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Modeling the Water-Energy-Food Nexus in ObR-E's: The Eight (8) Coordinates

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Abstract

The need to formulate quantifiers for water, energy and food (WEF) is necessary sequel to conservation issues worldwide. Existing methodologies on the WEF nexus appear less fitting in sustainability arguments because of incompleteness. This article analyzes the WEF nexus in open but restricted environments (ObR-E's) with completeness assumption in form of the known inter-intra dependence of nexus elements for sustainability and better conservation practice. The analysis leads to the discovery of the **Jalingo equation** whose any non simplistic solution is a solution to the WEF problem in some ObR-E's world wide. It is important to seek other non simplistic solutions for this equation under certain constraints known to affect WEF in ObR-E's of specialty.

Keywords: WEF; WEF nexus; ObR-E; ObR-E-Types; Jalingo equation

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1. Introduction

In Foran (2015), Chang et al. (2016) and most recently, Anne et al. (2018), the need to develop stronger models for the WEF nexus is motivated. Precisely, justification for a holistic consideration of the nexus in continuous time is provided. For completeness, emphasis on considering the natural inter and intra dependence of elements of the nexus is made; Hoff (2011), Bazilian (2011) and Foran (2015). In this respect, models with inter-intra assumption are needed at present because of functionality; Wolfe et al. (2016) and Tamee et al. (2018). This class of models uniquely can stand tall on challenges of adaptability posed by nexus elements because of completeness; Anne et al. (2018). A clear benefit is that derived measures are complete and therefore sustainable; Biggs et al. (2015). Admittedly, models with such per degree are most suitable for adoption in natural environments such as the open-but-restricted environment (ObR-E). On records, environments allowing the usage of water, energy and food without bounds amidst foreseen and unforeseen scarcities exist in the majority; United Nations (2014). We refer to these environments as ObR-E's.

Globally, ObR-E's are of varied forms depending on the rotation of the nexus relative to usage of elements; Harley et al. (2015). ObR-E-Type I evolves sequel to the rotation of the WEF nexus to usage of a single element (water/energy/food). This environment might have existed long time ago or exists at present during acute shortage periods of two elements. Additionally, it may be a future environment. Under low energy utilization, ObR-E-Type I could be theorized as Green-Naghdi environment; El-Karamany and Ezzat (2017). On the other hand, ObR-E-Type II evolves sequel to the rotation of the nexus to usage of two elements (water and energy/water and food/food and energy). This ObR-E appears to be the dominant form by geographical coverage at present. Most environments round the globe use more of two elements to date with methodologies on the use of the other element in progress. Finally, ObR-E-Type III evolves when the nexus rotates relative to usage of three elements (water, energy and food). In any form, research has shown that ObR-E's are known for misuse of restricted elements; Stephen et al. (2002), Hamed Al-H et al. (2010) and Chang et al. (2016).

Generally, the WEF nexus on ObR-E's especially ObR-E-Type II has been studied in the literature. For a survey, see Harley et al. (2015), Adongo and Cong (2016), Levy (2017) and more recently, Tamee et al. (2018). Clearly, even with the bulk of models for this nexus on ObR-E's, research on ObR-E-Type I is scarce in the literature. Additionally, developed models for ObR-E-Type II and ObR-E-Type III mostly are devoid of the inter-intra dependence assumption of elements. Moreover, models in this category are descriptive rather than analytic. We argue that such simplistic models cannot capture the true state of the nexus on any ObR-E because of incompleteness. Additionally, we claim that this incompleteness is the principal cause of issues of sustainability in development plans covering the nexus and low livelihood standards in ObR-E's globally today. Thus, there is need to provide enigmatic models on all forms of ObR-E's for conservation purposes.

This paper develops a model that predicts the state of the nexus relative to usage of any number of elements in ObR-E's in this respect. Most importantly, the governing equation for WEF dynamics in ObR-E's necessary for solving the global problem of water, energy and food is discovered and named the **Jalingo equation**. Additionally, basic properties of the said equation that guarantee smoothness of path to a sustainable WEF in ObR-E's are specified. We envisaged that the future of global WEF discourses on ObR-E's and ObR-E associated functions will depend to a large extent on the optimal solution of the said equation.

