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On Double Fuzzy M-open Mappings and Double Fuzzy M-closed Mappings

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Abstract

We introduce and investigate some new class of mappings called double fuzzy M-open map and double fuzzy M-closed map in double fuzzy topological spaces. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between double fuzzy open, double fuzzy θ semiopen, double fuzzy δ preopen, double fuzzy M open and double fuzzy e open and their respective closed mappings.

Keywords: Double fuzzy open map; Double fuzzy θ semiopen map; Double fuzzy δ preopen map; Double fuzzy M open map and double fuzzy e open map

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1. Introduction

"Intuitionistic fuzzy sets" were first introduced by Atanassov (1986). Then, Coker (1997) introduced the notion of "Intuitionistic fuzzy topological space." Garcia and Rodabaugh (2005) proved that the term "intuitionistic" is unsuitable in mathematics and applications. Also, they introduced the name 'double' for the term 'intuitionistic'. In the past two decades many researchers, Samanta and Mondal (2002), Zahran et al. (2010), Mohammed and Ghareeb (2016), Mohammed et al. (2016) and Mohammed et al. (2017) doing more applications on double fuzzy topological spaces. From 2011, El-Maghrabi and Al-Johany (2011), El-Maghrabi and Al-Johany (2013), El-Maghrabi and Al-Johany (2014b) and El-Maghrabi and Al-Johany (2014a) introduced and studied some properties on M-open sets and maps in topological spaces. In double fuzzy topological spaces, Sathiyaraj et al. (2019a) introduced (ι, κ)- fuzzy M closed sets. Using them double fuzzy M continuous functions were studied by Sathiyaraj et al. (2019b). In this paper we introduce double fuzzy M-open (resp. closed) functions and study some of their properties in double fuzzy topological spaces.

Here we use the notations from Periyasamy et al. (2019), Sathiyaraj et al. (2019a) and cited therein.

2. On double fuzzy M open and double fuzzy M closed mappings

In this section, we introduce the concept of double fuzzy M open (double fuzzy M closed) mappings in double fuzzy topological spaces and obtained some of their properties.

Definition 2.1.

A function f from a double fuzzy topological space (briefly, dfts) (X, τ, τ^*) to a dfts (Y, σ, σ^*), is called as a double fuzzy open (resp. double fuzzy θ semiopen, double fuzzy δ preopen, double fuzzy M open and double fuzzy e open) (briefly dfO, (resp. $df\theta sO$, $df\delta pO$, $dfMO$ and $dfeO$)) function if $f(\mu)$ is an (ι, κ)-fuzzy open (resp. (ι, κ)-fuzzy θopen, (ι, κ)-fuzzyθsemiopen, (ι, κ)-fuzzyδpreopen, (ι, κ)-fuzzy Mopen and (ι, κ)-fuzzy$e$open) (briefly, (ι, κ)-fo (resp. (ι, κ)-$f\theta o$, (ι, κ)-$f\theta so$, (ι, κ)-$f\delta po$, (ι, κ)- fMo and (ι, κ)-feo)) set in I^Y for every (ι, κ)-fo set $\mu \in I^X$ for all $\iota \in I_0$ and $\kappa \in I_1$.

Definition 2.2.

A function f from a dfts (X, τ, τ^*) to a dfts (Y, σ, σ^*), is called as a double fuzzy closed (resp. double fuzzy θ semiclosed, double fuzzy δ preclosed, double fuzzy M closed and double fuzzy e closed) (briefly dfC, (resp. $df\theta sC$, $df\delta pC$, $dfMC$ and $dfeC$)) function if $f(\mu)$ is an (ι, κ)-fuzzy closed (resp. (ι, κ)-fuzzy θclosed, (ι, κ)-fuzzy θsemiclosed, (ι, κ)-fuzzyδpreclosed, (ι, κ)-fuzzy Mclosed and (ι, κ)-fuzzy$e$closed) (briefly, (ι, κ)-fc (resp. (ι, κ)-$f\theta c$, (ι, κ)-$f\theta sc$, (ι, κ)-$f\delta pc$, (ι, κ)-fMc and (ι, κ)-fec)) set in I^Y for every (ι, κ)-fc set $\mu \in I^X$ for all $\iota \in I_0$ and $\kappa \in I_1$.

The theorems 2.1-2.5 give us relationships among the open (resp. closed) maps defined in the definitions 2.1 (resp. 2.2) and other existing open (resp. closed) maps in double fuzzy topological spaces.

Theorem 2.1.

Let f:$(X,\tau,\tau^*) \to (Y,\eta,\eta^*)$ be a mapping, then, every $df\delta pO$ (resp. $df\delta pC$) mapping is $dfMO$ (resp. $dfMC$) mapping.

Proof:

The proof follows from the definitions and fact that every $df\delta po$ (resp. $df\delta pc$) set is $dfMo$ (resp. $dfMc$ set). ■

The converse of the Theorem 2.1, in general, need not be true. It can be verified from the following example.

Example 2.1.

Let $X = Y = \{a, b, c\}$ and consider the double fuzzy topologies (X,τ,τ^*) and (Y,η,η^*) with

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.9}, \\ 0, & \text{otherwise}, \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.9}, \\ 1, & \text{otherwise}, \end{cases}$$

and

$$\eta(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.1}, \\ 0, & \text{otherwise}, \end{cases} \qquad \eta^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.1}, \\ 1, & \text{otherwise}. \end{cases}$$

Then, the identity function $f : (X,\tau,\tau^*) \to (Y,\eta,\eta^*)$ is a $dfMO$ (resp. $dfMC$) function but not a $df\delta pO$ (resp. $df\delta pC$) function, since the image of the fuzzy set $\underline{0.9}$ (resp. $\underline{0.1}$) is an $(\frac{1}{2},\frac{1}{2})$-$fMo$ (resp. $(\frac{1}{2},\frac{1}{2})$-$fMc$) set but not an $(\frac{1}{2},\frac{1}{2})$- $f\delta po$ (resp. $(\frac{1}{2},\frac{1}{2})$- $f\delta pc$) set in (Y,η,η^*).

