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On b-chromatic Number of Prism Graph Families

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Abstract

A b-coloring of graph G is a proper k-coloring C that verifies the following property: for every color class c_i, $1 \leq i \leq k$, there exists a vertex x_i, with color c_i, such that all the other colors in C are utilized in x_i neighbors. The b-chromatic number of a graph G, denoted by $\varphi(G)$, is the largest integer k such that G may have a b-coloring by k colors. In this paper we discuss the b-coloring of prism graph Y_n, central graph of prism graph $C(Y_n)$, middle graph of prism graph $M(Y_n)$ and the total graph of prism graph $T(Y_n)$ and we obtain the b-chromatic number for these graphs.

Keywords: b-coloring; prism graph; central graph; middle graph and total graph

AMS-MSC (2010): 05C15, 05C75, 05C76

1. Introduction

Graph coloring is one of the comprehensively researched and popular subject and applied in various disciplines such as channel assignment problems [Sohn (2013)], marketing and micro-economics [Talla Nobibon (2012)], timetabling [Burke et al. (2007), Sabar et al.

(2012), Badoni and Gupta (2014), Xu et al. (2014)] etc., which are studied by various mathematicians and computer scientists around the world.

Let G be a finite, undirected graph with no loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$. Definitions not given here may be found in [Harary (1969)]. A k-coloring of a graph $G = (V, E)$ is defined as a function c on $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ into a set of colors $C = \{1, 2, \dots, k\}$, such that $c(u) \neq c(v)$ for all $uv \in E(G)$. The chromatic number of G, denoted by $\chi(G)$, is the minimum cardinality k for which G has a proper k-coloring. A dominating proper k-coloring of G is a proper k-coloring that verifies the following property: for each color i, $1 \leq i \leq k$, there exists a vertex x, with color i, adjacent to vertices colored with every color j, $1 \leq j \neq i \leq k$. Such vertices are called dominating vertices and the set of vertices $\{x_1, x_2, x_3, \dots, x_n\}$ is called a b-dominating system. The b-chromatic number of G, denoted by $\varphi(G)$, is the maximum k such that a b-coloring with k colors exist for G.

This coloring was introduced by [Irving and Manlove (1999)]. They proved that determining b-chromatic number $\varphi(G)$ for an arbitrary graph G is NP-hard, but polynomial for trees.

The following upper bound for the b-chromatic number of a graph, presented in [Irving and Manlove (1999)], has been proved to be very useful. If G admits a b-coloring with m colors, then G must have at least m vertices with degree at least $m - 1$. The m-degree of a graph G, denoted by $m(G)$, is the largest integer m such that G has m vertices of degree at least $m - 1$. For a given graph G, it may be easily remarked that $\chi(G) \leq \varphi(G) \leq m(G)$.

It is an interesting problem to distinguish those graphs G such that $\varphi(G) = m(G)$. From this point of view, d-regular graphs are of special interest. [Kratochvil et al. (2002)] proved that if a d-regular graph G has at least d^4 vertices, then $\varphi(G) = d + 1$. Also, [Jakovac and Klavzar (2010)], proved that the b-chromatic number of cubic graphs is 4 except for the petersen graph, $K_{3,3}$ and the prism over K_3.

In recent years, there has been vigorous growth of interest in the study of b-coloring since the publication of [Irving and Manlove (1999)]. The b-chromatic number of some special graph classes has been discussed in [Alkhateeb and Kohl (2014), Havet et al. (2012), Kohl (2013), Kouider and Zamime (2017), Lisna and Sunitha (2015), Shaebani (2013), Vivin and Vekatachalam (2015)].

In the present work, we determine the exact value of $\varphi(G)$, where G are Y_n, $C(Y_n)$, $M(Y_n)$ and $T(Y_n)$ respectively.

2. Preliminaries

The central graph [Immanuel and Gella (2014), Kaliraj et al. (2014), Mahde and Mathad (2016)] of graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G.

The middle graph [Mohanapriya et al. (2016), Sutha et al. (2014), Vivin and Vekatachalam (2015)] of G, is defined with the vertex set $V(G) \cup E(G)$, where two vertices are adjacent, if and only if they are either adjacent edges of G or one is a vertex and the other is an edge incident with it and it is denoted by $M(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [Immanuel and Gella (2014), Mohanapriya et al. (2016), Vivin and Vekatachalam (2015)] of G, denoted by

$T(G)$ is defined as follows. The vertex set of $T(G)$ is $V(G) \cup E(G)$, where two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one the following holds:
(i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y are adjacent in G.
(iii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

A prism graph Y_n, is a graph corresponding to the skeleton of n-prism. A prism graph Y_n has $2n$ vertices and $3n$ edges.

