Chapter

Optimal Maintenance Policy for Second-Hand Equipments under Uncertainty

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Abstract

This chapter addresses a maintenance optimization problem for re-manufactured equipments that will be reintroduced into the market as second-hand equipments. The main difference of this work and the previous literature on the maintenance optimization of second-hand equipments is the influence of the uncertainties due to the indirect obsolescence concept. The uncertainty is herein about the spare parts availability to perform some maintenance actions on equipment due to technology vanishing. The maintenance policy involves in fact a minimal repair at failure and a preventive repair after some operating period. To deal with this shortcoming, the life cycle of technology or spare parts availability is defined and modeled as a random variable whose lifetimes distribution is well known and Weibull distributed. Accordingly, an optimal maintenance policy is discussed and derived for such equipment in order to overcome the uncertainty on reparation action. Moreover, experiments are then conducted and different life cycle of technologies are evaluated according to their obsolescence processes (accidental or progressive vanishing) on the optimal operating condition.

Keywords: optimal preventive period, indirect obsolescence, minimal repair, second-hand equipment, rejuvenation, virtual age, reliability, residual age

1. Introduction

In a variety of markets and with the rapid economic development, the number of second-hand equipments such as automobiles and high-priced electronic equipment is increasing significantly. Theses equipments tend to degrade with respect to their age and are more likely to fail during their warranty periods than are new equipments. This has generated a stream of parts and goods that can be reconditioned/refreshed to be reused in maintenance actions.

For several decades, many researchers have worked to model and optimize maintenance policies for stochastically degrading production and manufacturing systems. Many interesting and significant results appeared in the literature. The initial framework for preventive maintenance (PM) is due to Barlow and hunter in their seminal paper [1]. Subsequently, a large variety of mathematical models appeared in the literature for optimal maintenance policies design and implementation. For a review on the topic, the reader is referred to [2–5], and the references therein. Recently Nakagawa and his coauthors proposed new models dealing with finite time horizon [6, 7]. Lugtighei and his co-authors also achieved a review of maintenance models [8].

The recent expansion of transaction volume on second-hand market has therefore made grown the potential benefit and research interest for businesses and equipments on such market through a better modeling of additional services such as warranty and maintenance optimization. Accordingly, several researches are performed in order to fit adaptive warranty policy for second-hand equipments. In fact, an analysis of warranty cost was discussed in [9] while Shafiee and al. proposed an approach to determinate an optimal upgrade level for second-hand equipment according to the overhaul cost structure in [10]. In the same way, seminal research was also conducted on optimizing maintenance or replacement policy for second-hand equipment in recent decade. Therefore, optimal maintenance policies were adjusted for second-hand equipments in [11, 12]. In which the authors established maintenance policies which ensure an optimal preventive repair period for second-hand equipment based on the cost and hypotheses on its initial age (deterministic or random). However, the maintenance models proposed in above references assume that the technology and spare parts to repair equipment remains available during optimization period.

The recent economic downturn combined with the rapid development of technology drawn a new business ecosystem with a new relationship between the producers and the consumers. The new ecosystem requires handy equipments with more innovation and cheaper. This situation affects deeply the industrial design and involves additional research cost and reduce the cycle life of product in manufacturer industry. To deal with this cost, the producers have suited their policy by adjusting the sale volume through repeat purchase in order to keep their profit margin. The repeat purchase involves making product with deliberated short life or useful life. Short products life cycle implies premature obsolescence. The premature obsolescence can be ones or combination of planned obsolescence, indirect obsolescence, incompatibility obsolescence, Style obsolescence. In planned obsolescence, the equipment fails systematically after some durations and without possibility to repair. Moreover the indirect obsolescence involves a deliberate unavailability of spare parts. Without spare parts, the repair execution becomes impossible to do in practice. We note that the consequences of premature obsolescence reduce the chance to perform maintenance with the time in practice. This situation makes equipment unrepairable and the execution of older maintenance policy unlikely. Accordingly, this situation arouses issues about our manner to think the maintenance optimization approach regardless the type of equipment (new or second-hand). This chapter is therefore going to highlight the drawback and impact of indirect obsolescence on a maintenance policy optimizing through the random modeling of maintenance execution in practice.

Based on the previous notes that a stochastic maintenance strategy is herein proposed and discussed under indirect obsolescence. The maintenance cost is modeled and analyzed according to the residual age of technology and its vanishing process (accidental, progressive).

The remainder of chapter is organized as follows. Next section explains clearly the problematic of maintenance policy. This section defines the repair strategy and introduces the nature of each repair. In Section 3, a mathematical model of maintenance cost is proposed with some explanations on the cost. In section 4, we discuss the optimality condition with respect to each repair cost and cost parameters. We finish this section by numerical experiment in which the maintenance policy is analyzed through the technology vanishing process.

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2. Problem description

We consider a smart maintenance policy for second-hand equipments under an uncertainty on the execution of repair due to the unavailability of required spare parts or technologies. The equipment is bought on second-hand market and rejuvenated for a safe operation. The rejuvenation has a cost and allows to reach a required reliability for second-hand equipment.

The considered equipments are assumed to operates under some uncertainties. To perform their tasks correctly, the equipments are going to be repaired minimally at failure and preventively after some operating period. Each repair involves a cost which depends on the type of repair.

