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A Simulation Study on the Size and Power Properties of Some Ridge Regression Tests

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Abstract

Ridge regression techniques have been extensively used to solve the multicollinearity problem for both linear and non-linear regression models since its inception. This paper studied different ridge regression *t*-type tests of the individual coefficients of a linear regression model. A simulation study has been conducted to evaluate the performance of the proposed tests with respect to their sizes and powers under different settings of the linear regression model. Our simulation results demonstrated that most of the proposed tests have sizes close to the 5% nominal level and all tests except t_{AKS}, t_{kM2} and t_{kM9} have considerable gain in powers over the ordinary OLS *t*-type test. It is also observed that some of the proposed test statistics are performing better than the HK and HKB tests which are proposed some authors.

Keywords: Empirical power; Ridge regression; Size of the test; Simulation Study; t-test; Type I error

MSC 2010 No.: 62J07, 62J99

1. Introduction

The concept of ridge regression is pioneered by Hoerl and Kennard (1970) to handle multicollinearity problem for engineering data. They found that there is a nonzero value of k (ridge or shrinkage parameter) for which mean square error (MSE) for the ridge regression estimator is smaller than the variance of the ordinary least squares (OLS) estimator. Estimating the biasing parameter is a vital issue in the ridge regression model. Several researchers at different period of times worked in this area of research and proposed different estimators for k. To mention a few, Hoerl and Kennard (1970), Hoerl, Kennard and Baldwin (1975), McDonald and Galarneau (1975), Lawless and Wang (1976), Dempster et al. (1977), Gibbons (1981), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi and Shukur (2008), Muniz and Kibria (2009), Gruber (1998, 2010), Muniz et al. (2012), Mansson et al. (2010), Hefnawy and Farag (2013), Roozbeh and Arashi (2013,2014), Arashi and Valizadeh (2014), Aslam (2014), Asar and Karaibrahimoğlu (2014), Saleh et al. (2014), Asar and Erişoğlu (2016), Fallah et al. (2017), Norouzirad and Arashi (2017), and very recently Saleh et al. (2019) among others. Kibria and Banik (2016) have studied 28 different ridge regression estimators those are available in literature and based on their simulation studies they have proposed five new ridge estimators. They compared ridge regression estimators in the sense of smaller MSE criterion. Based on their empirical findings, the following 15 ridge estimators HSL, AM, GM, MED, KS_MAX, KM2, KM3, KM5, KM8, KM9, KHMO, CJH, FG and proposed KB3 performed better than the rest in the sense of smaller MSE and recommended to practitioners. For details, see Kibria and Banik (2016).

It is well known that to make inference about unknown population parameter, one may consider both confidence interval and hypothesis testing methods. However, the literature on the test statistics for testing the regression coefficients under the ridge regression model is very limited. First, Halawa and Bassiouni (2000) compared empirical sizes and powers of two tests, based on the estimator of k proposed by Hoerl and Kennard (1970) and Hoerl, Kennard and Baldwin (1975). Their results evident that for models with large standard errors, the ridge based t-tests have correct sizes with considerable gain in powers over those of the least squares t-test. For models with small standard errors, tests are found to be slightly exceeding the nominal level in few cases. Recently, Cule et al. (2011) evaluated the performance of tests proposed by Hoerl and Kennard (1970), Hoerl, Kennard and Baldwin (1975) and Lawless and Wang (1976) based on linear ridge and logistic ridge regression models.

Since aforementioned ridge regression estimators (Kibria and Banik (2016)) are considered by several researchers at different times and under different simulation conditions, testing regression coefficients based on the basis of size and power properties under the ridge regression model are not comparable as a whole. Therefore, the important contribution of this paper is to compare several t test statistics for testing regression coefficients those are recommended by Kibria and Banik (2016). Since a theoretical assessment among the test statistics is not possible, a simulation study has been conducted to evaluate the performances of the suggested test statistics. Thus, sixteen test statistics will be compared based on the empirical size and power properties following the testing procedure that are detailed given by Halawa and Bassiouni (2000).

The organization of the paper is as follows. The proposed test statistics are described in section 2. A Monte Carlo simulation study has been conducted and results are discussed in Section 3. Finally, some concluding remarks are given in Section 4.

2. Statistical Methodology

Consider the following linear regression model

$$y = X\beta + e, \quad (1)$$

where y is an n×1 vector of observations, β is an (p+1)×1 vector of unknown regression coefficients, X is an n×(p+1) observed matrix of the regressors and e is an n×1 vector of random errors, which is distributed as multivariate normal with mean 0 vector and covariance matrix $\sigma^2 I_n$ and I_n is an identity matrix of order n.

To test

$$H_0: \beta_i = 0 \quad (2)$$

$$H_1: \beta_i \neq 0 \quad (3)$$

the ordinary *t*-statistic for regression coefficients is defined as

$$t(0) = \hat{\beta}_i(0)/S(\hat{\beta}_i(0)), \quad (4)$$

where $\hat{\beta}_i(0)$ is the i^{th} components of $\hat{\beta}(0) = (X'X)^{-1}X'Y$ and $S(\hat{\beta}_i(0))$ is the square root of the i^{th} diagonal elements of $\hat{\sigma}^2(X'X)^{-1}$ with $\hat{\sigma}^2 = (Y - X\hat{\beta}(0))'(Y - X\hat{\beta}(0))/(n - p - 1)$.
The t-test statistic for testing (2) vs (3) under the ridge regression model is

$$t(k) = \hat{\beta}_i(k)/S(\hat{\beta}_i(k)), \quad (5)$$

where $\hat{\beta}_i(k)$ is the i^{th} element of $\hat{\beta}(k) = (X'X + kI_n)^{-1}X'y$ and $S(\hat{\beta}_i(k))$ is an estimate of the standard error obtained as the square root of the i^{th} element of the diagonal of the covariance matrix

$$\mathrm{var}\left(\hat{\beta}(k)\right) = \sigma^2(k)(X'X + kI)^{-1}X'X(X'X + kI)^{-1}$$

with

$$\hat{\sigma}^2(k) = (Y - X\hat{\beta}(k))'(Y - X\hat{\beta}(k))/(n - p).$$

Following Halawa and El Bassiouni (2000) and Kibria and Banik (2016), we will define the following test statistics for testing the individual regression coefficients.

2.1. Hoerl and Kennard (1970) Test

To test (2) vs. (3), the test statistic (denoted by t_{HK}) is defined in (5), where

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}^2_{max}}, \hat{\sigma}^2 = \sum_{i=1}^{n} \hat{e}_i^2/(n-p), \hat{e}_i = y_i - X_i^{*\prime}\hat{\alpha}$$

and $\hat{\alpha}_{max}$ is the maximum element of $\hat{\alpha} = D\hat{\beta}$ and D is an orthogonal matrix such that D′CD’=Λ, where Λ=diag($\lambda_1, \lambda_2, \ldots, \lambda_p$) contains eigenvalues of the matrix $C = X'X$.

2.2. Hoerl, Kennard and Baldwin (1975) Test

To test (2) vs. (3), the test statistic (denoted by t_{HKB}) is defined in (5), where

$$\hat{k}_{HKB} = \frac{p\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}}.$$

2.3. Hocking, Speed and Lynn (1976) test

To test (2) vs. (3), the test statistic (denoted by t_{HSL}) is defined in (5), where

$$\hat{k}_{HSL} = \hat{\sigma}^2 \frac{\sum_{i=1}^{p}(\lambda_i\hat{\alpha}_i)^2}{(\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2)^2}, \hat{\sigma}^2 = \sum_{i=1}^{n}\hat{e}_i^2/(n-p), \hat{e}_i = y - X^{*\prime}\hat{\alpha}, \lambda_i$$

are eigenvalues of the matrix $X'X$ and $\hat{\alpha}_i$ is the ith element of $\hat{\alpha}$ and $\hat{\alpha} = D\hat{\beta}$.

