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Abstract

In the present paper, rotation and radiation effects on unsteady MHD flow passed an inclined plate with variable wall temperature and mass diffusion in the presence of Hall current has been studied. The fluid considered is viscous, incompressible and electrically conducting. Earlier, we have studied unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion. In that study we had analyzed the effect of Hall current (2016). We obtained the results which were in agreement with the desired flow phenomenon. To study further, we are changing the model by considering radiation and rotation effect on fluid. The plate temperature and the concentration level near the plate increase linearly with time. The model contains equations of motion, diffusion equation and equation of energy. The governing system of partial differential equations is transformed to dimensionless equations by using non-dimensional variables. To analyze the solution of the model, desirable sets of the values of the parameters have been considered. The governing equations involved in the flow model are solved by the Laplace-transform technique. The results obtained have been analyzed with the help of graphs drawn for different parameters. The numerical values obtained for the drag at boundary and Nusselt number have been tabulated. We found that the values obtained for velocity, concentration and temperature are in concurrence with the actual flow of the fluid. The results of the study may find applications in the field related to the solar physics dealing with the solar cycle, the sunspot development, the structure of rotating magnetic stars etc.

Keywords: MHD flow; Rotation; Radiation; inclined plate; Mass diffusion; Hall current

MSC 2010 No: 76W05, 76D05, 80A20

NOMENCLATURE			
a^*	Absorption constant	Ω	Angular velocity
C	Species concentration in the fluid	β	Volumetric coefficient of thermal
$\overline{C}$	The dimensionless concentration	β^*	Volumetric coefficient of concentration
C_P	Specific heat at constant pressure	υ	The kinematic viscosity
C_w	Species concentration at the plate	ρ	The fluid density
C_∞	The concentration in the fluid	σ	Electrical conductivity
D	Mass diffusion	μ	The magnetic permeability
u,v	Velocity of the fluid in x & z- direction	θ	The dimensionless temperature
$\bar{u}, \bar{v}$	Dimensionless velocity in x & z-direction	μ	The coefficient of viscosity
T	Temperature of the fluid	G_m	Mass Grashof number
K_0	The chemical reaction parameter	G_r	Thermal Grashof number
M	The magnetic Field parameter	k	The thermal conductivity
M	The Hall current parameter	T_∞	The temperature of the fluid
Pr	Prandtl number	g	Gravity acceleration
R	Radiation parameter	T_w	Temperature of the plate
Sc	Schmidt number	t	Time

1. Introduction

The MHD flow past a flat plate is one of the classical problems in the fluid dynamics. Applications of the study arise in magnetic field controlled material processing systems, planetary and solar plasma fluid dynamical systems, rotating MHD induction machine energy generators etc.

Hall effect on free and forced convective flow in a rotating channel was analyzed by Prasada et al. (1982). Seth and Nandkeolya (2009) have considered MHD Couette flow in a rotating system in the presence of an inclined magnetic field. MHD flow over a moving plate in a rotating fluid with magnetic field, Hall currents and free stream velocity was developed by Takhar et al. (2002). Ghosh et al. (2009) have worked on Hall effect on MHD flow in a rotating system with heat transfer characteristics. Effect of rotation on MHD flow past an accelerated isothermal vertical plate with heat and mass diffusion was discussed by Muthucumaraswamy et al. (2010). Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature was investigated by Rajesh and Varma (2009). Garg (2012) has worked on combined effects of thermal radiations and Hall current on moving vertical porous plate in a rotating system with variable temperature. Further, Garg (2013) has studied magneto-hydrodynamics and radiation effects on the flow due to moving vertical porous plate with variable temperature. Hazarika (2014) has analyzed Hall current in a rotating channel on MHD flow with radiation and viscous dissipation. Zeeshan and Majeed (2016) have considered effect of magnetic dipole on radiative non-darcian mixed convective flow over a stretching sheet in porous medium.

