Applications and Applied Mathematics: An International Journal (AAM)

Volume 13 | Issue 2 | Article 1

12-2018

The analysis of M/M/1 queue with working vacation in fuzzy environment

G. Kannadasan
Annamalai University

N. Sathiyamoorth
Annamalai University

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 13, Issue 2 (December 2018), pp. 566 – 577

The analysis of $M/M/1$ queue with working vacation in fuzzy environment

[1]G. Kannadasan & [2*] N. Sathiyamoorth

[1]Department of Mathematics
Annamalai University
Annamalainagar - 608 002, India
klsk.g.21@gmail.com

[2]Department of Mathematics
Annamalai University
Annamalainagar - 608 002, India
n.satyamurthi@gmail.com

*Corresponding Author

Received: February 22, 2018; Accepted: July 20, 2018

Abstract

This study investigates the $FM/FM/1$ queue with working vacation. For this fuzzy queuing model, the researcher obtains some performance measure of interest such as the regular busy period, working vacation period, stationary queue length and waiting time. Finally, numerical results are presented to show the effects of system parameters.

Keywords: $FM/FM/1$ queue; the regular busy period; working vacation period; stationary queue length and waiting time

MSC 2010 No.: 68M20, 90B22

1. Introduction

The authors Servi and Finn (2002) introduced a class of semi-vacation policies: a customer is served at a lower rate during a vacation, such a vacation is called a working vacation (WV). They studied an $M/M/1$ queue with working vacations, and obtained the transform formulae for the distributions of the number of customers in the system and sojourn time in steady state, then applied these results to performance analysis of gateway router in fiber communication networks.

Subsequently, Kim et al. (2003), Wu and Takagi (2006) investigated $M/G/1$ queues with working vacations. Baba (2005) extended this study to a $GI/M/1$ queue with working vacations. In view of partly utilizing service ability during a vacation, the working vacation policy of the queue with single server is similar to the vacation policy in a multi server queue. The latter can be found in Tian and Zhang (2006).

It is well known that stochastic decomposition results have a probabilistic interpretation in a classical vacation queue, where the server completely stops service during a vacation. Fuhrmann and Cooper (1985), Shanthikumar (1988) established stochastic decomposition structures for a classical $M/G/1$ queue with general vacations. The queue with working vacations where the server serves customers at a lower rate during a vacation has similar stochastic decomposition properties studied by Liua et.al(2007). Also the same authors gave stochastic decomposition structures of the number of customers and sojourn time in an $M/M/1/WV$ queue, and obtain the distributions of the additional queue length and additional delay.

Kalayanaraman et.al (2009) introduced a single server vacation queue with fuzzy service time and vacation time distributions with some performance measures. Kalyanaraman et.al (2010) gave a single server fuzzy queue with group arrivals and server vacation.

Kalyanaraman et.al (2013) investigated a fuzzy bulk queue with modified Bernoulli vacation and restricted admissible customers. They obtained some performance measures in fuzzy parameters.

Kannadasan and Sathiyamoorthi (2017) investigate the $FM/FM/1$ queue with single working vacation. We obtain some system characteristic such as the number of customer in the system in study-state, the virtual time of a customer is in the system, the server is in idle period, the server is in regular busy period. Kannadasan et.al (2017) also gave analysis for the $FM/FM/1$ queue with multiple working vacation with N-Policy, using non-linear programming method,with mean queue length, mean waiting time, at N=2.

Mery And Gokilavani (2011) investigate the performance measure of a $M^X/M/1$ multiple working vacation (MWV) queuing model in a fuzzy environment. Mary George and Jayalakshmi (2013) studied on the analysis of $G/M(n)/1/k$ queuing system with multiple exponential vacations and vacations of fuzzy length. Ramesh et.al (2013) constructs the membership function of the system characteristics of a batch-arrival queuing system with multiple servers, in which the batch-arrival rate and customer service rate are all fuzzy numbers.