The minimal repair takes place at failure. This reparation does not impact equipment age and allows to maintain equipment failure risk at the level before it fails. Meanwhile, the preventive repair equipment is performed after some operating period. This repair consists of a soft overhaul of equipment and allows to reduce the equipment age and then the failure risk. The preventive repair requires higher cost and uses more spare parts than minimal repair. All repairs need new spare parts to replace or repair faulty components anywhere and anytime on equipment in order that the equipment gets a minimal required failure risk to operate in safe condition. The replacement of faulty or failed components involves new one available. However, this availability becomes uncertain with the indirect obsolescence.

This uncertainty is therefore considered in our works. Accordingly, the useful life of the technology or the availability period of spare parts behind equipments is considered as a random value with a continuous probability distribution. To derive a cost of maintenance policy for such equipments under uncertainty of technology, the chance to perform reparation is evaluated and integrated in the models. The appraisal of the probability to perform repair depends on the residual life distribution of technology and its probability distribution function.

The next section proposes a mathematical model for the maintenance policy optimization under uncertainty on the technology availability to ensure the repair.

3. Mathematical model of maintenance cost

The second-hand equipment with certain ages u is considered in this chapter. In fact an equipment is bought on second-hand market with an initial age at price $C_{ac}(u)$. The cost $C_{ac}(u)$ stands for the acquisition price whose value depends on the age at acquisition and the technology. if we assume that the new equipment costs C_{new} then the acquisition price function C_{ac} has to respect some mathematical properties such as

$$C_{ac}(0) = C_{new}, \tag{1}$$

$$\frac{d}{du} C_{ac}(u) \leq 0. \tag{2}$$

$$\lim_{u \to \infty} C_{ac}(u) = 0, \tag{3}$$

Roughly speaking, an equipment at age $u = 0$ is bought at the price of new Eq. (1). Moreover, the acquisition price C_{ac} remains non-increasing function with respect to acquisition age (u) (Eq. (2)) and becomes null for older equipment Eq. (3).

In addition, the bought equipment is overhauled before operating. This overhaul consists of deep repair for the equipment in order to get a required threshold in reliability by rejuvenation. The rejuvenation involves a safe operating condition costs C_{rej}. This cost C_{rej} is function of initial age u and the desired rejuvenation on the equipment age u_f. $C_{rej}(u, u_f)$ stands for the rejuvenation cost and respect also some mathematical properties defined as follows:

$$\begin{cases} C_{rej}(u, u_f = u) = 0, \\ C_{rej}(u, u_f = 0) = \infty, \end{cases} \tag{4}$$

Accordingly, the rejuvenation allow to reduce the age of equipment from u to u_f such as $u \geq u_f$. To fit reality, we assume that having new equipment by rejuvenation is impossible and non-rejuvenation costs anything in practice Eq. (4). In addition the rejuvenation cost increases in function of initial age u Eq. (5) while it decreases with the final age u_f Eq. (6)

$$\frac{\partial}{\partial u} C_{rej}(u, u_f) > 0, \tag{5}$$

$$\frac{\partial}{\partial u_f} C_{rej}(u, u_f) < 0. \tag{6}$$

3.1 Preventive repair cost

The preventive repair involves more spare parts and duration to diagnose and to replace failed components. This repair is performed after each operating period with T_k length. Each preventive repair is assumed imperfect but better than the minimal repair. Herein, the preventive repair is expressed by age reduction model with infinite memory [13]. The preventive action allows to reduce the virtual age of equipment by a multiplicative factor α_k. Each preventive repair involves a cost $C_p(k, \alpha_k)$ and also spends a duration which is negligible relatively to the length of operation period.

Moreover the preventive cost depends on the age reduction according to the factor α_k. The preventive repair cost is assumed non-decreasing function with respect k and denotes by $C_p(k)$.

$$C_p(k, \alpha_k) = C_p(k). \tag{7}$$

The virtual age reduction due to preventive repair is performed according to α_k multiplicative factor. Therefore in case of multiplicative factor the reduction after k^{th} preventive repair is $\alpha_k Age_{k-1}$. Therefore, the virtual age of equipment at the end of k^{th} period

$$Age_k = \alpha_k Age_{k-1} + T_k, \tag{8}$$

where Age_{k-1} represents the virtual age of equipment before the k^{th} preventive repair. Therefore k^{th} preventive impact the age of equipment by factor α_k. Accordingly, the age of equipment after the k^{th} preventive becomes Age_k.

Proposition 1. *The virtual age of equipment at beginning of the k^{th} operating period is therefore equal to:*

$$\begin{cases} Age_0 = \alpha_0 u, \quad \text{if } k = 1 \\ Age_{k-1} = \left(\prod_{j=0}^{k-1} \alpha_j\right) u + \sum_{m=1}^{k-1} \left(\prod_{j \geq m}^{k-1} \alpha_j\right) T_m, \quad \text{otherwise} \end{cases} \tag{9}$$

with $\alpha_{rej} = \alpha_0$.