3. Main results

In the main section, we determine the exact value of b-chromatic number of prism graph and its central, middle and total graph.

Theorem 3.1

If $n \geq 4$, then the b-chromatic number of prism graph is $\varphi\{\mathrm{Y_n}\} = 4$.

Proof:

Let Y_n be the prism graph on $2n$ vertices and $3n$ edges. Let $V(Y_n) = \{v_1, v_2, v_3, \dots, v_n\} \cup \{u_1, u_2, u_3, \dots, u_n\}$ (taken in order clockwise). We assign 4 colors to the vertices of Y_n as follows:
Consider color class $C = \{c_1, c_2, c_3, c_4\}$

Assign
$$c(v_i) = i, \text{ for } 1 \leq i \leq 4;$$
$$c(u_i) = i + 2, \text{ for } 1 \leq i \leq 2;$$
$$c(u_{i+2}) = i, \text{ for } 1 \leq i \leq 2.$$

Now for the remaining vertices we proceed with any proper coloring. We get b-coloring with b-vertices u_3, v_2, v_3 and u_2 for color classes $1, 2, 3$ and 4 respectively. It is maximal since Y_n has m-degree 4. Thus $\varphi\{\mathrm{Y_n}\} = 4$; for $n \geq 4$ (see Figure 1).

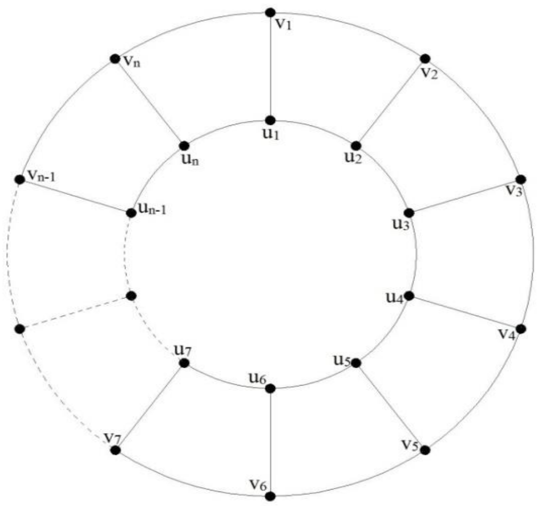

Figure 1. Prism Graph Y_n

Theorem 3.2

If $n \geq 4$, then the b-chromatic number of the central graph of prism graph is $\varphi\{C(Y_n)\} = n + \left\lfloor \frac{n}{2} \right\rfloor$.

Proof:

Let $V(Y_n) = \{v_1, v_2, v_3, \dots, v_n\} \cup \{u_1, u_2, u_3, \dots, u_n\}$ and $E(Y_n) = \{e_i : 1 \leq i \leq n-1\} \cup \{e_i' : 1 \leq i \leq n-1\} \cup \{e_i'' : 1 \leq i \leq n\} \cup \{e_n\} \cup \{e_n'\}$, where e_i is the edge $v_i v_{i+1}$ (for $1 \leq i \leq n-1$), e_i' is the edge $u_i u_{i+1}$ (for $1 \leq i \leq n-1$), e_i'' is the edge $v_i u_i$ (for $1 \leq i \leq n$), e_n is the edge $v_n v_1$ and e_n' is the edge $u_n u_1$. By definition of central graph, each edge of the prism graph is subdivided by a new vertex, then $V(C(Y_n)) = V(Y_n) \cup E(Y_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{v_i' : 1 \leq i \leq n\} \cup \{u_i' : 1 \leq i \leq n\} \cup \{v_i'' : 1 \leq i \leq n\}$, where v_i', u_i' and v_i'' represents the edge e_i, e_i' and e_i'' $(1 \leq i \leq n)$, respectively.