Kannadasan and Sathiyamoorthi (2017) established the $FM^X/FM/1$ queue with multiple working vacation and some performance measure of interest such as mean system length, mean system sojourn time, mean busy period for the server and working vacation. Kannadasan and Sathiyamoorthi (2018) worked in fuzzy analysis technique in the FM/FM/1 queue with single working vacation and set-up times.

In this paper, we investigate the $FM/FM/1$ queue with working vacation. In section 2, we describe the queue model. In section 3 and 4 we discuss the fuzzy model with the regular busy period, working vacation period, stationary queue length and waiting time are studied in fuzzy environment respectively. In section 5 includes numerical study about the performance measures.

2. The Crisp Model

Consider a classic $M/M/1$ queue with arrival rate λ and service rate μ_b. The server beings a working vacation of random length at the instants when the queue becomes empty, and the vacation duration V follows an exponential distribution with parameter θ. During a working vacation an arriving customer is served at a rate of μ_v. when a vacation ends, if there are no customers yet in the queue, another working vacation is taken; otherwise, the service switches service rate from μ_v to μ_b, and a regular busy period starts. Similar to (2002) this queue is referred to as an $M/M/1/MV$ queue.

We assume that inter arrival times, service times and working vacation times are mutually independent. In addition , the service discipline is first in first out (FIFO). Let $Q_v(t)$ be the number of customer in the system at time t and let

$$J(t) = \begin{cases} 0, & \text{the system is in a working vacation period at time } t, \\ \\ 1, & \text{the system is in a start-up period at time } t. \end{cases}$$

Then, $Q_v(t), J(t)$ is the QBD process with the state space,

$$\Omega = \{0, 0\} \cup \{(k, j) : k \geq 1, j = 0, 1\}.$$

3. The Model in Fuzzy Environment

In this section, the arrival rate, service rate, working vacation an customer is served at a rate, working vacation time are assumed to be fuzzy numbers $\bar{\lambda}, \bar{\beta}, \bar{v}, \bar{\theta}$ respectively. Now,

$$\bar{\lambda} = \{w, \mu_{\bar{\lambda}}(w); w \in S(\bar{\lambda})\},$$

$$\bar{\beta} = \{x, \mu_{\bar{b}}(x); x \in S(\bar{\beta})\},$$

$$\bar{v} = \{y, \mu_{\bar{v}}(y); y \in S(\bar{v})\}, \ and$$

$$\bar{\theta} = \{z, \mu_{\bar{\theta}}(z); z \in S(\bar{\theta})\},$$

where $S(\bar{\lambda})$, $S(\bar{\beta})$, $S(\bar{v})$ $S(\bar{\theta})$ are the universal set's of the arrival rate, service rate, working vacation a customer is served at a rate, working vacation time respectively. It define $f(w, x, y, z)$ as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\beta}, \bar{v}, \bar{\theta})$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $f(\bar{\lambda}, \bar{b}, \bar{v}, \bar{\theta})$ can be defined as;

$$\mu_{\bar{f}(\bar{\lambda},\bar{\beta},\bar{v},\bar{\theta})}(H) = \sup_{\substack{w \in S(\bar{\lambda}) \\ x \in S(\bar{\beta}) \\ y \in S(\bar{v}) \\ z \in S(\bar{\theta})}} \{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z) / H = f(w, x, y, z)\}. \tag{1}$$

If the α- cuts of $f(\bar{\lambda}, \bar{\beta}, \bar{v}, \bar{\theta})$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The regular busy period

$$P_0 = K_0(1-\rho) = \frac{(\mu_b - \lambda)[2\mu_v - N]}{(\mu_b - \lambda)[2\mu_v - N] + \theta \times N}, \tag{2}$$

where $N = \lambda + \theta + \mu_v - \sqrt{(\lambda + \theta + \mu_v)^2 - 4\lambda\mu_v}$.