2. Basic Assumptions

Exponential usage times and Poisson arrival mass transits are assumed in time and space. We assumed further that the states of the three elements (water, energy and food) are accessible from one another with period unity (interdependence) and invariant measure P. Consequently, the three states form a naturally coupled communicating Markov chain as expected; see Aldawody et al. (2018). Suppose that each state in the nexus has a natural tendency for self communication (intra-dependence) during idle periods in addition to the busy period communication specified above. Then the communication process is complete. Under this condition, a given set of differential equations is satisfied; see Medhi (2003). Suppose further that state transitions take place from the idle state (state 0) of an element to its busy state (state 1) or that of another element (complex interactions) similar to the layer-media problem of Ezzat and El-Bary (2018) in a probability sense. Then, there are eight (8) coordinates needed for a complete discussion of the WEF nexus for a conserved ObR-E (Hammack (2013)). These coordinates are obtained by solving a system of coupled differential equations under the probabilistic laws of dependence and independence of generating events in both time and space. The following notations are adopted henceforth:

λ: Arrival rate of a mass of element of the nexus.
μ: Usage rate of elements in an ObR-E.
ρ: Utilization/Occupation rate of elements in an ObR-E.
i: State of Water anytime in an ObR-E.
j: State of Energy anytime in an ObR-E.
k: State of Food anytime in an ObR-E.
$P_{i,j,k}$: Stationary probability that water, energy and food are in states $i,\ j,\ k$ in ObR-E.
$P_{i,0,0}$: Stationary probability that water is in state $i(\neq 0)$, energy and food in idle states.
$P_{0,j,0}$: Stationary probability that energy is in state $j(\neq 0)$, water and food in idle states.
$P_{0,0,k}$: Stationary probability that food is in state $k(\neq 0)$, water and energy in idle states.
$P_{i,j,0}$: Stationary probability that water is in state $i(\neq 0)$, energy in state $j(\neq 0)$, and food in idle state.
$P_{i,0,k}$: Stationary probability that water in state $i(\neq 0)$, food in state $j(\neq 0)$, and energy in idle state.

$P_{0,j,k}$: Stationary probability that energy is in state $j(\neq 0)$, food in state $k(\neq 0)$, and water in idle state.

This work formulates analytic solution that predicts the needed eight (8) coordinates for a complete discussion and analysis of the WEF nexus in any ObR-E.

3. Balanced Equations and Analysis

Under the rate equality principle common to Poisson mass movements, let $\rho < 1$ so that the WEF system is stable. Then, the global balanced equations derived from associated Kolmogrov forward and backward differential equations for the WEF system are given by

$$\lambda P_{0,0,0} = \mu(P_{1,0,0} + P_{0,1,0} + P_{0,0,1}), \tag{1}$$

$$(\lambda + \mu)(P_{1,0,0} + P_{0,1,0} + P_{0,0,1}) = \lambda P_{0,0,0} + \mu(P_{1,1,0} + P_{1,0,1} + P_{0,1,1}), \tag{2}$$

$$(\lambda + \mu)(P_{1,1,0} + P_{1,0,1} + P_{0,1,1}) = \lambda(P_{1,0,0} + P_{0,1,0} + P_{0,0,1}) + \mu P_{1,1,1}. \tag{3}$$

Let $\rho = \frac{\lambda}{\mu}$. Combining (1), (2) and (3) coupled with the Markov property of the states; we have

$$P_{1,0,0} + P_{0,1,0} + P_{0,0,1} = \rho P_{0,0,0}, \tag{4}$$

$$P_{1,1,0} + P_{1,0,1} + P_{0,1,1} = \rho(P_{1,0,0} + P_{0,1,0} + P_{0,0,1}), \tag{5}$$

$$P_{1,1,1} = \rho(P_{1,1,0} + P_{1,0,1} + P_{0,1,1}). \tag{6}$$

The idle state probability $P_{0,0,0}$ is obtained from the normalization condition that

$$P_{0,0,0} + P_{1,0,0} + P_{0,1,0} + P_{0,0,1} + P_{1,1,0} + P_{1,0,1} + P_{0,1,1} + P_{1,1,1} = 1. \tag{7}$$

In view of (4), (5) and (6), upon applying basic properties of geometric series on (7) therefore,

$$P_{0,0,0} = \frac{1}{(1+\rho)^3}. \tag{8}$$

For a numerical illustration of the behavior of $P_{0,0,0}$ with the utilization parameter ρ, suppose that ρ varies from $0.001\ to\ 0.9999$. The following numerical results corresponding to $P_{0,0,0}$ for such ρ are obtained.

From Table 1, it can be seen that the stationary idle state probability $P_{0,0,0}$ values decrease with increase in the utilization rate ρ of the nexus as expected.

Lemma 3.1.