Theorem 2.2.

Let f:$(X,\tau,\tau^*) \to (Y,\eta,\eta^*)$ be a mapping, then, every $df\theta sO$ (resp. $df\theta sC$) mapping is $dfMO$ (resp. $dfMC$) mapping.

Proof:

The proof follows from the definitions and fact that every $df\theta so$ (resp. $df\theta sc$) set is $dfMo$ (resp. $dfMc$) set. ■

The converse of the Theorem 2.2, in general, need not be true. It can be verified from the following example.

Example 2.2.

Let $X = Y = \{a, b, c\}$ and let the fuzzy sets α, β and γ are defined as $\alpha(a) = 0.3,\ \alpha(b) = 0.4,\ \alpha(c) = 0.5;\ \beta(a) = 0.6,\ \beta(b) = 0.9,\ \beta(c) = 0.5;\ \gamma(a) = 0.3,\ \gamma(b) = 0$ and $\gamma(c) = 0.5$. Consider the double fuzzy topologies (X, τ, τ^*) and (Y, η, η^*) with

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \underline{1} - \gamma, \\ 0, & \text{otherwise,} \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{2}{3}, & \text{if } \lambda = \underline{1} - \gamma, \\ 1, & \text{otherwise,} \end{cases}$$

and

$$\eta(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{2}{3}, & \text{if } \lambda = \alpha, \\ \frac{1}{3}, & \text{if } \lambda = \beta, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \alpha, \\ \frac{2}{3}, & \text{if } \lambda = \beta, \\ 1, & \text{otherwise.} \end{cases}$$

Then, the identity function $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ is a $dfMO$ (resp. $dfMC$) function but not a $df\theta sO$ (resp. $df\theta sC$) function, since the image of the fuzzy set $\underline{1} - \gamma$ (resp. γ) is an $(\frac{1}{3}, \frac{2}{3})$-$fMo$ (resp. $(\frac{1}{3}, \frac{2}{3})$-$fMc$) set but not an $(\frac{1}{3}, \frac{2}{3})$- $f\theta so$ (resp. $(\frac{1}{3}, \frac{2}{3})$- $f\theta sc$) set in (Y, η, η^*).

Theorem 2.3.

Let f:$(X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ be a mapping, then, every $df\theta O$ (resp. $df\theta C$) mapping is $df\theta sO$ (resp. $df\theta sC$) mapping.

Proof:

The proof follows from the definitions and fact that every $df\theta o$ (resp. $df\theta c$) set is $df\theta so$ (resp. $df\theta sc$) set. ∎

The converse of the Theorem 2.3, in general, need not be true. It can be verified from the following example.

Example 2.3.

Let $X = Y = \{a, b, c\}$ and let the fuzzy sets α and β are defined as $\alpha(a) = 0.3,\ \alpha(b) = 0.4,\ \alpha(c) = 0.5;\ \beta(a) = 0.6,\ \beta(b) = 0.5,$ and $\beta(c) = 0.5$. Consider the double fuzzy topologies (X, τ, τ^*) and (Y, η, η^*) with

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4}, & \text{if } \lambda = \underline{1} - \alpha, \\ 0, & \text{otherwise,} \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{3}{4}, & \text{if } \lambda = \underline{1} - \alpha, \\ 1, & \text{otherwise,} \end{cases}$$

and

$$\eta(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{3}{4}, & \text{if } \lambda=\alpha,\\ \frac{1}{4}, & \text{if } \lambda=\beta,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{4}, & \text{if } \lambda=\alpha,\\ \frac{3}{4}, & \text{if } \lambda=\beta,\\ 1, & \text{otherwise.}\end{cases}$$

Then, the identity function $f:(X,\tau,\tau^*)\to(Y,\eta,\eta^*)$ is a $df\theta sO$ (resp. $df\theta sC$) function but not a $df\theta O$ (resp. $df\theta C$) function, since the image of the fuzzy set $\underline{1}-\alpha$ (resp. α) is an $(\frac{1}{4},\frac{3}{4})$- $f\theta so$ (resp. $(\frac{1}{4},\frac{3}{4})$- $f\theta sc$) set but not an $(\frac{1}{4},\frac{3}{4})$- $f\theta o$ (resp. $(\frac{1}{4},\frac{3}{4})$- $f\theta c$) set in (Y,η,η^*).

Theorem 2.4.

Let f:$(X,\tau,\tau^*)\to(Y,\eta,\eta^*)$ be a mapping, then, every $df\theta O$ (resp. $df\theta C$) mapping is dfO (resp. dfC) mapping.

Proof:

The proof follows from the definitions and fact that every $df\theta o$ (resp. $df\theta c$) set is dfo (resp. dfc) set. ■

The converse of the Theorem 2.4, in general, need not be true. It can be verified from the following example.

Example 2.4.