2.4. Kibria (2003) Tests

Kibria (2003) proposed estimators for k based on arithmetic mean (AM), geometric mean (GM) and median of $\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$.

2.4.1. Test based on AM

To test (2) vs. (3), the test statistic (denoted by t_{AM}) is defined in (5), where

$$\hat{k}_{AM} = \frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}.$$

2.4.2. Test based on GM

To test (2) vs. (3), the test statistic (denoted by t_{GM}) is defined in (5), where

$$\hat{k}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p}\hat{\alpha}_i^2)^{\frac{1}{p}}}.$$

2.4.3. Test ased on Median

To test (2) vs. (3), the test statistic (denoted by t_{MED}) is defined in (5), where

$$\hat{k}_{MED} = \text{Median}\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right\}.$$

2.5. Alkhamisi, Kalaf and Shukur (2006) Test

To test (2) vs. (3), the test statistic (denoted by t_{AKS}) is defined in (5), where

$$\hat{k}_{max}^{KS} = \max\left(\frac{\lambda_i \hat{\sigma}_i^2}{(n-p)\hat{\sigma}_i^2 + \lambda_i \hat{\alpha}_i^2}\right).$$

2.5. Muniz and Kibria (2009) Tests

2.5.1. KM2 Test

To test (2) vs. (3), the test statistic (denoted by t_{KM2}) is defined in (5), where

$$\hat{k}_{kM2} = \max\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right).$$

2.5.2. KM3 Test

To test (2) vs. (3), the test statistic (denoted by t_{KM3}) is defined in (5), where

$$\hat{k}_{KM3} = \max\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right)$$

2.5.3. KM5T Test

To test (2) vs. (3), the test statistic (denoted by t_{KM5}) is defined in (5), where

$$\hat{k}_{KM5} = \left(\prod_{i=1}^{p} \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)^{\frac{1}{p}}.$$

2.6. Muniz, Kibria, Mansson and Shukur (2012) tests

2.6.1. KM8 Test

To test (2) vs. (3), the test statistic (denoted by t_{KM8}) is defined in (5), where

$$\hat{k}_{KM8} = \max\left(\frac{1}{q_i}\right),\ q_i = \frac{t_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + t_{max}\hat{\alpha}_i^2}.$$

2.6.2. KM9 Test

To test (2) vs. (3), the test statistic (denoted by t_{KM9}) is defined in (5), where

$$\hat{k}_{KM9} = \max(q_i),\ q_i = \frac{t_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + t_{max}\hat{\alpha}_i^2}.$$

2.7. Nomura (1988) Test

To test (2) vs. (3), the test statistic (denoted by t_{HMO}) is defined in (4), where

$$\hat{k}_{HMO} = p\hat{\sigma}^2 / \sum_{i=1}^{p}\left[\hat{\alpha}_i^2 / \left\{1 + \left(1 + \lambda_i(\hat{\alpha}_i^2/\hat{\sigma}^2)^{\frac{1}{2}}\right)\right\}\right].$$

2.8. Crouse, Jin and Hanumara (1995) Test

To test (2) vs. (3), the test statistic (denoted by t_{CJH}) is defined in (5), where

$$\hat{k}_{CJH} = \begin{cases} \dfrac{p\hat{\sigma}^2}{\left(\hat{\beta}_{OLS} - J\right)'\left(\hat{\beta}_{OLS} - J\right) - \hat{\sigma}^2 tr(X'X)^{-1}} & \text{if } \left(\hat{\beta}_{OLS} - J\right)'\left(\hat{\beta}_{OLS} - J\right) > \hat{\sigma}^2 tr(X'X)^{-1} \\ \dfrac{p\hat{\sigma}^2}{\left(\hat{\beta}_{OLS} - J\right)'\left(\hat{\beta}_{OLS} - J\right)}, & \text{otherwise} \end{cases}$$

where the estimated value of J will be $\hat{J} = (\sum_{i=1}^{p}\beta_i)I_p$ and tr(A) is the trace of the matrix A..

2.9. Feras and Gore (2009) Test

To test (2) vs. (3), the test statistic (denoted by t_{CJH}) is defined in (5), where

$$\hat{k}_{FG} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}[\hat{\alpha}_i^2 / [\left(\frac{\hat{\alpha}_i^4\lambda_i^2}{4\hat{\sigma}^4} + \frac{6\hat{\alpha}_i^2\lambda_i}{\hat{\sigma}^2}\right)^{\frac{1}{2}} - \frac{\lambda_i\hat{\alpha}_i^2}{2\hat{\sigma}^2}]]}.$$

2.10. Kibria and Banik (2016) test

To test (2) vs. (3), the test statistic (denoted by t_{KB3}) is defined in (5), where

$$\hat{k}_{KB3} = \text{Max}(\hat{k}_{GM}, \hat{k}_{MED}, \hat{k}_{KM3}, \hat{k}_{HMO}, \hat{k}_{CJH}, \hat{k}_{FG}).$$

Since a theoretical comparison among the above test statistics is not possible, a simulation study has been conducted to compare the performance of the test statistics in section follow.

3. Simulation study

Our interest is to compare the selected tests for testing (2) vs. (3) w.r.t their sizes and powers under different settings of the regression model (1) by a simulation study. The simulation design and simulation results are discussed in this section:

3.1. Simulation Design

MATLAB 2014 programming codes are used for all calculations of this paper. We consider sample sizes n=15, 30, 50, 80 and 100, the number of regressors p = 4, 6, 8, 10 and 25 and the standard deviation of the error term is chosen as σ=1. To see the effects of multicollinearity by stating the correlation matrix among the regressors, we assume ρ=0.6, 0.7, 0.8, 0.9 and 0.95. An n×p matrix X is created as $H\Lambda^{0.5}G'$, where H is any (n×p) matrix whose columns are orthogonal and Λ is the diagonal matrix of eigenvalues of the correlation matrix and G is the matrix of normalised eigenvectors of the correlation matrix respectively. The study is supported on the most favourable (MF) and least favourable (LF) directions of β of (1) by the normalised eigenvectors corresponding to the largest and smallest eigenvalues of X′X respectively. For a detailed explanations of MF and LF directions of β and whole the simulation procedure, please visit Halawa and Bassiouni (2000, pp. 346-348).

To estimate the 5% nominal size (α=0.05) for testing (2) vs (3) under different conditions of (1), 5000 pseudo random vectors from $N(0,\sigma^2)$ are created to compute the error term in (1). Without loss of any generality, we let zero intercept for (1). Under the null model, substituting i^{th} elements of the considered MF and LF directions of β by zero, model (1) is used to find 5000 simulated vectors of y. The estimated sizes are computed as the percentage of times of the absolute values of all selected test statistics greater than the critical value of $t_{0.025, (n-p-1)}$.