Proof: From Eq. (8) we deduce each virtual age as follows

$$Age_{k-1} = \alpha_{k-1}Age_{k-2} + T_{k-1} \tag{10}$$

for different values of k suxh as k in $\{k-2, k-3, \dots, 1\}$, we obtain then

$$Age_{k-2} = \alpha_{k-2}Age_{k-3} + T_{k-2} \tag{11}$$

$$\cdots = \cdots\cdots\cdots$$

$$Age_2 = \alpha_2 Age_1 + T_2 \tag{12}$$

$$Age_1 = \alpha_1 Age_0 + T_1 \tag{13}$$

$$Age_0 = \alpha_0 u \tag{14}$$

The result of proposition (1) is deduced by recursive replacement of each Age_i from bottom to top. Therefore we obtain

$$\begin{aligned} Age_{k-1} &= (\alpha_{k-1}\alpha_{k-2}\cdots\alpha_1\alpha_0)u \\ &\quad + (\alpha_{k-1}\alpha_{k-2}\cdots\alpha_1)T_1 + (\alpha_{k-1}\alpha_{k-2}\cdots\alpha_2)T_2 \\ &\quad + \cdots\cdots\cdots\cdots \\ &\quad + (\alpha_{k-1})T_{k-2} + T_{k-1} \end{aligned} \tag{15}$$

this can be rewritten as follows

$$Age_{k-1} = \left(\prod_{j=0}^{k-1}\alpha_j\right)u + \sum_{m=1}^{k-1}\left(\prod_{j\geq m}^{k-1}\alpha_j\right)T_m, \quad \text{with} \quad k > 1 \tag{16}$$

3.2 Minimal repair cost

The minimal repair takes place at failure. The equipment after such repairs is considered As Bad As Old (ABAO). The cost of such repair during the k^{th} operating period with T_k length is denoted by $C_m(k)$ and depends on the expected number of failures and the unit cost per repair. The minimal repair cost on k^{th} operating period is

$$C_m(k) = C_{m|k}\left(\int_{Age_{k-1}}^{Age_{k-1}+T_k}\lambda(t)dt\right), \tag{17}$$

where $C_{m|k}$ and $\int_{Age_{k-1}}^{Age_{k-1}+T_k}\lambda(t)dt$ stand respectively for the unit minimal repair cost and the expected number of failures. Moreover $C_{m|k}$ and $\lambda(t)$ are non-decreasing function with respect respectively to k and t. From Eq.(17), we deduce the mathematical formula of cost $C_m(k)$ by integration

$$C_m(k) = C_{m|k}\left(\Lambda\left(Age_{k-1} + T_k\right) - \Lambda\left(Age_{k-1}\right)\right), \tag{18}$$

with $\Lambda(t)$ the cumulative failure risk function of equipment. Eq.(18) depends on the age reduction Age_{k-1} process according to the effect of preventive repair.

3.3 Repair cost during period k

The repair cost during k^{th} period involves the both repairs (minimal and preventive) from Eq. (7) and Eq. (18). The repair cost due to operating on period k is derived as follows:

$$C(k) = C_m(k) + C_p(k-1). \tag{19}$$

Eq.(19) holds under hypothesis that the technologies or spare parts are always available during the k^{th}. The required technology to repair our equipment is available with uncertainty. This uncertainty is governed by the residual life cycle of the equipment technology and its probability distribution. In fact to perform reparation during period k^{th}, the technology has to be available. This availability is not certain and requires a probability due to uncertainty on the spare parts. Therefore for a given technology characterized by its life cycle Y, we define the probability to perform these repairs by p_k and compute it by use of the residual life cycle distribution as follows

$$p_k = Prob\left(Y \geq \tilde{T}_{k-1} + Y_{ac} | Y \geq Y_{ac}\right), \tag{20}$$

where Y_{ac} stands for the age of technology at acquisition date of equipment and $\tilde{T}_{k-1}$ represents the cumulative operating duration. Therefore

$$\tilde{T}_{k-1} = \sum_{j=1}^{k-1} T_j, \tag{21}$$

and

$$p_k = \frac{Prob\left(Y \geq \tilde{T}_{k-1} + Y_{ac}\right)}{Prob(Y \geq Y_{ac})}. \tag{22}$$

Based on Eq. (22), we derive the repair cost which takes into account of the uncertainty of technology and the probability to perform each repair during period k by

$$\tilde{C}(k) = \frac{C(k)}{p_k}. \tag{23}$$

The cost $\tilde{C}(k)$ remains more realistic to evaluate the expected repair cost. This later cost integrates the cost of repairs and the probability to perform its in practice under some uncertainties. Through Eq.(23), we ensure in addition that when the technology is certainly available both cost are the same. However for unavailable technology during period k the cost $\tilde{C}(k)$ tends toward infinity in order to highlight the unrepairable case of equipment due to unavailability of spare parts.

3.4 Total maintenance cost

The total cost due to maintenance policy is function of acquisition price $C_{ac}(u)$ of equipment. In addition, the total cost implies also all repairs costs regardless its nature (preventive or minimal). Indeed, the total maintenance cost represents the sum of all costs (acquisition, rejuvenation and reparation). For a given number of operating sequences n, the total maintenance cost of our maintenance policy is derived and written as follows:

$$C_T(n) = \sum_{k=1}^{n} \tilde{C}(k) + C_{ac}(u). \tag{24}$$

Based on Eq. (25), we deduce the total maintenance cost per unit time as the ratio between $C_T(n)$ and the length or duration of operating period $\sum_{k=1}^{n} T_k$.

$$\tilde{C}_T(n) = \frac{C_T(n)}{\sum_{k=1}^{n} T_k}. \tag{25}$$

Remaining of the chapter is going to make a discussion on the conditions which ensure the optimality of the total maintenance cost or the total maintenance cost per unit time defined. This analysis is going to be done through some propositions.

