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ENGINEERING SCIENCES

APPLICATION OF FUZZY ELECTRE METHOD WITH TRAPEZOIDAL FUZZY NUMBERS

Ph.D., *Salimov Vagif Hasan Oglu*, assoc. prof. of "Computer engineering department" Azerbaijan state oil and industry university, Baku, Azerbaijan Republic, ORCID ID <https://orcid.org/0000-0002-0590-5437>

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ABSTRACT

The article is devoted to the problem of multi-criteria decision-making. Methods for solving this problem can be divided into two large groups: methods using the aggregation of all alternatives according to all criteria and the solution of the resulting single-criterion problem. The second group is associated with the procedure of pairwise comparisons and stepwise aggregation. The first group includes methods: weighted average sum, product and their various modifications, the second group includes - AHP, ELECTRE, TOPSIS, PROMETHEE, ELECTRE. For many problems assessment of the criteria implemented by experts and presented in linguistic form. The effective approach for dealing with linguistic information is fuzzy set theory proposed by L. Zadeh. In this paper is proposed fuzzy ELECTRE method. This method is presented in details. As application problem is used the equipment selection problem The issues of practical implementation of this method are discussed in details. The results of the solution test problem at all stages are presented.

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1. Introduction. Multi Criteria Decision making – (MCDM) is one of the actual problem in the theory of decision-making [1-2]. From a mathematical point of view, it belongs to the class of vector optimization problems. The criteria can be divided into two groups: the criteria for which the maximum value is optimal and the criteria for which the minimum value is optimal. MCDM problems can be solved with an accuracy of many non- dominated alternatives or many trade-offs. Obtaining a single solution can only be implemented on the basis of some compromise scheme that reflects the preferences of the decision maker (DM). Methods for solving this problem can be divided into two large groups: methods using the aggregation of all alternatives according to all criteria and the solution of the resulting single-criterion problem, the second group is associated with the procedure of pairwise comparisons and stepwise aggregation. The first group includes methods: weighted average sum, weighted average product and their various modifications [3-4], the second group includes - Analytical Hierarchy Process (AHP), Elimination and Choice Translating Reality (ELECTRE). The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Preference Ranking Organization Method (PROMETHEE) [5-17]. The work [3] provides information about popularity of various methods of multi-criteria decision-making. This paper discusses the ELECTRE method.

The ELECTRE method was developed by group of the French scientists led by professor B. Rua at the end of 60th years This method was very popular for solving multi-criteria problem under certain conditions. In general the ELECTRE method is based on the approach of pairwise comparison of alternatives.

The fuzzy ELECTRE method was developed by Chen in 2006 [6] for problem with linguistic uncertainty.

2. Description of the method.

We consider the problem where decision DM makes decisions in linguistic form.

Consider all stages of fuzzy ELECTRE method:

1. First we define linguistic variables for criterion weight importance (Table 1) and the decisions with fuzzy trapezoidal numbers (Table 2).

Table 1. Linguistic variables of criterion weights

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very Low (VL)	(0,0.1,0.2,0.3)
Low (L)	(0.1,0.3,0.45,0.7)
Medium (ML)	(0.4,0.5,0.7,0.8)
High (H)	(0.5,0.6,0.75,0.85)
Very High (VH)	(0.6,0.7,0.8,0.9)

Table 2. Linguistic variables for the decision

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very Poor (VP)	(0,1,2,3)
Poor (P)	(1,3,4,7)
Medium Poor (MP)	(4,5,7,8)
Good (G)	(7,8,9,9.25)
Very Good (VG)	(9, 9.25, 9.5,10)

2. Present the linguistic decisions as the matrix of outcomes (alternatives - criteria) n - number of criteria m - number of alternatives (Table 3).

Table 3. MCDM problem representation

	C_1	C_2	C_3		C_n
A_1	\tilde{X}_{11}	\tilde{X}_{12}	\tilde{X}_{13}		\tilde{X}_{1n}
A_2	\tilde{X}_{21}	\tilde{X}_{22}	\tilde{X}_{23}		\tilde{X}_{2n}
A_3	\tilde{X}_{31}	\tilde{X}_{32}	\tilde{X}_{33}		\tilde{X}_{3n}
A_m	\tilde{X}_{m1}	\tilde{X}_{m2}	\tilde{X}_{m3}		\tilde{X}_{mn}

Where $\tilde{X}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$ is fuzzy trapezoidal representation of linguistic terms.