Further, Zeeshan et al. (2016) have studied the effect of magnetic dipole on viscous ferro-fluid passed a stretching surface with thermal radiation. Unsteady ferromagnetic liquid flow and heat transfer over a stretching sheet with the effect of dipole and heat flux was investigated by Majeed et al. (2016). Unsteady MHD flow passed an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current was studied by Rajput and Kumar (2016). Influence of magnetic field on nano-fluid with free convection in an open porous cavity by means of Lattice Boltzmann method was analyzed by Sheikholeslami (2017). Further, Sheikholeslami (2017) has considered numerical simulation of magnetic nano-fluid with natural convection in porous media. Effect of Lorentz forces on nano-fluid in a porous cylinder was also studied by Sheikholeslami (2017). Further, Sheikholeslami along with Rokni (2017) has discussed nano-fluid with two phase model in existence of induced magnetic field. Magnetohydrodynamic nano-fluid convective flow in a porous enclosure by means of LBM was explained by Sheikholeslami and Shedzad (2017). Sheikholeslami along with Bhatti (2017) have presented forced convection of nano-fluid in presence of constant magnetic field considering shape effects of nano-particles.

The present study is carried out to examine the effects of rotation and radiation on unsteady MHD flow passed an inclined plate with variable wall temperature and mass diffusion in the presence of Hall current. The problem is solved by the Laplace transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction and Nusselt number have been tabulated.

2. Mathematical Analysis

The geometrical model of the problem is shown in Figure 1.

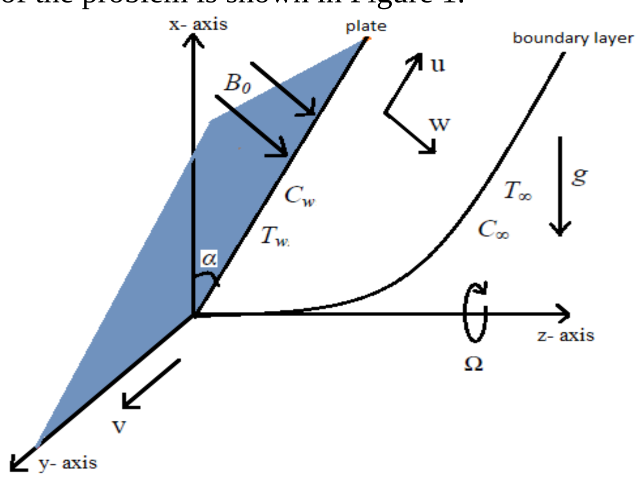

Figure 1. Physical model of the flow

The x-axis is taken along the vertical plane; and z-axis which is normal to it lies in the horizontal plane. The plate is inclined at an angle α from vertical. The plate and the boundary layer of the fluid rotate as a rigid body with a uniform angular velocity Ω about the z-axis. The magnetic field B_0 of uniform strength is applied perpendicular to the flow. The fluid is electrically conducting; however; its magnetic Reynolds number is very small. Therefore, the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Initially, it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At time $t > 0$, the plate starts moving with a velocity u_0 in its own plane, and temperature of the plate is raised to T_w. The concentration C_w near the plate is raised linearly with respect to time. So, under above assumptions, the governing equations are as follows:

$$\frac{\partial u}{\partial t} - 2\Omega v = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta Cos\alpha (T - T_\infty) + g\beta^* Cos\alpha\,(C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho(1+m^2)}. \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \upsilon \frac{\partial^2 v}{\partial z^2} + \frac{\sigma B_0^2 (mu - v)}{\rho(1+m^2)}. \tag{2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}. \tag{3}$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z}. \tag{4}$$

The following boundary conditions have been considered.

$$\left.\begin{array}{l} t \le 0 : u = 0,\ v = 0,\ T = T_\infty,\ C = C_\infty,\ \text{for every } z, \\ t > 0 : u = u_0,\ v = 0,\ T = T_\infty + (T_w - T_\infty)A_0,\ C = C_\infty + (C_w - C_\infty)A_0,\ \text{at } z=0, \\ u \to 0,\ v \to 0,\ T \to T_\infty,\ C \to C_\infty \ \text{ as } z \to \infty. \end{array}\right\} \tag{5}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z} = -4a^* \sigma (T_\infty^4 - T^4). \tag{6}$$

The temperature difference within the flow is considered sufficiently small, hence T^4 can be expressed as the linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Using Equations (6) and (7), Equation (4) becomes

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - 16a^* \sigma T_\infty^3 (T - T_\infty). \tag{8}$$

.

