Chapter

Resonant Optical Solitons in (3 + 1)-Dimensions Dominated by Kerr Law and Parabolic Law Nonlinearities

Khalil S. Al-Ghafri

Abstract

This study investigates the optical solitons of of (3+1)-dimensional resonant nonlinear Schrödinger (3D-RNLS) equation with the two laws of nonlinearity. The two forms of nonlinearity are represented by Kerr law and parabolic law. Based on complex transformation, the traveling wave reduction of the governing model is derived. The projective Riccati equations technique is applied to obtain the exact solutions of 3D-RNLS equation. Various types of waves that represent different structures of optical solitons are extracted. These structures include bright, dark, singular, dark-singular and combined singular solitons. Additionally, the obliquity effect on resonant solitons is illustrated graphically and is found to cause dramatic variations in soliton behaviors.

Keywords: Optical solitons, 3D-RNLS equation, Kerr law and parabolic law nonlinearities, Projective Riccati equations method, Obliquity influence

1. Introduction

Soliton is one of the important nonlinear waves that has been under intensive investigation in the physical and natural sciences. It has been noticed that solitons play a significant role on describing the physical phenomena in various branches of science, such as optical fibers, plasma physics, nonlinear optics, and many other fields [1–5]. For example, solitons in the field of nonlinear optics are known as optical solitons and have the capacity to transport information through optical fibers over transcontinental and transoceanic distances in a matter of a few femtoseconds [6, 7]. Moreover, it is found that the efficient physical properties of solitons may support the improvement on photonic and optoelectronic devices [8, 9]. Further to this, optical solitons can be exploited widely in optical communication and optical signal processing systems [10, 11].

The formation of solitons is essentially caused due to a delicate balance between dispersion and nonlinearity in the medium. Understanding the dynamics of solitons can be performed through focusing deeply on one model of the nonlinear Schrödinger family of equations with higher order nonlinear terms [12, 13]. Thus, various studies in literatures scrutinized the resonant nonlinear Schrödinger equation which is mainly the governing model that describes soliton propagation and

Madelung fluids in many nonlinear media. Several integration schemes have been implemented to examine the behavior of solitons such as ansatz method, semi-inverse variational principle, simplest equation approach, first integral method, functional variable method, sine-cosine function method, (G'/G)-expansion method, trial solution approach, generalized extended tanh method, modified simple equation method, and improved extended tanh-equation method. For more details, readers are referred to references [14–25].

The present study concentrates on the investigation of resonant optical solitons in (3 + 1)-dimensions with two types of nonlinear influences, namely, Kerr law and parabolic law nonlinearities. In particular, we shed light on the model of (3 + 1)-dimensional resonant nonlinear Schrödinger (3D-RNLS) equation given in the form

$$iQ_t + \eta\nabla^2 Q + \sigma F(s)|Q|^2 Q + s\delta\left(\frac{\nabla^2|Q|}{|Q|}\right)Q = 0, \quad i = \sqrt{-1}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \tag{1}$$

where the dependent variable $Q(x, y, z, t)$ is a complex-valued wave profile and the independent variables x, y and z stand for spatial coordinates while t indicates temporal coordinate. The non-zero constants η, σ, and δ account for the coefficients of the group velocity, non-Kerr nonlinearity, and resonant nonlinearity, respectively. The parameter s plays an important role on manipulating the physical properties of distinct media and consequently affecting the behaviors of constructed solitons, see [26].

Here, we will consider two specific cases for the function $F(s)$ that represent the effect of nonlinearity in the media. These two nonlinear influences include the Kerr law and parabolic law nonlinearities. Hence, Eq. (1) with the two laws of nonlinearity has the following forms

$$iQ_t + \eta\nabla^2 Q + \sigma|Q|^2 Q + \delta\left(\frac{\nabla^2|Q|}{|Q|}\right)Q = 0, \tag{2}$$

and

$$iQ_t + \eta\nabla^2 Q + \left(\sigma|Q|^2 + \rho|Q|^4\right)Q + \delta\left(\frac{\nabla^2|Q|}{|Q|}\right)Q = 0. \tag{3}$$

The first model given in Eq. (2) is the 3D-RNLS equation dominated by the Kerr law nonlinearity and is found to have applications in the optical fiber and water waves when the refractive index of the light is proportional to the intensity. The second model presented in Eq. (3) is the 3D-RNLS equation with the parabolic law nonlinearity which arises in the context of nonlinear fiber optics.

In literatures, there are some studies that dealt with the 3D-RNLS equation to find exact solutions. For example, Ferdous et al. [27] investigated the conformable time fractional 3D-RNLS equation with Kerr and parabolic law nonlinearities. Different structures of oblique resonant optical solitons have been obtained by using the generalized $\exp(-\Phi(\xi))$-expansion method. Furthermore, Sedeeg et al. [28] studied the two models of 3D-RNLS equation given in (2) and (3) by applying the modified extended tanh method. Optical soliton solutions including dark, singular and combo solitons are extracted in addition to periodic solutions. Moreover, the exact solutions of the 3D-RNLS equation with Kerr law nonlinearity given in (2) has been examined by Hosseini et al. [29] by exploiting the new expansion methods based on the Jacobi elliptic equation. Recently, Hosseini et al. [30] studied the optical solitons and modulation instability of the models given in (2) and (3).

Various forms of optical solitons are derived with the aid of the $\exp_a$ and hyperbolic function techniques.

The aim of current work is to derive the optical solitons of 3D-RNLS equation presented in (2) and (3). The mathematical technique applied to solve the models is based on a finite series expressed in terms of the solution of projective Riccati equations. The paper is organized as follows. In Section 2, we analyze the idea of implementing the proposed method. In Section 3, the traveling wave reduction of (2) and (3) is extracted. Then, Section 4 displays the derivation of resonant optical solitons in (3 + 1)-dimensions. In Section 5, the main outlook of results and remarks are presented. Finally, the conclusion of work is given in Section 6.

2. Elucidation of solution method

Consider a nonlinear partial differential equation (NLPDE) for $Q(x, y, z, t)$ in the form

$$P\Big(Q, Q_t, Q_x, Q_y, Q_z, Q_{xx}, Q_{yy}, Q_{zz}, \ldots\Big) = 0, \tag{4}$$

where P is a polynomial in its arguments.