4. Analysis of maintenance cost

The optimality of the maintenance is going to be deeply discussed throughout some parameters. In fact the cost model proposed in Eq. (24) and Eq. (25) depend on several parameters from equipment design, the efficiency of repair action and environment through the technology useful life. This section expects to derive optimal conditions.

Proposition 2. *The optimal age u to acquire a second-hand equipment according to our maintenance policy verifies*

$$-\frac{\partial C_{ac}(u)}{\partial u} = \sum_{k=1}^{n} \frac{1}{p_k} \left(C_{m|k} \left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right) \prod_{j=0}^{k-1} \alpha_j + \frac{\partial C_p(k-1)}{\partial u} \right) \tag{26}$$

Proof: The cost model is derived with respect to u. We note that the derivative function of the total cost per unit time with respect to u is equivalent to $\frac{\partial}{\partial u} C_T(n)$. Then

$$\begin{aligned} \frac{\partial}{\partial u} C_T(n) &= \frac{\partial}{\partial u} \left(\sum_{k=1}^{n} \tilde{C}(k) + C_{ac}(u) \right), \\ &= \left(\sum_{k=1}^{n} \frac{\partial \tilde{C}(k)}{\partial u} + \frac{\partial C_{ac}(u)}{\partial u} \right), \end{aligned} \tag{27}$$

from Eq. (23), we deduce $\frac{\partial \tilde{C}(k)}{\partial u} = \frac{1}{p_k} \frac{\partial C(k)}{\partial u}$. Then

$$\begin{aligned} \frac{\partial C(k)}{\partial u} &= \frac{\partial}{\partial u} \left(C_m(k) + C_p(k-1) \right), \\ &= \frac{\partial C_m(k)}{\partial u} + \frac{\partial C_p(k-1)}{\partial u}, \end{aligned}$$

with

$$\frac{\partial C_m(k)}{\partial u} = \left(C_{m|k} \frac{\partial Age_{k-1}}{\partial u} \right) \left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right) \tag{28}$$

Accordingly, the derivation of cost due to the k^{th} operating cycle is written as follows

$$\frac{\partial C(k)}{\partial u} = C_{m|k} \left(\frac{\partial Age_{k-1}}{\partial u} \right) \left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right) + \frac{\partial C_p(k-1)}{\partial u}, \tag{29}$$

and with uncertainty, we obtain then

$$\frac{\partial \tilde{C}(k)}{\partial u} = \frac{1}{p_k}\left(C_{m|k}\frac{\partial Age_{k-1}}{\partial u}\left(\lambda\left(Age_{k-1}+T_k\right)-\lambda\left(Age_{k-1}\right)\right)+\frac{\partial C_p(k-1)}{\partial u}\right) \tag{30}$$

with $\frac{\partial Age_{k-1}}{\partial u} = \prod_{j=0}^{k-1}\alpha_j$ according to Eq.(9). The derivative function of total cost $\frac{\partial C_T(n)}{\partial u} = 0$ or total cost per unit time $\frac{\partial \tilde{C}_T(n)}{\partial u} = 0$ are deduced as follows:

$$\frac{\partial C_T(n)}{\partial u} = 0 \Leftrightarrow \frac{\partial \tilde{C}_T(n)}{\partial u} = 0$$

and

$$\frac{\partial C_T(n)}{\partial u} = \sum_{k=1}^{n}\frac{1}{p_k}\left(C_{m|k}\left(\lambda\left(Age_{k-1}+T_k\right)-\lambda\left(Age_{k-1}\right)\right)\prod_{j=0}^{k-1}\alpha_j+\frac{\partial C_p(k-1)}{\partial u}\right) + \frac{\partial C_{ac}(u)}{\partial u} \tag{31}$$

therefore posing the right-hand term of Eq.(31) equals to zero allows to get Eq.(26).

Moreover the preventive repair cost is often independent of the acquisition age u of second-hand equipment. In this case, the optimal condition on acquisition age u is

$$\frac{\partial C_{ac}(u)}{\partial u} = -\sum_{k=1}^{n}\frac{1}{p_k}\left(C_{m|k}\left(\lambda\left(Age_{k-1}+T_k\right)-\lambda\left(Age_{k-1}\right)\right)\prod_{j=0}^{k-1}\alpha_j\right). \tag{32}$$

Another way, the proposed maintenance policy involves a sequence of preventive repairs. These repairs are performed after some operating periods $T_{i,i=1,\cdots,n}$ whose lengths ensure a minimal maintenance cost per unit time. The next proposition establishes then the conditions on each operation period $T_{i,i=1,\cdots,n}$ before preventive repair for an optimal use of equipment under our maintenance policy.