The $\{P_h : h = 1, 2, 3\}$ that the WEF nexus is in state h is given by

$$P_1 = \frac{\rho}{(1+\rho)^3}, \tag{9}$$

$$P_2 = \frac{\rho^2}{(1+\rho)^3}, \tag{10}$$

$$P_3 = \frac{\rho^3}{(1+\rho)^3}. \tag{11}$$

Table 1. $P_{0,0,0}$ for some ρ

ρ	$P_{0,0.0}$
0.0010	0.997005990
0.1250	0.702331961
0.2990	0.456218137
0.4753	0.311428943
0.8111	0.168334335
0.9000	0.145793847
0.9999	0.125018751

Proof:

This is consequence of (4), (5) and (6). ∎

Equations (9), (10) and (11) are strong equations for predicting integer-state based tendencies for the three elements combined anytime. For instance, (9) predicts the stationary tendency that either water, energy or food is utilized in an ObR-E. Unfortunately, (9) is less stronger in identifying which one of the three elements is indeed utilized. The same weakness applies for (10) and (11) respectively. Unfortunately, predicting impact at element levels are what is necessarily required for completeness. It is interesting to note that (11) and (6) are equivalent equations under free lunch assumption. Additionally, (4), (5) and (6) are coupled equations generated by complex interactions of elements in an ObR-E; Lindberg and Leflaive (2015) and Stijn et al. (2017). Therefore, it is only in rare cases closed form analytic solution exists for each component probability. This explains why incomplete analysis of the WEF nexus in ObR-E's dominates the literature. We seek to find a much more stronger solution for each component probability. We make the following proposition.

Proposition 3.2.

Suppose $\Im = \{(i, j, k), (l, m, n), (p, q, r)...\}$ denote a set of generalized coordinates for the WEF nexus in an ObR-E. Let $\oplus$ be a linear operator on $\Im$. Then $\oplus$ is that unique injector of $\Im$ to $\Im$.

Proof:

It suffices to show that for any two arbitrary coordinates $\Im_y, \Im_z \in \Im$, the function $\oplus$ such that $\oplus : \Im \to \Im$ is one to one. Consider $\Im_y, \Im_z$ coordinates in $\Im$. Define $\oplus$ such that

$$\Im_y \oplus \Im_z = \Im_{y+z}, \tag{12}$$

for some fixed states y and $z \in \Im$. Then, the proposition holds good. ∎

In view of Proposition 3.2 above, the complexity in solving the coupled system (4), (5) and (6) reduces to solving the three-parameter equation (4) a.s. Hence, (4) is called the **Jalingo equation** because of originality and uniqueness in solving the WEF nexus related problems in ObR-E's. This equation has the following properties:

1. It has infinitely many solutions to which the mean solution is simple.
2. Every non simple solution is a realistic solution.
3. A realistic solution (any feasible solution) generates a set of solutions for the entire WEF nexus in some ObR-E's including the optimal solution.

4. A Feasible Solution

Consider the unitary states whose tendencies are represented by the **Jalingo equation** above. Under the condition that water is energy and vice versa (Giulio (2014)). By extension, the same could be said for water and food. This implies the existence of at least a time point on the WEF nexus such that energy and food states are in the water state. Let this point be a chosen embedded point on the WEF nexus time axis relative to the states in (4) above. If one counts the number of angular rotations taking the unitary energy and food states of (4) to water state clockwise then, the said equation will have the representation that

$$\left(1 + \pi + \pi^2\right) P_{1,0,0} = \rho P_{0,0,0}. \tag{13}$$

In view of (8),

$$P_{1,0,0} = \frac{\rho}{(1+\rho)^3(1+\pi+\pi^2)}, \tag{14}$$

$$P_{0,1,0} = \frac{\pi\rho}{(1+\rho)^3(1+\pi+\pi^2)}, \tag{15}$$

$$P_{0,0,1} = \frac{\pi^2\rho}{(1+\rho)^3\,(1+\pi+\pi^2)}. \tag{16}$$

Additionally, by proposition 3.2,

$$P_{1,1,0} = \frac{\rho(\pi+1)}{(1+\rho)^3\,(1+\pi+\pi^2)}, \tag{17}$$

$$P_{1,0,1} = \frac{\rho(\pi^2+1)}{(1+\rho)^3\,(1+\pi+\pi^2)}, \tag{18}$$

$$P_{0,1,1} = \frac{\pi\rho(\pi+1)}{(1+\rho)^3\,(1+\pi+\pi^2)}. \tag{19}$$

5. Numerical Approximations

For a numerical consideration of the nexus relative to any ObR-E, we study the behavior of each $P_{i,j,k}$ relative to the utilization coefficient $\rho \in (0, 1)$ leading to a complete discussion and analysis. The following numerical results are obtained.