Since we seek for exact traveling wave solutions, we introduce the wave variables

$$Q(x, t) = q(\xi), \qquad \xi = x\cos\alpha + y\cos\beta + z\cos\gamma + ct. \tag{5}$$

Inserting (5) into Eq. (4), one can find the following ordinary differential equation (ODE)

$$H\Big(q, q', q'', q''', \ldots\Big) = 0, \tag{6}$$

where prime denotes the derivative with respect to ξ. Then, integrate Eq. (6), if possible, to reduce the order of differentiation.

Now, assume that the solution of Eq. (6) can be expressed in the finite series of the form

$$U(\xi) = a_0 + \sum_{l=1}^{m}\Big(a_l f^l(\xi) + b_l g^l(\xi)\Big), \tag{7}$$

where $a_0, a_l, b_l, (l = 1, 2, \ldots, m)$ are constants to be identified. The parameter m, which is a positive integer, can be determined by balancing the highest order derivative term with the highest order nonlinear term in Eq. (6).

The variables $f(\xi), g(\xi)$ satisfy the equations

$$\begin{gathered} f'(\xi) = \varepsilon A g^2(\xi), \qquad g'(\xi) = -Af(\xi)g(\xi) - \frac{B}{A}g(\xi)(R - Bf(\xi)), \\ g^2(\xi) = \varepsilon\left[\frac{1}{A^2}(R - Bf(\xi))^2 - f^2(\xi)\right], \end{gathered} \tag{8}$$

where A and B are arbitrary constants and $\varepsilon = \pm 1$. The third equation in the system (8) represents the first integral which gives the relation between the functions $f(\xi)$ and $g(\xi)$.

The set of Eqs. (8) is found to admit the following solutions

$$f_1(\xi) = \frac{R\tanh(R\xi)}{A + B\tanh(R\xi)}, \qquad g_1(\xi) = \frac{R\text{sech}(R\xi)}{A + B\tanh(R\xi)}, \tag{9}$$

demands $\varepsilon = 1$.

$$f_2(\xi) = \frac{R\text{coth}(R\xi)}{A + B\text{coth}(R\xi)}, \qquad g_2(\xi) = \frac{R\,\text{csch}(R\xi)}{A + B\text{coth}(R\xi)}, \tag{10}$$

implies $\varepsilon = -1$.

$$f_3(\xi) = \frac{A}{AC + (A^2 - B^2)\xi}, \qquad g_3(\xi) = \frac{\sqrt{-\varepsilon(A^2 - B^2)}}{AC + (A^2 - B^2)\xi}, \tag{11}$$

provided $R = 0$, where C is an arbitrary constant.

The substitution of (7) along with (8) into Eq. (6) generates a polynomial in $f^i(\xi)g^j(\xi)$. Equating each coefficient of $f^i(\xi)g^j(\xi)$ in this polynomial to zero, yields a set of algebraic equations for a_i, b_j. Solving this system of equations, we can obtain many exact solutions of Eq. (4) according to (9)–(11).

3. Traveling wave reduction for Eqs. (2) and (3)

In order to tackle the complex models of 3D-RNLS equation with Kerr law and parabolic law nonlinearities given in (2) and (3), we embark on analyzing their structures by using the wave transformation of the form

$$Q(x,t) = q(\xi)e^{i\varphi}, \tag{12}$$

where

$$\xi = x\cos\alpha + y\cos\beta + z\cos\gamma + \nu t, \; \varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \omega t. \tag{13}$$

3.1 Traveling wave reduction for Eq. (2)

Applying transformation (12), the 3D-RNLS equation with Kerr law nonlinearity given in (2) is broken down into real and imaginary parts as

$$\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)(\eta + \delta)q'' - \left(\omega + \eta\kappa^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)\right)q + \sigma q^3 = 0, \tag{14}$$

and

$$\left(\nu + 2\eta\kappa\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)\right)q' = 0. \tag{15}$$

From Eq. (15), we obtain

$$\nu = -2\lambda\eta\kappa, \tag{16}$$

where $\lambda = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$. Hence, Eq. (14) reduces to the form

$$\lambda(\eta + \delta)q'' - (\omega + \lambda\eta\kappa^2)q + \sigma q^3 = 0. \tag{17}$$

3.2 Traveling wave reduction for Eq. (3)

Similarly, we utilize the wave transformation (12) to the 3D-RNLS equation with parabolic law nonlinearity given in (3) which is decomposed to real and imaginary parts as

$$\lambda(\eta + \delta)q'' - (\omega + \lambda\eta\kappa^2)q + \sigma q^3 + \rho q^5 = 0, \tag{18}$$

and

$$(\nu + 2\lambda\eta\kappa)q' = 0. \tag{19}$$

From Eq. (19), we come by the expression given in (16). To seek a closed form solution, the structure of Eq. (18) has to be rearranged. Thus, we multiply Eq. (18) by q' and integrate with respect to ξ to arrive at

$$\lambda(\eta + \delta)q'^2 - (\omega + \lambda\eta\kappa^2)q^2 + \frac{\sigma}{2}q^4 + \frac{\rho}{3}q^6 + 2\mu = 0, \tag{20}$$

where μ is the integration constant. For convenience, we make use of the variable transformation given as

$$q^2 = V, \tag{21}$$

which leads to $q'^2 = V'^2/4V$. Thus, Eq. (20), after manipulating, becomes

$$\lambda(\eta + \delta)V'^2 + 8\mu V - 4(\omega + \lambda\eta\kappa^2)V^2 + 2\sigma V^3 + \frac{4}{3}\rho V^4 = 0. \tag{22}$$

4. Optical soliton solutions of 3D-RNLS equation with Kerr law and parabolic law nonlinearities

Now, we aim to employ the projective Riccati equations method given in Section 2 to extract the exact resonant optical soliton solutions with Kerr law and parabolic law nonlinearities for 3D-RNLS equations given in (2) and (3). Basically, the proposed technique will be implemented to Eqs. (17) and (20) and then their obtained solutions will be inserted into (12) so as to derive the optical solitons of the models discussed in this work.

