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Modeling the effects of insecticides and external efforts on crop production

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Abstract. In this paper a nonlinear mathematical model is proposed and analyzed to understand the effects of insects, insecticides and external efforts on the agricultural crop productions. In the modeling process, we have assumed that crops grow logistically and decrease due to insects, which are wholly dependent on crops. Insecticides and external efforts are applied to control the insect population and enhance the crop production, respectively. The external efforts affect the intrinsic growth rate and carrying capacity of crop production. The feasibility of equilibria and their stability properties are discussed. We have identified the key parameters for the formulation of effective control strategies necessary to combat the insect population and increase the crop production using the approach of global sensitivity analysis. Numerical simulation is performed, which supports the analytical findings. It is shown that periodic oscillations arise through Hopf bifurcation as spraying rate of insecticides decreases. Our findings suggest that to gain the desired crop production, the rate of spraying and the quality of insecticides with proper use of external efforts are much important.

Keywords: mathematical model, crop production, external efforts, stability, Hopf bifurcation.

1 Introduction

The rapid growth of world's human population has extensively demanded food, fiber and products of agricultural system. Sources that can provide the ability to fulfill the future demand of agricultural products are the extension of agricultural fields increasing the productivity of agricultural croplands. The expansion of cropland in the forest areas requires a large amount of cost due to poor soil fertility, and this extension of land causes risk on

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factors that affects biodiversity and health of ecosystem [4, 16]. Keeping the protection of environmental health in mind, we require to enhance the productivity per unit area of agricultural field to ensure the world food security, and for this, more sustainable and efficient cropping system should be adopted [3, 19]. Insects are responsible to reduce the production capacity of crops because they attack the growing crops and destroy their leaves and roots. Stalk borer on corn, codling moth on apples, boll weevil on cotton, etc. are some examples of insects. The crop damage by these insects plays an important role for reduction in the production of crop [12, 24]. Oliveira et al. [20] estimated 7.7% annual crop production loss due to insects in Brazil, which reduces approximately 25 million tones of fiber, food and biofuel. Gharde et al. [5] have estimated crop losses due to weeds in different major crops of India viz. wheat (18.6%), mustard (21.4%), rice (21.4%), maize (25.3%), soybean (31.4%), etc. Therefore, management of the insect population is an important part to enhance the production of crop.

There are several mechanisms to control the insect population, e.g., chemical, physical and biological control. In particular the chemical control of insect population by using chemical substances is one of the most widely adopted techniques in agriculture sector around the world. Farmers use the insecticides to control the insect population in agriculture fields as they are highly toxic towards a particular family of insects. There are several studies available in the literature showing the importance of insecticides to control the insect population in the crop fields [2, 17, 21, 23, 32]. In particular, Wang et al. [31] examined the combined effects of spraying of insecticides with the release of infected insects and concluded that this combination gives the good strategy of insect control. Venturino et al. [30] proposed an epidemic model to control the insect population (white-fly) in the Jatropha Curcas plant and concluded that spraying the insecticides is a continuous and impulsive strategy to control the white-fly. Kar et al. [7] discussed the insect control by introducing the infected pest along with an optimal use of pesticides. Misra et al. [13] proposed a mathematical model to enhance the production of crop by controlling the insect population using insecticides. Tang et al. [28] have examined that application of insecticides is beneficial to suppress the mass of insects in the predator-prey type dynamical system.

Although insecticides provide benefits in the battle of controlling the insect population, but the continuous use of insecticides may cause detrimental effects on crop production, for example, it may alter decomposition rates of soil organic matter [25]. This affects soil productivity and reduces the carrying capacity of crop fields. Plants require the precise combination of nutrients (nitrogen, potassium, phosphorus, zinc, etc.) to grow and develop, and these nutrients are absorbed by plants from soil. Due to continuous cropping, the concentration of nutrients in the soil of crop field decreases. As a result, plants suffer from nutrient deficiencies by which their growth affected. External efforts in terms of additional nutrients supply and use of different cropping systems are applied in the cultivation field to enhance the production. Different cropping system refers to the use of management techniques for crops and crop sequences on the same field over a period of time [18]. Intercropping is a type of cropping system that holds the potential to achieve a desired crop production because it is more stable than monocropping [6]. According to Perrin and Phillips [22], a mixture of crops can be a powerful technique to minimize the crop damage from insects i.e., this may enhance crop production. In the agricultural ecosystem, to increase the production of fruit trees, the concept of intercropping has been widely adopted [9]. Intercropping of plants is used as a strategy to control the insect population in many crops [26]. In a multi-cropping system, sometimes one crop may provide an alternative source of food for insect population, and subsequently, this may enhance the production of the desired crop [29]. Li et al. [10] have done the experiment on intercropping (Wheat/Soybean, Wheat/Maize) and found that the yields in intercropping system are higher than the monocropping system and has ensured that intercropping restored the partial soil fertility, which is lost in monocropping. Some evidences show that intercropping helps to improve the soil fertility of cultivated fields. This means that these external efforts increase the agricultural outputs by increasing the growth rate of crop production as well as increasing the carrying capacity of agricultural field.

In this paper, we analyze a crop production model by controlling the insect population using the insecticides and external efforts. This paper is organized as follows: In Section 2, we formulate the model system describing the interaction of dynamical variables. In Section 3, we find the feasible equilibria and analyze their stability. In Section 4, we discuss the existence of Hopf bifurcation. To identify the most influential parameters that have a significant impact on the crop production, we perform a global sensitivity analysis in Section 5. In Section 6, we perform the numerical simulation for the validation of analytical results. Finally, we discussed the model outcomes in Section 7.

