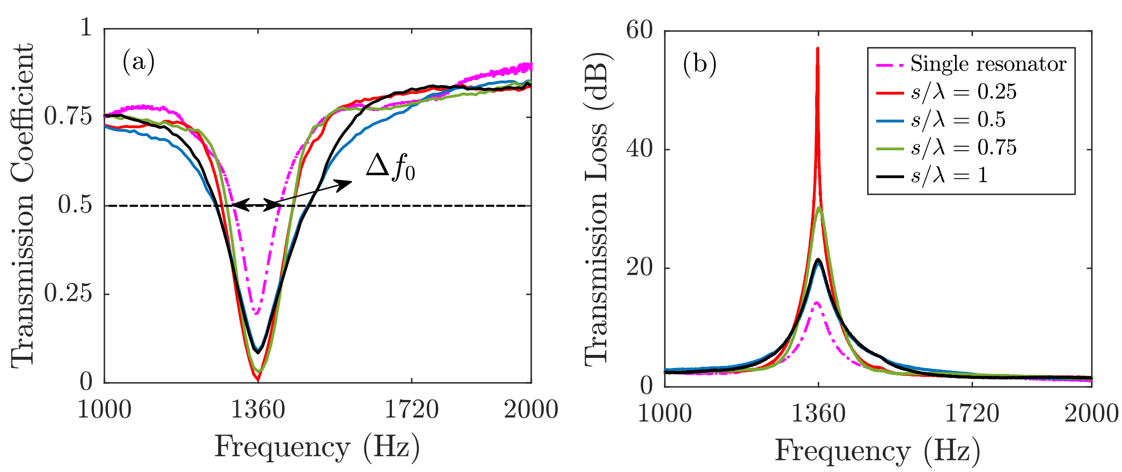


Figure 2: (**a**) Experimental results for transmission coefficient induced by different spacing (s/λ) configurations; (**b**) Experimental results for transmission loss induced by different spacing (s/λ) configurations.

The change in SPL (sound pressure level) is plotted against the location (x) of microphones G3-G18, as the wave propagates downstream, normalised by the distance between microphones G1 and G20 (d)

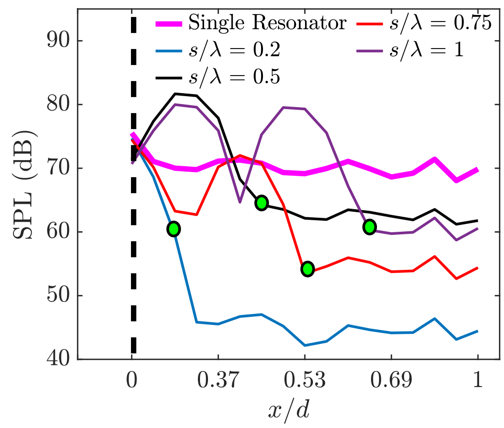


Figure 3: Change in the SPL value at resonance frequency (f_o) in the impedance tube for different spacing configurations between $0.2 < s/\lambda < 1$.