4.1 Oblique resonant solitons of 3D-RNLS equation with Kerr law nonlinearity

According to the expansion given in (7) and the balance between the terms q'' and q^3, the solution of Eq. (17) reads

$$q(\xi) = a_0 + a_1 f(\xi) + b_1 g(\xi). \tag{23}$$

Substituting (23) together with Eqs. (8) into Eq. (17) gives rise to an equation having different powers of $f^i g^j$. Collecting all the terms with the same power of $f^i g^j$

together and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations simultaneously leads to the following results.

Set I. If $\varepsilon = 1$, then the following cases of solutions are retrieved.

Case I1. $a_0 = a_1 = 0$, $b_1 = \pm\sqrt{\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)}{\sigma}}$, $\omega = \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)$.

$$Q(x,y,z,t) = \pm R\sqrt{\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)}{\sigma}}\ \frac{\text{sech}(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\, e^{i\varphi}, \tag{24}$$

where $\lambda\sigma(\eta+\delta)\left(A^2-B^2\right) > 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case I2. $a_0 = \pm\frac{BR}{A}\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}$, $b_1 = 0$, $\omega = -\lambda\left(2[\eta+\delta]R^2 + \eta\kappa^2\right)$.

$$Q(x,y,z,t) = \pm R\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}\ \frac{B + A\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\, e^{i\varphi}, \tag{25}$$

where $\lambda\sigma(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda\left(2[\eta+\delta]R^2 + \eta\kappa^2\right)t$.

Case I3. $a_0 = \pm\frac{BR}{A}\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}$, $b_1 = \pm\sqrt{\frac{\lambda(\eta+\delta)\left(A^2-B^2\right)}{2\sigma}}$,

$$\omega = -\lambda\left([\eta+\delta]\frac{R^2}{2} + \eta\kappa^2\right).$$

$$Q(x,y,z,t) = \pm R\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}\left\{\frac{B + A\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\right.$$
$$\left.\pm\frac{\sqrt{B^2-A^2}\,\text{sech}(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\right\}e^{i\varphi}, \tag{26}$$

where $\lambda\sigma(\eta+\delta) < 0$, $A^2 < B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda\left([\eta+\delta]\frac{R^2}{2} + \eta\kappa^2\right)t$.

Set II. If $\varepsilon = -1$, then the following cases of solutions are generated.

Case II1. $a_0 = a_1 = 0$, $b_1 = \pm\sqrt{-\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)}{\sigma}}$, $\omega = \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)$.

$$Q(x,y,z,t) = \pm R\sqrt{-\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)}{\sigma}}\ \frac{\text{csch}(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}\, e^{i\varphi}, \tag{27}$$

where $\lambda\sigma(\eta+\delta)\left(A^2-B^2\right) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case II2. $a_0 = \pm\frac{BR}{A}\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}$, $b_1 = 0$, $\omega = -\lambda\left(2[\eta+\delta]R^2 + \eta\kappa^2\right)$.

$$Q(x,y,z,t) = \pm R\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}\ \frac{B + A\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}e^{i\varphi}, \tag{28}$$

where $\lambda\sigma(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda(2[\eta+\delta]R^2 + \eta\kappa^2)t$.

Case II3. $a_0 = \pm\frac{BR}{A}\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}$, $b_1 = \pm\sqrt{-\frac{\lambda(\eta+\delta)(A^2-B^2)}{2\sigma}}$,

$$\omega = -\lambda\left([\eta+\delta]\frac{R^2}{2} + \eta\kappa^2\right).$$

$$Q(x,y,z,t) = \pm R\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}\left\{\frac{B + A\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}\right.$$
$$\left.\pm\frac{\sqrt{A^2-B^2}\operatorname{csch}(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}{A + B\coth(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t])}\right\}e^{i\varphi}, \tag{29}$$

where $\lambda\sigma(\eta+\delta) < 0$, $A^2 > B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda\left([\eta+\delta]\frac{R^2}{2} + \eta\kappa^2\right)t$.

Set III. If $R = 0$, then the following cases of solutions are created.

Case III1. $a_0 = a_1 = 0$, $b_1 = \pm\sqrt{\frac{2\lambda(\eta+\delta)(A^2-B^2)}{\varepsilon\sigma}}$, $\omega = -\lambda\eta\kappa^2$.

Case III2. $a_0 = 0$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}$, $b_1 = 0$, $\omega = -\lambda\eta\kappa^2$.

Case III3. $a_0 = 0$, $a_1 = \pm\frac{A^2-B^2}{A}\sqrt{-\frac{\lambda(\eta+\delta)}{2\sigma}}$, $b_1 = \pm\sqrt{\frac{\lambda(\eta+\delta)(A^2-B^2)}{2\varepsilon\sigma}}$, $\omega = -\lambda\eta\kappa^2$.

Herein, these three cases in the Set III provide the solution of the form

$$Q(x,y,z,t) = \pm\sqrt{-\frac{2\lambda(\eta+\delta)}{\sigma}}\ \frac{(A^2-B^2)}{AC + (A^2-B^2)(x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t)}e^{i\varphi}, \tag{30}$$

where $\lambda\sigma(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda\eta\kappa^2 t$.

4.2 Oblique resonant solitons of 3D-RNLS equation with parabolic law nonlinearity

Based on the expansion given in (7), we consider that the solution to Eq. (22) takes the form

$$q(\xi) = a_0 + \sum_{l=1}^{2}\left(a_l f^l(\xi) + b_l g^l(\xi)\right). \tag{31}$$

Inserting (31) together with Eqs. (8) into Eq. (22) gives rise to an equation having different powers of $f^i g^j$. Collecting all the terms with the same power of $f^i g^j$ together and equating each coefficient to zero, yields a set of algebraic equations. Solving these equations simultaneously leads to the following results.

Set I. If $\varepsilon = 1$, then the following cases of solutions are obtained.