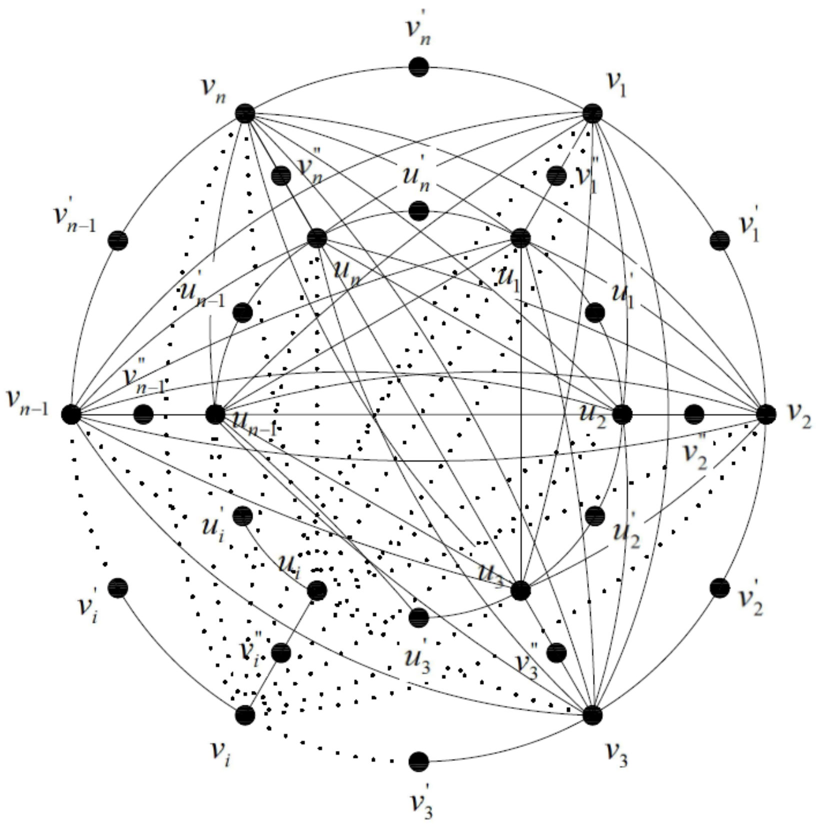

Figure 2. Central Graph of Prism Graph $C(Y_n)$

We assign the following coloring to the vertices of $C(Y_n)$ as follows:

$$c(v_i) = i \text{ for } 1 \leq i \leq n.$$
$$c(v_1') = n,\ c(v_{i+1}') = i \text{ for } 1 \leq i \leq n-1.$$

If n is odd, then assign the same color to u_n which we assigned to v_n (i.e. c_n).

Now assign

$$c(u_{2k-1}) = c(u_{2k}) = n + k \text{ for } 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor;$$
$$c(v_i'') = i + 1 \text{ for } 1 \leq i \leq n - 1 \text{ and } c(v_n'') = 1;$$
$$c(u_i') = i \text{ for } 1 \leq i \leq n - 1 \text{ and } c(u_n') = 1.$$

If we assign distinct colors to the vertices of u_i (for $1 \leq i \leq n$), then it does not produce b-coloring. Also, note that any relocation and combination of the colors to the graph fails to put up the new color. Therefore, this coloring is maximal, so that b-chromatic and has size $n + \left\lfloor \frac{n}{2} \right\rfloor$. Hence,

$$\varphi(\mathrm{C}(\mathrm{Y_n})) = n + \left\lfloor \frac{n}{2} \right\rfloor, \text{ for } n \geq 4 \text{ (see Figure 2).}$$

□

Theorem 3.3

If $\mathrm{M(Y_n)}$ is the middle graph of prism graph, then

$$\varphi\{\mathrm{M(Y_n)}\} = \begin{cases} 6; & for\ n = 3, 4 \\ 7; & for\ n \geq 5 \end{cases}$$

Proof:

Let $V(Y_n) = \{v_1, v_2, v_3, \dots, v_n\} \cup \{u_1, u_2, u_3, \dots, u_n\}$ and $E(Y_n) = \{e_i: 1 \leq i \leq n - 1\} \cup \{e_i': 1 \leq i \leq n - 1\} \cup \{e_i'': 1 \leq i \leq n\} \cup \{e_n\} \cup \{e_n'\}$, where e_i is the edge $v_i v_{i+1}$ (for $1 \leq i \leq n - 1$), e_i' is the edge $u_i u_{i+1}$ (for $1 \leq i \leq n - 1$), e_i'' is the edge $v_i u_i$ (for $1 \leq i \leq n$), e_n is the edge $v_n v_1$ and e_n' is the edge $u_n u_1$. By definition of middle graph, $V(M(Y_n)) = V(Y_n) \cup E(Y_n) = \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n\} \cup \{v_i': 1 \leq i \leq n\} \cup \{u_i': 1 \leq i \leq n\} \cup \{v_i'': 1 \leq i \leq n\}$, where v_i', u_i' and v_i'' represents the edge e_i, e_i' and e_i'' $(1 \leq i \leq n)$ respectively.