To estimate the power of the i^{th} components of β under both MF and LF directions, β_i is repeatedly placed by $J\omega(0)\sigma\beta_i$, J=1,2,3,...,40, where

$$\omega(0)= \sqrt{(1 + (p - 2)\rho)/(1 - \rho)(1 + (p - 1)\rho)}]$$

(see Halawa and Bassiouni (2000), pp.346-348). For each J, like size calculations, same error vectors are generated to calculate 5000 y vectors from the regression model (1) under the alternative model. The terms replaced β_i present a continuing shift which persist until the estimated power of t(0) achieves 0.85 or J attains its highest any occurs first. For detailed on the calculation of power and size of the test, we refer our readers to Halawa and Bassiouni (2000). The estimated powers are calculated like sizes as the percentage of times of the absolute values of all selected test statistics greater than the critical value of $t_{0.025, (n-p-1)}$. The estimated size of the test for different n, p, σ=1, most the favourable (MF) orientation and for ρ=0.60, 0.70, 0.80, 0.90 and 0.95 are presented in Tables 3.1 to 3.5 respectively and, least the favourable (LF) orientation and for ρ =0.60, 0.70, 0.80, 0.90 and 0.95 are presented in Tables 3.6 to 3.10 respectively.

3.2. Results discussion: Size of the tests

In Table 3.1, we have recorded the empirical size of the tests for given ρ=0.60 and different sample sizes and regressors and most the favourable (MF) orientation. For a better picture, estimated sizes are plotted in Figure 3.1a and Figure 3.1b for p = 4, 6 and 8 and p = 8 and 10 respectively. It is observed that except t_{GM}, t_{kM3}, t_{kM8}, t_{HMO}, t_{CHJ} and t_{K8} have sizes close to the 5% nominal level (see Figure 3.1a). From Figure 3.1b, it is noticeable that sizes of all tests are close to the 5% nominal level except t_{kM8}, t_{HMO} and t_{KB} tests specially when p = 4. For large sample sizes n = 80 and n=100 (see Table 3.1), the t_{KB} test performed the best as compare to other tests in the sense of attaining 5% nominal size. From Tables 3.2 to 3.5 and Figures 3.1a,b and 3.2, we observed the following:

(i) The tests t_{kM8}, t_{HMO} and t_{KB} has sizes observed close to the 5% nominal level when $\rho = 0.6$;
(ii) The tests t_{kM8} and t_{KB} has sizes observed close to the 5% nominal level when $\rho = 0.7$;

(iii) The tests t_{kM8}, t_{HMO} and t_{KB} has sizes observed close to the 5% nominal level when ρ=0.8;

(iv) The tests $t_{(0)}$, t_{HK}, t_{HSL}, t_{AKS}, t_{HMO} and t_{KB} has sizes observed close to the 5% nominal level when $\rho = 0.9$;

(v) The tests $t_{(0)}$, t_{AKS}, t_{HMO}, t_{FG} and t_{KB} has sizes observed close to the 5% nominal level when $\rho = 0.95$.

From Figure 3.2, it appears that for large or given n, as the value of ρ increases, the size of the test get closer to the 5% nominal level.

We have tabulated estimated sizes for least favourable (LF) orientation for ρ = 0.60, 0.70, 0.80, 0.90 and 0.95 in Table 3.6 to Table 3.10 respectively. It is observed from these tables, performances of the estimated sizes of the tests at the 5% nominal level are almost close to the 5% nominal level for MF orientation. Overall, our simulation results show that in all considered cases, t_{HK}, t_{HKB}, t_{HSL}, t_{AM}, t_{AKS}, t_{FG} and t_{KB} are in general acceptable that means have sizes close to the 5% nominal level.

Table 3.1: Simulated sizes of considered tests for σ=1 and ρ = 0.60 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0598	0.0550	0.0584	0.0626	0.0658	0.1008	0.0830
t_{HK}	0.0580	0.0534	0.0564	0.0620	0.0656	0.1002	0.0824
t_{HKB}	0.0570	0.0522	0.0540	0.0604	0.0644	0.0994	0.0814
t_{HSL}	0.0568	0.0522	0.0548	0.0600	0.0638	0.0990	0.0816
t_{AM}	0.0570	0.0522	0.0540	0.0604	0.0644	0.0994	0.0814
t_{GM}	0.0452	0.0450	0.0442	0.0660	0.0674	0.0972	0.0790
t_{MED}	0.0482	0.0486	0.0484	0.0672	0.0604	0.0982	0.0800
t_{AKS}	0.0598	0.0550	0.0584	0.0626	0.0658	0.1008	0.0830
t_{kM2}	0.0598	0.0550	0.0584	0.0626	0.0658	0.1008	0.0830
t_{kM3}	0.0486	0.0464	0.0464	0.0670	0.0604	0.0990	0.0806
t_{kM5}	0.0570	0.0536	0.0548	0.0616	0.0658	0.1006	0.0820

t_{kM8}	0.0312	0.0294	0.0314	0.0522	0.0640	0.0816	0.0656
t_{kM9}	0.0598	0.0550	0.0584	0.0626	0.0658	0.1008	0.0830
t_{HMO}	0.0282	0.0276	0.0260	0.0466	0.0562	0.0824	0.0628
t_{CHJ}	0.0452	0.0418	0.0430	0.0618	0.0632	0.0934	0.0740
t_{FG}	0.0578	0.0536	0.0544	0.0616	0.0656	0.1006	0.0822
t_{KB}	0.0224	0.0212	0.0208	0.0424	0.0506	0.0784	0.0588

Note: ρ = correlation coefficient, p = No. of explanatory variables

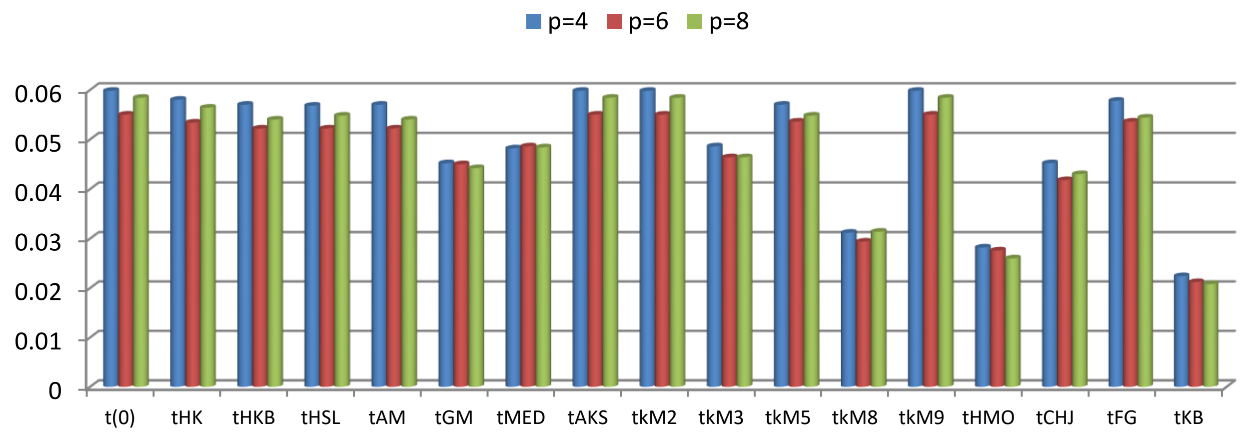

Figure 3.1a: Empirical sizes for various considered values of p when random sample size n=30