Case I1. $b_1 = a_2 = b_2 = 0$, $a_0 = \frac{-Ra_1}{(A+B)}$, $a_1 = \pm\frac{A^2-B^2}{2A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$,

$\omega = \frac{\lambda\left((\eta+\delta)(A+B)^2 a_0^2 - \eta\kappa^2 a_1^2\right)}{a_1^2}$, $\sigma = \frac{2\lambda(\eta+\delta)(A-B)(A+B)^2 a_0}{A a_1^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\mp\frac{R}{2}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\ \left\{1 - \frac{B + A\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{32}$$

where $\lambda\rho(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case I2. $b_1 = a_2 = b_2 = 0$, $a_0 = \frac{R a_1}{(A-B)}$, $a_1 = \pm\frac{A^2-B^2}{2A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$,
$\omega = \frac{\lambda\left((\eta+\delta)(A-B)^2 a_0^2 - \eta\kappa^2 a_1^2\right)}{a_1^2}$, $\sigma = \frac{2\lambda(\eta+\delta)(A+B)(A-B)^2 a_0}{A a_1^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\pm\frac{R}{2}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\ \left\{1 + \frac{B + A\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{33}$$

where $\lambda\rho(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case I3. $a_1 = a_2 = b_2 = 0$, $a_0 = \pm\frac{R b_1}{\sqrt{A^2-B^2}}$, $b_1 = \pm\frac{1}{2}\sqrt{\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)}{\rho}}$,
$\omega = -\frac{\lambda\left(5(\eta+\delta)\left(A^2-B^2\right)a_0^2 + 4\eta\kappa^2 b_1^2\right)}{4b_1^2}$, $\sigma = -\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)a_0}{b_1^2}$, $\mu = -\frac{\lambda(\eta+\delta)\left(A^2-B^2\right)a_0^3}{4b_1^2}$.

$$Q(x,y,z,t) = \left[\pm\frac{R}{2}\sqrt{\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1 \pm \frac{\sqrt{A^2-B^2}\operatorname{sech}\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{34}$$

where $\lambda\rho(\eta+\delta) > 0$, $A^2 > B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) - \lambda\left([\eta+\delta]\frac{5R^2}{4} + \eta\kappa^2\right)t$.

Case I4. $a_2 = b_2 = 0$, $a_0 = -\frac{R b_1}{A(A+B)}\sqrt{A^2-B^2}$, $a_1 = \pm\frac{b_1}{A}\sqrt{A^2-B^2}$, $b_1 = \pm\frac{1}{4}\sqrt{\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)}{\rho}}$, $\omega = -\frac{\lambda\left((\eta+\delta)(A+B)A^2 a_0^2 + 4\eta\kappa^2(A-B)b_1^2\right)}{4(A-B)b_1^2}$, $\sigma = -\frac{\lambda(\eta+\delta)(A+B)A a_0}{2b_1^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\pm\frac{R}{4A}\sqrt{\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)}{\rho}}\left\{\frac{\sqrt{-\left(A^2-B^2\right)}\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)} \pm \frac{A\operatorname{sech}\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)}{A + B\tanh\left(R[x\cos\alpha + y\cos\beta + z\cos\gamma - 2\lambda\eta\kappa t]\right)} - \frac{\sqrt{-\left(A^2-B^2\right)}}{(A+B)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{35}$$

where $\lambda\rho(\eta+\delta) < 0$, $A^2 < B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)t$.

Case I5. $a_2 = b_2 = 0$, $a_0 = \pm\frac{R b_1}{A(A-B)}\sqrt{-\left(A^2-B^2\right)}$, $a_1 = \pm\frac{b_1}{A}\sqrt{-\left(A^2-B^2\right)}$, $b_1 = \frac{1}{4}\sqrt{\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)}{\rho}}$, $\omega = -\frac{\lambda\left((\eta+\delta)(A-B)A^2 a_0^2 + 4\eta\kappa^2(A+B)b_1^2\right)}{4(A+B)b_1^2}$, $\sigma = -\frac{\lambda(\eta+\delta)(A-B)A a_0}{2b_1^2}$, $\mu = 0$.

$$Q(x,y,z,t)=\left[\pm\frac{R}{4A}\sqrt{\frac{3\lambda(\eta+\delta)(A^2-B^2)}{\rho}}\left\{\frac{\sqrt{-(A^2-B^2)}\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right.\right.$$
$$\left.\left.\pm\frac{A\,\mathrm{sech}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}+\frac{\sqrt{-(A^2-B^2)}}{(A-B)}\right\}\right]^{\frac{1}{2}}e^{i\varphi},\tag{36}$$

where $\lambda\rho(\eta+\delta)<0$, $A^2<B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)+\lambda\left([\eta+\delta]\frac{R^2}{4}-\eta\kappa^2\right)t$.

Case I6. $a_0=-\frac{\left(2\lambda(\eta+\delta)A^2+\sigma b_2\right)R^2}{\sigma A^2}$, $a_1=\frac{2BRb_2}{A^2}$, $b_1=\pm\frac{A^2a_0+R^2b_2}{RA^2}\sqrt{A^2-B^2}$, $a_2=\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega=-\lambda\left([\eta+\delta]\frac{5R^2}{4}+\eta\kappa^2\right)$, $\rho=\frac{3\sigma^2}{16\lambda(\eta+\delta)R^2}$, $\mu=\frac{\lambda^2(\eta+\delta)^2R^4}{2\sigma}$.

$$Q(x,y,z,t)=\left[-\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1\pm\frac{\sqrt{A^2-B^2}\mathrm{sech}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}}e^{i\varphi},\tag{37}$$

where $A^2>B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)-\lambda\left([\eta+\delta]\frac{5R^2}{4}+\eta\kappa^2\right)t$.

Case I7. $b_1=0$, $a_0=\frac{\left(A^2a_1-(A+B)Rb_2\right)R}{(A-B)A^2}$, $a_1=\frac{2\left(\lambda(\eta+\delta)\left(A^2-B^2\right)A+\sigma Bb_2\right)R}{\sigma A^2}$, $a_2=\frac{\left(A^2-B^2\right)b_2}{A^2}$,
$\omega=\lambda\left([\eta+\delta]R^2-\eta\kappa^2\right)$, $\rho=\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)^2A^2}{4\left(A^2a_1-2BRb_2\right)^2}$, $\mu=0$.

$$Q(x,y,z,t)=\left[\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1+\frac{B+A\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}}e^{i\varphi},\tag{38}$$

where $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)+\lambda\left([\eta+\delta]R^2-\eta\kappa^2\right)t$.

