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A Study of Transversely Isotropic Thermoelastic Beam with Green-Naghdi Type-II and Type-III Theories of Thermoelasticity

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Abstract

The present research deals with the study of transversely isotropic thermoelastic beam in the context of Green-Naghdi (GN) theory of thermoelasticity of Type-II and Type-III. The mathematical model is prepared for the thin beam in a closed form with the application of Euler Bernoulli beam theory. The Laplace Transform technique has been used to find the expressions for displacement component, lateral thermal moment, deflection and axial stress in transformed domain. The general algorithm of the inverse Laplace Transform is developed to compute the results numerically in physical domain. The effect of two theories of thermoelasticity Green-Naghdi-II and Green-Naghdi-III has been depicted on the various quantities. Some particular cases have also been deduced.

Keywords: Transversely Isotropic thermoelastic; Beam; Green-Naghdi theory of thermoelasticity of Type-II and Type-III; Lateral Deflection; Thermal Moment; Axial Stress

MSC 2010 No.: 74B15, 80A20, 74H15

1. Introduction

Beams as elementary structural components are usually used in bridges and beam-column system of constructions, piezoelectric devices, heat exchanger tubes in production equipment, titanium alloy artificial bones. Moreover, flying slender cylindrical structures like rockets, missiles are considered as a free-free beam to study the dynamical response and failure analysis under transient dynamical loads was given by Yu *et al.* (1996). Marin (1997) had proved the Cesaro means of the kinetic and strain energies of dipolar bodies with finite energy. Marin (1998) investigated and solved the initial-boundary value problem without recourse either to an energy conservation law or to any boundedness assumptions on the thermoelastic coefficients in thermoelastic bodies with voids. Bernoulli– Euler beam model is based on a modified couple stress theory studied by Park and Gao (2006). Sun *et al.* (2008) used Laplace transform technique to study the vibration occurrences due to pulsed laser heating of a microbeam under different boundary conditions. Marin (2010) discussed the problem of vibrations in thermoelasticity of dipolar bodies.

Thermoelastic beams with voids were studied by Li and Cheng (2010). Sharma (2011) investigated the governing equations of vibrations in a beam in a closed form based on the Euler-Bernoulli theory. The thermoelastic beams with voids was presented by Sharma and Grover (2011). Gupta (2011) investigated the propagation of waves in the transversely isotropic medium with GN theory of type-II and III. Ezzat *et al.* (2012) by considering the fractional order dual-phase-lag heat conduction law, built a mathematical model of two-temperature magneto-thermoelasticity. Zang and Fu (2012) established a new beam model for a viscoelastic micro-beam based on the modified couple stress theory.

The problem of thermoelastic damping in a micro-beam resonator by the modified couple stress theory was examined by Ghader *et al.* (2012). Guo *et al*. (2013) presented the problem of thermoelastic damping in vented MEMS beam with eigenvalue formulation and the Galerkin finite element method. Marin and Stan (2013) studied the micro stretch elastic bodies using Lesan and Quintanilla of dipolar bodies with stretch. Simsek and Reddy (2013) and Shaat *et al.* (2014) examined the bending and vibration of functionally graded microbeams using the modified couple stress theory and higher order beam theory. Allam and Abouelregal (2014) investigated the thermoelastic waves prompted by pulsed laser and varying heat of homogeneous microscale beam resonators using Laplace transform technique. Abouelregal and Zenkour (2014) discussed the problem of an axially moving microbeam and examined the effects of the pulse-width of thermal vibration, moving speed and the transverse excitation.

Sharma and Kaur (2014b) investigated the transverse vibrations in thermoelastic-diffusive thin micro beam based on Euler-Bernoulli theory under clamped-clamped boundary conditions. Zenkour and Abouelregal (2015) examined the problem of thermoelastic vibration of an axially moving microbeam subjected to sinusoidal pulse heating. Sharma *et al.* (2015) illustrated the two-dimensional deformation using Laplace and Fourier transforms in a homogeneous, transversely isotropic thermoelastic solids with two temperatures w.r.t. type-II Green-Naghdi. Deswal and Kalkal (2015) deal with the problem of thermo-viscoelastic interactions in a homogeneous, isotropic three-dimensional medium with surface suffers a time dependent thermal shock. The problem was treated based on three-phase-lag model with two temperatures.