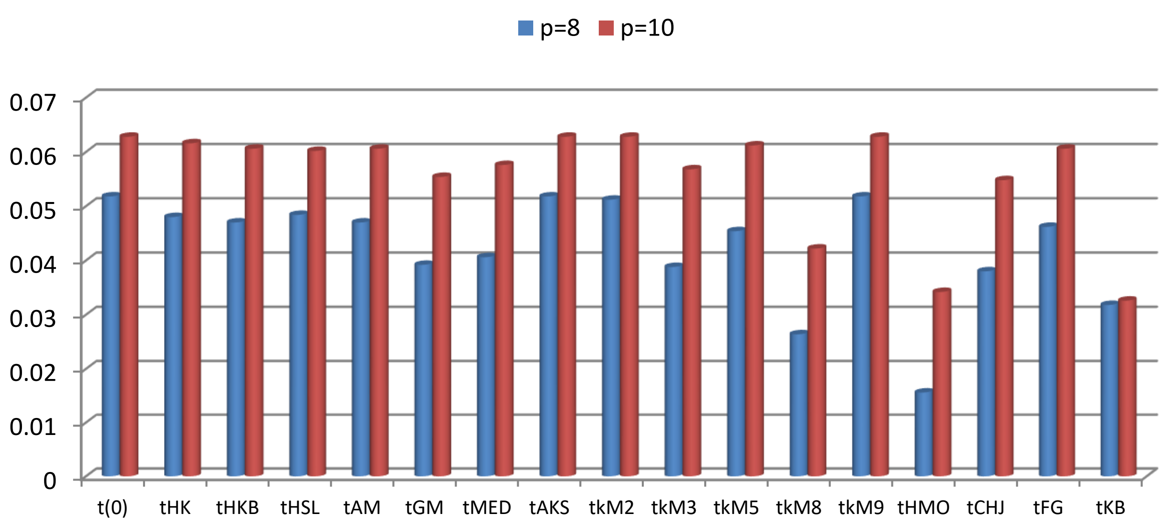

Figure 3.1b: Empirical sizes for various considered values of p when random sample size n=50

Table 3.2: Simulated sizes of considered tests for σ=1 and ρ = 0.70 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0634	0.0624	0.0656	0.0652	0.0696	0.0684	0.0620
t_{HK}	0.0604	0.0586	0.0632	0.0638	0.0686	0.0678	0.0608

t_{HKB}	0.0608	0.0584	0.0618	0.0624	0.0668	0.0660	0.0698
t_{HSL}	0.0596	0.0590	0.0606	0.0614	0.0656	0.0648	0.0680
t_{AM}	0.0608	0.0584	0.0618	0.0624	0.0668	0.0660	0.0698
t_{GM}	0.0488	0.0466	0.0518	0.0682	0.0612	0.0612	0.0626
t_{MED}	0.0510	0.0504	0.0550	0.0602	0.0640	0.0638	0.0658
t_{AKS}	0.0634	0.0624	0.0656	0.0652	0.0696	0.0684	0.0620
t_{kM2}	0.0634	0.0620	0.0652	0.0652	0.0696	0.0684	0.0620
t_{kM3}	0.0498	0.0476	0.0518	0.0696	0.0638	0.0622	0.0662
t_{kM5}	0.0610	0.0588	0.0616	0.0642	0.0686	0.0666	0.0616
t_{kM8}	0.0348	0.0296	0.0340	0.0530	0.0572	0.0504	0.0580
t_{kM9}	0.0634	0.0624	0.0656	0.0652	0.0696	0.0684	0.0620
t_{HMO}	0.0388	0.0350	0.0388	0.0572	0.0622	0.0510	0.0598
t_{CHJ}	0.0508	0.0474	0.0498	0.0646	0.0672	0.0580	0.0688
t_{FG}	0.0616	0.0600	0.0630	0.0642	0.0686	0.0670	0.0618
t_{KB}	0.0310	0.0270	0.0328	0.0526	0.0564	0.0466	0.0530

Note: ρ = correlation coefficient, p = No. of explanatory variables

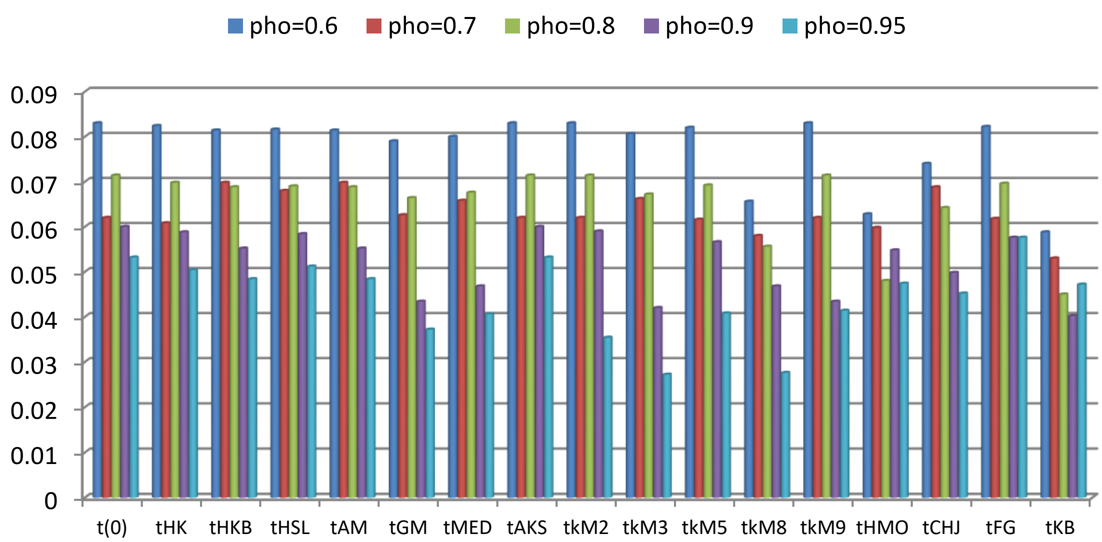

Figure 3.2: Empirical sizes for various considered values of ρ when random sample size n=100

Table 3.3: Simulated sizes of considered tests for σ=1 and ρ = 0.80 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0492	0.0472	0.0476	0.0518	0.0628	0.0624	0.0714
t_{HK}	0.0462	0.0446	0.0450	0.0480	0.0616	0.0612	0.0698
t_{HKB}	0.0450	0.0448	0.0444	0.0470	0.0606	0.0610	0.0688

t_{HSL}	0.0472	0.0456	0.0460	0.0484	0.0602	0.0614	0.0690
t_{AM}	0.0450	0.0448	0.0444	0.0470	0.0606	0.0610	0.0688
t_{GM}	0.0358	0.0354	0.0348	0.0392	0.0554	0.0574	0.0664
t_{MED}	0.0382	0.0396	0.0390	0.0406	0.0576	0.0582	0.0676
t_{AKS}	0.0492	0.0472	0.0476	0.0518	0.0628	0.0624	0.0714
t_{kM2}	0.0486	0.0466	0.0470	0.0512	0.0628	0.0624	0.0714
t_{kM3}	0.0366	0.0352	0.0344	0.0388	0.0568	0.0600	0.0672
t_{kM5}	0.0442	0.0446	0.0436	0.0454	0.0612	0.0618	0.0692
t_{kM8}	0.0234	0.0226	0.0240	0.0264	0.0422	0.0498	0.0556
t_{kM9}	0.0492	0.0472	0.0476	0.0518	0.0628	0.0624	0.0714
t_{HMO}	0.0164	0.0134	0.0154	0.0156	0.0342	0.0404	0.0480
t_{CHJ}	0.0380	0.0348	0.0382	0.0380	0.0548	0.0566	0.0642
t_{FG}	0.0448	0.0450	0.0438	0.0462	0.0606	0.0618	0.0696
t_{KB}	0.0126	0.0104	0.0122	0.0318	0.0326	0.0366	0.0450