The working vacation period

$$P_1 = \frac{(\mu_b - \lambda)[2\mu_v - N] + \theta[2\mu_v - N]}{\mu_b[2\mu_v.N]}. \tag{3}$$

The stationary queue length

$$E(L) = \frac{\mu_b - \lambda}{\mu_b - \lambda z}\left[\frac{\mu_b - \lambda}{\mu_b} + \frac{\theta.N}{\mu_b(2\mu_v - N)}\right]^{-1} \times \left[\frac{2\mu_v - N}{2\mu_v}\right] + \left[\frac{N}{2\mu_v}\right]\left(\frac{\mu_b - \mu_v}{\mu_b}\right)\frac{z(2\mu_v - N)}{2\mu_v - (N)z}. \tag{4}$$

The waiting time

$$E(W) = \frac{1}{(\mu_b - \lambda)} + \frac{2\mu_v[\mu_b - \mu_v]}{\lambda(2\mu_v\mu_b - N)} \times \frac{N}{2\mu_v - N}. \tag{5}$$

We obtain the membership function of some performance measures, namely the regular busy period, the working vacation period, the stationary queue length and the waiting time. For the system in terms of this membership function are:

$$\mu_{\overline{P_0}}(A) = \sup_{\substack{w \in S(\bar{\lambda}) \\ x \in S(\bar{\beta}) \\ y \in S(\bar{b}) \\ z \in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z)/A\right\}, \tag{6}$$

where

$$A = \frac{(x-w)[2y - M]}{(x-w)[2y-M] + z[M]}, \; M = w + z + y - \sqrt{(w+z+y)^2 - 4wy}.$$

$$\mu_{\overline{P_1}}(B) = \sup_{\substack{w \in S(\bar{\lambda}) \\ x \in S(\bar{\beta}) \\ y \in S(\bar{b}) \\ z \in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z)/B\right\}, \tag{7}$$

$$B = \frac{(x-w)[2y-M] + z[2y-M]}{2xyM}.$$

$$\mu_{\overline{E(L)}}(C) = \sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{b})\\ z\in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z)/C\right\}, \tag{8}$$

where

$$C = \left(\frac{x-w}{x-w.u}\right)\left[\frac{y-w}{y} + \frac{z.M}{(2xy-M)}\right]^{-1} \times \left[\frac{M}{2y}\right] + \left[\frac{M}{2y}\right]\left[\frac{x-y}{x}\right] \times \frac{u(2y-M)}{2y-(M)u}$$

$$\mu_{\overline{E(W)}}(D) = \sup_{\substack{w\in S(\bar{\lambda})\\ x\in S(\bar{\beta})\\ y\in S(\bar{b})\\ z\in S(\bar{\theta})}} \left\{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{v}}(y), \mu_{\bar{\theta}}(z)/D\right\}, \tag{9}$$

and

$$D = \frac{1}{x-w} + \frac{2y(x-y)}{w(2yx-M)} \times \frac{M}{2y-M}.$$

Using the fuzzy analysis technique explain, we can find the membership of $\overline{P_0},\ \overline{P_1},\ \overline{E(L)},\ \overline{E(W)}$ as a function of the parameter α. Thus the α-cut approach can be used to develop the membership function of $\overline{P_0},\ \overline{P_1},\ \overline{E(L)},\ \overline{E(W)}$.

4. Performance Measure of Interest

The following Performance measure are studied for this model in fuzzy environment.

The regular busy period

Based on Zadeh's extension principle, $\mu_{P_0}(A)$ is the superimum of minimum over $\mu_{\bar{\lambda}}(w),\ \mu_{\bar{\beta}}(x),\ \mu_{\bar{v}}(y),\ \mu_{\bar{\theta}}(z)\}$, $A = [A]$ to satisfying $\mu_{P_0}(A) = \alpha,\ \ 0 < \alpha \leq 1$.