Proposition 3. *The optimal operating duration before the i^{th} preventive repair is fulfilled for a T_i such as*

$$\sum_{k=1}^{n}\frac{\partial}{\partial T_i}\left(\frac{C(k)}{p_k}\right) = \tilde{C}_T(n) \Rightarrow \sum_{k=1}^{n}\left(\frac{1}{p_k}\frac{\partial C_m(k)}{\partial T_i} - \frac{C(k)}{p_k^2}\frac{\partial p_k}{\partial T_i}\right) = \tilde{C}_T(n) \tag{33}$$

with

$$\frac{\partial C_m(k)}{\partial T_i} = \begin{cases} 0, & \text{if } i>k, \\ C_{m|k}\lambda\left(Age_{k-1}+T_k\right), & \text{if } i=k, \\ C_{m|k}\left(\lambda\left(Age_{k-1}+T_k\right)-\lambda\left(Age_{k-1}\right)\right)\frac{\partial Age_{k-1}}{\partial T_i}, & \text{otherwise}, \end{cases} \tag{34}$$

where $\frac{\partial Age_{k-1}}{\partial T_i} = \prod_{j\geq i}^{k-1}\alpha_j$ *and*

$$\frac{\partial p_k}{\partial T_i} = \begin{cases} 0, & \text{if } i>k \\ -\frac{f_{tech}\left(\tilde{T}_{k-1}+Y_{ac}\right)}{\overline{F}_{tech}(Y_{ac})}, & \text{otherwise}. \end{cases} \tag{35}$$

where f_{tech} and $\overline{F}_{tech}$ stand for density and reliability functions of spare parts or technology availability during optimizing period.

Proof: Each optimal T_i verifies next equation

$$\frac{\partial \tilde{C}_T(n)}{\partial T_i} = 0, \tag{36}$$

this is equivalent to

$$\frac{\partial \tilde{C}_T(n)}{\partial T_i} = \frac{\partial}{\partial T_i}\left(\frac{C_T(n)}{\tilde{T}_n}\right), \tag{37}$$

then Eq.(36) becomes

$$\frac{\partial C_T(n)}{\partial T_i} = \tilde{C}_T(n), \tag{38}$$

therefore the left-hand term of Eq.(38) is equal to

$$\begin{aligned} \frac{\partial C_T(n)}{\partial T_i} &= \frac{\partial}{\partial T_i}\left(\sum_{k=1}^{n}\frac{C(k)}{p_k} + C_{ac}(u)\right), \\ &= \left(\frac{\frac{\partial C(k)}{\partial T_i}p_k - C(k)\frac{\partial p_k}{\partial T_i}}{p_k^2} + \frac{\partial C_{ac}(u)}{\partial T_i}\right), \\ &= \sum_{k=1}^{n}\left(\frac{1}{p_k}\frac{\partial C(k)}{\partial T_i} - \frac{C(k)}{p_k^2}\frac{\partial p_k}{\partial T_i}\right) \end{aligned}$$

the derivative function of $\frac{\partial \tilde{C}_T}{\partial T_i}$ depends on the $\frac{\partial C(k)}{\partial T_i}$ and $\frac{\partial p_k}{\partial T_i}$. In fact

$$\frac{\partial C(k)}{\partial T_i} = \frac{\partial C_m(k)}{\partial T_i} + \frac{\partial C_p(k-1)}{\partial T_i}$$

and

$$\frac{\partial C_p(k-1)}{\partial T_i} = 0$$

then Eq.(33) is deduced based on

$$\frac{\partial C(k)}{\partial T_i} = \frac{\partial C_m(k)}{\partial T_i}$$

with

$$\frac{\partial C_m(k)}{\partial T_i} = \begin{cases} 0, \quad \text{if } i > k, \\ C_{m|k}\lambda\left(Age_{k-1} + T_k\right), \quad \text{if } i = k, \\ C_{m|k}\left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right)\frac{\partial Age_{k-1}}{\partial T_i}, \quad \text{otherwise,} \end{cases} \tag{39}$$

where $\frac{\partial Age_{k-1}}{\partial T_i} = \prod_{j \geq i}^{k-1}\alpha_j$ is deduced from Eq.(9) of Proposition (1). In addition, the derivative $\frac{\partial p_k}{\partial T_i}$ is

$$\begin{aligned}\frac{\partial p_k}{\partial T_i} &= \frac{\partial}{\partial T_i} Prob\left(Y > \tilde{T}_{k-1} + Y_{ac} | Y > Y_{ac}\right), \\ &= \frac{\partial}{\partial T_i} \frac{1 - F_{tech}\left(\tilde{T}_{k-1} + Y_{ac}\right)}{1 - F_{tech}(Y_{ac})},\end{aligned} \tag{40}$$

the derivative of p_k with respect T_i is

$$\frac{\partial p_k}{\partial T_i} = \begin{cases} 0 \ \text{ si } \ i \geq k, \\ -\dfrac{f_{tech}\left(\tilde{T}_{k-1} + Y_{ac}\right)}{\overline{F}_{tech}(Y_{ac})} \ \ \text{otherwise.}\end{cases} \tag{41}$$

Therefore the derivative of total cost per unit time with respect is deduced from Eq.(39) and Eq.(40) by

$$\frac{\partial}{\partial T_i}\left(\frac{C(k)}{p_k}\right) = \begin{cases} 0, \quad \text{if } \ i > k, \\ \dfrac{C_{m|k}}{p_k}\lambda\left(Age_{k-1} + T_k\right), \quad \text{if } \ i = k, \\ \left(\dfrac{C_{m|k}}{p_k}\left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right)\prod_{j \geq i}^{k-1}\alpha_j \right. \\ \left. + \dfrac{f_{tech}\left(\tilde{T}_{k-1} + Y_{ac}\right)}{1 - \overline{F}_{tech}(Y_{ac})}\left(C_m(k) + C_p(k-1)\right)\dfrac{1}{p_k^2}\right), \quad \text{otherwise.}\end{cases} \tag{42}$$

The next proposition derives the optimal condition for α_i which stands for the age reduction factor due to the preventive repair if it takes place.