Let $X=Y=\{a,b,c\}$ and let the fuzzy set α is defined as $\alpha(a)=0.3$, $\alpha(b)=0.5$, and $\alpha(c)=0.5$, consider the double fuzzy topology (X,τ,τ^*) with

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{2}{5}, & \text{if } \lambda=\underline{0.5},\\ \frac{1}{5}, & \text{if } \lambda=\alpha,\\ 0, & \text{otherwise,}\end{cases}\qquad \tau^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{3}{5}, & \text{if } \lambda=\underline{0.5},\\ \frac{4}{5}, & \text{if } \lambda=\alpha,\\ 1, & \text{otherwise.}\end{cases}$$

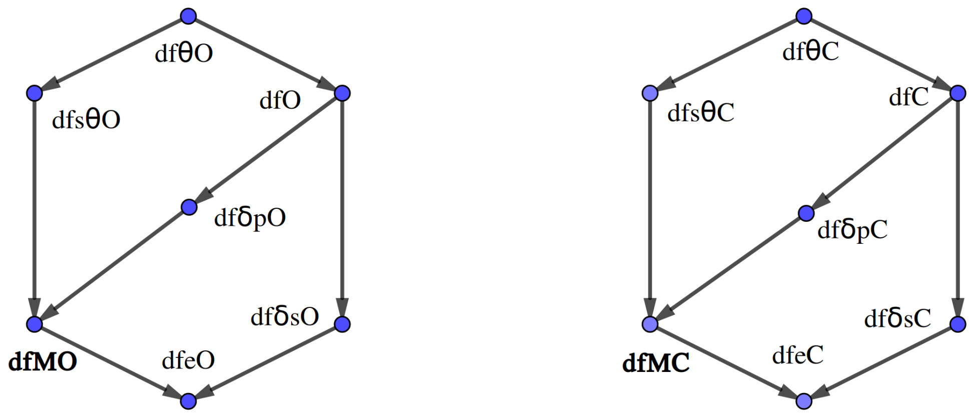
Then, the identity function $f:(X,\tau,\tau^*)\to(X,\tau,\tau^*)$ is a dfO (resp. dfC) function but not a $df\theta O$ (resp. $df\theta C$) function, since the image of the fuzzy set α (resp. $\underline{1}-\alpha$) is an $(\frac{1}{5},\frac{4}{5})$- fo (resp. $(\frac{1}{5},\frac{4}{5})$- fc) set but not an $(\frac{1}{5},\frac{4}{5})$- $f\theta o$ (resp. $(\frac{1}{5},\frac{4}{5})$- $f\theta c$) set.

Theorem 2.5.

Let f:$(X,\tau,\tau^*)\to(Y,\eta,\eta^*)$ be a mapping, then, every $dfMO$ (resp. $dfMC$) mapping is $dfeO$ (resp. $dfeC$) mapping.

Proof:

The proof follows from the definitions and fact that every $dfMo$ (resp. $dfMc$) set is $dfeo$ (resp. $dfec$) set. ■



Note: A → B denotes A implies B, but not conversely.

Figure 1. Relationships between $dfMO$ (resp. $dfMC$) maps and other existing maps

The converse of the Theorem 2.5, in general, need not be true. It can be verified from the following example.

Example 2.5.

Let $X = Y = \{a, b, c\}$ and let the fuzzy sets α and β defined as $\alpha(a) = 0.5,\ \alpha(b) = 0.3,\ \alpha(c) = 0.2;\ \beta(a) = 0.5,\ \beta(b) = 0.6$ and $\beta(c) = 0.6$, consider the double fuzzy topologies (X, τ, τ^*) and (Y, η, η^*) with

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{6}, & \text{if } \lambda = \underline{1} - \beta, \\ 0, & \text{otherwise,} \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{5}{6}, & \text{if } \lambda = \underline{1} - \beta, \\ 1, & \text{otherwise,} \end{cases}$$

and

$$\eta(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{6}, & \text{if } \lambda = \alpha, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{5}{6}, & \text{if } \lambda = \alpha, \\ 1, & \text{otherwise.} \end{cases}$$

Then, the identity function $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ is a $dfeO$ (resp. $dfeC$) function but not a $dfMO$ (resp. $dfMC$) function, since the image of the fuzzy set $\underline{1} - \beta$ (resp. β) is an $(\frac{1}{6}, \frac{5}{6})$- feo (resp. $(\frac{1}{6}, \frac{5}{6})$- fec) set but not an $(\frac{1}{6}, \frac{5}{6})$- fMo (resp. $(\frac{1}{6}, \frac{5}{6})$- fMc) set in (Y, η, η^*).

From the above discussion, the implications in Figure 1 are hold.

Now, we discuss some new kinds of neighbourhoods in double fuzzy topological spaces.

Definition 2.3.

A mapping $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ is called $dfMO$ at a fuzzy point x_r if the image of each (ι, κ)-Q neighbourhood of x_r is an (ι, κ)-MQ neighbourhood of $f(x_r) \in I^Y$.

We characterize $dfMO$ (resp. $dfMC$) maps. Some of the proofs are obvious and hence omitted.

Theorem 2.6.

A mapping $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ is $dfMO$ if and only if it is $dfMO$ at every fuzzy point $x_\iota \in I^X$.

Theorem 2.7.