The following Four case arise:

Case(i) : $\mu_{\bar{\lambda}}(w) = \alpha,\ \mu_{\bar{\beta}}(x) \geq \alpha,\ \mu_{\bar{v}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,$
Case(ii) : $\mu_{\bar{\lambda}}(w) \geq \alpha,\ \mu_{\bar{\beta}}(x) = \alpha,\ \mu_{\bar{v}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,$
Case(iii) : $\mu_{\bar{\lambda}}(w) \geq \alpha,\ \mu_{\bar{\beta}}(x) \geq \alpha,\ \mu_{\bar{v}}(y) = \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha,$
Case(iv) : $\mu_{\bar{\lambda}}(w) \geq \alpha,\ \mu_{\bar{\beta}}(x) \geq \alpha,\ \mu_{\bar{v}}(y) \geq \alpha,\ \mu_{\bar{\theta}}(z) = \alpha.$

For case (i), the lower and upper bound of α- cuts of $\overline{P_0}$ can be obtained through the corresponding parametric non-linear programs;

$$[\overline{P_0}]_{\alpha}^{L_1} = \min_{\Omega}\{[A]\}\ and\ [\overline{P_0}]_{\alpha}^{U_1} = \max_{\Omega}\{[A]\}.$$

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{E(L)}$ for the cases $(ii), (iii)\ and\ (iv)$.

By considering the cases, simultaneously the lower and upper bounds of the α-cuts of $\overline{P_0}$ can be written as

$$[\overline{P_0}]_\alpha^L = \min_{\Omega}\{[A]\}, \quad and\ [\overline{P_0}]_\alpha^U = \max_{\Omega}\{[A]\}.$$

Such that

$$w_\alpha^L \leq w \leq w_\alpha^U,\ x_\alpha^L \leq x \leq x_\alpha^U,\ y_\alpha^L \leq y \leq y_\alpha^U,\ z_\alpha^L \leq z \leq z_\alpha^U.$$

.

If both $(\overline{P_0})_\alpha^L$ and $(\overline{P_0})_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(A) = [(P_0)_\alpha^L]^{-1}$ and $R(A) = [(P_0)_\alpha^U]^{-1}$, can be derived from which the membership function $\mu_{\overline{P_0}}(A)$ can be constructed as

$$\mu_{\overline{P_0}}(A) = \begin{cases} L(A), (P_0)_{\alpha=0}^L \leq A \leq (P_0)_{\alpha=0}^U, \\ 1, \quad (P_0)_{\alpha=1}^L \leq A \leq (P_0)_{\alpha=1}^U, \\ R(A), (P_0)_{\alpha=1}^L \leq A \leq (P_0)_{\alpha=0}^U. \end{cases} \tag{10}$$

In the same way as we said before we get the following results.

The working vacation period

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), (P_1)_{\alpha=0}^L \leq B \leq (P_1)_{\alpha=0}^U, \\ 1, \quad (P_1)_{\alpha=1}^L \leq B \leq (P_1)_{\alpha=1}^U, \\ R(B), (P_1)_{\alpha=1}^L \leq B \leq (P_1)_{\alpha=0}^U. \end{cases} \tag{11}$$

The stationary queue length

$$\mu_{\overline{E(L)}}(C) = \begin{cases} L(C), (E(L))_{\alpha=0}^L \leq C \leq (E(L))_{\alpha=0}^U, \\ 1, \quad (E(L))_{\alpha=1}^L \leq C \leq (E(L))_{\alpha=1}^U, \\ R(C), (E(L))_{\alpha=1}^L \leq C \leq (E(L))_{\alpha=0}^U. \end{cases} \tag{12}$$

The waiting time

$$\mu_{\overline{E(W)}}(D) = \begin{cases} L(d), (E(W))_{\alpha=0}^L \leq D \leq (E(W))_{\alpha=0}^U, \\ 1, \quad (E(W))_{\alpha=1}^L \leq D \leq (E(W))_{\alpha=1}^U, \\ R(D), (E(W))_{\alpha=1}^L \leq D \leq (E(W))_{\alpha=0}^U. \end{cases} \tag{13}$$

