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q-Sumudu transforms pertaining to the product of family of q-polynomials and generalized basic hypergeometric functions

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Abstract

The prime objective of commenced article is to determine q-Sumudu transforms of a product of unified family of q-polynomials with basic (or q-) analog of Fox's H-function and q-analog of I-functions. Specialized cases of the leading outcome are further evaluated as q-Sumudu transform of general class of q-polynomials and q-Sumudu transforms of the basic analogs of Fox's H-function and I-functions.

Keywords: q-Analog of Sumudu transforms; basic hypergeometric functions; general class of q- polynomials; Fox's H-function; basic analog of I-function

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1. Introduction

In the past few decades, diverse techniques of q-calculus have been investigated and largely used in many other areas of mathematics. In recent articles, applications of q-calculus operators have been studied to define certain new classes of functions which are analytic in the open disk, see (Purohit and Raina 2011, 2015, 2015), (Murugusundaramoorthy et al. 2017) and (Govindaraj and Sivasubramanian 2017). About applications of q-calculus in the field of approximation theory, one may refer the current books by Aral et al. (2013) and Gupta et al. (2013). Recently in the field of adaptive filtering the utilization of q-calculus has also been discussed by Al-Saggaf et al. (2015), Ahmed et al. (2018) and Arif et al. (2018). In addition, for more applications q-calculus, one may see such type of works Annaby et al. (2012), Ernst (2012, 2017), and references there in. On the other hand, integral transform is also one of the major tools to solve differential equations. Laplace, Fourier, Mellin and Hankel transforms are frequently using for the same. Present-days, some new integral transforms like Sumudu, natural and Mangontarum etc. are used over and over in the literature.

The integral transform of Sumudu type was introduced through Watugala (1993), and he put forward to obtain the result of ordinary differential equations in problems of control engineering. Although, the Sumudu transform is not a new integral transform, even it is a simply s-multiplied Laplace transform, but the major influence of the Sumudu transform is that it is useful to obtain solution of problems beyond resorting to a different frequency domain, by cause of it conserve scale and unit properties. Thus a lot of work has been done on the theory and applications of Sumudu transforms. In 2003, Belgacem et al. given explanatory observation for the Sumudu transform, and investigated number of fundamental properties of it. Nowadays, the Sumudu transform is an important integral transform to solve ordinary differential equation, see Nisar and Belgacem (2017), Nisar et al. (2017), (2017) and Silambarasan et al. (2018).

In 2013, Albayrak et al. gives q-analog of the Sumudu transforms, and they also obtained q-Sumudu transforms of certain special functions, including q-polynomials. Certain inversion and representation formulas and their applications for q-Sumudu transforms were also discussed in 2014. More recently Purohit and Ucar (2018), using q-Sumudu transform gives an alternative solution for the q-kinetic equation involving the Riemann-Liouville fractional q-integral operators. Moreover, for q-analog of other integral transforms, q-image formulas and their recent applications one can see (Al-Omari 2016, 2017) and (Al-Omari et al. 2018) Motivated by these avenues of applications, a large number of workers have made use of the q-Sumudu transforms in the theory of special functions of one and more variables. In the present paper, we aimed to evaluate the q-Sumudu transforms for a product of general class of q-polynomials and basic analogue of some generalized special functions. Special cases of our main results have also been discussed.

2. Preliminaries

For our investigation, we need q-analogs of the Sumudu transform, introduced by Albayrak et al. (2013), as follow:

$$S_q\{f(t);s\}=\frac{1}{(1-q)s}\int_0^s E_q(\frac{q}{s}t)\,f(t)\,d_qt\ ,\ \ s>0\ , \tag{1}$$

preamble to the collection of functions

$$A=\{\,f(t)|\exists M,\tau_1,\tau_2>0,|\,f(t)|<ME_q(|t|/\tau_j),t\in(-1)^j\times[0,\infty)\} \tag{2}$$

and

$$\boldsymbol{S}_q\{f(t);s\}=\frac{1}{(1-q)s}\int_0^\infty e_q(-\frac{1}{s}t)\,f(t)\,d_qt\ ,\ \ s>0, \tag{3}$$

provided the functions belongs to the set

$$B=\{\,f(t)\,|\,\exists M,\tau_1,\tau_2>0,|\,f(t)|<Me_q(|t|/\tau_j),t\in(-1)^j\times[0,\infty)\}. \tag{4}$$