To write the Equations (1) - (3) and (8) in dimensionless from, we introduce the following non - dimensional quantities:

$$\begin{cases} \bar{z}=\dfrac{zu_0}{\upsilon},\ \bar{u}=\dfrac{u}{u_0},\ \bar{v}=\dfrac{v}{u_0},\ \theta=\dfrac{(T-T_\infty)}{(T_w-T_\infty)},\ S_c=\dfrac{\upsilon}{D}, \\ \mu=\rho\upsilon,\ P_r=\dfrac{\mu c_p}{k},\ R=\dfrac{16a^*\sigma\upsilon^2T_\infty^3}{ku_0^2}, \\ G_r=\dfrac{g\beta\upsilon(T_w-T_\infty)}{u_0^{\ 3}},\ M=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2},\ G_m=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^{\ 3}}, \\ \bar{C}=\dfrac{(C-C_\infty)}{(C_w-C_\infty)},\ \bar{t}=\dfrac{tu_0^2}{\upsilon},\bar{\Omega}=\dfrac{\upsilon\Omega}{u_0^2}. \end{cases} \tag{9}$$

Then the model becomes

$$\frac{\partial\bar{u}}{\partial\bar{t}}-2\bar{\Omega}\bar{v}=\frac{\partial^2\bar{u}}{\partial\bar{z}^2}+G_r\,\theta\,Cos\alpha+G_m\,\bar{C}\,Cos\alpha-\frac{M(\bar{u}+m\bar{v})}{(1+m^2)}. \tag{10}$$

$$\frac{\partial\bar{v}}{\partial\bar{t}}+2\bar{\Omega}\bar{u}=\frac{\partial^2\bar{u}}{\partial\bar{z}^2}+\frac{M(m\bar{u}-\bar{v})}{(1+m^2)}. \tag{11}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{S_c}\frac{\partial^2\bar{C}}{\partial\bar{z}^2}. \tag{12}$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial\bar{z}^2}-\frac{R\theta}{P_r}. \tag{13}$$

The corresponding boundary conditions (5) become:

$$\left.\begin{array}{l} \bar{t}\le 0:\bar{u}=0,\ \bar{v}=0,\ \theta=0,\ \bar{C}=0,\ \text{for every } \bar{z}, \\ \bar{t}>0:\bar{u}=1,\ \bar{v}=0,\ \theta=\bar{t},\ \bar{C}=\bar{t},\ \text{ at } \bar{z}=0, \\ \bar{u}\to 0,\ \bar{v}\to 0,\ \theta\to 0,\ \bar{C}\to 0,\ \ as\ \ \bar{z}\to\infty. \end{array}\right\} \tag{14}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t}-2\Omega v=\frac{\partial^2 u}{\partial z^2}+G_r\,\theta\,Cos\alpha+G_m\,C\,Cos\alpha-\frac{M(u+mv)}{(1+m^2)}. \tag{15}$$

$$\frac{\partial v}{\partial t}+2\Omega u=\frac{\partial^2 v}{\partial z^2}+\frac{M(mu-v)}{(1+m^2)}. \tag{16}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}. \tag{17}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}-\frac{R\theta}{P_r}. \tag{18}$$

The boundary conditions become

$$\left.\begin{array}{l} t\le 0: u=0, v=0, \theta=0, C=0, \text{ for every } z, \\ t>0: u=1, v=0,\ \theta=t,\ C=t, \text{ at } z=0, \\ u\to 0,\ v\to 0,\ \theta\to 0,\ C\to 0, \text{ as } z\to\infty. \end{array}\right\} \tag{19}$$