Let $(X,\ \tau,\ \tau^*)$ and $(Y,\ \eta,\ \eta^*)$ be dfts's and $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ be a mapping. Then, the following statements are equivalent:
(i) f is $dfMO$ function.
(ii) $f(\lambda)$ is an (ι, κ)-fMo set in (Y, η, η^*) for each (ι, κ)-fo set λ in (X, τ, τ^*).
(iii) f is $dfMC$ function.
(iv) $f(\lambda)$ is an (ι, κ)-fMc set in (Y, η, η^*) for each (ι, κ)-fc set λ in (X, τ, τ^*).
(v) $MC_{\tau,\tau^*}(f(\lambda),\ \iota, \kappa) \leq f(C_{\eta,\eta^*}(\lambda,\ \iota, \kappa)),\ \forall\ \lambda \in I^X$.
(vi) $I_{\tau,\tau^*}(\theta C_{\tau,\tau^*}(f(\lambda),\ \iota, \kappa),\ \iota, \kappa) \wedge C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(f(\lambda),\ \iota, \kappa),\ \iota, \kappa) \leq f(C_{\eta,\eta^*}(\lambda,\ \iota, \kappa)), \forall\ \lambda \in I^X$.
(vii) $f(I_{\tau,\tau^*}(\lambda,\ \iota, \kappa)) \leq C_{\eta,\eta^*}(\theta I_{\eta,\eta^*}(f(\lambda),\ \iota, \kappa),\ \iota, \kappa) \vee I_{\eta,\eta^*}(\delta C_{\eta,\eta^*}(f(\lambda), \iota, \kappa), \iota, \kappa)$ for each $\lambda \in I^X$.
(viii) $f(I_{\tau,\tau^*}(\lambda,\ \iota, \kappa)) \leq MI_{\eta,\eta^*}(f(\lambda),\ \iota, \kappa)$, for each $\lambda \in I^X$.
(ix) $I_{\tau_1,\tau_1^*}(f^{-1}(\lambda), \iota, \kappa) \leq f^{-1}(MI_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa))$ for each $\lambda \in I^Y$.

Proof:

(i)⇒(ii), (iii)⇒(iv), (v)⇒(vi), (vii)⇒(viii), are direct to prove, other results are provided here.

(ii)⇒(iii): Let $\underline{1} - \lambda$ be an (ι, κ)-fo set in (X, τ, τ^*), by (ii), we have $f(\underline{1} - \lambda)$ is an (ι, κ)-fMo set of (Y, η, η^*). But $f(\underline{1} - \lambda) = \underline{1} - f(\lambda)$. Therefore, $f(\lambda)$ is an (ι, κ)-fMc set of (Y, η, η^*) $\forall \lambda \in (X, \tau, \tau^*)$, (ι, κ)-fc set.

(iv) ⇒ (v): Since $C_{\tau,\tau^*}(\lambda, \iota, \kappa)$ is an (ι, κ)-fc set, then, $f(C_{\tau,\tau^*}(\lambda, \iota, \kappa))$ is an (ι, κ)-fMc set in Y. Hence, $MC_{\eta,\eta^*}(f(\lambda), \iota, \kappa) \leq MC_{\eta,\eta^*}(f(C_{\tau,\tau^*}(\lambda, \iota, \kappa)), \iota, \kappa) = f(C_{\tau,\tau^*}(\lambda, \iota, \kappa))$.

(vi) ⇒ (vii): Let $\underline{1} - \lambda$ instead of λ in (vi), then, (vii) will follows directly.

(viii)⇒(ix) Let $\lambda \in I^Y$, by (viii) we have $f(I_{\tau_1,\tau_1^*}(f^{-1}(\lambda), \iota, \kappa)) \leq MI_{\tau_2,\tau_2^*}(ff^{-1}(\lambda), \iota, \kappa) \leq MI_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa) \Rightarrow I_{\tau_1,\tau_1^*}(f^{-1}(\lambda), \iota, \kappa)) \leq f^{-1}(M_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa))$.

(ix)⇒(i) For each $\lambda \in I^X$, with $\tau_1(\lambda) \geq \iota$ and $\tau_1^*(\lambda) \leq \kappa$ since $I_{\tau_1,\tau_1^*}(\lambda, \iota, \kappa) = \lambda,\ f(\lambda) \leq MI_{\tau_2,\tau_2^*}(f(\lambda), \iota, \kappa) \leq f(\lambda)$. Thus, $f(\lambda) = MI_{\tau_2,\tau_2^*}(f(\lambda), \iota, \kappa)$. $f(\lambda)$ is (ι, κ)-fMo in Y. ■

Theorem 2.8.

Let $(X,\ \tau, \tau^*)$ and $(Y,\ \eta, \eta^*)$ be dfts's. Let $f : X \to Y$ be a $dfMC$ mapping iff f is surjective, then, for each subset μ of Y and each (ι, κ)-fuzzy open set α in X containing $f^{-1}(\mu)$, there exists an (ι, κ)-fMo set β of Y containing μ such that $f^{-1}(\beta) \leq \alpha$.

Proof:

Suppose that $\beta = \underline{1} - f(\underline{1} - \alpha)$ and α is an (ι, κ)-fo set of X containing $f^{-1}(\mu)$. Then, by hypothesis, β is (ι, κ)-fMo in Y. But $f^{-1}(\mu) \leq \alpha$, then, $\mu \leq f(\alpha)$ and $f(\underline{1} - \alpha) \leq \underline{1} - \mu$, i.e $\mu \leq \beta$ and $f^{-1}(\beta) \leq \alpha$.

Conversely, Let δ be a (ι, κ)-fc set and y be any point of $\underline{1} - f(\delta)$. Then, $f^{-1}(y) \in \underline{1} - \delta$ which is (ι, κ)-fo set in X. Hence, by hypothesis, there exists an (ι, κ)-fMo set β containing y such that $f^{-1}(\beta) \leq \underline{1} - \delta$. But f is surjective, then, $y \in \beta \leq \underline{1} - f(\delta)$ and $\underline{1} - f(\delta)$ is the union of (ι, κ)-fMo sets and hence, $f(\delta)$ is (ι, κ)-fMc set in Y. Therefore, f is $dfMC$ map. ■

Theorem 2.9.

Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's and $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be a $dfMO$ (resp. $df\delta sO$, $df\delta pO$) mapping. If $\mu \in I^Y$ and $\lambda \in I^X,\ \tau_1(\underline{1} - \lambda) \geq \iota,\ \tau_1^*(\underline{1} - \lambda) \leq \kappa,\ \iota \in I_0\ \kappa \in I_1$ such that $f^{-1}(\mu) \leq \lambda$, then, there exists an (ι, κ)-fMc (resp. (ι, κ)-fδsc, (ι, κ)-fδpc) set ν of Y such that $\mu \leq \nu, f^{-1}(\nu) \leq \lambda$.

Proof:

Let $\nu = \underline{1} - f(\underline{1} - \lambda)$. Since $f^{-1}(\mu) \leq \lambda$, we have $f(\underline{1} - \lambda) \leq \underline{1} - \mu$. Since f is $dfMO$ map, then, ν is (ι, κ)-fMc in Y and $f^{-1}(\nu) = \underline{1} - f^{-1}(f(\underline{1} - \lambda)) \leq \underline{1} - (\underline{1} - \lambda) = \lambda$. The other cases of the theorem can be proved in a same manner. ■

Theorem 2.10.

If $f : (X, \tau_1, \tau_1^*) \to (Y,\ \tau_2, \tau_2^*)$ be a $dfMO$ mapping. Then, for each $\mu \in I^Y$,

$$f^{-1}(C_{\tau_2,\tau_2^*}(\theta I_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\delta C_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \leq C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa).$$

Proof:

Since $\tau_1(\underline{1} - C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa)) \geq \iota,\ \tau_1^*(\underline{1} - C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa)) \leq \kappa$ and $f^{-1}(\mu) \leq C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa)$ for each $\mu \in I^Y$, it follows from Theorem 2.9, that there exists an (ι, κ)-fMc set λ of $Y,\ \mu \leq \lambda$ such that $f^{-1}(\lambda) \leq C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa)$. So $\lambda \geq C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa), \iota, \kappa) \wedge I_{\tau_2,\tau_2^*}(\theta C_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa), \iota, \kappa)$, hence,

$$\begin{aligned} f^{-1}(\lambda) &\geq f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa), \iota, \kappa)) \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\theta C_{\tau_2,\tau_2^*}(\lambda, \iota, \kappa), \iota, \kappa)) \\ &\geq f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\theta C_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)). \end{aligned}$$

Hence,
$f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\theta C_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \leq C_{\tau_2,\tau_2^*}(f^{-1}(\mu), \iota, \kappa)$. ■

Theorem 2.11.

If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be a bijective mapping such that

$$f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\theta C_{\tau_2,\tau_2^*}(\mu, \iota, \kappa), \iota, \kappa)) \leq C_{\tau_1,\tau_1^*}(f^{-1}(\mu), \iota, \kappa),$$

for each $\mu \in I^Y$, then, f is $dfMO$ map.

Proof:

Let $\lambda \in I^X, \iota \in I_0,\ \kappa \in I_1$ with $\tau_1(\lambda) \geq \iota,\ \tau_1^*(\lambda) \leq \kappa$. Then, hypothesis,

$$\begin{aligned} f^{-1}(C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(f(\underline{1}-\lambda),\iota,\kappa),\iota,\kappa))\ \wedge f^{-1}(I_{\tau_2,\tau_2^*}(\delta C_{\tau_2,\tau_2^*}(f(\underline{1}-\lambda),\iota,\kappa),\iota,\kappa)) &\leq C_{\tau_1,\tau_1^*}(f^{-1}(f(\underline{1}-\lambda)),\iota,\kappa) \\ &= C_{\tau_1,\tau_1^*}(\underline{1}-\lambda,\iota,\kappa) \\ &= \underline{1}-\lambda \end{aligned}$$

and so, $C_{\tau_2,\tau_2^*}(\delta I_{\tau_2,\tau_2^*}(f(\underline{1}-\lambda),\iota,\kappa),\iota,\kappa) \wedge I_{\tau_2,\tau_2^*}(\delta C_{\tau_2,\tau_2^*}(f(\underline{1}-\lambda),\iota,\kappa),\iota,\kappa) \leq f(\underline{1}-\lambda)$, which shows that, $f(\underline{1}-\lambda)$ is an (ι,κ)-fMc set of Y. Since f is bijective, then, $f(\lambda)$ is an (ι,κ)-fMo set of Y, therefore, f is $dfMO$ map. ∎

Theorem 2.12.

Let (X,τ,τ^*) and (Y,η,η^*) be dfts's. Let $f : X \to Y$ be a $dfMC$ mapping. Then, the following statements hold.
(i) If f is a surjective map and $f^{-1}(\alpha)\overline{q}f^{-1}(\beta)$ in X, then, there exists $\alpha,\ \beta \in I^Y$ such that $\alpha\overline{q}\beta$.
(ii) $MI_{\eta,\eta^*}(MC_{\eta,\eta^*}(f(\lambda),\iota,\kappa),\iota,\kappa) \leq f(C_{\tau,\tau^*}(\lambda,\iota,\kappa))$, for each $\lambda \in I^X$.