On the other hand, the q-version of exponential series are defined by

$$e_q(t)=\sum_{n=0}^{\infty}\frac{x^n}{(q;q)_n}=\frac{1}{(t;q)_\infty} \qquad |t|<1 \tag{5}$$

and

$$E_q(x)=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n(n-1)/2}\,x^n}{(q;q)_n}=(t;q)_\infty \qquad (t\in C). \tag{6}$$

The basic improper integration cf. (Jackson, 1905) and (De Sole and Kac 2005) are defined as

$$\int_0^x f(t)\,d_qt=x\,(1-q)\sum_{k=0}^{\infty}q^k f(xq^k)\,, \tag{7}$$

$$\int_0^{\infty/A} f(x)\,d_qx=(1-q)\sum_{k\in Z}\frac{q^k}{A}\,f(\frac{q^k}{A}). \tag{8}$$

By using the results of (7) and (8), the q-Sumudu transforms perhaps expressed as:

$$S_q\{f(t);s\}=(q;q)_\infty\sum_{k=0}^{\infty}\frac{q^k f(sq^k)}{(q;q)_k} \tag{9}$$

and

$$\boldsymbol{S}_q\{f(t);s\}=\frac{s^{-1}}{\left(-\frac{1}{s};q\right)_\infty}\sum_{k\in Z}q^k f(q^k)\left(-\frac{1}{s};q\right)_k. \tag{10}$$

For our purpose, we suppose α is real or complex and $|q|<1$, then the q-shifted factorial is expressed as under (cf. Gasper and Rahman, 1990)

$$(a;q)_n = \begin{cases} 1 & ; \quad n=0 \\ (1-a)(1-aq)\cdots(1-aq^{n-1}) & ; \quad n\in \mathbb{N}, \end{cases} \tag{11}$$

and its natural extension is given by

$$(a;q)_\alpha = \frac{(a;q)_\infty}{(aq^\alpha;q)_\infty}, \quad \alpha \in \mathbb{C}. \tag{12}$$

For $n=\infty$ the definition (1) remains useful as a convergent infinite by-product, provided $|q|<1$, as under:

$$(a;q)_\infty = \prod_{j=0}^{\infty}(1-aq^j). \tag{13}$$

Moreover, the (basic) q-analog of the binomial (power) function $(x\pm y)^n$ cf. Ernst (2003), is given by

$$(x\pm y)^{(n)} \equiv (x\pm y)_n \equiv x^n(\mp y/x;q)_n = x^n \sum_{k=0}^{n}\begin{bmatrix} n \\ k \end{bmatrix}_q q^{k(k-1)/2}(\pm y/x)^k, \tag{14}$$

so that

$$\underset{q\to 1^-}{Lt}(x\pm y)^{(n)} = (x\pm y)^n, \tag{15}$$

where the q-version of binomial coefficient is given as:

$$\begin{bmatrix} \alpha \\ k \end{bmatrix}_q = \frac{(q^{-\alpha};q)_k}{(q;q)_k}(-q^\alpha)^k q^{-k(k-1)/2} \quad (k\in\mathbb{N}, \alpha\in\mathbb{C}). \tag{16}$$

Consider

$$f(x) = \sum_{n=-\infty}^{+\infty} A_n x^n$$

be a power series in x, defined over a bounded sequence of real or complex numbers, (cf. Gasper and Rahman, 1990) thereupon we have

$$f[x\pm y]_q = \sum_{n=-\infty}^{+\infty} A_n\, x^n(\mp y/x;q)_n. \tag{17}$$

Further, the q-gamma function is defined as follows: (cf. De Sole and Kac, 2005)

$$\Gamma_q(\alpha)=\int_0^{1/(1-q)} x^{\alpha-1}E_q(q(1-q)x)d_qx \qquad (\alpha>0). \tag{18}$$