Combining Equations (15) and (16), we get

$$\frac{\partial q}{\partial t}=\frac{\partial^2 q}{\partial z^2}+G_r\,\theta\, Cos\alpha+G_m\, C\, Cos\alpha-\frac{M(1-im)q}{1+m^2}-2i\Omega q. \tag{20}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}. \tag{21}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}-\frac{R\theta}{P_r}. \tag{22}$$

Here $q=u+iw,$ and the corresponding boundary conditions are,

$$\left.\begin{array}{l} t\le 0: q=0,\ \theta=0,\ C=0, \text{ for all } z, \\ t>0: q=1,\ \theta=t,\ C=t, \text{ at } z=0, \\ q\to 0,\ \theta\to 0,\ C\to 0, \text{ as } z\to\infty. \end{array}\right\} \tag{23}$$

The dimensionless governing Equations (20) to (22), subject to the boundary conditions (23), are solved by the usual Laplace - transform technique. The solution obtained is as under:

$$C=t\left\{(1+\frac{z^2S_c}{2t})erfc\left(\frac{\sqrt{S_c}}{2\sqrt{t}}\right)-\frac{z\sqrt{S_c}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{z^2}{4t}S_c}\right\}$$

$$\theta=\frac{e^{-\sqrt{R}z}}{4\sqrt{R}}\{[\, erfc\left(\frac{-2\sqrt{R}t+zP_r}{\sqrt{P_r t}}\right)](2\sqrt{R}t-zP_r)+e^{2\sqrt{R}z}[erf\left(\frac{2\sqrt{R}t+zP_r}{\sqrt{P_r t}}\right)](2\sqrt{R}t+zP_r)\}$$

$$q=\frac{1}{2}e^{-\sqrt{a}z}A_{33}+\frac{G_r Cos\alpha}{4(a-R)^2}[2e^{-\sqrt{a}z}(A_1+P_rA_2)+2tA_2e^{-\sqrt{a}z}(a-R)+zA_3e^{-\sqrt{a}z}(\sqrt{a}-\frac{R}{\sqrt{a}})$$

$$+2A_{12}A_4(1-P_r)]+\frac{1}{4a^2}G_m Cos\alpha[e^{-\sqrt{a}z}(2A_1+2\sqrt{a}A_3)+2e^{-\sqrt{a}z}A_2(S_c+at)+2A_{13}A_5(1$$

$$-S_c)]-\frac{1}{2\sqrt{\pi}(a-R)^2A_{11}}P_rG_rCos\alpha[A_{16}A_6\sqrt{\pi}z(at-1-Rt+P_r)+A_{14}A_7\sqrt{\pi}z(1-P_r)$$

.

$$+\frac{1}{2}\sqrt{\frac{P_r}{R}}A_{16}A_8A_{11}\sqrt{\pi}z(a-R)]-\frac{G_m Cos\alpha}{2a^2\sqrt{\pi}}[2az\sqrt{S_c}e^{-\frac{z^2S_c}{4t}}\sqrt{t}+A_{15}\sqrt{\pi}(az^2S_c+2at+2S_c$$
$$-2)+A_{13}\sqrt{\pi}(A_9+A_{10}S_c)].$$

The expressions for the symbols involved in the above equations are given in the appendix.

2.1. Skin friction

The dimensionless skin friction at the plate $z = 0$ is obtained by

$$\left(\frac{dq}{dz}\right)_{z=0} = \tau_x + i\tau_y \ .$$

The numerical values of τ_x and τ_y for different parameters are given in Table 1.