Case I8. $b_1=0$, $a_0=-\frac{\left(A^2a_1+(A-B)Rb_2\right)R}{(A+B)A^2}$, $a_1=-\frac{2\left(\lambda(\eta+\delta)\left(A^2-B^2\right)A-\sigma Bb_2\right)R}{\sigma A^2}$,
$a_2=\frac{\left(A^2-B^2\right)b_2}{A^2}$,
$\omega=\lambda\left([\eta+\delta]R^2-\eta\kappa^2\right)$, $\rho=-\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)^2A^2}{4\left(A^2a_1-2BRb_2\right)^2}$, $\mu=0$.

$$Q(x,y,z,t)=\left[\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1-\frac{B+A\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}}e^{i\varphi},\tag{39}$$

where $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)+\lambda\left([\eta+\delta]R^2-\eta\kappa^2\right)t$.

Case I9. $a_0=\frac{(\lambda(\eta+\delta)(A+B)A-2\sigma b_2)R^2}{2\sigma A^2}$, $a_1=\frac{(A-B)A^2a_0+(A+B)R^2b_2}{A^2R}$, $b_1=\pm\frac{\lambda(\eta+\delta)R}{2\sigma}\sqrt{-(A^2-B^2)}$, $a_2=\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega=\lambda\left([\eta+\delta]\frac{R^2}{4}-\eta\kappa^2\right)$, $\rho=-\frac{3\sigma^2}{4\lambda(\eta+\delta)R^2}$, $\mu=0$.

$$Q(x,y,z,t) = \left[\frac{\lambda(\eta+\delta)R^2}{2\sigma}\left\{1+\frac{B+A\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right.\right.$$
$$\left.\left.\pm\frac{\sqrt{-\left(A^2-B^2\right)}\operatorname{sech}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{40}$$

where $A^2 < B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)t$.

Case I10. $a_0 = \frac{(\lambda(\eta+\delta)(A-B)A-2\sigma b_2)R^2}{2\sigma A^2}$, $a_1 = -\frac{(A+B)A^2 a_0 + (A-B)R^2 b_2}{A^2 R}$, $b_1 = \pm\frac{\lambda(\eta+\delta)R}{2\sigma}\sqrt{-\left(A^2-B^2\right)}$, $a_2 = \frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega = \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)$, $\rho = -\frac{3\sigma^2}{4\lambda(\eta+\delta)R^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\frac{\lambda(\eta+\delta)R^2}{2\sigma}\left\{1-\frac{B+A\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right.\right.$$
$$\left.\left.\pm\frac{\sqrt{-\left(A^2-B^2\right)}\operatorname{sech}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\tanh\left(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t]\right)}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{41}$$

where $A^2 < B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)t$.

Set II. If $\varepsilon = -1$, then the following cases of solutions are acquired.

Case II1. $b_1 = a_2 = b_2 = 0$, $a_0 = \frac{-Ra_1}{(A+B)}$, $a_1 = \pm\frac{A^2-B^2}{2A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$, $\omega = \frac{\lambda\left((\eta+\delta)(A+B)^2a_0^2 - \eta\kappa^2a_1^2\right)}{a_1^2}$, $\sigma = \frac{2\lambda(\eta+\delta)(A-B)(A+B)^2a_0}{Aa_1^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\mp\frac{R}{2}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1-\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{42}$$

where $\lambda\rho(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case II2. $b_1 = a_2 = b_2 = 0$, $a_0 = \frac{Ra_1}{(A-B)}$, $a_1 = \pm\frac{A^2-B^2}{2A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$, $\omega = \frac{\lambda\left((\eta+\delta)(A-B)^2a_0^2 - \eta\kappa^2a_1^2\right)}{a_1^2}$, $\sigma = \frac{2\lambda(\eta+\delta)(A+B)(A-B)^2a_0}{Aa_1^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\pm\frac{R}{2}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1+\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}} e^{i\varphi}, \tag{43}$$

where $\lambda\rho(\eta+\delta) < 0$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case II3. $a_1 = a_2 = b_2 = 0$, $a_0 = \pm\frac{Rb_1}{\sqrt{A^2-B^2}}$, $b_1 = \pm\frac{1}{2}\sqrt{\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)}{\rho}}$, $\omega = -\frac{\lambda\left(5(\eta+\delta)\left(A^2-B^2\right)a_0^2 + 4\eta\kappa^2b_1^2\right)}{4b_1^2}$, $\sigma = -\frac{2\lambda(\eta+\delta)\left(A^2-B^2\right)a_0}{b_1^2}$, $\mu = -\frac{\lambda(\eta+\delta)\left(A^2-B^2\right)a_0^3}{4b_1^2}$.

$$Q(x,y,z,t) = \left[\pm\frac{R}{2}\sqrt{\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1\pm\frac{\sqrt{-(A^2-B^2)}\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{44}$$

where $\lambda\rho(\eta+\delta)>0$, $A^2<B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)-\lambda\left([\eta+\delta]\frac{5R^2}{4}+\eta\kappa^2\right)t.$

Case II4. $a_2=b_2=0$, $a_0=-\frac{Rb_1}{(A+B)}$, $a_1=\pm\frac{A^2-B^2}{4A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$, $b_1=\pm\frac{Aa_1}{A^2-B^2}\sqrt{A^2-B^2}$, $\omega=\frac{\lambda\left((\eta+\delta)(A+B)^2a_0^2-4\eta\kappa^2a_1^2\right)}{4a_1^2}$, $\sigma=\frac{\lambda(\eta+\delta)(A+B)\left(A^2-B^2\right)a_0}{2Aa_1^2}$, $\mu=0.$

$$Q(x,y,z,t) = \left[\mp\frac{R}{4}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1-\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right.\right.$$
$$\left.\left.\pm\frac{\sqrt{A^2-B^2}\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{45}$$

where $\lambda\rho(\eta+\delta)<0$, $A^2>B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)+\lambda\left([\eta+\delta]\frac{R^2}{4}-\eta\kappa^2\right)t.$

Case II5. $a_2=b_2=0$, $a_0=\frac{Rb_1}{(A-B)}$, $a_1=\pm\frac{A^2-B^2}{4A}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}$, $b_1=\pm\frac{Aa_1}{A^2-B^2}\sqrt{A^2-B^2}$, $\omega=\frac{\lambda\left((\eta+\delta)(A-B)^2a_0^2-4\eta\kappa^2a_1^2\right)}{4a_1^2}$, $\sigma=\frac{\lambda(\eta+\delta)(A-B)\left(A^2-B^2\right)a_0}{2Aa_1^2}$, $\mu=0.$

$$Q(x,y,z,t) = \left[\pm\frac{R}{4}\sqrt{-\frac{3\lambda(\eta+\delta)}{\rho}}\left\{1+\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right.\right.$$
$$\left.\left.\pm\frac{\sqrt{A^2-B^2}\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{46}$$

where $\lambda\rho(\eta+\delta)<0$, $A^2>B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)+\lambda\left([\eta+\delta]\frac{R^2}{4}-\eta\kappa^2\right)t.$