3. Calculate normalized matrix

$$\tilde{R} = (r_{ij}), i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

4. The normalized fuzzy decision matrix is calculated with the formulas given below, where J and J_1 represent the maximization criteria set, and minimization criteria set respectively.

$$\tilde{r}_{ij} = (\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{a_j^*}, \frac{c_{ij}}{a_j^*}, \frac{d_{ij}}{d_j^*}), j \in J \tag{1}$$

$$\tilde{r}_{ij} = (\frac{a_j^*}{d_{ij}}, \frac{a_j^*}{c_{ij}}, \frac{a_j^*}{b_{ij}}, \frac{a_j^*}{a_{ij}}), j \in J_1 \tag{2}$$

$$d_j^* = \max_i d_{ij}, j \in J \tag{3}$$

$$a_j^* = \min_i a_{ij}, j \in J_1 \tag{4}$$

5. Calculate weighted decision matrix

$$\tilde{V} = (v_{ij}), i = 1, 2, \dots, n \tag{5}$$

Where

$$\tilde{v}_{ij} = \tilde{v}_{ij} \otimes \tilde{w}_j, i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

6. Determine concordance set J_c (set is all criteria in which alternative k is superior than alternative l) can be determined by following criteria

$$\tilde{c}_{kl} = \{j, \tilde{v}_{kj} \geq \tilde{v}_{lj}\} \tag{6}$$

7. Determine discordance set J_d (set is all criteria in which alternative k not is superior than alternative l) can be determined by following criteria

$$\tilde{d}_{kl} = \{j, \tilde{v}_{kj} < \tilde{v}_{lj}\} \tag{7}$$

8. Determine the concordance matrix where elements is calculated by formula

$$\tilde{c}_{kl} = \sum_{j \in \tilde{c}_{kl}} \tilde{w}_j \tag{8}$$

9. Determine the discordance matrix where elements is calculated by formula

$$\tilde{d}_{kl} = \frac{\max_j |\tilde{v}_{kj} - \tilde{v}_{lj}|}{\max_{j \in D_{kl}} |\tilde{v}_{kj} - \tilde{v}_{lj}|} \tag{9}$$

10. Determine average concordance index

$$\tilde{c}_* = \frac{1}{m(m-1)} \sum_{k=1}^m \sum_{l=1}^m \tilde{c}_{kl} \tag{10}$$

11. Determine average discordance index

$$\tilde{D}_* = \frac{1}{m(m-1)} \sum_{k=1}^m \sum_{l=1}^m \tilde{d}_{kl} \tag{11}$$

12. Determine Boolean concordance matrix F

$$\text{if } \tilde{c}_{kl} \geq \tilde{c}_* \text{ then } \tilde{f}_{kl} = 1 \text{ othewise } \tilde{f}_{kl} = 0 \tag{12}$$

13. Determine Boolean discordance matrix G

$$\text{if } \tilde{d}_{kl} \geq \tilde{D}_* \text{ then } \tilde{g}_{kl} = 1 \text{ othewise } \tilde{g}_{kl} = \tag{13}$$

14. Calculate global preference matrix E by multiplication $E = FG$

15. Determine alternative with max preference by calculation sum of preference indexes by row of global matrix

3. Practical example.

As practice problem we consider equipment selection problem with following 4 criteria and 3 alternatives:

- C1- price
- C2- noise level
- C3- usability
- C4- dimension

As seen for C3 optimal decision is maximum for other three criteria is minimum.

Consider application of fuzzy ELECTRE method for this problem. All computations were performed in Ms Excel.

1. Presentation of decisions in linguistic decision matrix (Table 4)

Table 4. Linguistic decision matrix

	C_1	C_2	C_3	C_4
A_1	VG	G	VG	MP
A_2	MP	G	G	VG
A_3	G	VG	MP	G

The vector of criteria importance is presented as follows

$$w = (ML, H, VH, H)$$

2. Convert linguistic presentation in trapezoidal fuzzy numbers (Table 5)

Table 5. linguistic presentation in trapezoidal fuzzy numbers

	C_1	C_2	C_3	C_4
A_1	(9, 9.25, 9.5,10)	(7,8,9,9.25)	(9, 9.25, 9.5,10)	(4,5,7,8)
A_2	(0.4,0.5,0.7,0.8)	(7,8,9,9.25)	(4,5,7,8)	(9, 9.25, 9.5,10)
A_3	(7,8,9,9.25)	(9, 9.25, 9.5,10)	(4,5,7,8)	(7,8,9,9.25)

$$w = (0.4,0.5,0.7,0.8) \quad (0.5,0.6,0.75,0.85) \quad (0.6,0.7,0.8,0.9) \quad (0.5,0.6,0.75,0.85)$$