Proposition 4. *Each optimal reduction factor $\alpha_{i, i=1,\cdots,n-1}$ is derived as a solution of next equation*

$$\sum_{k=1}^{n} \frac{\partial C_m(k)}{\partial \alpha_i} = -\frac{\partial C_p(i)}{\partial \alpha_i} \tag{43}$$

Proof: The proof of this proposition results of the calculation of the derivative function with respect to α_i and posing equal to zero. This implies

$$\frac{\partial}{\partial \alpha_i}\left(\frac{\tilde{C}_T(n)}{\tilde{T}_n}\right) = 0, \quad \Rightarrow$$

$$\frac{\partial C_T(n)}{\partial \alpha_i} = 0,$$

$$\begin{aligned}\frac{\partial C_T(n)}{\partial \alpha_i} &= \frac{\partial}{\partial \alpha_i}\sum_{k=1}^{n}\left(C_m(k) + C_p(k-1)\right) \\ &= \sum_{k=1}^{n}\frac{\partial C_m(k)}{\partial \alpha_i} + \frac{\partial C_p(k-1)}{\partial \alpha_i} \\ &= \sum_{k=1}^{n}\left(\frac{\partial C_m(k)}{\partial \alpha_i}\right) + \frac{\partial C_p(i)}{\partial \alpha_i}\end{aligned}$$

therefore

$$\frac{\partial C_T(n)}{\partial \alpha_i} = 0, \quad \Rightarrow$$

$$\sum_{k=1}^{n}\left(\frac{\partial C_m(k)}{\partial \alpha_i}\right) = -\frac{\partial C_p(i)}{\partial \alpha_i}$$

with

$$\frac{\partial C_m(k)}{\partial \alpha_i} = C_{m|k}\left(\lambda\left(Age_{k-1} + T_k\right) - \lambda\left(Age_{k-1}\right)\right)\frac{\partial Age_{k-1}}{\partial \alpha_i} \tag{44}$$

and

$$\frac{\partial Age_{k-1}}{\partial \alpha_i} = \begin{cases} 0, \quad i > k-1, \\ \left(\prod_{j=0, j\neq i}^{k-1} \alpha_j\right)u + \sum_{m=1}^{k-1}\prod_{j\geq m, j\neq i}^{k-1} \alpha_j T_m, \quad i \leq k-1. \end{cases} \tag{45}$$

The last proposition deals with the optimality according to the number n of operating periods. We note that n is integer and the derivative with respect to n is not possible. However to make comparison, the maintenance cost per unit time is going to be evaluate for $n-1,\ n,$ and $n+1$ number of operating periods.

Proposition 5. *A number of maintenance periods n is optimal according to our maintenance policy if it verifies*

$$\begin{cases} U_n > \dfrac{\tilde{T}_{n+1}}{\tilde{T}_n}, \\ U_{n-1} < \dfrac{\tilde{T}_n}{\tilde{T}_{n-1}}. \end{cases} \tag{46}$$

where U_n is sequence such as $U_n = \frac{\tilde{C}_T(n+1)}{\tilde{C}_T(n)}$..

Proof: n is integer and at the optimal the total maintenance cost ensure the next inequalities

$$\begin{cases} \dfrac{\tilde{C}_T(n+1)}{\tilde{T}_{n+1}} > \dfrac{\tilde{C}_T(n)}{\tilde{T}_n} \quad (a) \\ \dfrac{\tilde{C}_T(n-1)}{\tilde{T}_{n-1}} > \dfrac{\tilde{C}_T(n)}{\tilde{T}_n} \quad (b) \end{cases} \tag{47}$$

we derive then the optimal condition for n as follows

$$\begin{cases} \dfrac{\tilde{C}(n+1)}{\tilde{C}(n)} > \dfrac{\tilde{T}_{n+1}}{\tilde{T}_n} \quad (a') \\ \dfrac{\tilde{C}(n)}{\tilde{C}(n-1)} < \dfrac{\tilde{T}_n}{\tilde{T}_{n-1}} \quad (b') \end{cases} \tag{48}$$

Eq.(48) depicts that when U_n is increasing sequence such $U_0 < 1$ then the optimal n is unique.

$$\begin{cases} \dfrac{\tilde{T}_{n+1}}{\tilde{T}_n} = 1 + \dfrac{T_{n+1}}{\tilde{T}_n} \approx 1, \\ \dfrac{\tilde{T}_n}{\tilde{T}_{n-1}} = 1 + \dfrac{T_n}{\tilde{T}_{n-1}} \approx 1. \end{cases} \tag{49}$$

In case of Eq.(49), the optimal n verifies both conditions $U_n > 1$ and $U_{n-1} < 1$. However, the complexity of the cost function structure makes the monotonicity analysis of U_n difficult in practice. Therefore the optimization in numerical experiment is going to perform for given number n of operating periods.