Case II6. $a_0=-\frac{\left(2\lambda(\eta+\delta)A^2-\sigma b_2\right)R^2}{\sigma A^2}$, $a_1=-\frac{2BRb_2}{A^2}$, $b_1=\pm\frac{2\lambda(\eta+\delta)R}{\sigma}\sqrt{-\left(A^2-B^2\right)}$, $a_2=-\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega=-\lambda\left([\eta+\delta]\frac{5R^2}{4}+\eta\kappa^2\right)$, $\rho=\frac{3\sigma^2}{16\lambda(\eta+\delta)R^2}$, $\mu=\frac{\lambda^2(\eta+\delta)^2R^4}{2\sigma}$.

$$Q(x,y,z,t) = \left[-\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1\pm\frac{\sqrt{-\left(A^2-B^2\right)}\,\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{47}$$

where $A^2<B^2$ and $\varphi=\kappa(x\cos\alpha+y\cos\beta+z\cos\gamma)-\lambda\left([\eta+\delta]\frac{5R^2}{4}+\eta\kappa^2\right)t.$

Case II7. $b_1=0$, $a_0=\frac{\left(A^2a_1+(A+B)Rb_2\right)R}{(A-B)A^2}$, $a_1=\frac{2\left(\lambda(\eta+\delta)\left(A^2-B^2\right)A-\sigma Bb_2\right)R}{\sigma A^2}$, $a_2=-\frac{\left(A^2-B^2\right)b_2}{A^2}$,

$\omega = \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)$, $\rho = -\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)^2A^2}{4\left(A^2a_1+2BRb_2\right)^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1+\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{48}$$

where $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case II8. $b_1 = 0$, $a_0 = -\frac{\left(A^2a_1-(A-B)Rb_2\right)R}{(A+B)A^2}$, $a_1 = -\frac{2\left(\lambda(\eta+\delta)\left(A^2-B^2\right)A+\sigma Bb_2\right)R}{\sigma A^2}$, $a_2 = -\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega = \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)$, $\rho = -\frac{3\lambda(\eta+\delta)\left(A^2-B^2\right)^2A^2}{4\left(A^2a_1+2BRb_2\right)^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\frac{2\lambda(\eta+\delta)R^2}{\sigma}\left\{1-\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{49}$$

where $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]R^2 - \eta\kappa^2\right)t$.

Case II9. $a_0 = \frac{(\lambda(\eta+\delta)(A+B)A+2\sigma b_2)R^2}{2\sigma A^2}$, $a_1 = \frac{(A-B)A^2a_0-(A+B)R^2b_2}{A^2R}$, $b_1 = \pm\frac{\lambda(\eta+\delta)R}{2\sigma}\sqrt{A^2-B^2}$, $a_2 = -\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega = \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)$, $\rho = -\frac{3\sigma^2}{4\lambda(\eta+\delta)R^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\frac{\lambda(\eta+\delta)R^2}{2\sigma}\left\{1+\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])} \pm\frac{\sqrt{A^2-B^2}\,\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{50}$$

where $A^2 > B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)t$.

Case II10. $a_0 = \frac{(\lambda(\eta+\delta)(A-B)A+2\sigma b_2)R^2}{2\sigma A^2}$, $a_1 = -\frac{(A+B)A^2a_0-(A-B)R^2b_2}{A^2R}$, $b_1 = \pm\frac{\lambda(\eta+\delta)R}{2\sigma}\sqrt{A^2-B^2}$, $a_2 = -\frac{\left(A^2-B^2\right)b_2}{A^2}$, $\omega = \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)$, $\rho = -\frac{3\sigma^2}{4\lambda(\eta+\delta)R^2}$, $\mu = 0$.

$$Q(x,y,z,t) = \left[\frac{\lambda(\eta+\delta)R^2}{2\sigma}\left\{1-\frac{B+A\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])} \pm\frac{\sqrt{A^2-B^2}\,\mathrm{csch}(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}{A+B\coth(R[x\cos\alpha+y\cos\beta+z\cos\gamma-2\lambda\eta\kappa t])}\right\}\right]^{\frac{1}{2}}e^{i\varphi}, \tag{51}$$

where $A^2 > B^2$ and $\varphi = \kappa(x\cos\alpha + y\cos\beta + z\cos\gamma) + \lambda\left([\eta+\delta]\frac{R^2}{4} - \eta\kappa^2\right)t$.

5. Results and remarks

To give a clear insight into the behavior of resonant optical solitons, the graphical representations for some of the extracted soliton solutions are presented. Besides, the obliqueness influence on the resonant solitons is examined. Thus, we display the 3D and 2D plots of the absolute of these solutions by selecting different values of the model parameters. For example, **Figure 1(a)**-**(b)** present the 3D and 2D plots of resonant soliton for the solution given in (24) of 3D-RNLS equation with Kerr-law nonlinearity. It is clear from the graph that the wave profile represents bright soliton. **Figure 1(c)**-**(d)** display the 3D plot for the effect of obliquity on the resonant soliton given in (24), where **Figure 1(c)** shows the relation between x and α while **Figure 1(d)** illustrates the relation between x and γ. **Figure 2(a)**-**(b)** exhibit the 3D and 2D plots of resonant dark soliton given in the solution (29) of 3D-RNLS equation with Kerr-law nonlinearity. The obliqueness influence on the solution (29) is shown in **Figure 2(c)**-**(d)**. Additionally, **Figure 3(a)**-**(b)** demonstrate the 3D and 2D plots of resonant soliton given in the solution (40) of 3D-RNLS equation with parabolic-law nonlinearity, where the wave profile describes a kink-shape soliton. It can be seen that **Figure 3(c)**-**(d)** present the obliquity impact on the solution (40). **Figure 4(a)**-**(b)** depict the 3D and 2D plots of resonant singular soliton given in the solution (48) of 3D-RNLS equation with parabolic-law nonlinearity, where the effect of obliqueness on this wave is illustrated in **Figure 4(c)**-**(d)**.