2.2. Nusselt number

The dimensionless Nusselt number is given by

$$Nu = \left(\frac{\partial\theta}{\partial z}\right)_{z=0}$$
$$= erfc[\frac{\sqrt{Rt}}{\sqrt{tP_r}}](\sqrt{Rt}-\frac{\sqrt{R}}{2}t+\frac{P_r}{4\sqrt{R}})-erfc[-\frac{\sqrt{Rt}}{\sqrt{tP_r}}](\frac{\sqrt{R}}{2}t+\frac{P_r}{4\sqrt{R}})-\frac{e^{-\frac{Rt}{P_r}}\sqrt{tP_r}}{\sqrt{\pi}}$$

The numerical values of Nu are given in Table 2 for different parameters.

3. Result and Discussions

In this present paper we have studied the effects of rotation and radiation on flow. The behavior of other parameters like magnetic field, Hall current and thermal buoyancy is almost similar to the earlier model studied by Rajput and Kumar (2016). The analytical results are shown graphically in Figures 2 to 11. The numerical values of skin-friction and Nusselt number are presented in Table 1 and Table 2, respectively. Effect of rotation on fluid flow behavior is shown by Figures 3 and 5. It is observed that an increase in rotation parameter, primary velocity decreases throughout the boundary layer region. However, it is observed that secondary velocity increases continuously with increase in rotation parameter near the surface of the plate. This implies that rotation tends to accelerate secondary velocity whereas it retards primary velocity in the boundary layer region. Figures 2 and 4 indicate the effect of radiation parameter R on both components of the velocity and it is observed that it retards the flow. This is because as the radiation parameter increases, the temperature of the system decreases as a result of which fluid flow becomes slow. Further, it is observed that the temperature decreases when Prandtl number and radiation parameter are increased (Figures 6 and 7). From Figure 8 it is observed that the temperature increases with time. This is due to the fact that heat is transported to the system continuously. Skin friction is given in Table 1. The values of τ_x increase with the increase in radiation and rotation parameters. The values of τ_y increase with the increase in rotation parameter and decrease with radiation parameter. Nusselt number is given in Table 2. The value of Nusselt number (Nu) decreases with increase in Prandtl number, radiation parameter and time.