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.4: Simulated sizes of considered tests for σ=1 and ρ = 0.90 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0476	0.0392	0.0574	0.0548	0.0538	0.0600	0.0600
t_{HK}	0.0422	0.0330	0.0534	0.0504	0.0498	0.0572	0.0588
t_{HKB}	0.0408	0.0288	0.0532	0.0480	0.0432	0.0558	0.0552
t_{HSL}	0.0434	0.0346	0.0546	0.0508	0.0488	0.0568	0.0584
t_{AM}	0.0408	0.0288	0.0532	0.0480	0.0432	0.0558	0.0552
t_{GM}	0.0248	0.0110	0.0458	0.0372	0.0266	0.0486	0.0434
t_{MED}	0.0334	0.0390	0.0504	0.0396	0.0310	0.0500	0.0468
t_{AKS}	0.0474	0.0378	0.0574	0.0548	0.0536	0.0600	0.0600
t_{kM2}	0.0398	0.0168	0.0550	0.0510	0.0476	0.0590	0.0590
t_{kM3}	0.0168	0.0028	0.0424	0.0314	0.0182	0.0460	0.0420
t_{kM5}	0.0350	0.0202	0.0524	0.0456	0.0392	0.0554	0.0566
t_{kM8}	0.0180	0.0054	0.0402	0.0344	0.0300	0.0462	0.0468
t_{kM9}	0.0470	0.0354	0.0574	0.0548	0.0300	0.0600	0.0434
t_{HMO}	0.0376	0.0250	0.0524	0.0456	0.0398	0.0544	0.0548
t_{CHJ}	0.0272	0.0194	0.0472	0.0398	0.0336	0.0506	0.0498
t_{FG}	0.0376	0.0242	0.0534	0.0470	0.0426	0.0558	0.0576
t_{KB}	0.0444	0.0522	0.0506	0.0488	0.0466	0.0434	0.0402

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.5: Simulated sizes of considered tests for σ=1 and ρ = 0.95 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0442	0.0408	0.0514	0.0476	0.0510	0.0512	0.0532
t_{HK}	0.0335	0.0308	0.0448	0.0420	0.0448	0.0494	0.0504
t_{HKB}	0.0370	0.0276	0.0458	0.0410	0.0410	0.0468	0.0484
t_{HSL}	0.0433	0.0332	0.0486	0.0454	0.0454	0.0494	0.0512
t_{AM}	0.0375	0.0276	0.0458	0.0410	0.0410	0.0468	0.0484
t_{GM}	0.0181	0.0064	0.0338	0.0272	0.0224	0.0386	0.0372
t_{MED}	0.0316	0.0154	0.0408	0.0324	0.0262	0.0424	0.0406
t_{AKS}	0.0372	0.0330	0.0512	0.0460	0.0482	0.0510	0.0532
t_{kM2}	0.0045	0.0000	0.0278	0.0214	0.0134	0.0378	0.0354
t_{kM3}	0.0052	0.0002	0.0260	0.0188	0.0098	0.0332	0.0272
t_{kM5}	0.0139	0.0030	0.0340	0.0282	0.0236	0.0408	0.0408
t_{kM8}	0.0036	0.0000	0.0252	0.0186	0.0092	0.0318	0.0276
t_{kM9}	0.0275	0.0098	0.0498	0.0444	0.2620	0.0506	0.0414
t_{HMO}	0.0332	0.0222	0.0442	0.0386	0.0372	0.0466	0.0474
t_{CHJ}	0.0293	0.0172	0.0386	0.0358	0.0344	0.0438	0.0452
t_{FG}	0.0376	0.0242	0.0534	0.0470	0.0426	0.0558	0.0576
t_{KB}	0.0544	0.0526	0.0501	0.0490	0.0468	0.0439	0.0406

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.6: Simulated sizes of considered tests for σ=1 and ρ = 0.60 for LF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0596	0.0554	0.0580	0.0623	0.0656	0.1009	0.0833
t_{HK}	0.0583	0.0538	0.0562	0.0621	0.0655	0.1000	0.0822
t_{HKB}	0.0575	0.0523	0.0543	0.0602	0.0643	0.0992	0.0812
t_{HSL}	0.0562	0.0524	0.0546	0.0603	0.0636	0.0993	0.0814
t_{AM}	0.0572	0.0520	0.0547	0.0601	0.0644	0.0994	0.0815
t_{GM}	0.0457	0.0457	0.0444	0.0668	0.0672	0.0970	0.0792
t_{MED}	0.0481	0.0483	0.0482	0.0676	0.0603	0.0981	0.0805
t_{AKS}	0.0590	0.0554	0.0581	0.0628	0.0654	0.1003	0.0831
t_{kM2}	0.0595	0.0552	0.0583	0.0628	0.0656	0.1006	0.0832
t_{kM3}	0.0489	0.0469	0.0465	0.0672	0.0600	0.0992	0.0804
t_{kM5}	0.0571	0.0535	0.0544	0.0613	0.0654	0.1004	0.0825
t_{kM8}	0.0315	0.0294	0.0318	0.0522	0.0643	0.0812	0.0653
t_{kM9}	0.0593	0.0553	0.0583	0.0625	0.0655	0.1006	0.0831
t_{HMO}	0.0280	0.0279	0.0264	0.0467	0.0560	0.0823	0.0624
t_{CHJ}	0.0450	0.0419	0.0432	0.0616	0.0631	0.0934	0.0743
t_{FG}	0.0579	0.0537	0.0543	0.0614	0.0655	0.1007	0.0821
t_{KB}	0.0226	0.0215	0.0207	0.0426	0.0503	0.0782	0.0584

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.7: Simulated sizes of considered tests for σ=1 and ρ = 0.70 for LF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0638	0.0622	0.0654	0.0650	0.0698	0.0684	0.0626
t_{HK}	0.0605	0.0584	0.0633	0.0638	0.0682	0.0675	0.0606
t_{HKB}	0.0607	0.0585	0.0616	0.0624	0.0663	0.0663	0.0698
t_{HSL}	0.0590	0.0593	0.0604	0.0615	0.0655	0.0646	0.0683
t_{AM}	0.0603	0.0582	0.0617	0.0623	0.0663	0.0665	0.0690
t_{GM}	0.0485	0.0464	0.0513	0.0680	0.0615	0.0614	0.0623
t_{MED}	0.0513	0.0502	0.0548	0.0601	0.0641	0.0636	0.0656
t_{AKS}	0.0637	0.0620	0.0650	0.0654	0.0693	0.0680	0.0625
t_{kM2}	0.0638	0.0623	0.0654	0.0656	0.0692	0.0684	0.0629
t_{kM3}	0.0490	0.0474	0.0515	0.0692	0.0635	0.0627	0.0660
t_{kM5}	0.0612	0.0587	0.0617	0.0643	0.0681	0.0668	0.0618
t_{kM8}	0.0346	0.0290	0.0343	0.0534	0.0573	0.0500	0.0583
t_{kM9}	0.0633	0.0626	0.0650	0.0657	0.0690	0.0683	0.0624
t_{HMO}	0.0387	0.0355	0.0383	0.0572	0.0623	0.0515	0.0590
t_{CHJ}	0.0504	0.0476	0.0494	0.0644	0.0670	0.0586	0.0687
t_{FG}	0.0613	0.0603	0.0632	0.0641	0.0683	0.0677	0.0616
t_{KB}	0.0315	0.0278	0.0327	0.0527	0.0563	0.0469	0.0535