3. Calculate normalized fuzzy decision matrix by corresponding formulas (Table 6)

Table 6. Normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4
A_1	(0.40,0.42,0.43,0.44)	(0.76, 0.78,0.88,1)	(0.9,0.93,0.95,1)	(0.5,0.57,0.8,1)
A_2	(0.5, 0.57, 0.8, 1)	(0.76,0.78,0.88, 1)	(0.7,0.8,0.9,0.93)	(0.4,0.42,0.43,0.44)
A_3	(0.43,0.44,0.5,0.57)	(0.7,0.74,0.76, 0.78)	(0.4,0.5,0.7,0.8)	(0.43,0.44,0.5,0.57)

4. Calculate weighted normalized fuzzy decision matrix (Table 7)

Table 7. Weighted normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4
A_1	(0.16,0.21,0.3, 0.36)	(0.38,0.47,0.66,0.8)	(0.54,0.65,0.76,0.9)	(0.25,0.34,0.60,0.85))
A_2	(0.2,0.29,0.56,0.8)	(0.38,0.47,0.66,0.8)	(0.42,0.56,0.72,0.83)	(0.2,0.25,0.32,0.38)
A_3	(0.17,0.22,0.35,0.46)	(0.35,0.44,0.57,0.62)	(0.24,0.35,0.56,0.72)	(0.22,0.27,0.38,0.49)

For ranking alternatives we have used following method / 7/

$$R(\tilde{A}) = \frac{1}{2}(a + b + \frac{1}{2}(d - c))$$

Let \tilde{A}_i and \tilde{A}_j two fuzzy numbers,

- (i) $R(\tilde{A}_i) > R(\tilde{A}_j)$ then $\tilde{A}_i > \tilde{A}_j$
 - (ii) $R(\tilde{A}_i) < R(\tilde{A}_j)$ then $\tilde{A}_i < \tilde{A}_j$
 - (iii) $R(\tilde{A}_i) = R(\tilde{A}_j)$ then $\tilde{A}_i = \tilde{A}_j$
- (13)

5. Determine concordance and discordance sets

For determine concordance and discordance sets we use formulas (6) and (7)

As result we have got set of concordance and discordance sets (Table 8)

Table 8. Concordance and discordance sets

Concordance set	Discordance set
C(1,2)=(2,3,4)	D(1,2)=(1)
C(1,3)=(2,3,4)	D(1,3)=(1)
C(2,1)=(1,2)	D(2,1)=(3,4)
C(2,3)=(1,2,3)	D(2,3)=(4)
C(3,1)=(1)	D(3,1)=(2,3,4)
C(3,2)=(4)	D(3,2)=(1,2,3)

Calculate concordance indexes matrix by formula (8). (Table 9)

Table 9. Concordance indexes matrix

	A_1	A_2	A_3
A_1	(0,0,0)	(1.6,1.9,2.3,2.6)	1.6,1.9,2.3,2.6)
A_2	(1.5,1.8,2.25,2.55)	(0,0,0,0)	(0.42,0.56,0.72,0.83)
A_3	(0.4,0.5,0.7,0.8)	(0.5,0.6,0.75,0.85)	(0,0,0,0)

and a discordance indexes matrix by formula (9) (Table 10)

Table 10. Discordance indexes matrix

	A_1	A_2	A_3
A_1	(0,0,0,0)	(0.8,0.89,0.93,0.94)	(0.033,0.033,0.25,0.56)
A_2	(1,1,1,1)	(0,0,0,0)	(0.111,0.095,0.375,1)
A_3	(1,1,1,1)	(1,1,1,1)	(0,0,0,0)

Next we calculate average concordance index by formula (10) and average discordance index (11) respectively. Calculate Boolean preference concordance matrix F (Table 11)

Table 11. Boolean preference concordance matrix

	A_1	A_2	A_3
A_1	0	1	1
A_2	1	0	0
A_3	0	0	0

and Boolean preference discordance matrix G (Table 12).

Table 12. Boolean preference discordance matrix

	A_1	A_2	A_3
A_1	0	1	0
A_2	1	0	0
A_3	1	1	0

Finally calculate and global preferences matrix E (Table 13)

Table 13. Global preferences matrix

	A_1	A_2	A_3
A_1	0	1	0
A_2	1	0	0
A_3	0	0	0

As we see two alternatives result we have alternatives A_1 and A_2 have same preference, it means that problem have two solutions.

Conclusions. The article is devoted to the problem of multi-criteria decision making for equipment selection. The analysis of existing methods for solving this problem is given. The fuzzy ELECTRE is used as a method for solving this problem. The issues of practical implementation of this method are discussed in details.

As practical problem the equipment selection problem with 4 criteria and 3 alternatives is considered. The results of the solution at all stages are presented.

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