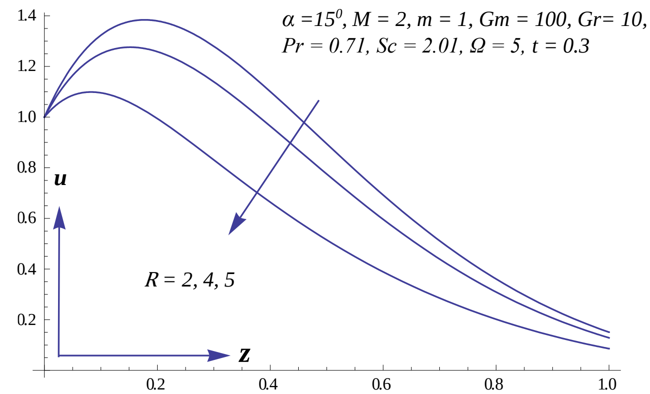

Figure 2. Velocity u for different values of R

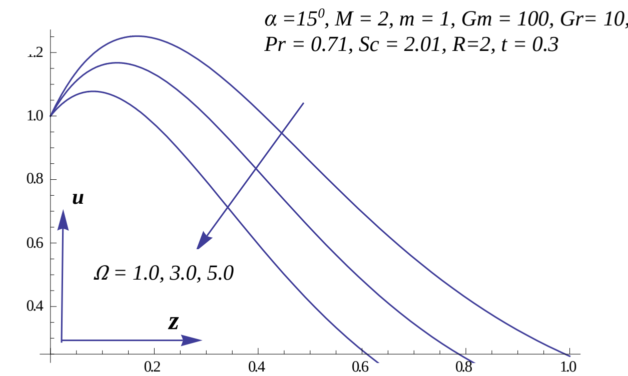

Figure 3. Velocity u for different values of Ω

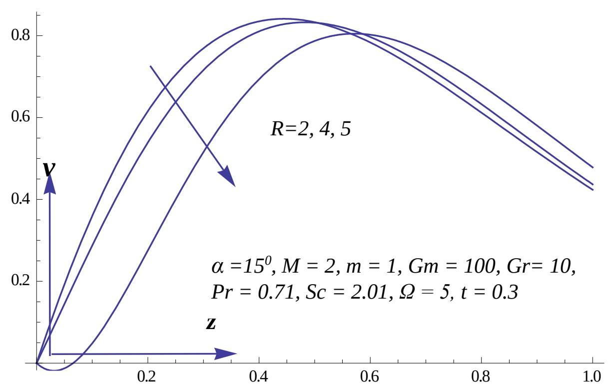

Figure 4. Velocity v for different values of R

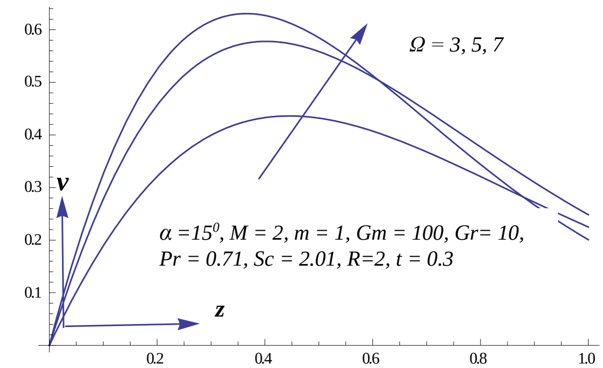

Figure 5. Velocity v for different values of Ω

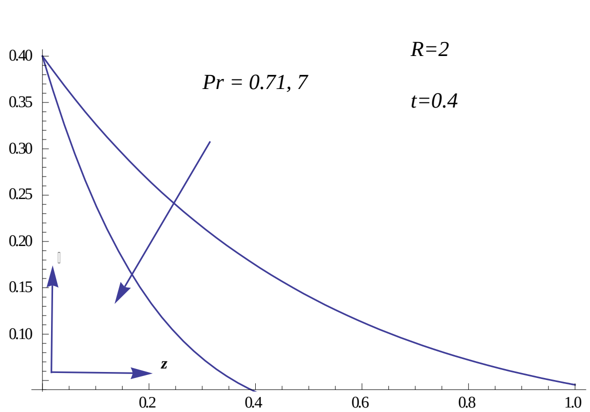

Figure 6. Temperature profile for different values of Pr

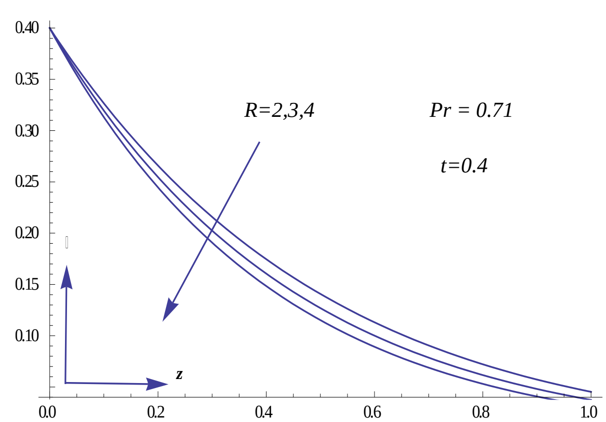

Figure 7. Temperature profile for different values of R

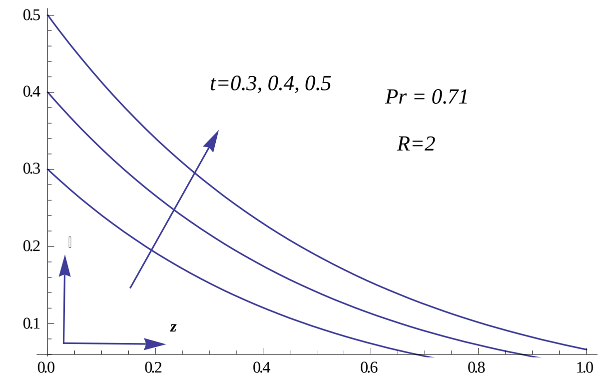

Figure 8. Temperature profile for different values of t

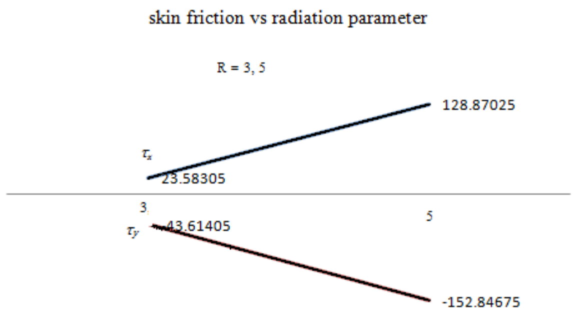

Figure 9. Skin friction for radiation parameter

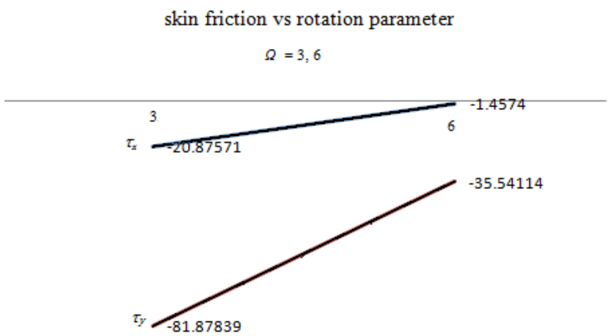

Figure 10. skin friction for rotation parameter

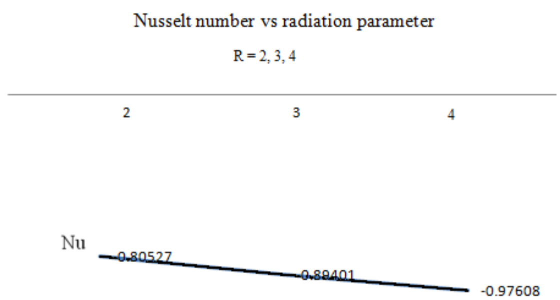

Figure 11. Nusselt number for radiation parameter

Table 1. Skin friction for different parameter (α=15^0)

M	m	Pr	Sc	Gm	Gr	R	Ω	T	τ_x	τ_y
2	1	0.71	2.01	100	10	3	5	0.3	023.58305	-043.61405
2	1	0.71	2.01	100	10	5	5	0.3	128.87025	-152.84675
2	1	0.71	2.01	100	10	2	3	0.3	-020.87571	-081.87839
2	1	0.71	2.01	100	10	2	6	0.3	-001.45740	-035.54114

Table 2. Nusselt number for different parameter

Pr	*R*	*t*	*Nu*
0.71	2	0.4	-0.80527
7.00	2	0.4	-1.95926
0.71	3	0.4	-0.89401
0.71	4	0.4	-0.97608
0.71	2	0.5	-0.95095
0.71	2	0.6	-1.09494

4. Conclusion

In this paper a theoretical analysis has been done to study the rotation and radiation effects on MHD flow past an inclined plate with variable wall temperature and mass diffusion in the presence of Hall current. The results obtained are in agreement with the usual flow. It has been found that the velocity in the boundary layer region decreases with the values of radiation parameter. It is observed that the radiation and the rotation parameters increase the drag at the plate surface. Nusselt number decreases with increase in radiation parameter, Prandtl number and time. The results obtained will have applications in the research related to solar physics dealing with the sunspot development, the structure of rotating magnetic stars, cooling of electronic components of a nuclear reactor, bed thermal storage and heat sink in the turbine blades. This work can further be extended by considering some more relevant fluid parameters like, heat generation or absorption, radiation with chemical reaction, etc.