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.8: Simulated sizes of considered tests for σ=1 and ρ = 0.80 for LF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0495	0.0475	0.0478	0.0516	0.0627	0.0626	0.0714
t_{HK}	0.0463	0.0443	0.0459	0.0483	0.0615	0.0617	0.0695
t_{HKB}	0.0454	0.0446	0.0440	0.0475	0.0603	0.0612	0.0688
t_{HSL}	0.0470	0.0456	0.0465	0.0483	0.0600	0.0614	0.0690
t_{AM}	0.0453	0.0447	0.0443	0.0473	0.0605	0.0615	0.0685
t_{GM}	0.0355	0.0356	0.0345	0.0399	0.0554	0.0576	0.0664
t_{MED}	0.0380	0.0390	0.0393	0.0405	0.0576	0.0580	0.0676
t_{AKS}	0.0493	0.0474	0.0474	0.0516	0.0627	0.0621	0.0712
t_{kM2}	0.0488	0.0467	0.0476	0.0518	0.0626	0.0626	0.0714
t_{kM3}	0.0369	0.0350	0.0346	0.0384	0.0568	0.0603	0.0672
t_{kM5}	0.0445	0.0446	0.0435	0.0455	0.0614	0.0615	0.0693
t_{kM8}	0.0237	0.0227	0.0244	0.0263	0.0425	0.0495	0.0556
t_{kM9}	0.0490	0.0475	0.0475	0.0516	0.0626	0.0623	0.0715
t_{HMO}	0.0166	0.0138	0.0156	0.0154	0.0345	0.0404	0.0480
t_{CHJ}	0.0388	0.0344	0.0380	0.0382	0.0547	0.0567	0.0645
t_{FG}	0.0446	0.0455	0.0436	0.0466	0.0605	0.0612	0.0696
t_{KB}	0.0129	0.0103	0.0127	0.0315	0.0326	0.0364	0.0453

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.9: Simulated sizes of considered tests for σ=1 and ρ = 0.90 for LF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0474	0.0393	0.0576	0.0546	0.0536	0.0607	0.0607
t_{HK}	0.0426	0.0334	0.0537	0.0505	0.0497	0.0574	0.0585
t_{HKB}	0.0405	0.0283	0.0536	0.0484	0.0434	0.0553	0.0554
t_{HSL}	0.0434	0.0344	0.0545	0.0506	0.0487	0.0565	0.0583
t_{AM}	0.0405	0.0286	0.0537	0.0484	0.0435	0.0557	0.0558
t_{GM}	0.0243	0.0113	0.0455	0.0376	0.0263	0.0482	0.0434
t_{MED}	0.0335	0.0394	0.0507	0.0398	0.0316	0.0505	0.0467
t_{AKS}	0.0473	0.0375	0.0578	0.0542	0.0534	0.0604	0.0608
t_{kM2}	0.0395	0.0166	0.0554	0.0515	0.0475	0.0590	0.0592
t_{kM3}	0.0164	0.0027	0.0426	0.0313	0.0180	0.0463	0.0423
t_{kM5}	0.0353	0.0208	0.0527	0.0458	0.0391	0.0552	0.0567
t_{kM8}	0.0185	0.0057	0.0408	0.0346	0.0307	0.0467	0.0463
t_{kM9}	0.0473	0.0354	0.0573	0.0547	0.0304	0.0603	0.0433
t_{HMO}	0.0374	0.0255	0.0528	0.0455	0.0392	0.0544	0.0545
t_{CHJ}	0.0275	0.0197	0.0475	0.0397	0.0337	0.0503	0.0490
t_{FG}	0.0376	0.0246	0.0537	0.0475	0.0424	0.0552	0.0575
t_{KB}	0.0445	0.0524	0.0508	0.0485	0.0464	0.0436	0.0403

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.10: Simulated sizes of considered tests for σ=1 and ρ = 0.95 for LF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t(0)	0.0445	0.0406	0.0515	0.0474	0.0515	0.0511	0.0536
t_{HK}	0.0336	0.0304	0.0447	0.0422	0.0448	0.0494	0.0504
t_{HKB}	0.0374	0.0277	0.0454	0.0415	0.0413	0.0468	0.0484
t_{HSL}	0.0433	0.03324	0.0482	0.0453	0.0454	0.0494	0.0512
t_{AM}	0.0375	0.0276	0.0457	0.0413	0.0410	0.0466	0.0484
t_{GM}	0.0187	0.0065	0.0332	0.0275	0.0224	0.0386	0.0377
t_{MED}	0.0315	0.0156	0.0406	0.0324	0.0262	0.0424	0.0406
t_{AKS}	0.0376	0.0333	0.0515	0.0462	0.0482	0.0510	0.0532
t_{kM2}	0.0049	0.0007	0.0273	0.0217	0.0134	0.0378	0.0354
t_{kM3}	0.0053	0.0006	0.0265	0.0182	0.0098	0.0333	0.0272
t_{kM5}	0.0135	0.0039	0.0348	0.0282	0.0234	0.0408	0.0408
t_{kM8}	0.0035	0.0005	0.0251	0.0188	0.0092	0.0318	0.0276
t_{kM9}	0.0277	0.0093	0.0497	0.0443	0.2620	0.0506	0.0414
t_{HMO}	0.0335	0.0226	0.0446	0.0384	0.0372	0.0466	0.0474
t_{CHJ}	0.0291	0.0175	0.0384	0.0352	0.0344	0.0439	0.0457
t_{FG}	0.0376	0.0246	0.0535	0.0475	0.0424	0.0558	0.0576
t_{KB}	0.0445	0.0521	0.0502	0.0483	0.0462	0.0439	0.0409

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.11. Simulated powers, maximum gain of chosen tests over $t(0)$ for $\sigma=1$ and $\rho = 0.60$ for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t_{HK}	0.2477	0.2366	0.2363	0.2288	0.2477	0.1450	0.2279
t_{HKB}	0.3173	0.3061	0.3101	0.2977	0.3197	0.1842	0.2972
t_{HSL}	0.3112	0.2980	0.3005	0.2869	0.3125	0.1817	0.2891
t_{AM}	0.3173	0.3061	0.3101	0.2977	0.3197	0.1842	0.2972
t_{GM}	0.3608	0.3467	0.3537	0.3322	0.3668	0.2032	0.3333
t_{MED}	0.3456	0.3330	0.3420	0.3213	0.3491	0.1974	0.3199
t_{AKS}	0.0003	0.0002	0.0003	0.0003	0.0000	0.0001	0.0004
t_{kM2}	0.0194	0.0177	0.0194	0.0186	0.0200	0.0116	0.0198
t_{kM3}	0.3414	0.3314	0.3383	0.3174	0.3499	0.1925	0.3145
t_{kM5}	0.2597	0.2532	0.2562	0.2448	0.2632	0.1509	0.2384
t_{kM8}	0.3718	0.3542	0.3618	0.3405	0.3764	0.2059	0.3431
t_{kM9}	0.0003	0.0002	0.0003	0.0003	0.0000	0.0001	0.0004
t_{HMO}	0.1398	0.1243	0.1216	0.1172	0.1688	0.0403	0.1140
t_{CHJ}	0.3643	0.3478	0.3542	0.3347	0.3695	0.2042	0.3368
t_{FG}	0.2813	0.2689	0.2744	0.2604	0.2821	0.1623	0.2582
t_{KB}	0.1436	0.1282	0.1260	0.1205	0.1734	0.0414	0.1168