2 The mathematical model

In this section, we formulate a nonlinear mathematical model to study the effects of controlling insect population using insecticides and applying external efforts to increase crop production. Here $A(t)$, $S(t)$, $P(t)$ and $F(t)$ denote the crop production, insect's density, amount of insecticides and applied external efforts, respectively, in a unit area of crop field at any time $t > 0$. The crop production follows the logistic growth with intrinsic growth rate r and the environmental carrying capacity K. Insects attack the agricultural crop and harm it, by which crop production decreases at a rate αAS. Insect population increases in proportion to this reduction rate of crop production, i.e., $\theta\alpha AS$ (θ is the proportionality constant). Due to intra-specific competition, growth rate of insect population decreases at a rate δS^2. Farmers spray insecticides on crop to kill the insects, and therefore, we have assumed the growth rate of insecticides proportional to the density of insect population (i.e., ϕS). Insecticides naturally deplete at a rate $\phi_0 P$. Insects uptake the insecticides by which insecticides decrease at a rate $\phi_1 SP$, and insect population decreases in proportion to this decrease in insecticides i.e., $\lambda\phi_1 SP$ (λ is the proportionality constant). Continuous use of insecticides reduces the carrying capacity of agricultural field for which we assume that crop production decreases at a rate $\alpha_1 PA^2$. Farmers apply external efforts in terms of additional nutrients supply and multiple crop system in the cultivation field to enhance the production. This increases the agricultural outputs in terms of the growth rate as well as carrying capacity of crop. It is assumed that external efforts are increased proportional to the difference of carrying capacity and actual crop

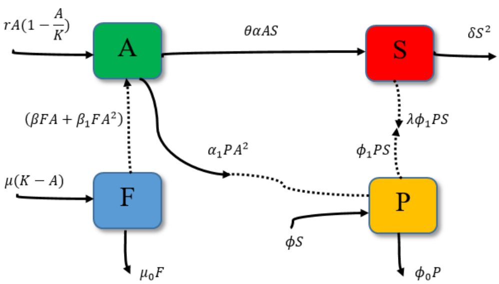

Figure 1. Schematic diagram for system 1.

production (i.e., $\mu(K-A)$), where μ is proportionality constant, and these external efforts naturally deplete (due to economic decpreciation and older technology) proportional to itself (i.e., μ_0F), where μ_0 is the natural depletion rate of external efforts. It is assumed that applied external efforts increase the intrinsic growth rate of crop production as well carrying capacity. To capture the effects of applied external efforts on the intrinsic growth and carrying capacity, we incorporate the terms βFA, and β_1FA^2, respectively, where β and β_1 are proportionality constants. In view of above assumptions, we have the following nonlinear mathematical model:

$$\begin{aligned}
\frac{\mathrm{d}A}{\mathrm{d}t} &= rA\left(1-\frac{A}{K}\right) - \alpha AS - \alpha_1PA^2 + \beta FA + \beta_1FA^2,\\
\frac{\mathrm{d}S}{\mathrm{d}t} &= \theta\alpha AS - \delta S^2 - \lambda\phi_1PS,\\
\frac{\mathrm{d}P}{\mathrm{d}t} &= \phi S - \phi_0P - \phi_1PS,\\
\frac{\mathrm{d}F}{\mathrm{d}t} &= \mu(K-A) - \mu_0F
\end{aligned} \tag{1}$$

with initial conditions $A(0)=A_0\geqslant 0, S(0)=S_0\geqslant 0$, $P(0)=P_0\geqslant 0$ and $F(0)=F_0\geqslant 0$.

Since the model system (1) governs the dynamics of agricultural crop production, insect population, amount of insecticides and external efforts, therefore, all the dynamical variables and parameters are assumed to be nonnegative.

The region of attraction for model system (1) is given in the following lemma.

Lemma 1. *The set*

$$\begin{aligned}
\Omega := \biggl\{&(A,S,P,F)\in\mathbb{R}^4_+\colon\ 0\leqslant A\leqslant K,\\
&0\leqslant S\leqslant \frac{\theta\alpha}{\delta}K,\ 0\leqslant P\leqslant \frac{\phi}{\phi_0}\frac{\theta\alpha}{\delta}K,\ 0\leqslant F\leqslant \frac{\mu}{\mu_0}K\biggr\},
\end{aligned}$$

contains the region of attraction of system (1) *and attracts all solutions initiating inside the interior of the positive orthant. For more detail*; *see* [8, 15].

3 Mathematical analysis

To determine the long-term behavior of the model system (1), we analyze it qualitatively. We examine its qualitative behavior using the stability theory of differential equations. We find the equilibrium points and check the stability behavior of these equilibrium points.

3.1 Equilibrium analysis

In this section, we obtain the feasible equilibria of the model system (1) by setting the rate of change of all the dynamical variables with respect to time t to zero. In this way, we see that model system (1) has three nonnegative equilibria, which are listed as follows:

1. The *trivial equilibrium* $E_0(0, 0, 0, \mu K/\mu_0)$, which always exists. This equilibrium depicts the situation when there is no agricultural crop. Since there is no agricultural crop, therefore, the insect population is also absent and hence there is no need to use insecticides. Since there is no agricultural crop, so maximum external efforts are needed to make the land fertile for the crop.
2. The *insect free equilibrium* $E_1(K, 0, 0, 0)$ always exists. This equilibrium describes the situation when the agricultural crop is present but the insect population, which damage the crop, is not present. As the insect population is absent, there is no use of insecticides in the agricultural field. As production of crop is maximum, so there is no need to apply external efforts.
3. The *interior equilibrium* $E^*(A^*, S^*, P^*, F^*)$ exists, provide $r - \alpha S^* > 0$. In this equilibrium, all the system's variables are nonzero and this describes the dynamics of proposed model.