Proof:

(i) Let $\gamma_1,\gamma_2 \in I^X$ such that $f^{-1}(\alpha) \leq \gamma_1$ and $f^{-1}(\beta) \leq \gamma_2$ such that $\gamma_1\overline{q}\gamma_2$. Then, there exists two (ι,κ)-fMo sets μ_1 and μ_2 such that $f^{-1}(\alpha) \leq \mu_1 \leq \gamma_1,\ f^{-1}(\beta) \leq \mu_2 \leq \gamma_2$. But f is a surjective map, then, $ff^{-1}(\alpha) = \alpha \leq f(\mu_1) \leq f(\gamma_1)$ and $ff^{-1}(\beta) = \beta \leq f(\mu_2) \leq f(\gamma_2)$. Since $\gamma_1\overline{q}\gamma_2$, then, $f(\gamma_1 \wedge \gamma_2) = \underline{0}$. Hence $\alpha \wedge \beta \leq f(\mu_1 \wedge \mu_2) \leq f(\gamma_1 \wedge \gamma_2) = \underline{0}$. Therefore, $\alpha\overline{q}\beta$ in Y. that is $\alpha \wedge \beta = \underline{0}$.

(ii) Since $\lambda \leq C_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq \underline{1}$ and f is a $dfMC$ mapping, then, $f(C_{\tau,\tau^*}(\lambda,\iota,\kappa))$ is (ι,κ)-fMc set in Y. Hence,

$$\begin{aligned} f(\lambda) &\leq MC_{\tau,\tau^*}(\lambda,\iota,\kappa) \\ &\leq f(C_{\tau,\tau^*}(\lambda,\iota,\kappa)). \end{aligned}$$

So, $MI_{\eta,\eta^*}(MC_{\eta,\eta^*}(f(\lambda),\iota,\kappa),\iota,\kappa) \leq f(C_{\tau,\tau^*}(\lambda,\iota,\kappa))$. ∎

Proposition 2.1.

Let $f : (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ $dfMO$ mapping and if for any fuzzy subset λ of Y is (ι,κ)-fuzzy nowhere dense then, f is $df\delta pO$ map.

Proof:

Let $\tau_1(\mu) \geq \iota,\ \tau_1^*(\mu) \leq \kappa$. Since f is an $dfMO$ mapping, then, $f(\mu)$ is an (ι,κ)-fMo set in (Y,τ_2,τ_2^*). Put $f(\mu) = \lambda$ is an (ι,κ)-fMo set in Y. Hence,

$$\lambda \leq C_{\tau,\tau^*}(\theta I_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa) \vee I_{\tau,\tau^*}(\delta C_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa).$$

But $\theta I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq C_{\tau,\tau^*}(\lambda,\iota,\kappa)$, and since λ is (ι,κ)-fuzzy nowhere dense, then, $\theta I_{\tau,\tau^*}(\lambda,\iota,\kappa) \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,\iota,\kappa),\iota,\kappa)$, we have $\theta I_{\tau,\tau^*}(\lambda,\iota,\kappa) = \underline{0}$. Using Lemma 3.1 in

Sathiyaraj et al. (2019b), f is $df\delta pO$ map. ∎

Theorem 2.13.

If $f:(X,\tau_1,\tau_1^*)\to(Y,\tau_2,\tau_2^*)$ be a $df\theta biCts$ mapping, then, the image of each (ι,κ)-fMo set in (X,τ_1,τ_1^*) under f is (ι,κ)-fMo set in (Y,τ_2,τ_2^*).

Proof:

Let f be a $df\theta biCts$ and μ be a (ι,κ)-fMo set in $(X\tau_1,\tau_1^*)$. Then,

$$\mu\le C_{\tau_1,\tau_1^*}(\theta I_{\tau_1,\tau_1^*}(\mu,\iota,\kappa),\iota,\kappa)\vee I_{\tau_1,\tau_1^*}(\delta C_{\tau_1,\tau_1^*}(\mu,\iota,\kappa),\iota,\kappa).$$

This implies that,

$$\begin{aligned}f(\mu)&\le f(C_{\tau_1,\tau_1^*}(\theta I_{\tau_1,\tau_1^*}(\mu,\iota,\kappa),\iota,\kappa))\vee f(I_{\tau_1,\tau_1^*}(\delta C_{\tau_1,\tau_1^*}(\mu,\iota,\kappa),\iota,\kappa))\\&\le C_{\tau_2,\tau_2^*}(f(\theta I_{\tau_1,\tau_1^*}(\mu,\iota,\kappa)),\iota,\kappa)\vee f(I_{\tau_1,\tau_1^*}(\delta C_{\tau_1,\tau_1^*}(\mu,\iota,\kappa),\iota,\kappa)).\end{aligned}$$

Since f is an $df\theta biCts$ mapping, then, f is $df\theta O$ map and $df\theta Cts$ map. Then, f is $df\theta sCts$ map and $df\theta pCts$ map. Hence $f(\mu)\le C_{\tau_2,\tau_2^*}\theta I_{\tau_2,\tau_2^*}(f(\mu),\iota,\kappa),\iota,\kappa)\vee I_{\tau_2,\tau_2^*}\delta C_{\tau_2,\tau_2^*}(f(\mu),\iota,\kappa),\iota,\kappa)$. This shows that, $f(\mu)$ is (ι,κ)-fMo set in (Y,τ_2,τ_2^*). ∎

The composition of two $dfMO$ mappings need not be $dfMO$ as shown by the following example.

Example 2.6.