Note: ρ = correlation coefficient, p = No. of explanatory variables

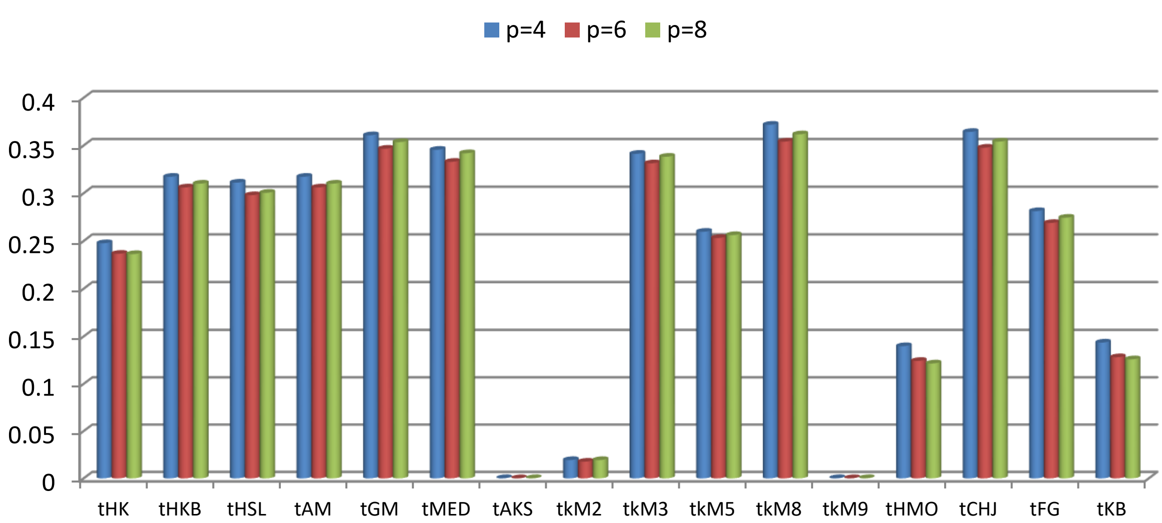

Figure 3.3a: Simulated powers, maximum gain of chosen tests over t(0) for selected values of p and n= 30

Table 3.12: Simulated powers, maximum gain of chosen tests over t(0) for σ=1 and ρ = 0.70 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t_{HK}	0.1695	0.1983	0.1965	0.1738	0.1684	0.1363	0.1732
t_{HKB}	0.2152	0.2485	0.2531	0.2206	0.2117	0.1712	0.2202
t_{HSL}	0.2165	0.2492	0.2541	0.2221	0.2141	0.1702	0.2226
t_{AM}	0.2152	0.2485	0.2531	0.2206	0.2117	0.1712	0.2202
t_{GM}	0.2339	0.2755	0.2798	0.2444	0.2315	0.1838	0.2415
t_{MED}	0.2284	0.2677	0.2712	0.2389	0.2265	0.1808	0.2355
t_{AKS}	0.0001	0.0003	0.0004	0.0002	0.0002	0.0003	0.0003
t_{kM2}	0.0199	0.0223	0.0216	0.0175	0.0179	0.0134	0.0187
t_{kM3}	0.2282	0.2690	0.2736	0.2394	0.2258	0.1790	0.2366
t_{kM5}	0.1889	0.2239	0.2255	0.1992	0.1882	0.1526	0.1951
t_{kM8}	0.2367	0.2780	0.2838	0.2472	0.2341	0.1848	0.2445
t_{kM9}	0.0001	0.0003	0.0004	0.0002	0.0002	0.0003	0.0003
t_{HMO}	0.1093	0.1152	0.1286	0.0895	0.0924	0.0634	0.1162
t_{CHJ}	0.2342	0.2740	0.2793	0.2429	0.2305	0.1832	0.2413
t_{FG}	0.1880	0.2222	0.2243	0.1987	0.1878	0.1522	0.1948
t_{KB}	0.1115	0.1175	0.1313	0.0913	0.0941	0.0643	0.1182

Note: ρ = correlation coefficient, p = No. of explanatory variables

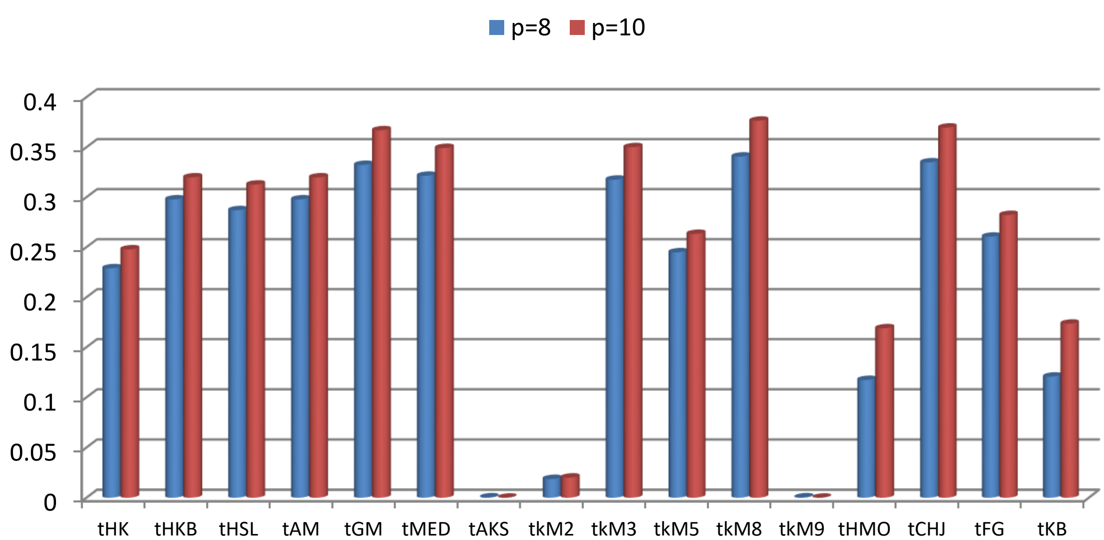

Figure 3.3b: Simulated powers, maximum gain of chosen tests over t(0) for selected values of p and n= 50

Table 3.13: Simulated powers, maximum gain of chosen tests over t(0) for σ=1 and ρ = 0.80 for MF orientation

Statistics	n:30			n=50		n=80	n=100
	p:4	6	8	8	10	10	25
t_{HK}	0.2273	0.1790	0.2160	0.2059	0.2283	0.2253	0.2055
t_{HKB}	0.2929	0.2312	0.2788	0.2674	0.2973	0.2934	0.2621
t_{HSL}	0.2850	0.2271	0.2735	0.2610	0.2902	0.2862	0.2576
t_{AM}	0.2929	0.2312	0.2788	0.2674	0.2973	0.2934	0.2621
t_{GM}	0.3306	0.2543	0.3157	0.2974	0.3375	0.3293	0.2918
t_{MED}	0.3186	0.2477	0.3030	0.2887	0.3221	0.3171	0.2834
t_{AKS}	0.0005	0.0003	0.0003	0.0002	0.0004	0.0001	0.0002
t_{kM2}	0.0185	0.0139	0.0174	0.0161	0.0201	0.0175	0.0151
t_{kM3}	0.3149	0.2414	0.2984	0.2826	0.3213	0.3093	0.2773
t_{kM5}	0.2403	0.1889	0.2266	0.2195	0.2383	0.2367	0.2135
t_{kM8}	0.3374	0.2591	0.3214	0.3027	0.3468	0.3368	0.2975
t_{kM9}	0.0005	0.0003	0.0003	0.0002	0.0004	0.0001	0.0002
t_{HMO}	0.1125	0.0871	0.1157	0.0969	0.1171	0.1176	0.0904
t_{CHJ}	0.3317	0.2549	0.3167	0.2982	0.3402	0.3313	0.2936
t_{FG}	0.2585	0.2017	0.2433	0.2335	0.2571	0.2549	0.2310
t_{KB}	0.1159	0.0891	0.1177	0.0995	0.1209	0.1210	0.0929