One can obviously see from **Figures 1**–**4** that the obliqueness influences the behavior of resonant solitons, where the structure of solitons is changed remarkably with the variation of obliqueness parameters. Further to this, it is noticed that the

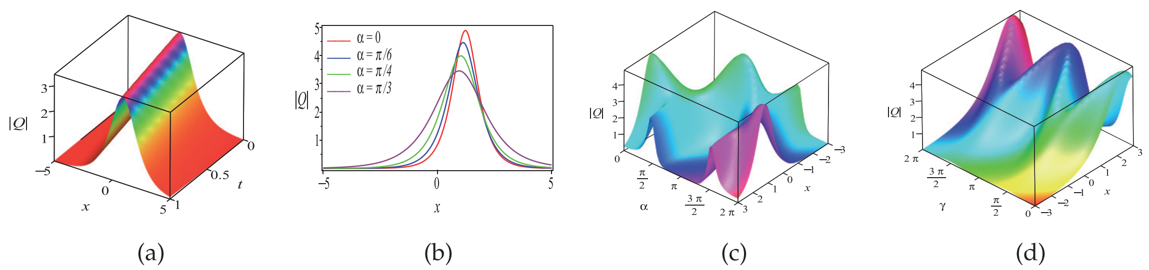

Figure 1.
(a)-(b) Resonant soliton and (c)-(d) obliqueness effect on resonant soliton corresponding to solution (24) with $\kappa = 0.5, \eta = \delta = \sigma = 1, R = A = 2, B = 1, \alpha = \beta = \gamma = \pi/3, y = z = 0, t = 1$.

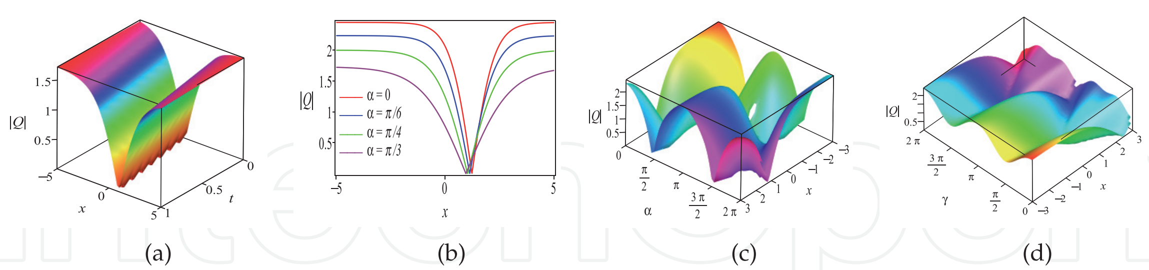

Figure 2.
*(a)-(b) Resonant soliton and (c)-(d) obliqueness effect on resonant soliton corresponding to solution (29) with the same values of parameters in **Figure 1** except* $\sigma = -1$.

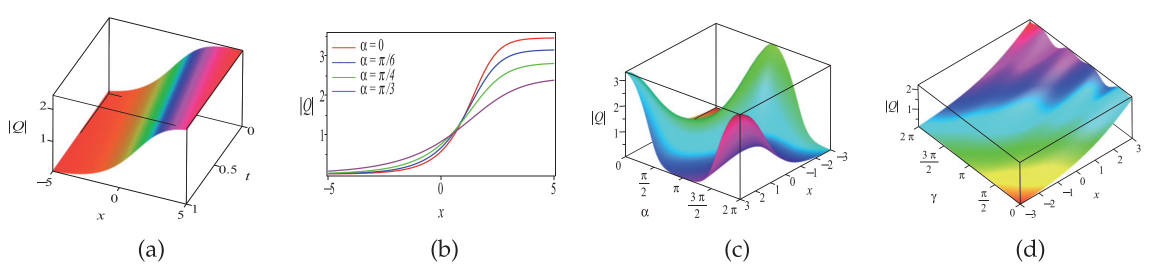

Figure 3.
*(a)-(b) Resonant soliton and (c)-(d) obliqueness effect on resonant soliton corresponding to solution (40) with the same values of parameters in **Figure 1** except* $A = 1, B = -2$.

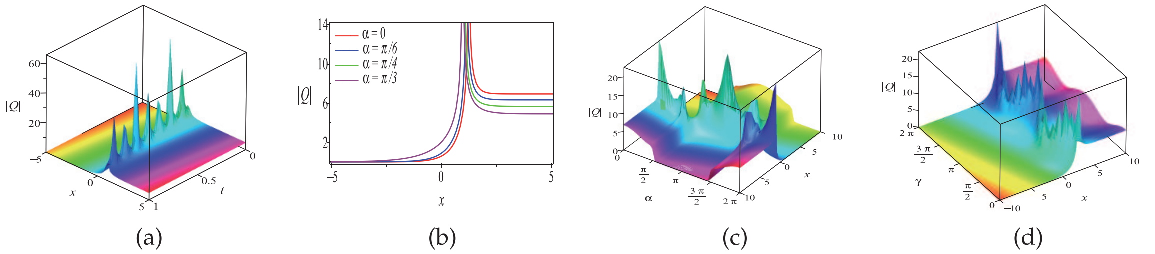

Figure 4.
(a)-(b) Resonant soliton and (c)-(d) obliqueness effect on resonant soliton corresponding to solution (48) with the same values of parameters in ***Figure 1****.*

amplitude of the resonant solitons decreases and the width rises with the increase of obliqueness as shown in the 2D graphs.

6. Conclusions

This work scoped the behavior of optical solitons of 3D-RNLS equation. The dominant nonlinearity in the model is considered to have two forms which are Kerr law and parabolic law. The resonant solitons are derived with the aid of projective Riccati equations method. Various forms of wave structures are retrieved such as bright, dark, singular, kink, dark-singular and combined singular solitons. The influence of obliquity on resonant solitons is also examined. It is found that the change in the obliqueness parameters leads to a noticeable variation on the behavior of optical soliton waves. In addition to this, the amplitude of the resonant solitons undergoes a reduction, but their width is enhanced as the obliqueness is increased. The results obtained in this work are entirely new and it may be useful to understand the dynamics of resonant solitons affected by obliqueness in different nonlinear media such as optical fiber and Madelung fluids.