Or

$$\Gamma_q(\alpha)=K(A;\alpha)\int_0^{\infty/A(1-q)} x^{\alpha-1}E_q(-(1-q)x)d_qx \qquad (\alpha>0), \tag{19}$$

where

$$K(A;t)=A^{t-1}\frac{\left(-q/A;q\right)_\infty}{\left(-q^t/A;q\right)_\infty}\cdot\frac{(-A;q)_\infty}{\left(-Aq^{1-t};q\right)_\infty} \qquad (t\in R). \tag{20}$$

For the variable t, the above function K (A; t) gives the subsequent interesting relation:

$$K(x;t+1)=q^tK(x;t). \tag{21}$$

Here the q-gamma function can also be written in the following form

$$\Gamma_q(\alpha)=\frac{(q;q)_\infty(1-q)^{1-\alpha}}{(q^\alpha;q)_\infty}=\frac{(1-q)_{\alpha-1}}{(1-q)^{\alpha-1}}=\frac{(q;q)_{\alpha-1}}{(1-q)^{\alpha-1}}\ , \tag{22}$$

where $\alpha\neq 0,-1,-2,\cdots$.

Now, we lead by looking back on a system of q-polynomials $f_{n,N}(x;q)$ in terms of a bounded complex sequence $\{S_{j,q}\}_{j=0}^{\infty}$, given as (cf. Srivastava and Agarwal,1989)

$$f_{n,N}(x,q)=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix}S_{j,q}x^j \qquad (n=0,1,2...), \tag{23}$$

fixed up with positive integer N.

By virtue of the Mellin-Barnes category q-contour integral, Saxena and Kumar (1995) made known a basic analog of the I-function as under:

$$I_{A_i,B_i}^{m,n}\left[x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}\\ \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j)_{m+1,B_i}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi\,\omega}\int_C \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\ \pi\, x^s}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji} s})\prod_{j=n+1}^{A_i} G(q^{a_{ji}-\alpha_{ji} s})\right\} G(q^{1-s})\sin \pi s}\, ds, \tag{24}$$

where $0 \le m \le B_i; 0 \le n \le A_i$; $i = 1, 2, \cdots, r$; r is finite; $\omega = \sqrt{-1}$; and

$$G(q^a) = \left\{\prod_{n=0}^{\infty}(1-q^{a+n})\right\}^{-1} = \frac{1}{(q^a;q)_\infty}.$$

Also $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and positive and a_j, b_j, a_{ji}, b_{ji} are arbitrary numbers of complex type.

The contour of integration C runs from $-i\infty$ to $+i\infty$ chosen so that all the poles of $G(q^{b_j-\beta_j s})$; $1 \le j \le m$, are to its right, and those of $G(q^{1-a_j+\alpha_j s})$, $1 \le j \le n$, are to its left and at least some $\varepsilon > 0$ distance away from the contour C. If $\mathrm{Re}[s\log(x) - \log\sin \pi s] < 0$, for huge amount of $|s|$ on the contour, in other words if $|\arg x| < \pi$, the basic integral defined above converges. It may be observed that the contour of integration C can be replaced by other suitably indented contours parallel to the imaginary axis.