In interior equilibrium E^*, all the system variables are present, and feasibility can be shown by analyzing the following set of algebraic equations:

$$r\left(1 - \frac{A}{K}\right) - \alpha S - \alpha_1 PA + \beta F + \beta_1 FA = 0, \tag{2}$$

$$\theta\alpha A - \delta S - \lambda\phi_1 P = 0, \tag{3}$$

$$\phi S - \phi_0 P - \phi_1 PS = 0, \tag{4}$$

$$\mu(K - A) - \mu_0 F = 0. \tag{5}$$

From equations (4) and (5) we find the values of P and F in terms of S and A, respectively, as follows:

$$P = \frac{\phi S}{\phi_0 + \phi_1 S}, \qquad F = \frac{\mu(K - A)}{\mu_0}. \tag{6}$$

Using above values of P and F in equation (3), we get

$$A = \frac{1}{\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{(\phi_0 + \phi_1 S)}\right)S, \tag{7}$$

Finally, using equations (6) and (7) in equation (2), we have

$$G(S) \equiv r\left[1 - \frac{1}{\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S}\right)S\right] - \alpha S - \frac{\alpha_1\phi S^2}{\theta\alpha(\phi_0 + \phi_1 S)}\left[\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S}\right] + \left[\frac{\beta\mu}{\mu_0} + \frac{\beta_1\mu S}{\mu_0\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S}\right)\right]\left[K - \frac{1}{\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S}\right)S\right]. \tag{8}$$

From equation (8) we can easily note that

1. $G(0) = r + \beta\mu K/\mu_0 > 0$,
2. $G(\theta\alpha K/\delta) < 0$.

These two points together imply that $G(S) = 0$ has at least one positive value S (say S^*) in $(0, \theta\alpha K/\delta)$. For uniqueness, we differentiate equation (8) with respect to S and obtain

$$\begin{aligned} G'(S^*) = &-\left[\frac{\beta\mu}{\mu_0} + \frac{\beta_1\mu}{\mu_0\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S^*}\right)S^*\right. \\ &\left.+ (r - \alpha S^*)\frac{\theta\alpha}{(\delta + \frac{\lambda\phi\phi_1}{\phi_0+\phi_1 S^*})S^*} + \frac{\beta\mu K}{\mu_0}\right]\left[\frac{\theta\alpha}{\delta + \frac{\lambda\phi\phi_0\phi_1}{(\phi_0+\phi_1 S^*)^2}}\right] \\ &- \alpha - \frac{\alpha_1\phi\phi_1}{(\phi_0 + \phi_1 S^*)\theta\alpha}\left(\delta + \frac{\lambda\phi\phi_1}{\phi_0 + \phi_1 S^*}\right)S^*. \end{aligned}$$

3. $G'(S^*) < 0$ if $r - \alpha S^* > 0$ i.e., derivative of G at S^* is negative.

Thus, $G(S) = 0$ has a unique positive root in $(0, \theta\alpha K/\delta)$. Using this value of S^* in equations (6) and (7), we find the positive values of P^*, F^* and A^*, respectively. Thus, we find the interior equilibrium point $E^*(A^*, S^*, P^*, F^*)$.

3.2 Stability analysis

Now, we present the local stability behavior of the feasible equilibria of system (1) using the Lyapunov's theory of stability [27]. We have the following result regarding the local stability of equilibrium E^*:

Theorem 1.

(i) *The trivial equilibrium E_0, and the insect-free equilibrium E_1, are always unstable.*

(ii) *The interior equilibrium E^*, whenever exists, is locally asymptotically stable, provided condition* (10) *holds.*

Proof. The Jacobian matrix J for system (1) is obtained as follows:

$$J = \begin{bmatrix} a_{11} & -\alpha A & -\alpha_1 A^2 & \beta A + \beta_1 A^2 \\ \theta\alpha S & \theta\alpha A - 2\delta S - \lambda\phi_1 P & -\lambda\phi_1 S & 0 \\ 0 & \phi - \phi_1 P & -(\phi_0 + \phi_1 S) & 0 \\ -\mu & 0 & 0 & -\mu_0 \end{bmatrix},$$

where $a_{11} = r - 2rA/K - \alpha S - 2\alpha_1 PA + \beta F + 2\beta_1 FA$.

(i) Eigenvalues of the Jacobian J at the equilibrium E_0 are obtained as $r + \beta\mu K/\mu_0$, 0, $-\phi_0$ and $-\mu_0$. Since one eigenvalue is always positive, therefore, equilibrium E_0 is unstable.

Evaluating the Jacobian J at the equilibrium E_1, we get the eigenvalues as $-\phi_0$, $\theta\alpha K$, and $(-(\mu_0 + r) \pm \sqrt{(\mu_0 + r)^2 - 4(\mu_0 + r)(r\mu_0 + \mu(\beta K + \beta_1 K^2))}\,)/2$. The positive sign of one eigenvalue implies the instability of the equilibrium E_1.

(ii) At the interior equilibrium E^*, the Jacobian matrix is

$$J_{E^*} = \begin{bmatrix} -\frac{rA^*}{K} - \alpha_1 P^* A^* + \beta_1 F^* A^* & -\alpha A^* & -\alpha_1 {A^*}^2 & \beta A^* + \beta_1 {A^*}^2 \\ \theta\alpha S^* & -\delta S^* & -\lambda\phi_1 S^* & 0 \\ 0 & \phi - \phi_1 P^* & -(\phi_0 + \phi_1 S^*) & 0 \\ -\mu & 0 & 0 & -\mu_0 \end{bmatrix}.$$

The characteristic polynomial for J_{E^*} is obtained as

$$X^4 + A_1 X^3 + A_2 X^2 + A_3 X + A_4 = 0, \tag{9}$$

where

$$A_1 = \mu_0 + \frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^* + \delta S^* + \phi_0 + \phi_1 S^*,$$

$$\begin{aligned} A_2 &= \mu A^*(\beta + \beta_1 A^*) + \mu_0\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \mu_0\delta S^* + \mu_0(\phi_0 + \phi_1 S^*) + (\delta S^* + \phi_0 + \phi_1 S^*)\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \delta S^*(\phi_0 + \phi_1 S^*) + \lambda\phi_1 S^*(\phi - \phi_1 P^*) + \theta\alpha^2 A^* S^*, \end{aligned}$$

$$\begin{aligned} A_3 &= \mu A^*(\beta + \beta_1 A^*)(\delta S^* + \phi_0 + \phi_1 S^*) + \mu_0\delta S^*\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \mu_0(\phi_0 + \phi_1 S^*)\left(\delta S^* + \frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \delta S^*(\phi_0 + \phi_1 S^*)\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) + \alpha_1\alpha\theta(\phi - \phi_1 P^*) S^* {A^*}^2 \\ &\quad + \lambda\phi_1 S^*(\phi - \phi_1 P^*)\left(\mu_0 + \frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \theta\alpha^2 A^* S^*(\mu_0 + \phi_0 + \phi_1 S^*), \end{aligned}$$