Conflict of interest

The author declares no conflict of interest.

Author details

Khalil S. Al-Ghafri
University of Technology and Applied Sciences, Ibri, Oman

*Address all correspondence to: khalil.ibr@cas.edu.om

References

[1] Lonngren KE. Soliton experiments in plasmas. Plasma Physics. 1983;25(9): 943–982.

[2] Agrawal GP. Nonlinear fiber optics: quantum electronics–principles and applications. Academic Press, New York; 1995.

[3] Mollenauer LF, Gordon JP. Solitons in optical fibers: fundamentals and applications. Elsevier; 2006.

[4] Biswas A, Konar S. Introduction to non-Kerr law optical solitons. CRC Press; 2006.

[5] Bao W. The nonlinear Schrödinger equation and applications in Bose-Einstein condensation and plasma physics. Dynamics in Models of Coarsening, Coagulation, Condensation and Quantization" (IMS Lecture Notes Series, World Scientific). 2007;9:141–240.

[6] Gedalin M, Scott T, Band Y. Optical solitary waves in the higher order nonlinear schrödinger equation. Physical review letters. 1997;78(3):448.

[7] Hong WP. Optical solitary wave solutions for the higher order nonlinear schrödinger equation with cubic-quintic non-kerr terms. Optics Communications. 2001;194(1-3):217–223.

[8] Liu W, Pang L, Han H, et al. Tungsten disulphide for ultrashort pulse generation in all-fiber lasers. Nanoscale. 2017;9(18):5806–5811.

[9] Liu W, Zhu YN, Liu M, et al. Optical properties and applications for MoS 2-Sb 2 Te 3-MoS 2 heterostructure materials. Photonics Research. 2018;6 (3):220–227.

[10] Singer A. Signal processing and communication with solitons [dissertation]. Massachusetts Institute of Technology; 1996.

[11] Song Y, Shi X, Wu C, et al. Recent progress of study on optical solitons in fiber lasers. Applied Physics Reviews. 2019;6(2):021313.

[12] Porsezian K. Soliton propagation in nonlinear optics with higher-order effects. Journal of Modern Optics. 1997; 44(2):387–394.

[13] Radhakrishnan R, Kundu A, Lakshmanan M. Coupled nonlinear Schrödinger equations with cubic-quintic nonlinearity: integrability and soliton interaction in non-kerr media. Physical Review E. 1999;60(3): 3314.

[14] Triki H, Hayat T, Aldossary OM, et al. Bright and dark solitons for the resonant nonlinear Schrödinger's equation with time-dependent coefficients. Optics & Laser Technology. 2012;44(7):2223–2231.

[15] Triki H, Yildirim A, Hayat T, et al. 1-soliton solution of the generalized resonant nonlinear dispersive Schrödinger's equation with time-dependent coefficients. Advanced Science Letters. 2012;16(1):309–312.

[16] Biswas A. Soliton solutions of the perturbed resonant nonlinear Schrodinger's equation with full nonlinearity by semi-inverse variational principle. Quantum Phys Lett. 2012;1 (2):79–89.

[17] Eslami M, Mirzazadeh M, Biswas A. Soliton solutions of the resonant nonlinear Schrödinger's equation in optical fibers with time-dependent coefficients by simplest equation approach. Journal of Modern Optics. 2013;60(19):1627–1636.

[18] Eslami M, Mirzazadeh M, Vajargah BF, et al. Optical solitons for the resonant nonlinear Schrödinger's equation with time-dependent

coefficients by the first integral method. Optik. 2014;125(13):3107–3116.

[19] Mirzazadeh M, Eslami M, Vajargah BF, et al. Optical solitons and optical rogons of generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity. Optik. 2014;125(16):4246–4256.

[20] Mirzazadeh M, Eslami M, Milovic D, et al. Topological solitons of resonant nonlinear Schödinger's equation with dual-power law nonlinearity by G'/G-expansion technique. Optik. 2014;125(19): 5480–5489.

[21] Mirzazadeh M, Arnous A, Mahmood M, et al. Soliton solutions to resonant nonlinear Schrödinger's equation with time-dependent coefficients by trial solution approach. Nonlinear Dynamics. 2015;81(1):277–282.

[22] Kilic B, Inc M. On optical solitons of the resonant Schrödinger's equation in optical fibers with dual-power law nonlinearity and time-dependent coefficients. Waves in Random and Complex Media. 2015;25(3):334–341.

[23] Arnous AH, Mirzazadeh M, Zhou Q, et al. Soliton solutions to resonant nonlinear schrodinger's equation with time-dependent coefficients by modified simple equation method. Optik. 2016;127(23): 11450–11459.

[24] Mirzazadeh M, Ekici M, Zhou Q, et al. Exact solitons to generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity. Optik. 2017;130:178–183.

[25] Al-Ghafri KS. Different physical structures of solutions for a generalized resonant dispersive nonlinear schrödinger equation with power law nonlinearity. Journal of Applied Mathematics. 2019;2019:1–8.

[26] Pashaev OK, Lee JH. Resonance solitons as black holes in madelung fluid. Modern Physics Letters A. 2002;17 (24):1601–1619.

[27] Ferdous F, Hafez MG, Biswas A, et al. Oblique resonant optical solitons with Kerr and parabolic law nonlinearities and fractional temporal evolution by generalized exp $(-\phi(\xi))$-expansion. Optik. 2019;178:439–448.

[28] Sedeeg AKH, Nuruddeen R, Gomez-Aguilar J. Generalized optical soliton solutions to the (3+ 1)-dimensional resonant nonlinear schrödinger equation with kerr and parabolic law nonlinearities. Optical and Quantum Electronics. 2019;51(6):1–15.

[29] Hosseini K, Matinfar M, Mirzazadeh M. A (3+ 1)-dimensional resonant nonlinear Schrödinger equation and its jacobi elliptic and exponential function solutions. Optik. 2020;207:164458.

[30] Hosseini K, Ansari R, Zabihi A, et al. Optical solitons and modulation instability of the resonant nonlinear Schrödinger equations in (3+ 1)-dimensions. Optik. 2020;209:164584.