Table 2. $P_{i,j,k}$ for some ρ in ObR-E Type I

ρ	$P_{1,0,0}$	$P_{0,1,0}$	$P_{0,0,1}$
0.0010	0.000071229	0.000005508	0.000991467
0.1250	0.000704904	0.000485012	0.087301243
0.2990	0.012646712	0.000753622	0.135657349
0.4753	0.015585902	0.000817815	0.147200031
0.8111	0.017648764	0.000754350	0.135782257
0.9000	0.017793446	0.000724923	0.130490012
0.9999	0.017842872	0.000690621	0.1243155768

Table 3. $P_{i,j,k}$ for some ρ in ObR-E Type II

ρ	$P_{1,1,0}$	$P_{1,0,1}$	$P_{0,1,1}$
0.0010	0.000005539	0.000991498	0.0009970101
0.1250	0.000487723	0.087306467	0.087790012
0.2990	0.000757810	0.135655604	0.136411002
0.4753	0.000822321	0.147204411	0.148021101
0.8111	0.000758511	0.135781661	0.136531101
0.9000	0.000728950	0.130489543	0.131214321
0.9999	0.000694460	0.124315628	0.125001234

Table 4. $P_{i,j,k}$ for ρ in ObR-E Type III

ρ	$P_{1,1,1}$
0.0010	0.000002992
0.1250	0.034290020
0.2990	0.134601243
0.4753	0.244500123
0.8111	0.422100137
0.9000	0.460611224
0.9999	0.500000000

6. Discussion and Scope

Remark 6.1.

ObR-E's are equivalent environments only under low occupation rate of the WEF nexus. This is clear from tables 2, 3 and 4 above. One can see that if the occupation rate ρ of the nexus is in the neighborhood of $(0, 0.001]$, the stationary $P_{i,j,k}$ values do not significantly differ in all the three ObR-E's. Consequently, it can be concluded that within such spheres of occupation rates of the WEF nexus, the three ObR-E's specified above are similar environments. On the other hand, if $\rho \to 1$, the independence of each ObR-E becomes more pronounced and distinct relative to the occupation rate of the nexus. This opens yet another discourse on the WEF nexus relative to the global issue of climate change in terms of financing.

Remark 6.2.

In ObR-E Type I, Water usage is significantly connected to energy usage only at low occupation rate of the nexus. Additionally, water usage is significantly connected to food usage only at high occupation rate of the nexus. This remark is clear from table 2 above. It can be seen that when ρ is in the neighborhood of $(0, 0.125]$, $P_{1,0,0}$ and $P_{0,1,0}$ values do not differ significantly from each other. This implies a strong connection between having water and having energy at the idle states of the other element in this respective ObR-E. On the other hand, as $\rho \to 1$ from below, the connection tendency between water and other elements moves in favor of food. It can be concluded then that in ObR-E-Type I, water usage is synonymous to food usage but not energy.

Remark 6.3.

In ObR-E-Type II, occupying water and food is similar to occupying energy and food. This is evident from table 3 above. One can see that if $\rho \to 1$ from below, the values of $P_{1,0,1}$ and $P_{0,1,1}$ are approximately the same. Consequently, the busy periods of the two states are equivalent in this ObR-E. Again, since occupying water and food is equivalent to occupying energy and food, a significant nexus in this regard is that of water and energy at idle and busy states of food in this ObR-E.

Remark 6.4.

In ObR-E Type III, high occupation rates implies high busy rate of the nexus. This is in view of the stationary values for $P_{1,1,1}$ relative to $\rho \in (0, 1)$ as in table 4.

7. Conclusion

In this article, water-energy-food nexus in ObR-E's is analyzed. The analysis led to the identification of the eight (8) coordinates required for a complete discussion of the nexus in ObR-E's. Additionally, a difference-differential equation named; the **Jalingo equation** whose any realistic solution is a solution to the lingering problem of water-energy and food in some ObR-E's is identified. Under the stability condition of the nexus in ObR-E's, a feasible solution for this equation together with associated coupled equations are presented. Finally, some numerical results and discussions for classified ObR-E's under various utilization rates of the nexus are given. There is a scope in understanding the operational research and management of several functions of finance, business, geography, environment and engineering in relation to the discovered equation of this work for optimization purposes.

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