Ezzat *et al.* (2015) discovered a new model of linear thermo-viscoelasticity for isotropic media considering the rheological properties of the volume with fractional relaxation operators. Ezzat *et al.* (2016) discussed a generalized model of two-temperature thermoelasticity theory with time-delay and Kernel function and Taylor theorem with memory-dependent derivatives involving two temperatures. Ezzat and El-Bary (2016) presented the mathematical model of fractional magneto-thermo-viscoelasticity for isotropic perfectly conducting media. Ezzat and El-Bary (2017) gave a unified mathematical model of phase-lag Green-Naghdi magneto-thermoelasticty theories depending upon fractional derivative heat transfer for perfectly conducting media in the presence of a constant magnetic field. Ezzat *et al*. (2017) defined a new mathematical model of two-temperature phase-lag Green–Naghdi thermoelasticty theories based on fractional derivative of heat transfer. Marin *et al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Kumar and Devi (2017) studied the vibrations in a homogeneous isotropic thin beam in modified couple stress theory and heat conduction equation for non-classical process in a closed form by employing the Euler Bernoulli beam theory. Despite of this several researchers as Kumar *et al.* (2017), Bhad *et al.* (2016), Tripathi *et al.* (2017) worked on different theory of thermoelasticity.

El-Karamany and Ezzat (2017) discussed the fractional phase-lag Green–Naghdi thermoelasticity theories with Maxwell–Cattaneo heat conduction law using the Caputo fractional derivative and the fractional order heat transport equation. Ezzat and El-Bary (2018) constructed an incorporated GN mathematical model of electro-thermoelasticity with consideration of heat conduction law with fractional order derivative. Ezzat *et al.* (2018) proposed *a* new mathematical model of generalized magneto-thermo-viscoelasticity theories with memory-dependent derivatives (MDD) of dual-phase-lag heat conduction law.

The present investigation deals with the problem of Transversely Isotropic thermoelastic beam in the context of GN-II and III Types theories of thermoelasticity. The Laplace Transform technique has been used to find the expressions for displacement component, lateral thermal moment, deflection and axial stress. The effect of two theories of thermoelasticity GN-II and GN-III has been depicted on the various quantities.

2. Basic Equations

Following Chandrasekharaiah (1998), Youssef (2011) and Green and Naghdi (1992), the constitutive relations and field equations for an anisotropic thermoelastic medium with GN theory of type-III in absence of body forces and heat sources are:

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\, \ddot{u}_i, \tag{2}$$

$$K_{ij}T_{,ij} + K^*_{ij}\dot{T}_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T}, \tag{3}$$

where

$$T = \varphi - a_{ij}\varphi_{,ij},$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \tag{4}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \tag{5}$$

$\beta_{ij} = \beta_i\delta_{ij},\ \ K_{ij} = K_i\delta_{ij},\ K^*_{ij} = K^*_i\delta_{ij}$, i is not summed, δ_{ij} is kronecker delta.

Here, $C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal elastic coupling tensor, T is the absolute temperature, T_0 is the reference temperature, φ is the conductive temperature, t_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, K^*_{ij} is the thermal conductivity, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

3. Formulation of the problem

Let us consider a homogeneous transversely isotropic rectangular thermoelastic thin beam (Figure 1) of length $(0 \le x \le L)$, width $\left(-\frac{b}{2} \le y \le \frac{b}{2}\right)$ and thickness $\left(-\frac{h}{2} \le z \le \frac{h}{2}\right)$, where x, y and z are the cartesian axes lying along the length, width and thickness of the beam so that x-axis coincides with the beam axis and y, z axis coincide with the end (x=0) with the origin located at the axis of the beam.