It is readable to note that as $r = 1, A_1 = A; B_1 = B;$ definition (24) yields the basic equivalent (q-analog) of the Fox's H-function due to Saxena et al. (1983), namely

$$H_{A,B}^{m\ ,n}\left[x;q\left|\begin{matrix}(a,\alpha)\\ \\(b,\beta)\end{matrix}\right.\right] = \frac{1}{2\pi\,\omega}\int_C \frac{\prod_{j=1}^{m} G(q^{b_j-\beta_j s})\prod_{j=1}^{n} G(q^{1-a_j+\alpha_j s})\ \pi\, x^s}{\prod_{j=m+1}^{B} G(q^{1-b_j+\beta_j s})\prod_{j=n+1}^{A} G(q^{a_j-\alpha_j s}) G(q^{1-s})\sin \pi s}\, ds, \tag{25}$$

Again, if we consider $\alpha_i = \beta_j = 1, \forall\, i$ and j in the definition (25), it reduces to a basic analog of the Meijer's G-function defined by Saxena et al. (1983), namely

$$H_{A,B}^{m,n}\left[x;q\left|\begin{matrix}(a,1)\\ \\(b,1)\end{matrix}\right.\right] \equiv G_{A,B}^{m,n}\left[x;q\left|\begin{matrix}a_1,\cdots,a_A\\ \\ b_1,\cdots,b_B\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\omega}\int_C \frac{\prod_{j=1}^{m} G(q^{b_j - s})\prod_{j=1}^{n} G(q^{1-a_j+s})\ \pi x^s}{\prod_{j=m+1}^{B} G(q^{1-b_j+s})\ \prod_{j=n+1}^{A} G(q^{a_j-s}) G(q^{1-s})\sin \pi s}\, ds, \tag{26}$$

where $0 \le m_1 \le B, 0 \le n_1 \le A$ and $\operatorname{Re}[s\log(x) - \log\sin \pi s] < 0.$

Moreover, if we take $n = 0,\ m = B$ in the definition (26), we obtain the basic analog of *E*-function (MacRobert's function) as Agarwal (1960):

$$G_{A,B}^{B,0}\left[x;q\left|\begin{matrix} a_1,\cdots,a_A \\ b_1,\cdots,b_B\end{matrix}\right.\right] = \mathrm{E_q}[\mathrm{B};\mathrm{b_j}:\mathrm{A};\mathrm{a_j}:x]. \tag{27}$$

For remarkable fundamental properties, along with numerous applications of the Meijer's *G*-function or Fox *H*-functions, one is allowed to refer the research treatise by Mathai and Saxena (1973, 1978) and Mathai et al. (2010)

3. Main Results

In this segment, we shall investigate the q-Sumudu transforms of a product of the universal system of q-polynomials and q-analogs of the *H*- function and *I*-functions. We state our results in terms of the following theorems:

Theorem 3.1.

If $\{S_{j,q}\}_{j=0}^{\infty}$ be a bounded complex sequence, let $m_1, n_1; A, B$ be positive integers such that $0 \le m_1 \le B, 0 \le n_1 \le A$ and N be an arbitrary positive integer. Then the following q-Sumudu transform holds:

$$S_q\left\{x^{\lambda} f_{n,N}(x,q)\, H_{A,B}^{m_1,n_1}\left[x^k;q\left|\begin{matrix}(a_1,\alpha_1),\ldots,(a_A,\alpha_A)\\(b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right]\right\}$$

$$=\frac{s^{\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j\end{bmatrix} S_{j,q}\, s^j\, H_{A+1,B}^{m_1,n_1+1}\left[s^k;q\left|\begin{matrix}(-\lambda - j,k),(a_1,\alpha_1),\ldots,(a_A,\alpha_A)\\(b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right]\ (k>0), \tag{28}$$

provided $\operatorname{Re}[s\log(x) - \log\sin \pi s] < 0.$

Proof:

On making use of definitions (23) and (25), the left hand side (let L) of the main result (28) can be represented as

$$L = S_q \left\{ x^{\lambda} \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, x^j \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j - \beta_j s}) \prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\, \pi (x^k)^z}{\prod_{j=m_1+1}^{B} G(q^{1-b_{ji}+\beta_{ji} s}) \prod_{j=n_1+1}^{A} G(q^{a_{ji}-\alpha_{ji} s}) G(q^{1-s}) \sin \pi s} ds \right\}$$

or

$$L = \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j - \beta_j s}) \prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\, \pi}{\prod_{j=m_1+1}^{B} G(q^{1-b_{ji}+\beta_{ji} s}) \prod_{j=n_1+1}^{A} G(q^{a_{ji}-\alpha_{ji} s}) G(q^{1-s}) \sin \pi s} S_q\left(x^{j+\lambda+kz}\right) ds\, .$$