Let $X=Y=Z=\{a,b,c\}$ and let the fuzzy sets α and β defined as $\alpha(a)=0.5$, $\alpha(b)=0.4$, $\alpha(c)=0.4$; $\beta(a)=0.5$, $\beta(b)=0.7$ and $\beta(c)=0.8$, consider the double fuzzy topologies (X,τ,τ^*) and (Y,η,η^*) with

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{6}, & \text{if } \lambda=\alpha,\\ 0, & \text{otherwise,}\end{cases}\qquad \tau^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{5}{6}, & \text{if } \lambda=\alpha,\\ 1, & \text{otherwise,}\end{cases}$$

$$\sigma(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{6}, & \text{if } \lambda=\beta,\\ 0, & \text{otherwise,}\end{cases}\qquad \sigma^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{5}{6}, & \text{if } \lambda=\beta,\\ 1, & \text{otherwise,}\end{cases}$$

and

$$\eta(\lambda)=\begin{cases}1, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{1}{6}, & \text{if } \lambda=\underline{1}-\beta,\\ 0, & \text{otherwise,}\end{cases}\qquad \eta^*(\lambda)=\begin{cases}0, & \text{if } \lambda\in\{\underline{0},\underline{1}\},\\ \frac{5}{6}, & \text{if } \lambda=\underline{1}-\beta,\\ 1, & \text{otherwise.}\end{cases}$$

Then, the identity function $f:(X,\tau,\tau^*)\to(Y,\sigma,\sigma^*)$ & $g:(Y,\sigma,\sigma^*)\to(Z,\eta,\eta^*)$ are $dfMO$ functions. But $g\circ f$ is not $dfMO$ function, since the image under $g\circ f$ of the fuzzy set $\underline{1}-\alpha$ is not an $(\frac{1}{6},\frac{5}{6})$- fMo set in (Z,η,η^*).

The next theorem gives the conditions under which the composition of $dfMO$ mapping is $dfMO$.

Theorem 2.14.

Let (X, τ_1, τ_1^*), (Y, τ_2, τ_2^*) and (Z, τ_3, τ_3^*) be dfts's. If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ and $g : (Y, \tau_2, \tau_2^*) \to (Z, \tau_3, \tau_2^*)$ are mappings, then, $g \circ f$ is $dfMO$ mapping if
(i) f is dfO and g is $dfMO$.
(ii) f is $dfMO$ and g is $df\theta biCts$ mapping.

Proof:

(i) Let $\tau_1(\mu) \geq \iota$ & $\tau_1^*(\mu) \leq \kappa$. Since f is dfO, then, $\tau_2(f(\mu)) \geq \iota$ & $\tau_2^* f(\mu)) \leq \kappa$. Since g is $dfMO$, then, $g(f(\mu)) = (g \circ f)(\mu)$ is (ι, κ)-fMo set in (Z, τ_3, τ_3^*). Hence, $g \circ f$ is $dfMO$.

(ii) Let $\tau_1(\mu) \geq \iota$ & $\tau_1^*(\mu) \leq \kappa$. Since f is $dfMO$, then, $f(\mu)$ is an (ι, κ)-fMo set in (Y, τ_2, τ_2^*). Since g is $df\theta biCts$, by Theorem 2.13, $(g \circ f)(\mu)$ is (ι, κ)-fMo set in (Z, τ_3, τ_3^*). Hence, $g \circ f$ is $dfMO$. ■

Theorem 2.15.

Let (X, τ_1, τ_1^*), (Y, τ_2, τ_2^*) and (Z, τ_3, τ_3^*) be dfts's. If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ and $g : (Y, \tau_2, \tau_2^*) \to (Z, \tau_3, \tau_3^*)$ are mappings, then,
(i) If $g \circ f$ is $dfMO$ mapping and f is a surjective $dfCts$ map, then, g is $dfMO$ map.
(ii) If $g \circ f$ is dfO mapping and g is an injective $dfMCts$ map, then, f is $dfMO$ map.

Proof:

(i) Let $\tau_2(\mu) \geq \iota,\ \tau_2^*(\mu) \leq \kappa$. Since f is $dfCts$, then, $f^{-1}(\mu)$ is an (ι, κ)-fo set in (X, τ_1, τ_1^*). But $g \circ f$ is $dfMO$ map, then, $(g \circ f)(f^{-1}(\mu))$ is (ι, κ)-fMo set in (Z, τ_3, τ_3^*). Hence, by surjective of f, we have $g(\mu)$ is (ι, κ)-fMo set of (Z, τ_3, τ_3^*). Hence, g is $dfMO$ map.
(ii) Let μ is an (ι, κ)-fo set in (X, τ_1, τ_1^*). and $g \circ f$ be an dfO. Then, $(g \circ f)(\mu) = g(f(\mu))$ is an (ι, κ)-fo set in (Z, τ_3, τ_3^*). Since g is an injective $dfMCts$ map, hence, $f(\mu)$ is fMo set in (Y, τ_2, τ_2^*). Therefore, f is $dfMO$. ■

3. (ι, κ) - fuzzy M - compactness and (ι, κ) - fuzzy M - connectedness

In this section, we study the properties (compactness and connectedness) of image (resp. pre image) under bijective (resp. surjective) $dfMO$ mappings.

Definition 3.1.

A dfts (X, τ, τ^*) is called
(i) (ι, κ)-fuzzy M-T_1 (resp. (ι, κ)-fuzzy T_1) if for every two distinct fuzzy points x_r, y_s of X, there exists two (ι, κ)-fMo (resp. (ι, κ)-fo) sets λ, μ such that $x_r \in \lambda$, $y_s \notin \lambda$ and $y_s \in \mu$, $x_r \notin \mu$.
(ii) (ι, κ)-fuzzy M-T_2 (resp. (ι, κ)-fuzzy T_2) if for every two distinct fuzzy points x_r, y_s of X, there exists two disjoint (ι, κ)-fMo (resp. (ι, κ)-fo) sets λ, μ such that $x_r \in \lambda$, $y_r \in \mu$.