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APPENDIX

$$a=\frac{M(1-im)}{1+m^2}+2i\Omega,\ A_0=\frac{u_0^2 t}{\upsilon},\ A_1=1+e^{2\sqrt{az}}(1-A_{18})-A_{17},\ A_2=-A_1,\ A_3=1-e^{2\sqrt{az}}(1-A_{18})-A_{17},$$

$$A_4=-1+A_{19}+A_{30}(A_{20}-1),\ A_5=-1+A_{21}+A_{28}(A_{22}-1),\ A_6=-1+A_{23}+A_{26}(A_{31}-1),\ A_7=-1+A_{29}+A_{27}(A_{30}-1),$$

$$A_8=-1+A_{23}+A_{26}(A_{31}-1),\ A_9=-1-A_{24}-A_{28}(1-A_{25}),\ A_{10}=-A_9,\ A_{11}=|z|.|P_r|,\ A_{12}=e^{\frac{at}{P_r-1}-\frac{Rt}{P_r-1}-z\sqrt{\frac{aP_r-R}{P_r-1}}},$$

$$A_{13}=e^{\frac{at}{S_c-1}-z\sqrt{\frac{aS_c}{S_c-1}}},\ A_{14}=e^{\frac{at}{P_r-1}-\frac{Rt}{P_r-1}-|z|.\sqrt{\frac{P_r(aP_r-R)}{P_r-1}}},$$

$$A_{15}=-1+erf\left[\frac{z\sqrt{S_c}}{2\sqrt{t}}\right]\quad A_{16}=e^{|z|.\sqrt{P_rR}},\ A_{17}=erf\left[\frac{2\sqrt{a}t-z}{2\sqrt{t}}\right],\ A_{18}=erf\left[\frac{2\sqrt{a}t+z}{2\sqrt{t}}\right],$$

$$A_{19}=erf\left(\frac{z-2t\sqrt{\frac{aP_r-R}{P_r-1}}}{2t}\right),\ A_{20}=erf\left(\frac{z+2t\sqrt{\frac{aP_r-R}{P_r-1}}}{2t}\right),\ A_{21}=erf\left(\frac{z-2t\sqrt{\frac{aS_c}{S_c-1}}}{2t}\right),\ A_{22}=erf\left(\frac{z+2t\sqrt{\frac{aS_c}{S_c-1}}}{2t}\right),$$

$$A_{23}=erf\left[\frac{|z|.|P_r|}{2\sqrt{t}}-\sqrt{\frac{tR}{P_r}}\right],\ A_{24}=erf\left[\frac{2t\sqrt{\frac{a}{S_c-1}}-2\sqrt{S_c}}{2t}\right],\ A_{25}=erf\left(\frac{2t\sqrt{\frac{a}{S_c-1}}+2\sqrt{S_c}}{2t}\right),\ A_{16}=e^{|z|.\sqrt{P_rR}},$$

$$A_{27}=e^{2|z|.\sqrt{\frac{P_r(aP_r-R)}{P_r-1}}},\ A_{28}=e^{-2z\sqrt{\frac{aS_c}{S_c-1}}},\ A_{29}=erf\left(\frac{|z|.|P_r|}{2\sqrt{t}}-\sqrt{\frac{t(R-aP_r)}{P_r(1-P_r)}}\right),\ A_{30}=e^{-2z\sqrt{\frac{aP_r-R}{P_r-1}}},$$

$$A_{31}=erf\left(\frac{|z|.|P_r|}{2\sqrt{t}}+\sqrt{\frac{tR}{P_r}}\right),\ A_{32}=erf\left(\frac{|z|.|P_r|}{2\sqrt{t}}+\sqrt{\frac{t(R-aP_r)}{P_r(1-P_r)}}\right),\ A_{33}=1+A_{17}+e^{2\sqrt{az}}A_{34},\ A_{34}=erfc\left(\frac{2\sqrt{a}t+z}{2\sqrt{t}}\right),$$

Biographical notes

Dr. Uday Singh Rajput is a faculty member in the department of mathematics and astronomy, University of Lucknow, India. He has more than 25 years of teaching experience at UG and PG levels and also guided students for Ph D degree. He has published more than 80 research articles. His research areas include MHD flows, Graph Theory and Operations Research.

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