On using the known result due Albayrak et al. (2013), namely

$$S_q(x^{\alpha-1}) = s^{\alpha-1}(1-q)^{\alpha-1}\Gamma_q(\alpha),$$

the above expression reduce to

$$L = \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \frac{s^{\lambda+j}}{2\pi\omega} \int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j - \beta_j s}) \prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\, \pi (s^k)^z (1-q)^{j+\lambda+ks} \Gamma_q(j+\lambda+ks+1)}{\prod_{j=m_1+1}^{B} G(q^{1-b_{ji}+\beta_{ji} s}) \prod_{j=n_1+1}^{A} G(q^{a_{ji}-\alpha_{ji} s}) G(q^{1-s}) \sin \pi s}\, .$$

The desired right-hand side of (28) may be obtained by further simplification, as under

$$S_q\left\{ x^{\lambda} f_{n,N}(x,q)\, H_{A,B}^{m_1,n_1}\left[x^k ; q \middle| \begin{matrix} (a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B) \end{matrix} \right]; s \right\}$$
$$= \frac{s^{\lambda}}{G(q)} \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j\, H_{A+1,B}^{m_1,n_1+1}\left[s^k ; q \middle| \begin{matrix} (-\lambda-j,k),(a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B) \end{matrix} \right] \ (k>0),$$

provided $\mathrm{Re}[s\log(x) - \log\sin\pi s] < 0$.

In similar fashion, we derive another result as under:

Theorem 3.2.

Consider $\mathrm{Re}(\mu) > 0$ and $\mathrm{Re}[s\log(x) - \log\sin\pi s] < 0$, then the q-Sumudu transform for a product of q-analog of I-function and q-polynomials family $f_{n,N}(x;q)$ is given by the subsequent formula:

$$S_q\left\{x^{\lambda} f_{n,N}(x,q) I_{A_i,B_i}^{m_1,n_1}\left[\rho x^k;q \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n_1},(a_{ji},\alpha_{ji})_{n_1+1,A_i} \\ (b_j,\beta_j)_{1,m_1},(b_j,\beta_j)_{m_1+1,B_i} \end{matrix}\right]\right\}$$

$$=\frac{s^{\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j I_{A_i+1,B_i}^{m_1,n_1+1}\left[\rho x^k;q \,\middle|\, \begin{matrix} (-j-\lambda,k),(a_j,\alpha_j)_{1,n_1},(a_{ji},\alpha_{ji})_{n_1+1,A_i} \\ (b_j,\beta_j)_{1,m_1},(b_j,\beta_j)_{m_1+1,B_i} \end{matrix}\right], \tag{29}$$

where $0\le m_1\le B_i; 0\le n_1\le A_i$; $i=1,2,\cdots,r$; r is finite, $|q|<1$, $\{S_{j,q}\}_{j=0}^{\infty}$ be a bounded complex sequence and λ is any arbitrary.

Proof:

On making use of definitions (23) and (24), the left hand side (say L) of the main result (29) becomes

$$L=S_q\left\{x^{\lambda-1}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, x^j \frac{1}{2\pi\omega}\int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j-\beta_j s})\prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\,\pi(\rho x^k)^s}{\sum_{i=1}^{r}\left\{\prod_{j=m_1+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n_1+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})\right\} G(q^{1-s})\sin\pi s}\,ds\right\}$$

$$L=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j \frac{1}{2\pi\omega}\int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j-\beta_j s})\prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\,\pi\rho^s}{\sum_{i=1}^{r}\left\{\prod_{j=m_1+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n_1+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})\right\} G(q^{1-s})\sin\pi s} S_q\left(x^{j+\lambda+ks}\right)ds.$$