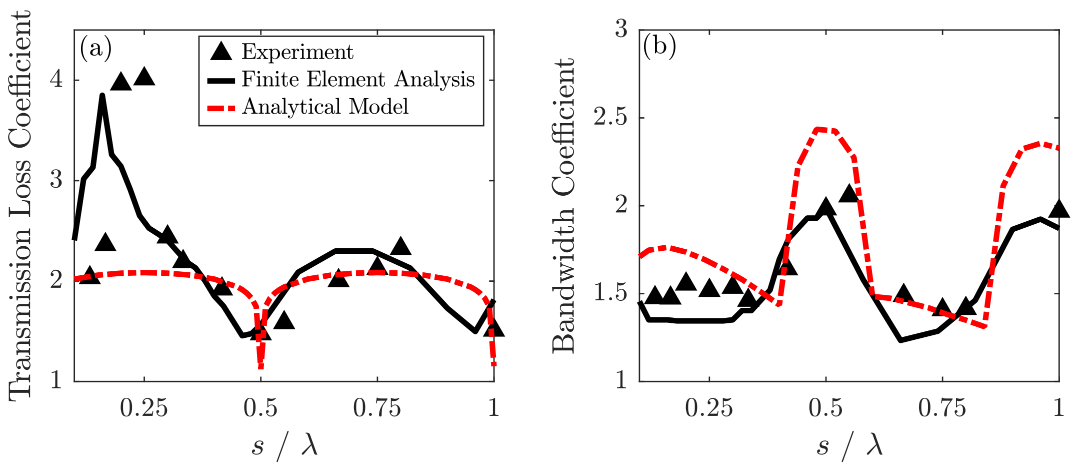


Figure 4: (**a**) Normalised transmission loss comparison for different spacing configurations; (**b**) Bandwidth Coefficient $\Delta f/\Delta f_o$ comparison for different spacing configurations.

and presented in Fig. 3, for different spacing configurations. Positions of the first and second resonators within the test section are shown with a black dotted line and green dots respectively. The sound pressure level is calculated at the resonators' resonance frequency of 1360 Hz.

As the spacing between the resonators is decreased, the slope of SPL increases with downstream microphone location (x/d). This indicates that reducing the spacing between two Helmholtz resonators within an acoustic liner may be beneficial when it comes to overall sound pressure level reduction. However, the overall change in sound pressure level (Δ SPL) i.e. the difference between SPL values of the first and last microphone locations has a sinusoidal behaviour. ΔSPL is maximum at $s/\lambda = 0.2$ and then reduces to a minimum at $s/\lambda = 0.5$ after which it peaks again at $s/\lambda = 0.75$ and finally having another minimum at $s/\lambda = 1$. This behaviour indicates that there may exist an "optimum" location to position resonators within an acoustic liner body for maximum sound attenuation.

A study was carried out to investigate the effects of changing resonator spacing on the transmission loss introduced in the impedance tube due to the presence of the resonators. Figure 4a shows a comparison of the transmission loss peak coefficient, with the different spacing configurations. The transmission loss peak coefficient is defined as a ratio of the peak transmission loss value for a particular spacing configuration to the peak transmission loss value for a single resonator case. Changing the spacing between two resonators also has a sinusoidal response when it comes to the transmission loss induced. There exists a local minimum at a spacing s/λ =0.5. The transmission loss increases either side of this spacing configuration. Decreasing the spacing has a more favourable effect on transmission loss, compared to increasing the spacing. The first peak transmission loss coefficient is obtained at s/λ =0.25 and the second peak is obtained at s/λ =0.75. The trend matches quite well between the experimental results, numerical simulations and the analytical model. The analytical model was based on a study by Cai and Mak [13], to develop an expression to calculate the transmission loss of a lined Helmholtz resonator array i.e. two identical Helmholtz resonators separated by a fixed distance.

The bandwidth coefficient (η) is a measure of the range of frequencies attenuated by a particular Helmholtz resonator spacing configuration. The bandwidth coefficient is given by $\eta = \Delta f/\Delta f_0$, where Δf is the bandwidth of frequencies attenuated for a transmission coefficient of 0.5, and Δf_0 is the bandwidth of frequencies attenuated with a single resonator configuration, as shown in Fig. 2. A study was carried out to investigate how the bandwidth coefficient (η) changes with different spacing configurations. As seen in Fig. 4b, the bandwidth coefficient has a maximum at s/λ=0.5, which indicates that this particular spacing configuration attenuates the broadest bandwidth of frequencies. Figure 4a showed how a spacing s/λ=0.5, leads to the lowest transmission loss peak. Therefore, the spacing, transmission loss and transmission coefficient are related in a way that the spacing configurations that lead to the lowest transmission loss, results in the broadest bandwidth range of frequencies being attenuated and vice-versa. The scales of the curves being non-similar in both Figs. 4a and 4b, might be due to the experimental conditions not being accurately represented in the analytical and FEM analysis. The COMSOL simulations assume a sound hard wall for the resonators. However, since the resonators were manufactured using acrylic sheets, the walls may not be perfectly sound hard. Similarly, thermal and viscous losses are ignored in the analytical model. All these factors may lead to the discrepancies in scaling for the numerical and analytical results compared to experiments. However, the trends of the analytical, experimental and numerical results match quite well.