$$\begin{aligned} A_4 &= \mu A^*(\beta + \beta_1 A^*)\big(\delta S^*(\phi_0 + \phi_1 S^*) + \lambda\phi_1 S^*(\phi - \phi_1 P)\big) \\ &\quad + \mu_0\delta S^*(\phi_0 + \phi_1 S^*)\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) + \mu_0\alpha_1\theta\alpha {A^*}^2 S^*(\phi - \phi_1 P^*) \\ &\quad + \lambda\phi_1\mu_0 S^*(\phi - \phi_1 P^*)\left(\frac{rA^*}{K} + \alpha_1 P^* A^* - \beta_1 F^* A^*\right) \\ &\quad + \theta\alpha^2 A^* S^* \mu_0(\phi_0 + \phi_1 S^*). \end{aligned}$$

It is apparent that A_1, A_2, A_3 and A_4 are always positive. Thus, using *Routh–Hurwitz criterion*, we can say that if the condition

$$A_3(A_1A_2 - A_3) - A_1^2A_4 > 0 \tag{10}$$

is satisfied, then the roots of equation (9) are either negative or with negative real part, and in this case, all the solution trajectories starting nearby the equilibrium E^* approach to E^* as t tends to infinity. □

Now, we discuss the global stability of the equilibrium E^* inside the region of attraction. The equilibrium point E^* is globally asymptotically stable if it is asymptotic stable for all initial start inside the region of attraction. Here, we use the Lyapunov's stability theory to determine the global stability of E^*. For this, we have obtained the following result regarding the global stability of E^*.

Theorem 2. *The interior equilibrium E^*, whenever exists, is globally asymptotically stable inside the region of attraction Ω, provided the following inequalities are satisfied*:

$$\alpha_1^2K^2 < \frac{2\lambda\phi_1S^*}{\theta P^*}\left(\frac{r}{K} + \alpha_1P^* - \beta_1F^*\right), \tag{11}$$

$$\beta_1^2K^2 < \frac{2\beta\mu_0}{\mu}\left(\frac{r}{K} + \alpha_1P^* - \beta_1F^*\right). \tag{12}$$

Proof. We consider a positive definite function as

$$W = \left(A - A^* - A^*\ln\frac{A}{A^*}\right) + m_1\left(S - S^* - S^*\ln\frac{S}{S^*}\right) + m_2(P - P^*)^2 + m_3(F - F^*)^2,$$

where m_1, m_2 and m_3 are positive constants to be chosen suitably later on.

Differentiating W with respect to time t along the solutions of model system (1), we get

$$\begin{aligned}\frac{dW}{\mathrm{d}t} &= (A - A^*)\left[-\frac{r}{K}(A - A^*) - \alpha(S - S^*) - \alpha_1(PA - P^*A^*)\right.\\&\quad\left. + \beta(F - F^*) + \beta_1(FA - F^*A^*)\right]\\&\quad + m_1(S - S^*)\big[\alpha\theta(A - A^*) - \delta(S - S^*) - \lambda\phi_1(P - P^*)\big]\\&\quad + m_2(P - P^*)\big[\phi(S - S^*) - \phi_0(S - S^*) - \phi_1(PS - P^*S^*)\big]\\&\quad + m_3(F - F^*)\big[-\mu(A - A^*) - \mu_0(F - F^*)\big],\\&\leqslant -\left(\frac{r}{K} + \alpha_1P^* - \beta_1F^*\right)(A - A^*)^2 - m_1\delta(S - S^*)^2\\&\quad - m_2\phi_0(P - P^*)^2 - m_3\mu_0(F - F^*)^2\\&\quad - \alpha(1 - m_1\theta)(A - A^*)(S - S^*) - \alpha_1A(A - A^*)(P - P^*)\\&\quad + (\beta + \beta_1A - m_3\mu)(A - A^*)(F - F^*).\end{aligned}$$

Now choosing $m_1 = 1/\theta$, $m_2 = \lambda\phi_1 S^*/(\theta\phi_0 P^*)$ (as $\phi - \phi_1 P^* = \phi_0 P^*/S^*$) and $m_3 = \beta/\mu$, $\mathrm{d}W/\mathrm{d}t$ can be made negative definite inside the region of attraction Ω, provided inequalities (11) and (12) are satisfied. □

Remark. The global stability condition (11) may be violated for small values of depletion rate of insecticides ϕ_1; however, on increasing the value of ϕ_1, this condition may be easily satisfied, and thus, the depletion rate of insecticides ϕ_1 has stabilizing effect on the system. Further, from condition (12) it is noted that for small values of β_1 and μ, this condition may be easily satisfied; however, on increasing these parameter values, the condition may be violated, and thus, these parameters have destabilizing effect on the system.

4 Existence of Hopf bifurcation

In this section, we study the Hopf bifurcation around the equilibrium point $E^*(A^*, S^*, P^*, F^*)$ by taking the spraying rate of insecticides ϕ as bifurcation parameter. Since all the coefficients of characteristic polynomial can be written as a function of ϕ, we have

$$X^4 + A_1(\phi)X^3 + A_2(\phi)X^2 + A_3(\phi)X + A_4(\phi) = 0. \tag{13}$$

It is clear that $A_j > 0$ ($j = 1, 2, 3, 4$) for any $\phi > 0$. Let at $\phi = \phi_c$,

$$A_3(\phi_c)\big(A_1(\phi_c)A_2(\phi_c) - A_3(\phi_c)\big) - A_1^2(\phi_c)A_4(\phi_c) = 0. \tag{14}$$

Then, at $\phi = \phi_c$, the characteristic polynomial can be written as

$$\left(X^2 + \frac{A_3}{A_1}\right)\left(X^2 + A_1 X + \frac{A_1 A_4}{A_3}\right) = 0.$$