Next section propose a numerical experiment in order to analyze the optimal solution under uncertainty on the technology or spare parts availability.

4.1 Numerical experiments

To make experiment, a second-hand equipment is considered with Weibull distribution as it lifetime distribution. The parameter of the Weibull distribution function are $\beta = 2.1$ and $\eta = 100$ which respectively stand for the shape and scale parameters. Additional, the second-hand equipment is assumed from technology whose vanishing may be accidental or progressive. In case of accidental vanishing, the technology life cycle is distributed according to a exponential distribution function while the technology life cycle follows Weibull distribution function with a shape parameter $\beta_{tech} > 1$ in case of progressive [14]. **Table 1** depicts the all parameters for a numerical experiment such as the acquisition price of second-hand equipment and the unit costs to perform repair.

To compare technology effect on our maintenance policy, two hypotheses are made. First, equipments are made from newer technology at beginning of its life cycle. Second, we assume that the equipment technology is at the end of its life cycle. In fact the life cycle of technology distribution are also modeled by Weibull probability distribution with $\eta_{tech} = 1000$ as the scale parameter and the shape parameters equal to $\beta_{tech} = 1$ and $\beta_{tech} = 4.1$ respectively for accidental and progressive vanishing. We note solving the optimization issues remains complex due to the number and the nature of policy parameters.

To deal with the complexity of problem, we focus on optimization and analysis of the first operating period. The results of this analysis are displayed in the next figures. **Figure 1** depicts the evolution of optimal rejuvenation ratio α_0 applied on equipment before operating. This figure highlights a period of acquisition age in which the equipment does not need rejuvenation. The length of this period depends on the age of technology and the vanishing process. Therefore, the length of this period is longer for equipments based on older technology with progressive

Equipment lifetime	**Weibull distribution**	$\boldsymbol{W(\beta = 100, \eta = 2.1)}$
Technology life cycle	Accidental vanishing	$W(\beta = 1, \eta = 1000)$
Technology life cycle	Progressive vanishing	$W(\beta = 4.01, \eta = 1000)$
Acquisition price of new	$C_{new} = 100000$	
Acquisition price function	$C_{ac}(u) = \frac{C_{new}}{1+u}$	
Cm_0	500	
Cp_0	1000	

Table 1.
Numerical Experiments parameters.

vanishing while its remains short for equipments based on newer technology. Moreover when vanishing process is accidental, the length of this period reach 15.79 unit of time. From **Figure 1**, we deduce that the optimal α_0 decreases with the initial acquisition age. For progressive vanishing technology, the required rejuvenation to optimally operate remains more important when the life cycle of technology is at beginning than ending. This results from the high probability of technology unavailability. Therefore, when the spare parts availability is uncertain the maintenance policy recommends a soft rejuvenation while a deep rejuvenation is needed otherwise. Another way, for the same age of technology at acquisition the rejuvenation in accidental vanishing less important than in progressive vanishing according to **Figure 1**.

Figure 2 presents the optimal operating period length versus initial acquisition age. **Figure 2** underlines that the optimal operating period decreases also with the acquisition age regardless the life cycle distribution of technology. Therefore the maintenance policy recommends short operating length with ending technology. In addition, the maintenance policy requires early preventive repair in case of progressive vanishing than accidental. This allows to reduce the uncertainty effect on the chance to perform repairs.

Figure 3 presents and allows to derive the optimal acquisition age to get the minimal operating cost per unit time during the first period of operation. The optimal cost per unit time reach minimal for accidental, progressive with $Y_{ac} = 100$ and $Y_{ac} = 1000$ at respectively to 45.71, 42.89, and $55, 63$. This involves that the use

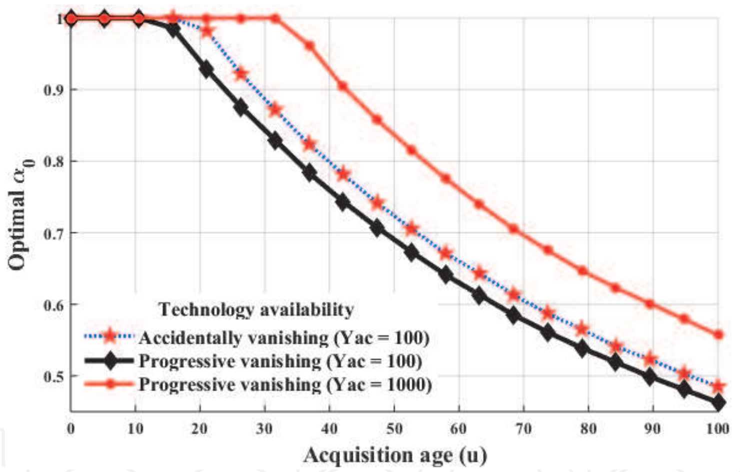

Figure 1.
Optimal rejuvenation ratio α_0.