To pursue a better understanding of the physical meaning of the effect observed in Figs. 3 and 4, the resonating cavity internal pressure and particle velocity are studied through transient finite element simulations. A snapshot of the pressure and velocity inside the resonating cavities as well as the impedance tube duct is shown in Fig. 5. Figure 5a shows the overall acoustic pressure and particle velocity snapshot for a resonator spacing s/λ=0.25, whereas Fig. 5b shows the same information for s/λ=0.5. In the case of s/λ=0.25, both resonators resonate in phase, as they experience high and low pressure cycles at the same time. The opposite is true for s/λ=0.5. In addition, s/λ=0.25 has a stronger effect on the incoming plane wave, leading to a significant distortion of wavefront as compared to s/λ=0.5. To further analyse both the acoustic pressure and velocity fields over two time periods, data captured with 50 domain probes evenly spread along the resonator centre line L1 and L2 with 1mm spacing, was used to present the contour plots, shown in the Fig. 6. Figures 6a and 6b describe the acoustic pressure inside Resonator 1 cavity, at two different resonator spacings i.e. s/λ = 0.25 and s/λ = 0.5. Similarly acoustic pressure inside Resonator 2 cavity is shown in Figs. 6e and 6f. Resonator 1 cavity particle velocities are shown in Figs. 6c and 6d, at 2 different resonator spacings i.e. s/λ = 0.25 and s/λ = 0.5. Similarly, Figs. 6g and 6h show the particle velocity inside Resonator 2 cavity. All results in Fig. 6 are subject to a source which was excited at f_0 over two time periods. At all separation distances, the magnitude of acoustic pressure in the Resonator 2 is always lower than acoustic pressure observed inside the Resonator 1 cavity. At a spacing s/λ = 0.25, the acoustic pressure seems to be concentrated in Resonator 1 whereas at s/λ = 0.5, both Resonator 1 and 2 seem to resonate in tandem. These results suggests that when pressure is concentrated in one resonator, the overall performance exhibits a higher transmission loss, and when pressure is more evenly spread across two resonators, a broader band of frequencies is attenuated.

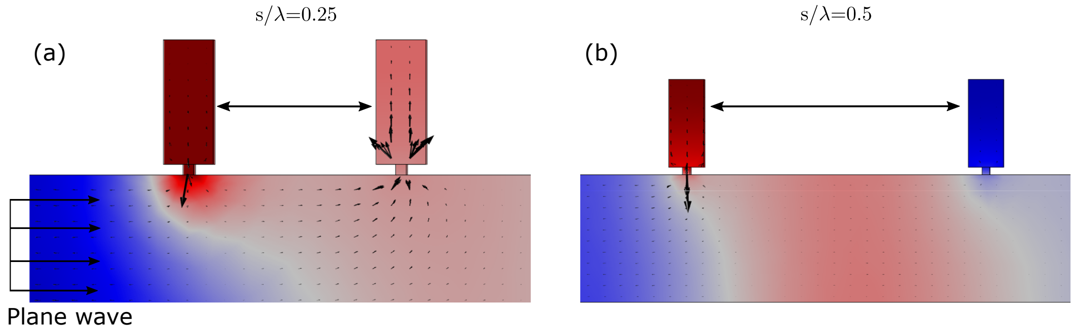


Figure 5: Contours of acoustic pressure and vector field of velocity within the resonator cavities and impedance tube duct for (**a**) s/λ = 0.25; (**b**) s/λ = 0.5.