Above equation has four roots, say X_i ($i = 1, 2, 3, 4$) with a pair of pure imaginary roots $X_{1,2} = \pm \mathrm{i} w_0$, where $w_0 = (A_3/A_1)^{1/2}$. For the existence of Hopf bifurcation, all the roots except $\pm \mathrm{i} w_0$ (i.e., X_3 and X_4) should lie in the left half of the complex plane. To identify the nature of remaining two roots, we have

$$X_3 + X_4 = -A_1, \tag{15}$$
$$w_0^2 + X_3 X_4 = A_2,$$
$$w_0^2(X_3 + X_4) = -A_3,$$
$$w_0^2 X_3 X_4 = A_4. \tag{16}$$

If X_3 and X_4 are complex conjugates, then from equation (15), we have $2\,\mathrm{Re}(X_3) = -A_1$, i.e., X_3 and X_4 have negative real parts. If X_3 and X_4 are real roots, then from equations (15) and (16) we find that X_3 and X_4 are negative. Thus, the roots X_3 and X_4 lie in the left half of the complex plane. This ensures the presence of Hopf bifurcation.

Now, we find the transversality condition under which Hopf bifurcation occurs. Let us consider a point ϕ in a neighborhood of ϕ_c, i.e., $\phi \in (\phi_c - \epsilon,\ \phi_c + \epsilon)$, the above roots become a function of ϕ, namely, $X_{1,2} = \eta(\phi) \pm \mathrm{i}\xi(\phi)$. Putting this value in equation (13) and separating real and imaginary parts, we get

$$\eta^4 + A_1\eta^3 + A_2\eta^2 + A_3\eta + A_4 + \xi^4 - 6\eta^2\xi^2 - 3A_1\eta\xi^2 - A_2\xi^2 = 0, \tag{17}$$

$$4\eta\xi\left(\eta^2 - \xi^2\right) - A_1\xi^3 + 3A_1\xi\eta^2 + 2A_2\eta\xi + A_3\xi = 0. \tag{18}$$

As $\xi(\phi) \neq 0$, from equation (18) it follows that

$$-(4\eta + A_1)\xi^2 + 4\eta^3 + 3A_1\eta^2 + 2A_2\eta + A_3 = 0.$$

Using the value of ξ^2 in equation (17), we find

$$\begin{aligned}&-64\eta^6 - 96A_1\eta^5 - 16\left(3A_1^2 + 2A_2\right)\eta^4 - 8\left(A_1^3 + 4A_1A_2\right)\eta^3\\&\quad - 4\left(A_2^2 + 2A_1^2A_2 + A_1A_3 - 4A_4\right)\eta^2 - 2A_1\left(A_1A_3 + A_2^2 - 4A_4\right)\eta\\&\quad - \left(A_3(A_1A_2 - A_3) - A_1^2A_4\right) = 0.\end{aligned}$$

We differentiate above with respect to ϕ, and recalling $\eta(\phi_c) = 0$, we get

$$\left[\frac{\mathrm{d}\eta}{\mathrm{d}\phi}\right]_{\phi=\phi_c} = \left[\frac{\frac{\mathrm{d}}{\mathrm{d}\phi}\left(A_3(A_1A_2 - A_3) - A_1^2A_4\right)}{-2A_1(A_1A_3 + A_2^2 - 4A_4)}\right]_{\phi=\phi_c}.$$

Using the value of $A_4(\phi_c) = (A_1(\phi_c)A_2(\phi_c)A_3(\phi_c) - A_3^2(\phi_c))/A_1^2(\phi_c)$ from equation (14) in above, we have

$$\left[\frac{\mathrm{d}\eta}{\mathrm{d}\phi}\right]_{\phi=\phi_c} = \left[\frac{\frac{\mathrm{d}}{\mathrm{d}\phi}(A_3(A_1A_2 - A_3) - A_1^2A_4)}{-2A_1(A_1A_3 + A_2^2 - 2A_2(\frac{2A_3}{A_1}) + (\frac{2A_3}{A_1})^2)}\right]_{\phi=\phi_c}.$$

This implies

$$\left[\frac{\mathrm{d}\eta}{\mathrm{d}\phi}\right]_{\phi=\phi_c} = \left[\frac{\frac{\mathrm{d}}{\mathrm{d}\phi}(A_3(A_1A_2 - A_3) - A_1^2A_4)}{-2A_1(A_1A_3 + (A_2 - 2\frac{A_3}{A_1})^2)}\right]_{\phi=\phi_c} \neq 0,$$

provided that $[(\mathrm{d}/\mathrm{d}\phi)(A_3(A_1A_2 - A_3) - A_1^2A_4)]_{\phi=\phi_c} \neq 0$. Hence, we have the following result for existence of Hopf bifurcation.

Theorem 3. *System* (1) *undergoes Hopf bifurcation around the interior equilibrium* E^* *if there exists* $\phi = \phi_c$ *such that*

$$A_3(\phi_c)\left(A_1(\phi_c)A_2(\phi_c) - A_3(\phi_c)\right) - A_1^2(\phi_c)A_4(\phi_c) = 0,$$

$$\left[\mathrm{Re}\,\frac{\mathrm{d}X_j(\phi)}{\mathrm{d}\phi}\right]_{\phi=\phi_c} \neq 0 \quad j = 1, 2,$$

i.e.,

$$\left[\frac{\mathrm{d}}{\mathrm{d}\phi}\left(A_3(A_1A_2 - A_3) - A_1^2A_4\right)\right]_{\phi=\phi_c} \neq 0.$$