To prove this result we divide the proof into three parts. In the first and second parts we will prove that the b-chromatic number of middle graph of prism graph is $\varphi\{\mathrm{M(Y_n)}\} = 6$, if $n = 3, 4$ and in the third part, we will prove that b-chromatic number of middle graph of prism graph is $\mathrm{M(Y_n)} = 7$, if $n \geq 5$.

Case I: $n = 3$. We assign 6 colors, $C = \{c_1, c_2, c_3, \dots, c_6\}$ to the vertices of $M(Y_3)$ as follows:

$$c(v_1) = 5;\ c(v_2) = 6 \text{ and } c(v_3) = 4;$$
$$c(v_i') = i \text{ for } 1 \leq i \leq 3;$$
$$c(v_i'') = 3 + i \text{ for } 1 \leq i \leq 3;$$
$$c(u_1) = 6;\ c(u_2) = 4;\ c(u_3) = 5;$$
$$c(u_1') = 2;\ c(u_2') = 3 \text{ and } c(u_3') = 1.$$

In $M(Y_3)$, v_i' and v_i'' (for $1 \leq i \leq 3$) are b-chromatic vertices for color c_i (for $1 \leq i \leq 6$).

Without loss of generality let $c(u_1') = 7$, then v_i' (for $1 \leq i \leq 3$) cannot be adjacent to a vertex of color 7. So by introducing the new color to u_1', the coloring is not a b-coloring. Also, note that any relocation and combination of the colors to the graph $M(Y_3)$ fails to put up the new color. Therefore, this coloring is b-chromatic with maximum coloring possible and has size 6. Thus, $\varphi\big(\mathrm{M}(\mathrm{Y}_3)\big) = 6$ (see Figure 3).

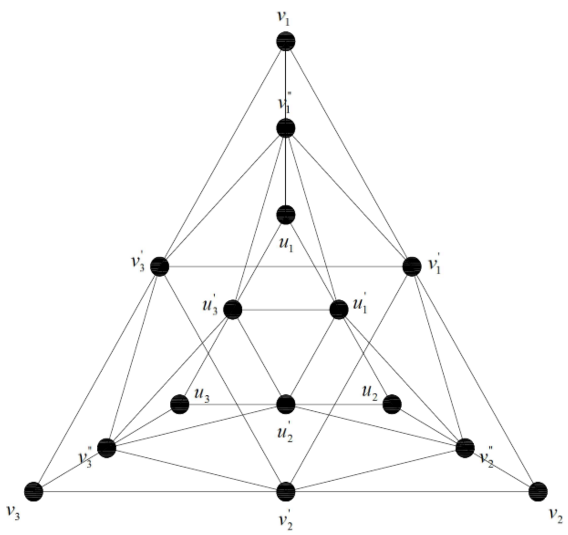

Figure 3. Middle Graph of Prism Graph $M(Y_3)$

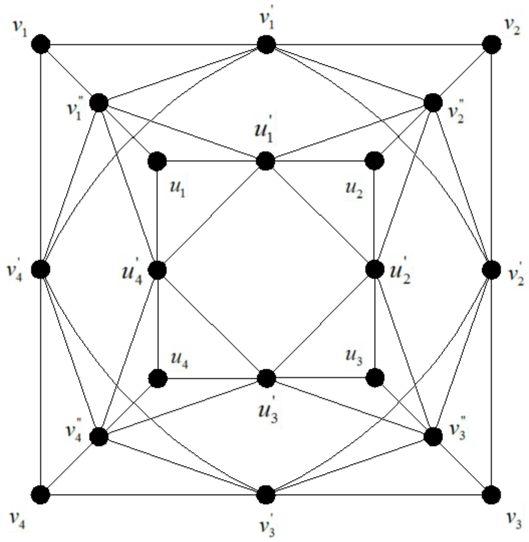

Figure 4. Middle Graph of Prism Graph $M(Y_4)$

Case II: $n = 4$. We assign 6 colors, $C = \{c_1, c_2, c_3, \dots, c_6\}$ to the vertices of $M(Y_4)$ as follows:

$$c(v_i') = i \text{ for } 1 \leq i \leq 4;$$
$$c(v_i) = 1 + i \text{ for } 1 \leq i \leq 3;\ c(v_4) = 1;$$
$$c(v_1'') = c(v_3'') = 5;$$
$$c\,(v_2'') = c\,(v_4'') = 6.$$