To view this mechanism in a more dynamic perspective, cavity particle velocities are analysed. The particle velocities are concentrated around the neck of the cavity, as is expected from a mass-spring model. The magnitude of velocities is showing the same pattern as seen with the acoustic pressure. However, an unexpected second velocity peak occurs in Resonator 2, and its magnitude is associated with the first peak. These results indicate that in multi-resonator situations, performance of the downstream resonators are affected, which may be a reason for higher transmission loss induced or wider bandwidth of frequencies attenuated depending on the resonator spacing.

To extend the understanding of the physics to the frequency-energy content of the acoustic pressure field, the resonator cavity SPL was calculated from experimental data. Recall that the experiments were conducted with a white noise signal and data was obtained using microphones M1 and M2, flush-mounted surface microphones on the resonator wall opposite to the neck. Figure 7 presents the SPL of the same

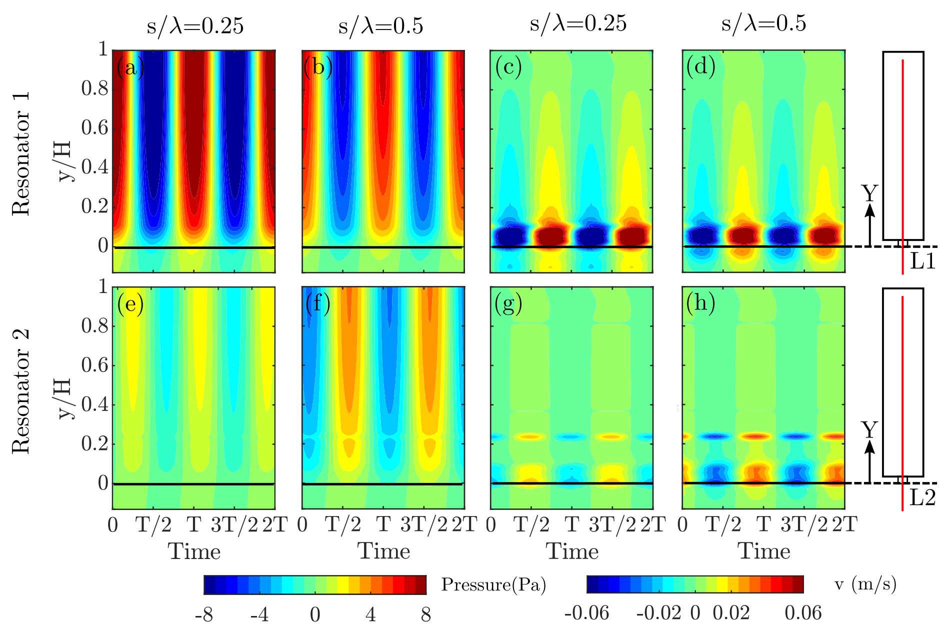


Figure 6: Contour plots of pressure inside the resonators for different spacing configurations between $0.25 < s/\lambda < 0.5$. (**a**) Acoustic pressure Resonator 1 $s/\lambda = 0.25$; (**b**) Acoustic pressure Resonator 2 $s/\lambda = 0.5$; (**c**) Particle velocity Resonator 1 $s/\lambda = 0.25$; (**d**) Particle velocity Resonator 1 $s/\lambda = 0.5$; (**e**) Acoustic pressure Resonator 2 $s/\lambda = 0.25$; (**f**) Acoustic pressure Resonator 2 $s/\lambda = 0.5$; (**g**) Particle velocity Resonator 2 $s/\lambda = 0.25$; (**h**) Particle velocity Resonator 2 $s/\lambda = 0.5$.

separation distances as above. Resonator 1 SPL shares a similar trend at all separations, with a peak at f_0. The SPL in Resonator 2 mostly follows the corresponding Resonator 1, however, at s/λ = 0.25, Resonator 2 shows an anti-resonance-like behaviour around f_0, leading to a dual resonance peak, with the magnitude at other frequencies slightly higher than in Resonator 1. A coherence study between microphones M1 and M2, and external microphones G3-G18, shows the same behaviour inside the impedance tube, with the effect being magnified moving downstream in the impedance tube [14]. This change in eigenfrequency of the downstream resonator at certain separation distances may be responsible for the increased transmission loss behaviour noticed in Figs. 3 and 4.