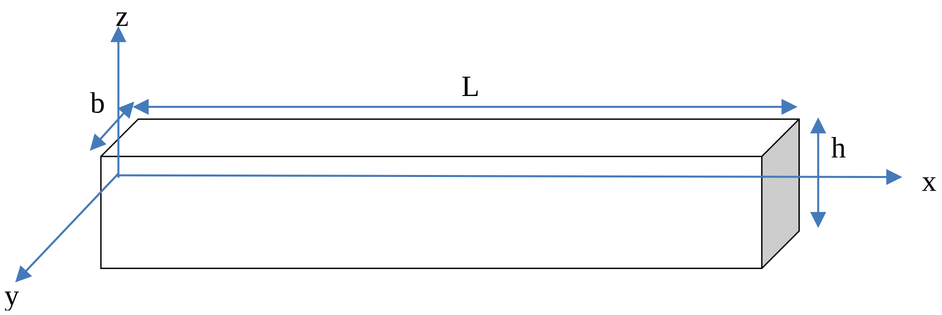

Figure 1. Considered design of the beam

The beam is assumed to be transversely isotropic in the sense that its mechanical and thermal properties are different along the thickness to that in a plane, transverse to it. In equilibrium, the beam is unstrained, unstressed and also kept at uniform temperature T_0 . Moreover, there is no flow of heat along the upper and lower surface of the beam so that

$$\frac{\partial T}{\partial z} = 0, \text{ at } z = \pm\frac{h}{2}. \tag{6}$$

and its axial ends are assumed to be at isothermal conditions. The beam undergoes bending vibrations of small amplitude about the x-axis such that the deformation is consistent with the linear Euler-Bernoulli theory. Thus, any plane cross-section initially perpendicular to the axis of the beam remains plane and perpendicular to the neutral surface during bending.

According to the fundamental Euler-Bernoulli theory for small deflection of a simple bending problem, the displacement components are given by Rao (2007)

$$u(x,y,z,t) = -z\frac{\partial w}{\partial x}, \qquad v(x,y,z,t) = 0, \qquad w(x,y,z,t) = w(x,t), \tag{7}$$

where $w(x,t)$ is the lateral deflection of the beam and t is the time.

The one dimensional constitutive equation obtained from equation (1) with the help of equation (7) becomes

$$t_{11} = -C_{11}z\frac{\partial^2 w}{\partial x^2} - \beta_1 T, \tag{8}$$

where $\beta_1 = (C_{11} + C_{13})\alpha_1 + C_{13}\alpha_3$ is the thermoelastic coupling parameter and α_1 , α_3 are the coefficient of linear thermal expansion along and perpendicular to plane of isotropy. The thermoelastic parameter $\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3$ along z-axis does not appear due to Euler-Bernoulli hypothesis.

The flexural moment of the cross-section of the beam following Sharma and Kaur (2014a) is given by

$$M(x,t) = -\int_{\frac{-h}{2}}^{\frac{h}{2}} b t_{11} z dz = C_{11} I \frac{\partial^2 w}{\partial x^2} + \beta_1 M_T \,, \tag{9}$$

where $M_T = b\int_{\frac{-h}{2}}^{\frac{h}{2}} Tz dz$ is the thermal moment of inertia of the beam and $\beta_1 M_T$ is the thermal moment of the beam. In addition, $I = \frac{bh^3}{12}$ is the moment of inertia of cross-section and $C_{11} I$ is the flexural rigidity of the beam.

The equation of transverse motion of the beam is given by Rao (2007)

$$\frac{\partial^2 M}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = q(x,t), \tag{10}$$

where $A = bh$ is the area of cross-section and $q(x,t)$ represents the load acting on the beam along the thickness direction. Using equation (9) in equation (10), we get

$$C_{11} I \frac{\partial^4 w}{\partial x^4} + \beta_1 \frac{\partial^2 M_T}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = q(x,t). \tag{11}$$

According to Lifshitz and Roukes (2000) no thermal gradient exists in the y-direction. Moreover, due to geometry, the thermal gradients in the plane of cross-section along the thickness direction of the beam are much larger than those along its axis and therefore, the heat conduction equation (1) under such situation with the help of displacement field becomes

$$\left(K_1 + K_1^* \frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2} + \left(K_3 + K_3^* \frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial z^2} = -\beta_1 T_0 \frac{\partial^4 w}{\partial x^2 \partial t^2} + \rho C_E \ddot{T}. \tag{12}$$