(iii) (ι, κ)-fuzzy M-connected (resp. (ι, κ)-fuzzy connected) if it cannot be expressed as the union of two disjoint non-empty (ι, κ)-fMo (resp. (ι, κ)-fo) sets of X. If X is not (ι, κ)-fuzzy M-connected (resp. not (ι, κ)-fuzzy connected), then, it is (ι, κ)-fuzzy M-disconnected (resp. (ι, κ)-fuzzy disconnected).
(iv) (ι, κ)-fuzzy M-Lindel$\ddot{o}$ff ((ι, κ)-fuzzy Lindel$\ddot{o}$ff) if every (ι, κ)-fuzzy M-open cover (resp. (ι, κ)-fuzzy open cover) of X has a countable subcover.
(v) (ι, κ)-fuzzy M-compact (resp. (ι, κ)-fuzzy compact) if for every (ι, κ)-fuzzy M-open cover (resp. (ι, κ)-fuzzy open cover) of X has a finite subcover.

Theorem 3.1.

Let (X, τ, τ^*) and (Y, η, η^*) be dfts's. Let $f : X \to Y$ be a bijective $dfMO$ mapping. Then, the following statements hold.
(i)] If X is a (ι, κ)-fuzzy T_i-space, then, Y is (ι, κ)-fuzzy M-T_i where i=1,2.
(ii) If Y is an (ι, κ)-fuzzy M-compact (resp. (ι, κ)-fuzzy M-Lindel$\ddot{o}$ff) space, then, X is (ι, κ)-fuzzy compact (resp. (ι, κ)-fuzzy Lindel$\ddot{o}$ff).

Proof:

(i) We prove that, for the case of a (ι, κ)-fuzzy T_1-space. Let y_{s_1}, y_{s_2} be two distinct points of Y. Then, there exists $x_{r_1}, x_{r_2} \in X$ such that $f(x_{r_1}) = y_{s_1}$ and $f(x_{r_2}) = y_{s_2}$. Since X is a (ι, κ)-fuzzy T_1-space, then, there exists two (ι, κ)-fo sets λ, μ of X such that $x_{r_1} \in \lambda$, $x_{r_2} \notin \lambda$ and $x_{r_2} \in \mu$, $x_{r_1} \notin \mu$. But, f is an $dfMO$ map, then, $f(\lambda)$, $f(\mu)$ are (ι, κ)-fMo sets of Y with $y_{s_1} \in f(\lambda)$, $y_{s_2} \notin f(\lambda)$ and $y_{s_2} \in f(\mu)$, $y_{s_1} \notin f(\mu)$. Therefore, Y is (ι, κ)-fuzzy M-T_1.

(ii) We prove that, the theorem for (ι, κ)-fuzzy M-compact. Let $\{\lambda_i : i \in I\}$ be a family of (ι, κ)-fuzzy open cover of X and f be a surjective $dfMO$ mapping. Then, $\{f(\lambda_i) : i \in I\}$ is an (ι, κ)-fuzzy M-open cover of Y. But, Y is (ι, κ)-fuzzy M-compact space, hence, there exists a finite subset I_0 of I such that $Y = \bigvee\{f(\lambda_i) : i \in I_0\}$. Then, by injective of f, $\{\lambda_i : i \in I_0\}$ is a finite subfamily of X. Therefore, X is (ι, κ)-fuzzy compact. ■

Theorem 3.2.

Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. If $f : X \to Y$ is a surjective $dfMO$ mapping and Y is (ι, κ)-fuzzy M-connected space, then, X is (ι, κ)-fuzzy connected.

Proof:

Suppose that, X is a (ι, κ)-fuzzy disconnected space. Then, there exists two non-empty disjoint (ι, κ)-fo sets λ, μ of X such that $X = \lambda \vee \mu$. But f is a surjective $dfMO$ map, then, $f(\lambda)$ and $f(\mu)$ are non-empty disjoint (ι, κ)-fMo sets of Y with $Y = f(\lambda) \vee f(\mu)$ which is a contradiction with the fact Y is (ι, κ)-fuzzy M-connected. ■

Example 3.1.

Let $X = \{a, b, c\}$ and let the fuzzy sets α β and γ defined as $\alpha(a) = 0,\ \alpha(b) = 1,\ \alpha(c) = 0;\ \beta(a) = 0,\ \beta(b) = 1\ \beta(c) = 0;\ \gamma(a) = 0,\ \gamma(b) = 0,$ and $\gamma(c) = 1$, consider the double fuzzy topologies (X, τ, τ^*) with

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{6}, & \text{if } \lambda = \alpha, \beta, \gamma, \\ 0, & \text{otherwise,} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{5}{6}, & \text{if } \lambda = \alpha, \beta, \gamma, \\ 1, & \text{otherwise.} \end{cases}$$

Then, the dfts (X, τ, τ^*) is (ι, κ)-fuzzy M-T_1 as well as (ι, κ)-fuzzy T_1.

Conclusion

Maps have always been tremendous importance in all branches of mathematics and the whole science. In the other hand, topology plays a significant role in quantum physics, high energy and super string theory. Thus, we introduced and investigated the classes of mappings called double fuzzy M-open map and double fuzzy M-closed map to the double fuzzy topological spaces. Also, some of their fundamental properties were studied. Some interesting properties and characterizations of the concepts introduced are studied. The relationship with other kinds of functions is studied. We could know that double fuzzy topological spaces are a generalization of some other kinds of topological spaces; therefore, our results can be considered as a generalization of the same results in other kinds of topological spaces. Also, it is possible to study this topic for a completely distributive DeMorgan algebra. Since double fuzzy topology forms an extension of fuzzy topology and general topology, we think that our results can be applied in quantum physics, modren physics, high energy, super string theory and GIS Problems and also, we hope these investigations will further encourage other researchers to explore the interesting connections between this area of topology and fuzzy set.

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