5. Numerical Study

The regular busy period

Suppose the arrival rate $\overline{\lambda}$, the service rate $\overline{\beta}$, working vacation an customer is served at a rate $\overline{\nu}$, and working vacation time $\overline{\theta}$ are assumed to be trapezoidal fuzzy numbers described by:
$\overline{\lambda} = [41, 42, 43, 44]$, $\overline{\beta} = [51, 52, 53, 54]$, $\overline{\nu} = [61, 62, 63, 64]$ and $\overline{\theta} = [71, 72, 73, 74]$ per hours, respectively. Then,

$$\lambda(\alpha) = \min_{w \in s(\bar{\lambda})} \{w \in s(\bar{\lambda}), G(x) \geq \alpha\}, \max_{w \in s(\bar{\lambda})} \{w \in s(\bar{\lambda}), \geq \alpha\},$$

where

$$G(x) = \begin{cases} w - 41, & 41 \leq w \leq 42, \\ 1 \quad , & 42 \leq w \leq 43, \\ 44 - w, & 43 \leq w \leq 44. \end{cases}$$

That is,

$\lambda(\alpha) = [41 + \alpha, 44 - \alpha], \; \mu(\alpha) = [51 + \alpha, 54 - \alpha],$

$\nu(\alpha) = [61 + \alpha, 64 - \alpha]$ and $\theta(\alpha) = [71 + \alpha, 74 - \alpha]$.

It is clear that, when $w = w_\alpha^U, \; x = x_\alpha^U, \; y = y_\alpha^U$ and $z = z_\alpha^U$, A attains its maximum value and, when $w = w_\alpha^L, \; x = x_\alpha^L, \; y = y_\alpha^L$ and $z = z_\alpha^L$, A attains its minimum value.

From the generated, for the given input value of $\overline{\lambda}, \; \overline{\mu}, \; \overline{\theta} \; and \; \overline{\beta}$.

i) For fixed values of w, x and y, A decreases as z increase.
ii) For fixed values of x, y and s, A decreases as w increase.
iii) For fixed values of y, z and w, A decreases as x increase.
iv) For fixed values of z, w and x, A decreases as y increase.

The smallest value of occurs when w-takes its lower bound. That is, $w = 41 + \alpha$ and x, y and z, take their upper bounds given by $x = 54 - \alpha, \; y = 64 - \alpha$ and $z = 74 - \alpha$ respectively. The maximum value of P_0 occurs when $w = 44 - \alpha, \; x = 51 + \alpha, \; y = 61 + \alpha, \; z = 71 + \alpha$.
If both $[P_0]_\alpha^L$ and $[P_0]_U^\alpha$ are invertible with respect to α, then the left shape function $L(A) = [(P_0)_\alpha^L]^{-1}$ and right shape function $R(A) = [(P_0)_\alpha^U]^{-1}$ can be obtained, from which the membership function $\mu_{\overline{P_0}}(A)$ can be constructed as;

$$\mu_{\overline{P_0}}(A) = \begin{cases} L(A), A_1 \leq A \leq A_2, \\ 1, \quad A_2 \leq A \leq B_3 \, , \\ R(A), A_3 \leq A \leq A_4. \end{cases} \tag{14}$$

The values of A_1, A_2, A_3 and A_4 as obtained from (14) are;

$$\mu_{\overline{P_0}}(A) = \begin{cases} L(A), & 0.2524 \leq A \leq 0.2938, \\ 1, & 0.2938 \leq A \leq 0.3755 \ , \\ R(A), & 0.3755 \leq A \leq 0.4112. \end{cases}$$

In the same way as we said before we get the following results.