Again, on using the known result, namely

$$S_q(x^{\alpha})=s^{\alpha}(1-q)^{\alpha}\Gamma_q(\alpha+1),$$

the above expression reduce to

$$L=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q}\, s^j \frac{x^{\lambda-1}}{2\pi\omega}\int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j-\beta_j s})\prod_{j=1}^{n_1} G(q^{1-a_j+\alpha_j s})\,\pi(x^k)^z(1-q)^{j+\lambda+ks-1}\Gamma_q(j+\lambda+ks)}{\sum_{i=1}^{r}\left\{\prod_{j=m_1+1}^{B_i} G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n_1+1}^{A_i} G(q^{a_{ji}-\alpha_{ji}s})\right\} G(q^{1-s})\sin\pi s}.$$

On further simplification in above relation, we easily obtain the right hand side of the result (29).

4. Extraordinary Cases

In the indicated segment, we shall deal with certain particular cases of our main sequel. For example, **i**f we set $r=1, A_1 = A;$ and $B_1 = B,$ in the main result (29), it yields to result (28). Also, if we fixed $\alpha_i = \beta_j = 1, \forall\, i$ and j in the result (28), we arrive at the coming result:

$$S_q\left\{x^{\lambda} f_{n,N}(x,q) G_{A,B}^{m,n}\left[x^k;q\left|\begin{matrix} a_1,\cdots,a_A \\ \\ b_1,\cdots,b_B\end{matrix}\right.\right]\right\}$$

$$=\frac{s^{\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N_j\end{bmatrix} S_{j,q}\, s^j\, G_{A+1,B}^{m\;\;,n\;\;+1}\left[s^k;q\left|\begin{matrix} -\lambda-j,k;a_1,\ldots a_A, \\ b_1,\ldots,b_B\end{matrix}\right.\right]\;(k>0). \tag{30}$$

By conveying particular values to the sequence $\{S_{j,q}\}_{j=0}^{\infty}$, our main result (28) can be brought to bear certain image formulas under q-Sumudu transforms involving orthogonal q-polynomials and the basic analog of Fox's H-function. To illustrate the same, we deal with the following case. By setting $N=1$ and

$$S_{j,q} = \frac{(q^{\alpha};q)_n}{(q;q)_n (q^{\alpha+1};q)_j} q^{j(j+1)/2},$$

we have

$$f_{n,1}(x,q) = L_n^{(\alpha)}(x,q).$$

Thereupon, the result (28) yields to

$$S_q\left\{x^{\lambda} L_n^{(\alpha)}(x,q)\, H_{A,B}^{m_1,n_1}\left[x^k;q\left|\begin{matrix}(a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right]\right\}$$

$$=\frac{(q^{\alpha};q)_n\, s^{\lambda}}{G(q)(q;q)_n}\sum_{j=0}^{n}\frac{q^{j(j+1)/2}\, s^j}{(q^{\alpha+1};q)_j} H_{A+1,B}^{m_1,n_1+1}\left[s^k;q\left|\begin{matrix}(-\lambda-j,k),(a_1,\alpha_1),\ldots,(a_A,\alpha_A) \\ (b_1,\beta_1),\ldots,(b_B,\beta_B)\end{matrix}\right.\right]\quad (k>0)\;. \tag{31}$$

A detailed account of various hypergeometric orthogonal q-polynomials can be found in the research monograph by Koekoek et al. (2010). Therefore, one can derive similar type of results by taking into consideration the definitions of the q-polynomials given in same paper.

5. Concluding Remarks

We conclude this paper with the remark that, by virtue of the unified nature of q-analog of I-function and general class of family of q-polynomials, the q-image formulas given by the relations (28) and (29) being are of general nature, and will lead to several q-Sumudu transforms for the product of orthogonal q-polynomials and q-special functions. Moreover, they are expected to find some importance in establishing solutions of differential equations involving q-special functions.

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