Now assign

$$c(u_1) = c(u_3) = 6;$$
$$c(u_2) = c(u_4) = 5;$$
$$c(u_1') = 4 \text{ and } c(u_{i+1}') = i \text{ for } 1 \leq i \leq 3.$$

In $M(Y_4)$; v_i' (for $1 \leq i \leq 4$), v_1'' and v_2'' are b-chromatic vertex. If we use 7 colors to meet the requirement of b-coloring, so let $c(v_3'') = 7$, then v_1', v_1'' and v_2'' cannot be adjacent to a vertex of color 7. So by introducing the new color to v_3'', the coloring is not a b-coloring. Also, note that any relocation and combination of the colors to the graph $M(Y_4)$ fails to put up the new color. Therefore, this coloring is b-chromatic with maximum coloring possible and has size 6. Thus, $\varphi\big(\mathrm{M}(\mathrm{Y}_4)\big) = 6$ (see Figure 4).

Case III: $n \geq 5$. We assign 7 colors to the vertices of $M(Y_n)$ as follows:
Consider color class $C = \{c_1, c_2, c_3, \dots, c_7\}$.

Assign

$$c(v_1) = c(v_3) = 6;$$
$$c(v_2) = c(v_4) = 7;$$

if $n > 5$, then assign

$$c\,(v_5) = 5.$$

{Special case: If $n = 5$ (i.e. For $M(Y_5)$), then $c(v_5) = 6$}

Now assign

$$c\,(v_i') = i; \text{ for } 1 \leq i \leq 4 \text{ and } c\,(v_n') = 5;$$
$$c\,(v_i'') = 2 + i \text{ for } 1 \leq i \leq 3;\ c\,(v_4'') = 1 \text{ and } c\,(v_i'') = 2 \text{ for } 5 \leq i \leq n;$$
$$c(u_1) = c(u_3) = 1;\ c\,(u_2) = c\,(u_4) = 2 \text{ and } c(u_i) = 7 \text{ for } 5 \leq i \leq n;$$
$$c\,(u_1') = 6;\ c\,(u_2') = 7;\ c\,(u_3') = 3;\ c\,(u_{n-1}') = 4 \text{ and } c\,(u_n') = 5.$$

If $n = 5$, then this coloring is completed. If $n > 5$, then for the remaining vertices of $M(Y_n)$, we further apply the following coloring:

$$c(v_i) = 1 \text{ for } 6 \leq i \leq n;$$
$$c(v_{2k-1}') = 6 \text{ for } n > 5;\ 3 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor;$$
$$c(v_{2k}') = 4 \text{ for } n > 6;\ 3 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor;$$
$$c(u_{2k}') = 5 \text{ for n} > 5;\ 2 \leq k \leq \left\lfloor \frac{n-2}{2} \right\rfloor;$$
$$c(u_{2k-1}') = 3 \text{ for n} > 6;\ 3 \leq \text{k} \leq \left\lfloor \frac{\text{n}-1}{2} \right\rfloor.$$

v_i' (for $1 \leq i \leq 4$), u_n', u_1' and u_2' are b-chromatic vertex. Obviously, this coloring is b-chromatic and has size 7. Then $\varphi\{M(Y_n)\} = 7$, for $n \geq 5$ (see Figure 5). It is maximal since $M(Y_n)$ has m-degree 7.