The working vacation period

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), B_1 \leq B \leq B_2, \\ 1, \quad B_2 \leq B \leq B_3 \ , \\ R(B), B_3 \leq B \leq B_4. \end{cases} \tag{15}$$

The values of B_1, B_2, B_3 and B_4 as obtained from (15) are

$$\mu_{\overline{P_1}}(B) = \begin{cases} L(B), & 1.3203 \leq B \leq 1.3773, \\ 1, & 1.3773 \leq B \leq 1.413 \ , \\ R(B), & 1.4133 \leq B \leq 1.4628. \end{cases}$$

The stationary queue length

$$\mu_{\overline{E(L)}}(C) = \begin{cases} L(C), C_1 \leq C \leq C_2, \\ 1, \quad C_2 \leq C \leq C_3 \ , \\ R(C), C_3 \leq C \leq C_4. \end{cases} \tag{16}$$

The values of C_1, C_2, C_3 and C_4 as obtained from (16) are

$$\mu_{\overline{E(L)}}(C) = \begin{cases} L(C), & 0.1198 \leq C \leq 0.1657, \\ 1, & 0.1657 \leq C \leq 0.2054 \ , \\ R(C), & 0.2054 \leq C \leq 0.2245. \end{cases}$$

The waiting time

$$\mu_{\overline{E(W)}}(D) = \begin{cases} L(D), D_1 \leq D \leq D_2, \\ 1, \quad D_2 \leq D \leq D_3 \ , \\ R(D), D_3 \leq D \leq D_4. \end{cases} \tag{17}$$

The values of D_1, D_2, D_3 and D_4 as obtained from (17) are

$$\mu_{\overline{E(W)}}(D) = \begin{cases} L(D), & 0.0751 \leq D \leq 0.0987, \\ 1, & 0.0987 \leq D \leq 0.1175 \ , \\ R(D), & 0.1175 \leq D \leq 0.1412. \end{cases}$$

Further, by fixed the vacation rate by a crisp value $\bar{\theta} = 72.4$ and $\bar{\nu} = 61.3$ taking arrival rate $\overline{\lambda} = [41, 42, 43, 44]$, service rate $\overline{\beta} = [51, 52, 53, 54]$ both trapezoidal fuzzy numbers the values of the regular busy period are generated and are plotted in the figure 1, it can be observed that as $\overline{\lambda}$ increases the the regular busy period increases for the fixed value of the service rate, where as for fixed value of arrival rate, the regular busy period decreases as service rate increases.

Similar conclusion can be obtained for the case $\bar{\theta} = 72.6$, $\bar{\nu} = 62.3$. Again for fixed values of taking $\overline{\lambda} = [41, 42, 43, 44]$, $\overline{\beta} = [51, 52, 53, 54]$, the graphs of the working vacation period are drawn in figure 2 respectively. These figure show that as arrival rate increases the working vacation period also increases, while the working vacation period decreases as the service rate increases in both the case.

It is also observed from the data generated that the membership value of the regular busy period is 0.41 and the membership value of the working vacation period 0.46 when the ranges of arrival rate, service rate, and the vacation rate lie in the intervals$(41, 42.4)$, $(52, 54.6)$, and $(72.8, 73.4)$ respectively.