Multiplying by zdz and integrating from $-\frac{h}{2}$ to $\frac{h}{2}$

$$\left(K_1 + K_1^* \frac{\partial}{\partial t}\right)\frac{1}{\beta_1 b}\frac{\partial^2 M_T}{\partial x^2} + \left(K_3 + K_3^* \frac{\partial}{\partial t}\right)\left[\frac{h}{2}\frac{\partial T}{\partial z}\left(x, \frac{h}{2}, t\right) + \frac{h}{2}\frac{\partial T}{\partial z}\left(x, \frac{-h}{2}, t\right) + \frac{12}{\beta_1 b h^2} M_T(x,t)\right]$$
$$= -\beta_1 T_0 \frac{h^3}{12}\frac{\partial^4 w}{\partial x^2 \partial t^2} + \rho C_E \frac{\partial^2 M_T}{\partial t^2}\,. \tag{13}$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$x' = \frac{x}{L},\ z' = \frac{z}{h},\ w' = \frac{w}{h},\ t' = \frac{vt}{L},\ \beta_1^* = \frac{\beta_1 T_0}{C_{11}},\ M_T' = \frac{M_T}{T_0 A h}, T' = \frac{T}{T_0},\quad a_R = \frac{L}{h},$$
$$C_{11} = \rho v^2,\ q_1(x',t') = \frac{L^2}{C_{11} A h} q(x,t),\ t_x' = \frac{t_x}{C_{11}}. \tag{14}$$

Now applying the dimensionless quantities from (14) on equation (11) and (13), after, suppressing the prime we get

$$\frac{1}{12 a_R^2}\frac{\partial^4 w}{\partial x^4} + \beta_1^* \frac{\partial^2 M_T}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} = q_1(x,t), \tag{15}$$

$$\frac{1}{a_R^2\beta_1}\left(K_1+K_1^*\frac{v}{L}\frac{\partial}{\partial t}\right)\left[\frac{\partial^2 M_T}{\partial x^2}\right]+\left(K_3+K_3^*\frac{v}{L}\frac{\partial}{\partial t}\right)\left[\frac{\partial T}{\partial z}\left(x,\frac{h}{2},t\right)+\frac{\partial T}{\partial z}\left(x,\frac{-h}{2},t\right)\right]+\frac{L}{\beta_1}M_T$$
$$=-\beta_1\frac{h^3v^2}{12a_R^4}\frac{\partial^4 w}{\partial x^2\partial t^2}+\rho C_E\frac{v^2}{a_R^2}\frac{\partial^2 M_T}{\partial t^2}. \quad (16)$$

Let us take the Laplace transform defined by

$$\mathcal{L}[f(t)]=\int_0^\infty e^{-st}f(t)dt=\bar{f}(s). \quad (17)$$

By applying (17) on (15)-(16) we get

$$(a_1D^4+s^2)\bar{w}(x,s)+\beta_1^*D^2\bar{M}_T(x,s)=\bar{Q}(x,s), \quad (18)$$

$$[a_6(s)D^2-a_3s^2-\ a_5a_0(s)]\bar{M}_T(x,s)+a_4s^2D^2\bar{w}(x,s)=-\,a_0(s)\xi_1(s)\,, \quad (19)$$

where

$$D=\frac{d}{dx}\,,$$
$$\xi_1(s)=\frac{h}{2}\left[\frac{d\bar{T}}{dz}\left(x,\frac{h}{2},s\right)+\frac{d\bar{T}}{dz}\left(x,\frac{-h}{2},s\right)\right], \quad (20)$$
$$a_0(s)=K_3+K_3^*\frac{v}{L}s\,,$$
$$a_1=\frac{1}{12a_R^2}\,,$$
$$a_3=\frac{\rho C_E v^2}{\beta_1 b a_R^2}\,,$$
$$a_4=\frac{\beta_1 v^2 h^3}{12a_R^4}\,,$$
$$a_5=\frac{12}{bh^2\beta_1},$$
$$a_6\,(s)=\frac{1}{b\beta_1a_R^2}(K_1+K_1^*\frac{v}{L}\ s).$$