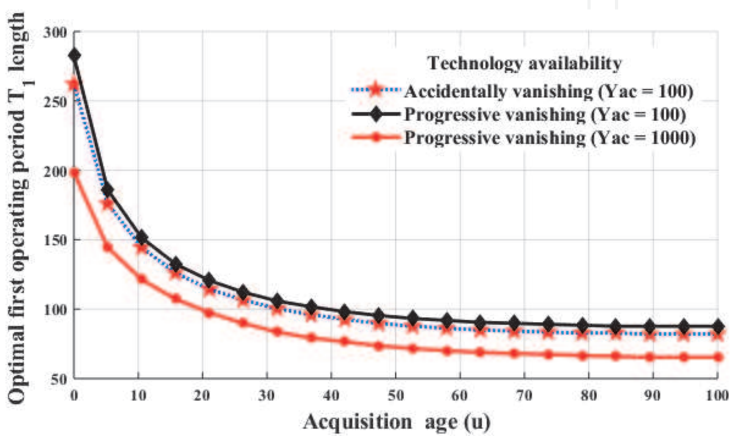

Figure 2.
Optimal operating period length T_1.

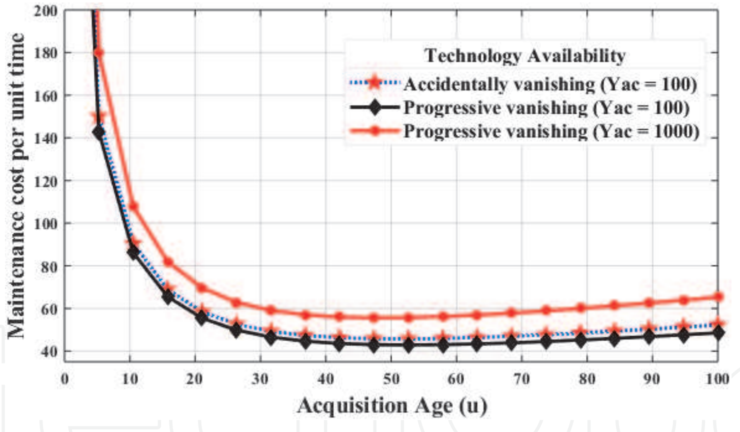

Figure 3.
Optimal maintenance cost per unit time.

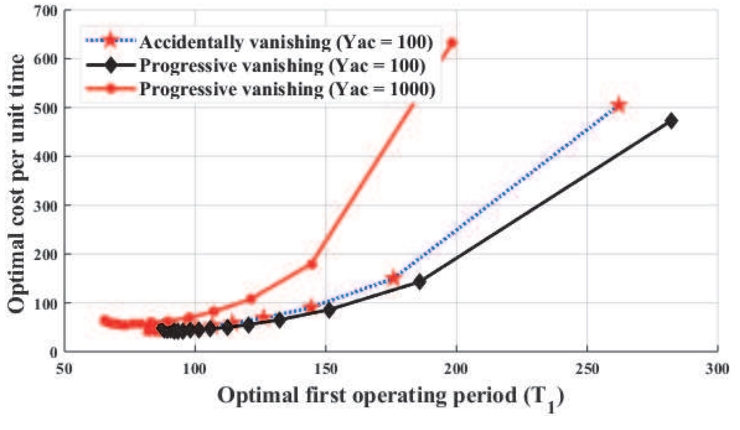

Figure 4.
Optimal period length T_k v.s maintenance cost per unit time.

of equipment with older technology remains expensive. Additional the uncertainty to predict the vanishing of technology in case of accidental makes the repairs performing uncertain and the cost per unit more expensive than progressive for a given age of technology. To finish we show through the **Figure 4** the evolution of optimal cost per unit time in function of operating period length. From **Figure 4**, we note that at optimal, the cost per unit time increases with the operating length. Therefore operating equipments with accidentally technology vanishing remains more expensive than progressive.

Table 2 resumes information and allows making comparison between parameters at optimal for each case. This table highlights some important information. Such as the use of equipments based on old technology remains expensive when the

Parameters	Accidental vanishing	Progressive vanishing	Progressive vanishing
	$Y_{AC} = 100$	$Y_{AC} = 100$	$Y_{AC} = 1000$
u	52.63	52.63	47.36
α_0	0.8054	0.6733	0.8577
T_1	87.67	93.27	73.66
$\tilde{C}(1)$	45.71	42.89	55.63
$C(1)$	4008.04	4001.04	4097.94

Table 2.
Optimal parameters for one operating cycle.

vanishing is progressive. In case of accidental vanishing process, the age of technology does not impact the optimal condition. However for the same age of technology, operating equipment with progressive vanishing is better than accidental because the prediction of pare parts availability remains easier for progressive than accidental vanishing technology.

5. Conclusion

This chapter analyzed an optimal maintenance policy strategy to operate second-hand equipment under uncertainty due to the indirect obsolescence concept. The uncertainty used, in the present work, is about the spare parts availability to perform some maintenance actions on equipment due to technology vanishing. The maintenance policy was defined and discussed with respect to several parameters relatively to equipment, technology and nature of repair. Therefore the optimality of maintenance policy was discussed and optimal conditions were derived mathematically. However finding these optimal parameters analytically remain complex. To reduce the complexity, a first optimal operating length was derived and compared according to the technology age and vanishing process. We highlight that the use of older technology requires less rejuvenation but remains more expensive than newer use. According to obtained results, we can note that the technology vanishing shows also the optimality of maintenance policy. Therefore the operating of equipment with technology whose vanishing process is accidental remains expensive. In the future works, we are going to focus on the development of algorithm to solve the complex optimization problem. This optimization will be made without restriction on the parameters.

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