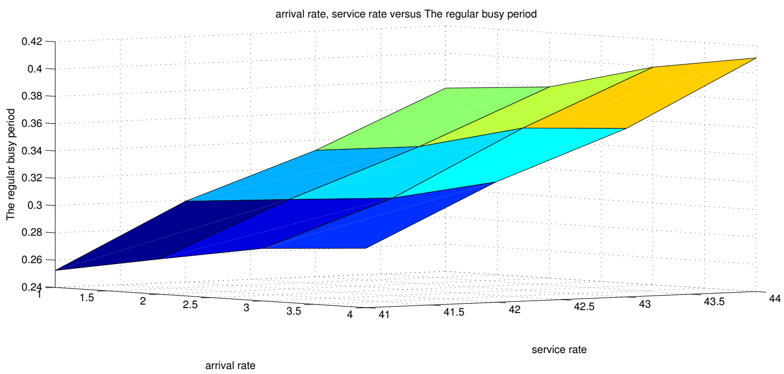

Figure 1: Arrival rate, service rate versus the regular busy period

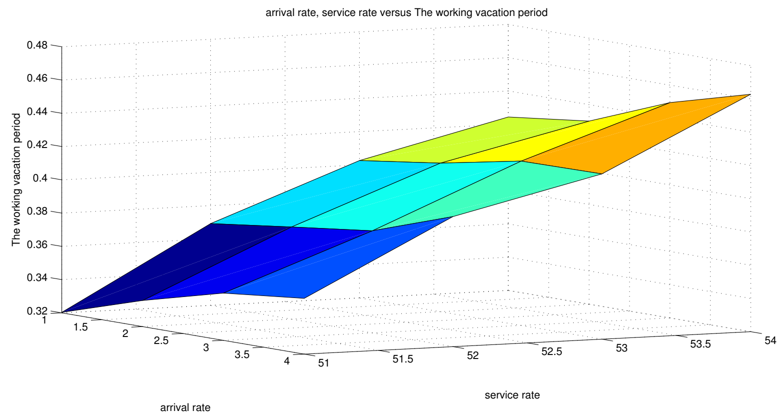

Figure 2: Arrival rate, service rate versus the working vacation period

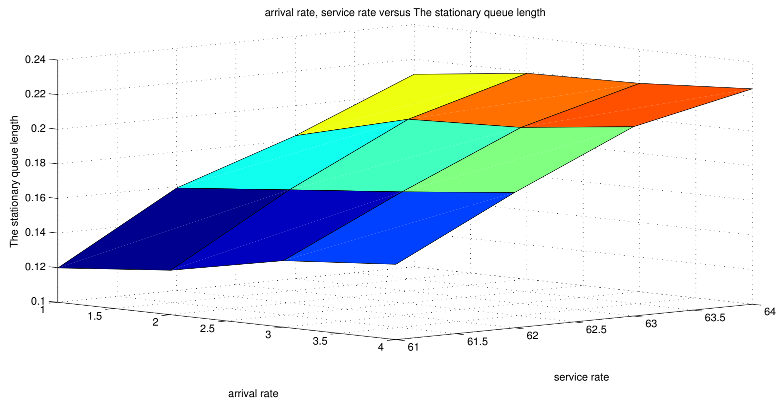

Figure 3: Arrival rate, service rate versus the stationary queue length

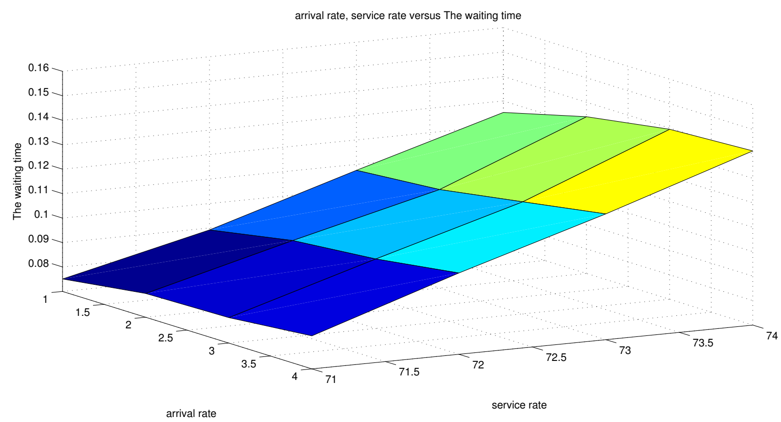

Figure 4: Arrival rate, service rate versus the waiting time

6. Conclusion

For the batch arrival queues, some researchers studied a batch arrival $M^X/M/1$ queue with single working vacation. Furthermore, they found the upper bound and lower bound of the stationary waiting time in the Laplace transform order, from which they got the upper bound and lower bound of the waiting time. In this research paper, we have studied the analysis of $M/M/1$ queue with working vacation in fuzzy environment. We have obtained the performance measures such as the regular busy period, working vacation period, stationary queue length and waiting time. We have obtained numerical results to all the performance measures for this fuzzy Queues. The application of this fuzzy queues, there are situations particularly in transportation system (Bus service, trains, and express elevators) where the service provided is a group that, a group of customers can be served simulataneously, batch servicing in this process.