From equation (18) and (19), we get

$$[D^6-pD^4+qD^2-r]\bar{w}(x,s)=\Gamma_1\bar{Q}(x,s), \quad (21)$$

where

$$p=\frac{a_3s^2a_1+\beta_1^*a_4s^2+a_5a_0(s)a_1}{a_6(s)a_1},\ q=\frac{s^2}{a_1},r=\frac{a_3s^4+a_5a_0(s)s^2}{a_6(s)a_1},\Gamma_1=\frac{a_6(s)D^2-a_3s^2-a_5a_0(s)}{a_6(s)a_1}.$$

For simplification of solution let us take $\bar{Q}(x,s)=0$ i.e., load on beam is assumed to be zero. The differential equation governing the lateral deflection $\bar{w}(x,s)$, equation (21) can take the form

$$(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)(D^2 - \lambda_3^2)\bar{w}(x, s) = 0, \tag{22}$$

where $\pm\lambda_1, \pm\lambda_2$ *and* $\pm\lambda_3$ are the characteristics roots of the equation $\lambda^6 - p\lambda^4 + q\lambda^2 - r = 0$ and hence,

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = p,$$
$$\lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2 = q,$$
$$\lambda_1^2\lambda_2^2\lambda_3^2 = \mathrm{r},$$

where p, q, r are the sum of all the roots, sum of roots taken two at a time and product of all the roots respectively.

Let the lateral deflection $\bar{w}(x, s)$ is given by

$$\bar{w}(x, s) = \sum_{i=1}^{3}\left[A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right], \tag{23}$$

where A_i and $B_i, i = 1,2,3$ are the constant coefficients and are dependent on the Laplace variable s and the thermal moment is given by

$$[a_6 K_1 \lambda_i^2 + \Gamma_1]\bar{M}_T(x, s) - a_3 s^2 D^2 \bar{w}(x, s) = \Gamma_2,$$

By putting the value of $\bar{w}(x, s)$, and we get $\bar{M}_T(x, s)$ as

$$\bar{M}_T(x, s) = \frac{a_3 s^2 \lambda_i^2}{a_6 K_1 \lambda_1^2 + \Gamma_1}\sum_{i=1}^{3}\left[A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right],$$

$$\bar{M}_T(x, s) = -\sum_{i=1}^{3} \varsigma_i\left[A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right] + \Gamma'_2, \tag{24}$$

where

$$\Gamma_2 = -[K_3 + K_3^* a_1 s],$$
$$\varsigma_i = \frac{a_4 s^2 \lambda_i^2}{a_6(s)\lambda_i^2 - a_3 s^2 - a_5 a_0(s)},$$

$$\Gamma_2' = \frac{a_0(s)\xi_1(s)}{a_6(s) - a_3 s^2 - a_5 a_0(s)}.$$

Using (8) and (15) and with the aid of (24), the axial stress $\bar{t}_{11}(x, s)$ can be written as

$$\bar{t}_{11}(x, s) = R_i - \sum_{i=1}^{3}\left[A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right] N_i, \tag{25}$$

where

$$R_i = -\frac{12\beta_1^* a_0(s)\xi_1(s)}{b\beta_1 h^2\left(a_6(s) - a_3 s^2 - a_5 a_0(s)\right)},$$

$$\varrho_i = \frac{h\lambda_i^2}{a_R^2} + \frac{12\varsigma_i\beta_1^*}{b\beta_1 h^2}.$$

4. Applications

We will examine thermal loads over the upper surface of the beam. The constant heat flux (-q_0) is normal to the upper surface ($z = \frac{h}{2}$) of the beam and the bottom surface ($z = \frac{-h}{2}$) is at zero temperature gradient. The boundary condition on the upper and bottom surface, the heat conduction equation is

$$q_0 = K_3 \frac{\partial T}{\partial z}\left(x, \frac{h}{2}, t\right), \frac{\partial T}{\partial z}\left(x, \frac{-h}{2}, t\right) = 0. \tag{26}$$