5 Global sensitivity analysis

In this section, we use a global sensitivity analysis to identify the most influential parameters that have significant impact on the variables of system (1) by giving a number between -1 and $+1$ $[1, 11]$. The positive sign of number depicts the positive correlation between the parameter and variable, while negative sign as negative correlation. The value of number depicts the strength of correlation. Assuming a uniform distribution for each parameter, we calculate the Partial Rank Correlation Coefficients (PRCCs) for the parameters r, K, α, θ, δ, ϕ, ϕ_0, ϕ_1 λ, μ, μ_0 α_1, β_1 and β_1 with the crop production by running 30 simulations per LHS, and baseline values of parameters are taken from Table 1. We deviate the parameter values $\pm 25\%$ from the baseline values. The bar diagram of the Partial Rank Correlation Coefficients (PRCCs) of crop production against these parameters are depicted in Fig. 2. The highest PRCC values of parameters have the largest impact on the crop production. The PRCC values of these parameters suggest that the intrinsic growth rate of agricultural crop r, carrying capacity of agricultural crop K, mortality rate δ, spraying rate of insecticides ϕ, uptake rate of insecticides by insects ϕ_1, depletion rate of insects due to insecticides λ, rate of application of external efforts μ, growth rate coefficient of crop production due to external efforts β, increasing rate of carrying capacity due to external efforts β_1, have positive correlation with crop production. The agricultural crop consumption rate by insects α, conversion efficiency θ, natural depletion rate of insecticides ϕ_0, declination rate of carrying capacity due to insecticides α_1 and natural depletion rate of external efforts μ_0 have significant negative correlations with crop production. Further, from this figure it may be noted that the parameters K, ϕ_1 and β_1 have large positive impact to increase the crop production; however, the parameters α_1, θ and ϕ_0 have much negative impact on the crop production.

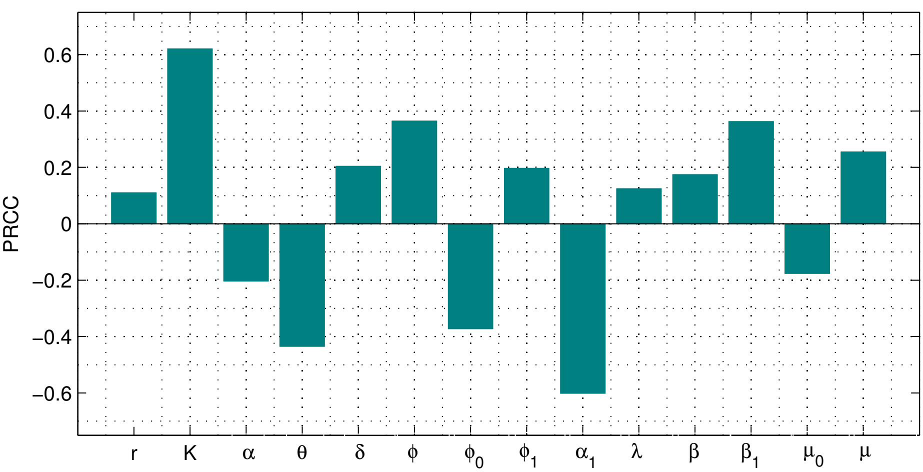

Figure 2. Effect of uncertainty of model (1) on the crop production $A(t)$. The mean value of parameters is chosen from Table 1.

Table 1. Biological meanings of parameters in system (1) and their value used in numerical simulations.

Parameters	Parameter's descriptions	Value	Sources
r	Intrinsic growth rate of agricultural crop	0.2	[14]
K	Carrying capacity of agricultural crop	50	[14]
α	Agricultural crop consumption rate by insects	0.025	[14]
θ	Conversion efficiency	0.6	[14]
δ	Mortality rate of insects due to intra-specific competition	0.05	[14]
ϕ	Rate of spraying insecticides	0.08	[14]
ϕ_0	Natural depletion rate of insecticides	0.01	[14]
ϕ_1	Uptake rate of insecticides by insects	0.05	[14]
λ	Death rate of insects due to insecticides	8	Assumed
μ	Rate of application of external efforts	0.1	[14]
μ_0	Natural depletion rate of external efforts	0.01	[14]
α_1	Declination rate of crop production due to insecticides	0.01	Assumed
β	Growth rate coefficient of crop production due to external efforts	0.001	Assumed
β_1	Increasing rate of crop production due to external efforts	0.01	Assumed

6 Numerical simulation

To simulate system (1), we consider a set of parameter values given in Table 1. For the chosen set of parameter values, the components of equilibria of system (1) are obtained as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 50 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad E^* = \begin{bmatrix} 49.83 \\ 2.96 \\ 1.50 \\ 1.64 \end{bmatrix}.$$

The eigenvalues corresponding to the equilibrium E_0, E_1 and E^* are calculated as $\Lambda_0 = (0.7, 0, -0.01, -0.01)$, $\Lambda_1 = (-0.01, 0.075, -0.105 \pm 1.579\mathrm{i})$, $\Lambda^* = (-0.069 \pm 1.594\mathrm{i}, -0.153 \pm 0.074\mathrm{i})$, respectively.

Here we see that one eigenvalue for equilibrium E_0 is positive, this means E_0 is unstable. Similarly, one eigenvalue of the Jacobian matrix evaluated at equilibrium E_1 is positive, implying the instability of E_1. All eigenvalues of the Jacobian matrix at E^* are with negative real part. This implies that the interior equilibrium E^* is locally asymptotically stable. Further, we checked the stability condition stated in Theorem 1, for this, we calculate the values of $A_1 = 0.44$, $A_2 = 2.62$, $A_3 = 0.78$ and $A_4 = 0.073$, which are positive. Also the value of $(A_3(A_1A_2 - A_3) - A_1^2A_4) = 0.28$, which is also positive, this means the stability condition stated in Theorem 1 is satisfied, and the interior equilibrium E^* is locally asymptotically stable.

To visualize the global stability of interior equilibrium E^*, we plot the solution trajectories of system (1) with different initial starts in S,A,F-space for $\alpha_1 = 0.00001$, $\beta = 0.00001$, $\beta_1 = 0.0000001$, $\phi_1 = 0.0005$, and rest of parameter values are same as given in Table 1, and check that the global stability conditions stated in the Theorem 2 are also satisfied for these parameter values. We may observe that the solution trajectories starting inside region of attraction approach towards the (S^*, A^*, F^*) in S,A,F-space, which shows that E^* is globally asymptotically stable in this space; see Fig. 3.