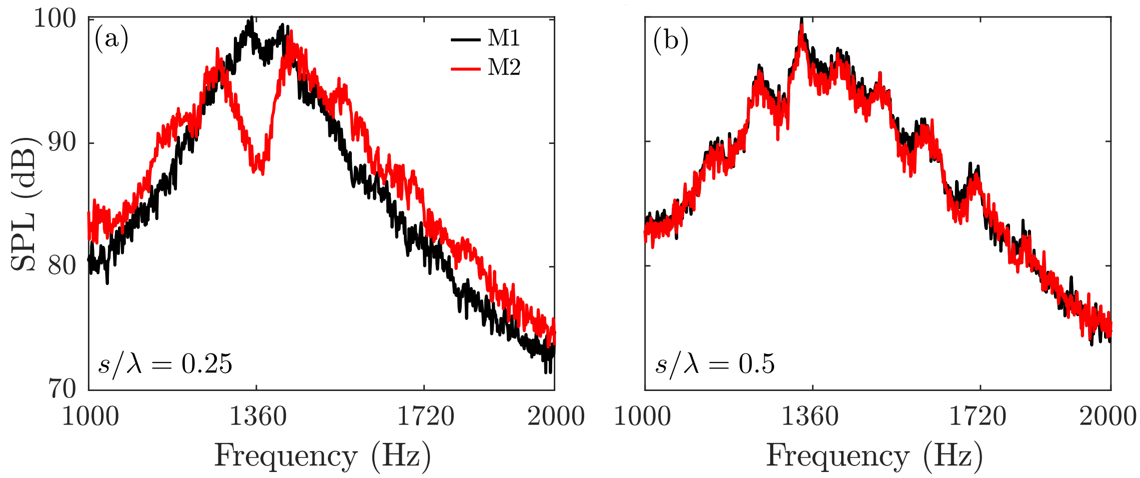


Figure 7: SPL of acoustic signal from microphone M1 and M2 for different spacing configurations between 0.25 <s/λ <0.5. (**a**) $s/\lambda = 0.25$; (**b**) s/$\lambda = 0.5$

5. Conclusion

The effect of altering the spacing between two Helmholtz resonators mounted in a grazing impedance tube is studied to investigate how the transmission loss and sound pressure level change. It is found that decreasing the spacing between the two resonators leads to a steeper reduction in sound pressure level (SPL) downstream in the impedance tube duct. Therefore, the spacing between Helmholtz resonators in a liner can be optimised to give maximum sound pressure level reduction whilst minimizing the number of resonators used to do so, thereby saving weight and added complexities associated with liner manufacture.

For spacing configurations between s/λ=0.13 and 1, it was found that the normalised transmission loss follows a sinusoidal pattern. Maximum transmission loss increases as the spacing between resonators is increased reaching a maximum at s/λ=0.25 after which it drops when the spacing is further increased and reaches a minimum at a spacing of s/λ=0.5. The transmission loss induced peaks again at a spacing of s/λ=0.75 and reduces finally at s/λ=1. The bandwidth coefficient (η) study illustrated that the spacing, transmission loss and transmission coefficient are related in a way that the spacing configurations that lead to the lowest transmission loss value result in the broadest bandwidth of frequencies being attenuated and vice-versa.

Further investigation on the acoustic pressure and particle velocities suggests Resonator 1 resonating with a high acoustic pressure magnitude at f_0, whereas Resonator 2 exhibiting an anti-resonance behaviour, for s/λ=0.25. Both resonators are almost equal in terms of cavity acoustic pressure magnitude, for s/λ=0.5. A second particle velocity peak occurs in the downstream resonator for all spacing configurations. The findings indicate that when energy is mostly concentrated in one resonator, the overall transmission loss increases, whereas, when energy is evenly spread between resonators, a broader band of frequencies is attenuated.

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