Applying (14) and (17) on (26), we get

$$\frac{d\bar{T}}{dz}\left(x, \frac{h}{2}, s\right) = \frac{q_0}{K_3}, \frac{d\bar{T}}{dz}\left(x, \frac{-h}{2}, s\right) = 0. \tag{27}$$

5. Boundary equations

Mechanical and thermal conditions are:

$$w(0,t) = 0, \quad \frac{\partial^2 w(0,t)}{\partial x^2} = 0, M_T(0,t) = 0, \tag{28}$$

$$w(L,t) = 0, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = 0, M_T(L,t) = 0. \tag{29}$$

From (20) and (27), the thermal influence is given by

$$\xi_1(s) = \frac{hq_0}{2K_3}. \tag{30}$$

using the dimensionless quantities(14) and equation (17) in the boundary conditions (28) and (29), yields

$$\bar{w}(0,s) = 0, \quad \frac{\partial^2 \bar{w}(0,s)}{\partial x^2} = 0, \bar{M}_T(0,s) = 0, \tag{31}$$

$$\bar{w}(l,s) = 0, \quad \frac{\partial^2 \bar{w}(l,s)}{\partial x^2} = 0, \bar{M}_T(l,s) = 0. \tag{32}$$

Substituting the values of $\bar{w}$ and $\bar{M}_T$ from equation (23) and (24) in the boundary conditions (28) and (29), we obtain the value of A_i and B_i as

$$A_i = \frac{\Delta_i}{\Delta}, B_i = \frac{\Delta_{i+3}}{\Delta}, \ i = 1,2,3. \tag{33}$$

and

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} & e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} \\ \lambda_1^2 e^{\lambda_1} & \lambda_2^2 e^{\lambda_2} & \lambda_3^2 e^{\lambda_3} & \lambda_1^2 e^{\lambda_1} & \lambda_2^2 e^{\lambda_2} & \lambda_3^2 e^{\lambda_3} \\ \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_1 & \varsigma_2 & \varsigma_3 \\ \varsigma_1 e^{\lambda_1} & \varsigma_2 e^{\lambda_2} & \varsigma_3 e^{\lambda_3} & \varsigma_1 e^{\lambda_1} & \varsigma_2 e^{\lambda_2} & \varsigma_3 e^{\lambda_3} \end{vmatrix},$$

$\Delta_i (i = 1,2,3,\dots,6)$ are obtained by replacing the columns by $[0, \quad 0, \quad 0, \quad 0, \quad \Gamma_2', \quad \Gamma_2']$ in Δ_i.

6. Inversion of Laplace Transform

To find the solution of the problem in physical domain, we must invert the transforms in equations (23)-(25). These equations are functions of t, the parameter of Laplace transform s and hence, are of the form $\widetilde{f}(x,s)$. To get the function $f(x,t)$ in the physical domain, first we invert the Laplace transform using

$$f(x,t) = \frac{1}{2\pi i} \int_{e-i\infty}^{e+i\infty} \tilde{f}(x,s) e^{-st} ds. \tag{34}$$

The last step is to calculate the integral in equation (34). The method for evaluating this integral is described in Press *et al.* (1986), which uses Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Particular Cases

i. We obtain a transversely isotropic thermoelastic beam without energy dissipation i.e., GN-II theory if $K_{ij}^* = 0$,
ii. We obtain a transversely isotropic thermoelastic beam with and without energy dissipation i.e. GN-III theory if $K_{ij} \neq 0 \neq K_{ij}^*$,
iii. We obtain a transversely isotropic thermoelastic beam with the classical theory of thermoelasticity if we take $K_{ij} = 0$

8. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of frequency, we now present some numerical results. Cobalt material is chosen from Dhaliwal (1980) for the purpose of numerical calculation, which is transversely isotropic. Physical data for a single crystal of cobalt is given by:

$$c_{11} = 3.07 \times 10^{11} Nm^{-2}, \qquad c_{12} = 1.650 \times 10^{11} Nm^{-2},$$
$$c_{13} = 1.027 \times 10^{10} Nm^{-2}, \qquad c_{33} = 3.581 \times 10^{11} Nm^{-2},$$
$$c_{44} = 1.510 \times 10^{11} Nm^{-2}, \qquad C_E = 4.27 \times 10^{2} jKg^{-1} deg^{-1},$$
$$\beta_1 = 7.04 \times 10^{6} Nm^{-2} deg^{-1}, \qquad \beta_3 = 6.90 \times 10^{6} Nm^{-2} deg^{-1},$$
$$K_1 = 0.690 \times 10^{2} Wm^{-1} Kdeg^{-1}, \quad K_3 = 0.690 \times 10^{2} Wm^{-1} K^{-1},$$
$$K_1^* = 0.02 \times 10^{2} NSec^{-2} deg^{-1}, \qquad K_3^* = 0.04 \times 10^{2} NSec^{-2} deg^{-1},$$

$$L/h = 10, b/h = 0.5, \qquad \rho = 8.836 \times 10^3 Kgm^{-3}.$$

In the graphs, the solid red line with centre symbol circle represents GN-II theory and solid black line represents GN-III theory.

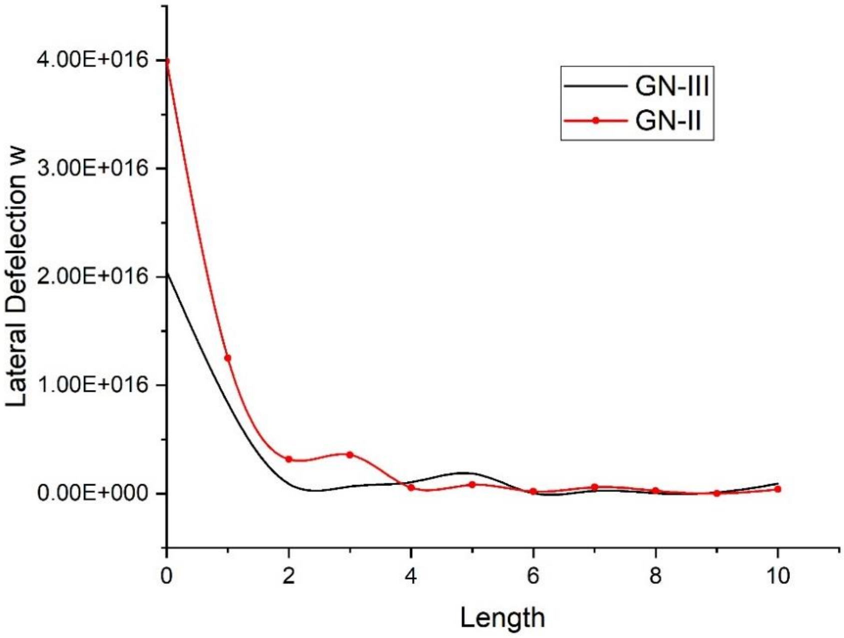

Figure 2. The lateral deflection *w* with length of beam

Figure 2 shows the variation in the lateral deflection *w* with respect to length. It is found that the lateral deflection *w* decreases sharply for the range $0 \leq Length \leq 2$ and then, oscillates in the remaining range for both the cases of GN-II and GN-III theories.