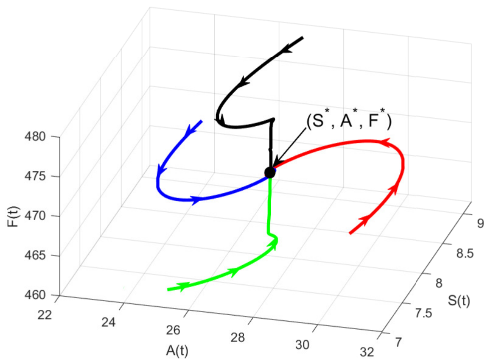

Figure 3. Global stability of equilibrium point E^* in S,A,F-space, this shows that solution trajectories starting inside the region of attraction approach the equilibrium point E^*.

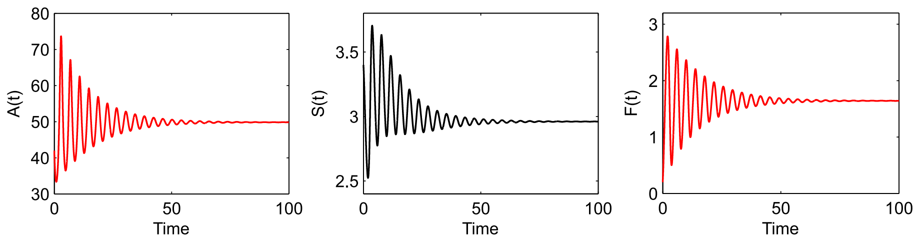

Figure 4. Variation of $A(t)$, $S(t)$ and $F(t)$ with respect to time t for $\phi = 0.08$, this shows that after damped oscillations, the solution trajectories settle to their respective equilibrium values, and thus, the equilibrium E^* is stable.

For the given set of parameter values in Table 1, the dynamics of system (1) changes as the spraying rate of insecticides ϕ decreases. For large value of spraying rate of insecticides, interior equilibrium E^* is stable, while a decrease in spraying rate of insecticides destabilizes the system and oscillations arise, which indicates the occurrence of Hopf bifurcation as the value of ϕ decreases below a threshold. We have numerically calculated the critical value of spray rate of insecticides (i.e., $\phi = \phi_c = 0.034332$) at which stability of system changes. It may be noted that for $\phi \in [0, \phi_c)$, two of the eigenvalues of Jacobian matrix calculated at E^* lie in the right half of Argand plane, which shows that the interior equilibrium of system (1) is unstable, while if the value of $\phi > \phi_c$, the interior equilibrium becomes stable. This shows that system (1) undergoes Hopf bifurcation around the interior equilibrium at $\phi = \phi_c$.

We see the variation of crop production $A(t)$, insect population $S(t)$ and external efforts $F(t)$ with respect to time t for $\phi = 0.08 > \phi_c$, which is shown in Fig. 4. This figure reveals that for $\phi > \phi_c$, all the dynamical variables attain their equilibrium values.

Further, we plot the solution trajectories in A,S,F-space for $\phi = 0.08$, which demonstrates that interior equilibrium is locally stable, i.e., the solution trajectories staring in the neighbourhood of interior equilibrium point in A,S,F-space approach towards interior equilibrium in A,S,F-space; see Fig. 5.

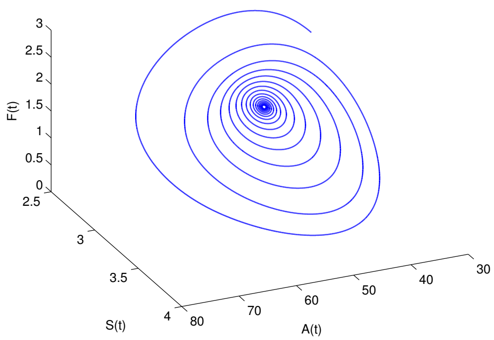

Figure 5. Phase portrait of system (1) for $\phi = 0.08$ in A,S,F-space showing the stability of equilibrium E^*.

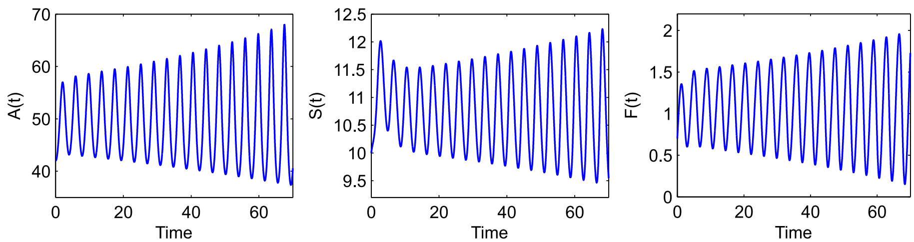

Figure 6. Variation of $A(t)$, $S(t)$ and $F(t)$ with respect to time t for $\phi = 0.027$, appearance of undamped sustained oscillations showing the instability of equilibrium E^*.

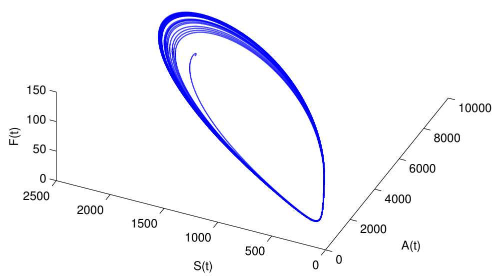

Figure 7. Appearance of limit cycle of system (1) for $\phi = 0.027$ in A,S,F-space showing the instability of equilibrium E^*.

Further, we plot the variation of $A(t)$, $S(t)$, and $F(t)$ with respect to time t for $\phi = 0.027 < \phi_c$ and see that dynamical variables have undamped sustained oscillations showing the instability of equilibrium E^*; see Fig. 6.

In Fig. 7, we plot a solution trajectories for $\phi = 0.027$ in A,S,F-space starting near the interior equilibrium in A,S,F-space and see that it goes further away from interior equilibrium, shows that the interior equilibrium is unstable.