Acknowledgment

The authors would like to thank the referees for their valuable suggestions and comments for improving this paper.

REFERENCES

Baba,Y., Analysis of a GI/M/1 queue with multiple working vacations, Oper. Res. Lett. 33 (2005) 201-209.

Fuhrmann,S., Cooper,R., Stochastic decompositions in the M/G/1 queue with generalized vacations, Oper. Res. 32 (1985)1117-1129.

Julia Rose Mary, K. and Gokilavani, T., Analysis of $M^X/M/1/MWV$ with Fuzzy parameter, International Journal of Computer Application, Vol.2, 2250-1797., 2014.

Kalyanaraman, R. Thillaigovidan, N, and Kannadasan, G., A single server vacation queue with fuzzy service time and vacation time distributions. International Journal of Applied Mathematics, Vol (1), 39–46 , 2009.

Kalyanaraman, R. Thillaigovidan, N, and Kannadasan, G., A single server fuzzy queue with group arrivals and server vacation., Annamalai University Science Journal, Vol 46 , 99–102, 2010.

Kalyanaraman, R. Thillaigovidan, N, and Kannadasan, G., A fuzzy bulk queue with modified bernolli vacation and restricted admissible customers. International Journal of intelligent and fuzzy systems, vol.24, No,1, pp,837–845, 2013.

Kannadasan, G., and Sathiyamoorthi, N.,the $FM^X/FM/1$ queue with multiple working vacation. International Journal of Application and Applied Mathematics (AAM). vol.12, Issue.2, ISSN:1932-9466, PP.695-709, 2017.

Kannadasan, G., and Sathiyamoorthi, N., The FM/FM/1 queue with single working vacation. International Journal on Recent and Innovation Trends in Computing and Communication. vol.5, Issue.7, ISSN:2321-8169, PP.648-654,2017.

Kannadasan, G., Sathiyamoorthi, N., and Murugsdas, P., Analysis for the FM/FM/1 Queue with Multiple Working Vacation with N-Policy.,Annals of Pure and Applied Mathematics.,Vol.14, No. 1, 2017, 133-141.

Kannadasan, G., and Sathiyamoorthi, N., The FM/FM/1 queue with swv and set-up times., International Journal of Pure and Applied Mathematics., Vol.118., No.6., 115–126, 2018.

Kim,J., Choi,D., Chae,K., Analysis of queue-length distribution of the M/G/1 queue with working vacations, in:International Conference on Statistics and Related Fields,Hawaii, 2003.

Mary Goerge and Jayalekshmi, Queuing System with vacations of fuzzy length, International Journal of Mathematics Research., 0976-5840, Vol.3,pp 89-103, 2011.

Ramesh.R., and Kumara G Ghuru.S., A batch-Arrival queue with multiple servers and fuzzy parameter: Parametric Programming Approach, 2319-7064,Vol.2, 2013.

Servi,L., Finn,S., M/M/1 queue with working vacations (M/M/1/WV), Perform. Eval. 50 (2002) 41-52.

Shanthikumar,J., On stochastic decomposition in M/G/1 type queues with generalized server vacations, Oper. Res. 36 (1988) 566-569.

Tian,N., Zhang,Z.G., A two threshold vacation policy in multiserver queueing systems, Eur. J. Oper. Res. 168 (1) (2006) 153-163.

Wen-yuan Liu., Xiu-li Xu., Nai-shuo Tian., Stochastic decomposition in the M/M/1 queue with working vacation. Operations Research Letters 35 (2007) 595-600.

Wu,D., Takagi,H., M/G/1 queue with multiple working vacations, Perform. Eval. 63 (7) (2006) 654-681.

X. Xu, Z. Zhang and N. Tian, ï¿œAnalysis for the M[X]/M/1 Working Vacation Queue,ï¿œ International Journal of In-formation and Management Sciences, Vol. 20, 2009, pp. 379-394.

Zadeh,L.A., Fuzzy sets as a basis a theory of possibility, Fuzzy sets and systems.1 (1978) pp.3-28 .