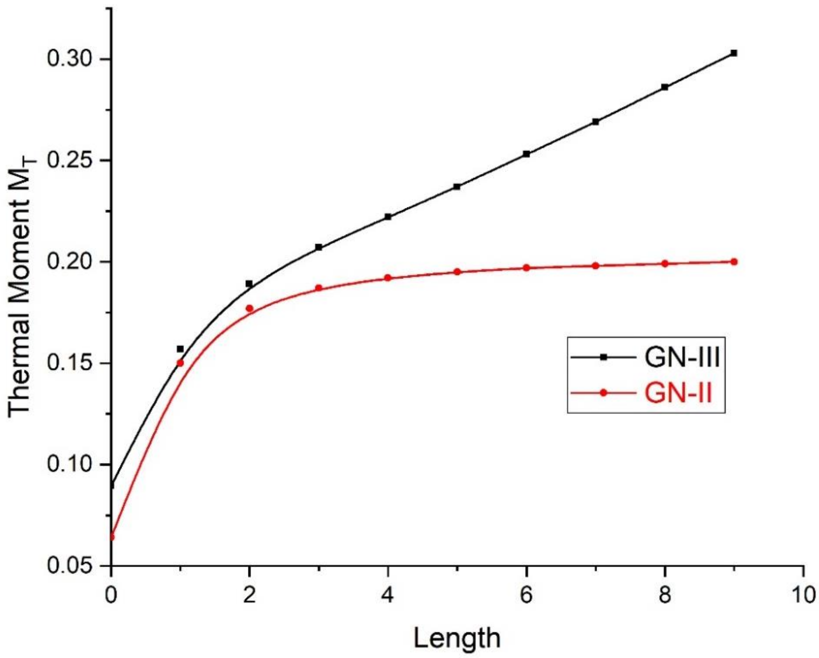

Figure 3. The variation of Thermal Moment M_T with length of the beam

Figure 3 illustrates the variation of Thermal Moment M_T with length of the beam. It is observed that thermal moment M_T increases sharply in range $0 \leq Length \leq 3$ for both GN-II and GN-III theories. However, there is sharp increase in GN-III theory as compared to GN-II theory.

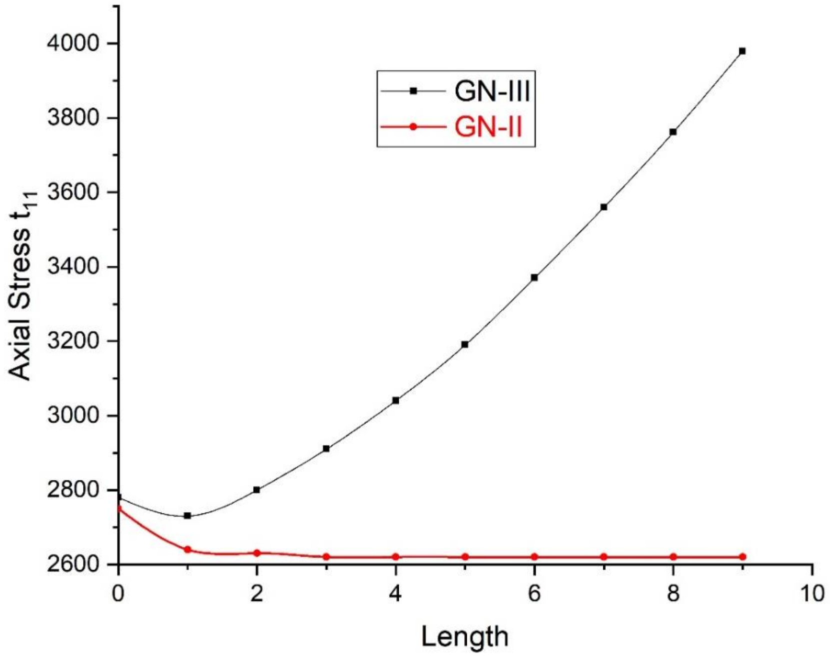

Figure 4. The variation in the axial stress t_{11} with respect to length

Figure 4 represents the variation in the axial stress t_{11} with respect to length of the beam. The axial stress t_{11} increase gradually in GN-III case, however, axial stress t_{11} decrease in the range $0 \leq Length \leq 1$ and then, become stable for rest of the length.

9. Conclusions

Thermomechanical response of transversely isotropic thermoelastic thin beam in the context of GN-II and GN-III theories of thermoelasticity have been investigated by using Euler-Bernoulli theory and Laplace transform technique. A numerical inversion technique has been used to find the solutions in the physical domain. The expressions for lateral deflection, thermal moment and axial stress have been derived and shown graphically to depict the effects successfully. It is observed that the behaviour and variation of lateral deflection *w* is oscillatory for the GN-II and GN-III theories. The value of thermal moment and axial stress is more in GN-III as compared to GN-II. This research help in design and construction of beam type accelerometers, sensors, resonators and other branches of engineering. The study of lateral deflection, thermal moment and axial stress is a significant problem of continuum mechanics.

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