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.14: Simulated powers, maximum gain of chosen tests over t(0) for σ=1 and ρ = 0.90 for MF orientation

Statistics	n:15	15	30	30	30	50	50
	p:4	7	4	7	10	7	10
t_{HK}	0.1991	0.0674	0.2471	0.154	0.0842	0.1718	0.1058
t_{HKB}	0.0692	0.0214	0.0942	0.0383	0.0258	0.0485	0.0305
t_{HSL}	0.1259	0.0827	0.1155	0.0449	0.0885	0.0371	0.0391
t_{AM}	0.1401	0.1056	0.1338	0.0518	0.2187	0.0426	0.1440
t_{GM}	0.1457	0.1145	0.1439	0.0521	0.1280	0.0431	0.0442
t_{MED}	0.1451	0.1142	0.1419	0.0528	0.2280	0.0437	0.1442
t_{AKS}	0.0116	0.0584	0.0139	0.0153	0.0343	0.0055	0.0079
t_{kM2}	0.1036	0.1142	0.1233	0.0521	0.1287	0.0428	0.0442
t_{kM3}	0.1458	0.1145	0.1440	0.0524	0.1280	0.0431	0.0446
t_{kM5}	0.1456	0.1149	0.1432	0.0529	0.1283	0.0434	0.0442
t_{kM8}	0.1454	0.1147	0.1440	0.0520	0.1286	0.0431	0.0448
t_{kM9}	0.0118	0.0871	0.0141	0.0164	0.0404	0.0055	0.0082
t_{HMO}	0.1437	0.1122	0.1389	0.0523	0.1263	0.0433	0.0442
t_{CHJ}	0.1450	0.1103	0.1410	0.0524	0.1243	0.0429	0.0443
t_{FG}	0.1442	0.1136	0.1400	0.0527	0.1273	0.0431	0.0441
t_{KB}	0.1458	0.1145	0.1440	0.0529	0.1280	0.0439	0.0442

Note: ρ = correlation coefficient, p = No. of explanatory variables

Table 3.15: Simulated powers, maximum gain of chosen tests over t(0) for σ=1 and ρ = 0.95 for MF orientation

Statistics	n:15	15	30	30	30	50	50
	p:4	7	4	7	10	7	10
t_{HK}	0.2513	0.0572	0.2970	0.1233	0.0600	0.1542	0.0981
t_{HKB}	0.0690	0.0219	0.0941	0.0384	0.0258	0.0484	0.0308
t_{HSL}	0.1427	0.1190	0.1414	0.0541	0.0859	0.0561	0.0552
t_{AM}	0.1606	0.1589	0.1583	0.0603	0.1991	0.0658	0.1667
t_{GM}	0.1696	0.1823	0.1671	0.0605	0.1002	0.0674	0.0682
t_{MED}	0.1690	0.1821	0.1666	0.0605	0.2002	0.0670	0.1682
t_{AKS}	0.0997	0.1243	0.0956	0.0485	0.0769	0.0367	0.0384
t_{kM2}	0.1690	0.1823	0.1679	0.0607	0.1002	0.0670	0.0682
t_{kM3}	0.1696	0.1829	0.1671	0.0605	0.1006	0.0677	0.0685
t_{kM5}	0.1693	0.1827	0.1676	0.0609	0.1002	0.0670	0.0682
t_{kM8}	0.1696	0.1823	0.1671	0.0605	0.1008	0.0678	0.0685
t_{kM9}	0.1201	0.1821	0.1172	0.0607	0.1001	0.0454	0.0525
t_{HMO}	0.1658	0.1765	0.1627	0.0605	0.1003	0.0667	0.0679
t_{CHJ}	0.1676	0.1720	0.1643	0.0604	0.0999	0.0666	0.0677
t_{FG}	0.1688	0.1808	0.1663	0.0605	0.1002	0.0675	0.0681
t_{KB}	0.1696	0.1823	0.1671	0.0605	0.1002	0.0670	0.0682

Note: ρ = correlation coefficient, p = No. of explanatory variables

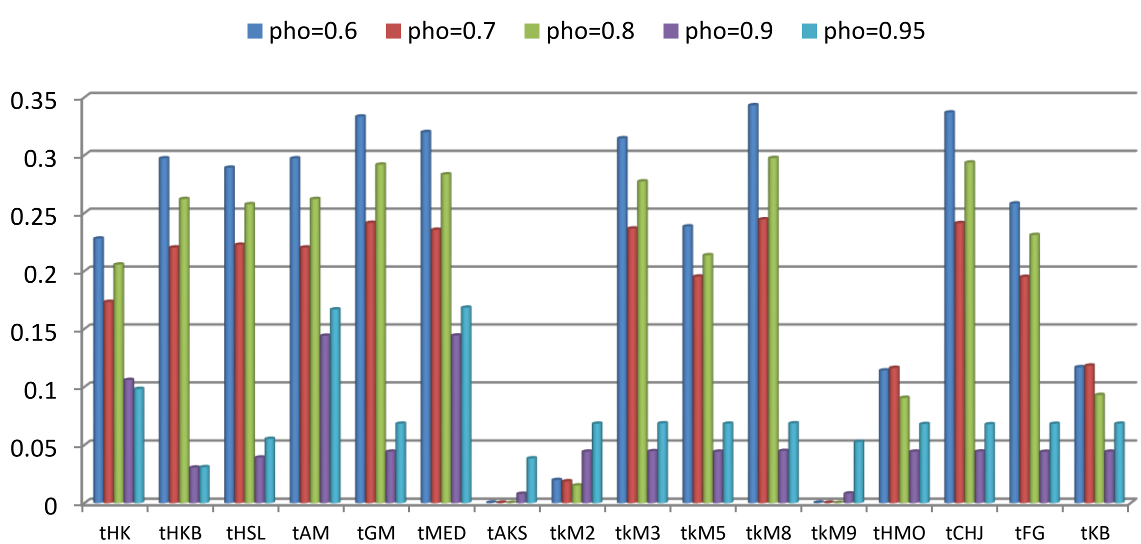

Figure 3.4: Simulated powers, maximum gain of chosen tests over t(0) for selected values of ρ and n= 100

4. Concluding Remarks

This paper considers several test statistics for testing the regression coefficients when multicolinearity exists in the linear regression model. A simulation study has been conducted to compare the performance of the test statistics based on the empirical size and power of the test. It is observed from our simulation study that the following test statistics, $t_{HK,}$ t_{HKB}, t_{HSL}, t_{AM}, t_{GM}, t_{MED}, t_{AKS}, t_{FG} and t_{KB} are in general acceptable as they have close to the 5% nominal level with high power. Overall we found that the substantial gain in powers of all the tests, except $t_{AKS,,}$ t_{KM2} and t_{KM9} over t(0). It is also noted that some of the proposed test statistics are performing better than the usual *t* test for linear regression coefficient and better than HK and HKB tests proposed by Halawa and Bassiouni (2000). We believe that the outcome of this research will be valuable assets for the researchers and practitioners in the area of engineering, physical, social and medical sciences.

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Dedication

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