We have drawn a bifurcation diagram by taking ϕ as a bifurcation parameter; see Fig. 8. From this figure it can be easily seen that for a small value of the spraying rate of insecticides, periodic solutions of increasing amplitude are observed, this shows the

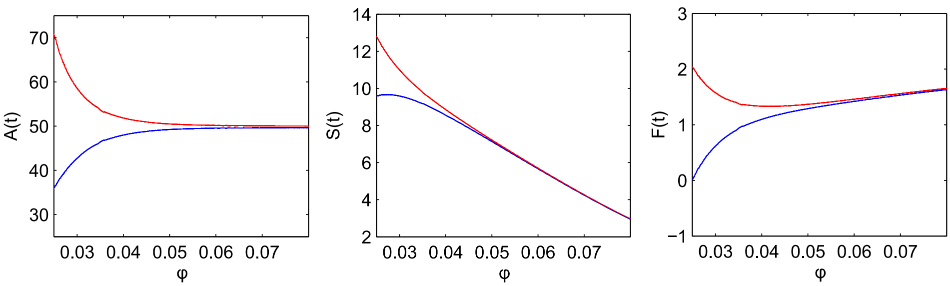

Figure 8. Bifurcation diagram of crop production $A(t)$, insect population $S(t)$ and external efforts $F(t)$ with respect to ϕ, other parameters are same as given in Table 1.

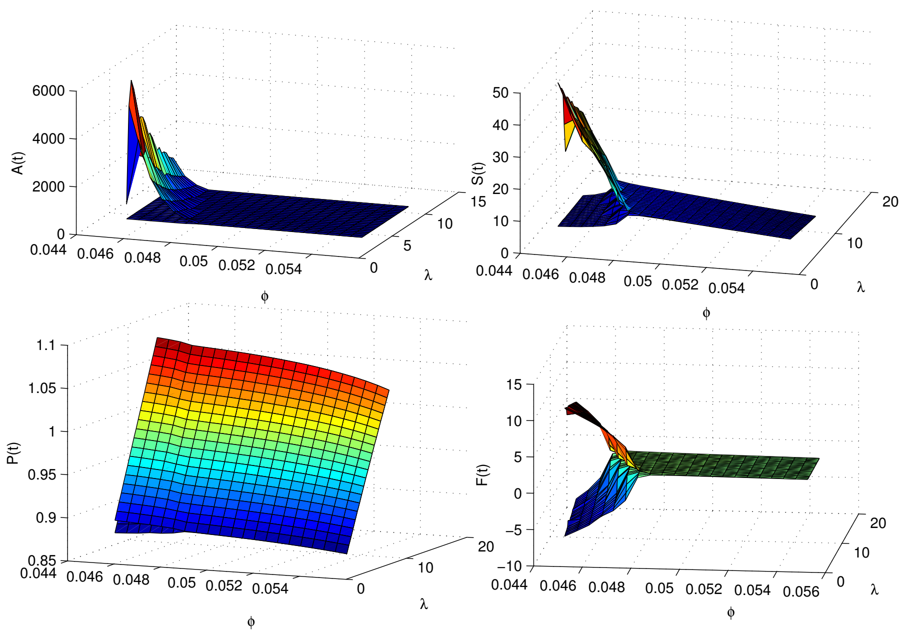

Figure 9. Bifurcation diagram of crop production $A(t)$, insect population $S(t)$, insecticide $P(t)$ and external efforts $F(t)$ with respect to ϕ and λ, other parameters are same as given in Table 1.

unstable behavior of equilibrium point. However, as we increase the value of spraying rate of insecticides after a threshold, all variables settle down to their equilibrium values, and interior equilibrium changes its stability from unstable to stable. Thus, we have observed that the interior equilibrium of system (1) goes from instability to stability as spraying rate of insecticides increases.

We have drawn a bifurcation diagram using surface plot by simultaneously varying the parameter values ϕ and λ; see Fig. 9. In this figure, it can be easily seen that for small values of the spraying rate of insecticides and depletion rate of insects due to insecticides, periodic solutions of increasing amplitude are observed, this shows the unstable behavior of equilibrium point. However, as we increase the values of ϕ and λ, after a threshold all variables settle down to their equilibrium values and interior equilibrium changes its stability from unstable to stable. Thus, we have observed that the interior equilibrium of system (1) goes from instability to stability as spraying rate of insecticides and death rate of insects due to insecticides increases.

7 Discussion

Adequate crop production plays a crucial role to fulfill the basic needs of livelihood and also it helps to strengthen the economy of a country by providing raw materials. To fulfill the demand for future food, it is requisite to nurture the agricultural crops to achieve better quality and quantity of production. Insect population play a destructive role and cause a loss in production of crop. Due to this, it is important to control the insect population for minimizing the loss and enhance the production of the crop by adopting an efficient cropping system. In this paper, we have proposed and analyzed a mathematical model to enhance the crop production via controlling the insects and applying the external efforts in terms of nutrients and multi cropping system. In the model formulation, we have assumed that insecticides are applied proportional to the insect population and continuous use of insecticides affects the carrying capacity of crop production. It is also assumed that external efforts are applied proportional to the difference of the carrying capacity and actual crop production, and they increase the intrinsic growth rate as well as carrying capacity of crop production. The proposed model has three nonnegative equilibrium points in which two are boundary and one is interior equilibrium. The analysis reveals that both boundary equilibrium points are unstable, and local as well as global asymptotic stability behavior of interior equilibrium are discussed. The condition for existence of Hopf bifurcation is obtained. Model analysis shows that the value of spraying rate of insecticides below a threshold value destabilizes the system, and periodic oscillations arise through Hopf bifurcation. It is also noted that for a large value of spraying rate (above the threshold value), the interior equilibrium is stable. This means that the interior equilibrium changes its stability from unstable to stable as the spraying rate of insecticides upon agricultural crop increases. To achieve the stability and minimize the harmful effects, the proposed model suggests to keep the spraying rate of insecticides just above (not much above) the threshold value. Further, efforts should be made to neutralize the harmful effects of insecticides and increase the crop production. The result of sensitivity analysis suggest that a strategy, which increases the parameters with negative PRCC values (i.e., θ, α and α_1), will adequately reduce the crop production, whereas positive PRCC values (i.e., δ, ϕ, λ, β and β_1) will help to increase the crop production. Our results support the assertion that for higher crop production, one should increase the rate of spraying insecticides with increasing population of insects, and rate of application of external efforts will effectively increase the crop production.

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