Hence, we can conclude from the first, second, and third case,

$$\varphi\{\mathrm{M(Y_n)}\} = \begin{cases} 6; & for\ n = 3, 4, \\ 7; & for\ n \geq 5. \end{cases} \qquad \square$$

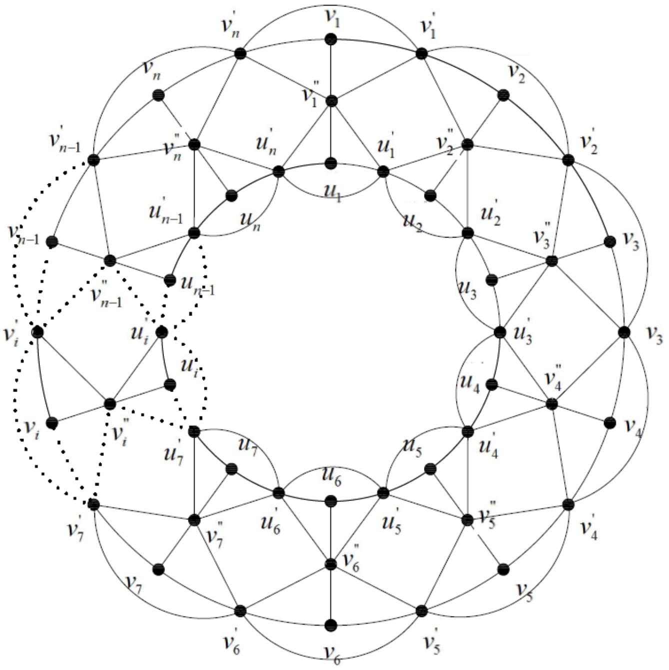

Figure 5. Middle Graph of Prism Graph $M(Y_n)$

Theorem 3.4

If $n \geq 4$, then b-chromatic number of total graph $T(Y_n)$ of prism graph is $\varphi\{T(Y_n)\} = 7$.

Proof:

We get the total graph of graph G from middle graph by adding edges that join the adjacent vertices of G. We assign 7 colors $C = \{c_1, c_2, c_3, \dots, c_7\}$ to the vertices of $T(Y_n)$ as follows:

$$c(v_n') = 1, c(u_n') = 3;$$
$$c(v_1) = 2, c(v_2) = 5, c(v_3) = 1, c(v_4) = 7;$$
$$c(v_1') = 7, c(v_2') = 6, c(v_3') = 4;$$
$$c(v_1'') = 4, c(v_2'') = 3, c(v_3'') = 2, c(v_4'') = 2;$$
$$c(u_1) = 5, c(u_2) = 4, c(u_3) = 3, c(u_4) = 6;$$
$$c(u_1') = 6, c(u_2') = 7, c(u_3') = 5.$$

These vertices are sufficient to produce a b-chromatic coloring of $T(Y_n)$, for $n \geq 4$. If $n > 4$, then for the remaining vertices of $T(Y_n)$, we further apply following coloring:

$$\mathrm{c}(\mathrm{v}_{2\mathrm{k}-1}) = 4, \text{ for n} > 4,\ 3 \leq \mathrm{k} \leq \left\lfloor \frac{\mathrm{n}+1}{2} \right\rfloor;$$
$$c(v_{2k}) = 7, \text{ for } n > 5,\ 3 \leq \mathrm{k} \leq \left\lfloor \frac{\mathrm{n}}{2} \right\rfloor;$$
$$c(v_{2k}') = 3, \text{ for } n > 4,\ 2 \leq \mathrm{k} \leq \left\lfloor \frac{n-1}{2} \right\rfloor;$$
$$c(v_{2k-1}') = 6, \text{ for } n > 5,\ 3 \leq \mathrm{k} \leq \left\lfloor \frac{\mathrm{n}}{2} \right\rfloor;$$

$$c(v_k'') = 5, \text{ for } n > 4, \ 5 \le k \le n;$$
$$c(u_{2k-1}) = 7, \text{ for } n > 4, \ 3 \le k \le \left\lfloor \frac{n+1}{2} \right\rfloor;$$
$$c(u_{2k}) = 2, \text{ for } n > 5, \ 3 \le k \le \left\lfloor \frac{n}{2} \right\rfloor;$$
$$c(u_{2k}') = 1, \text{ for } n > 4, \ 2 \le k \le \left\lfloor \frac{n-1}{2} \right\rfloor;$$
$$c(u_{2k-1}') = 4, \text{ for } n > 5, \ 3 \le k \le \left\lfloor \frac{n}{2} \right\rfloor.$$

We get b-coloring with b-vertices $v_3, v_3'', u_3, v_1'', v_2, v_2'$ and v_1' for color classes 1, 2, 3, 4, 5, 6, and 7, respectively. It is maximal since $T(Y_n)$ has m-degree 7.

Hence, $\varphi\{T(Y_n)\} = 7$, for $n \ge 4$ (see Figure 6). □

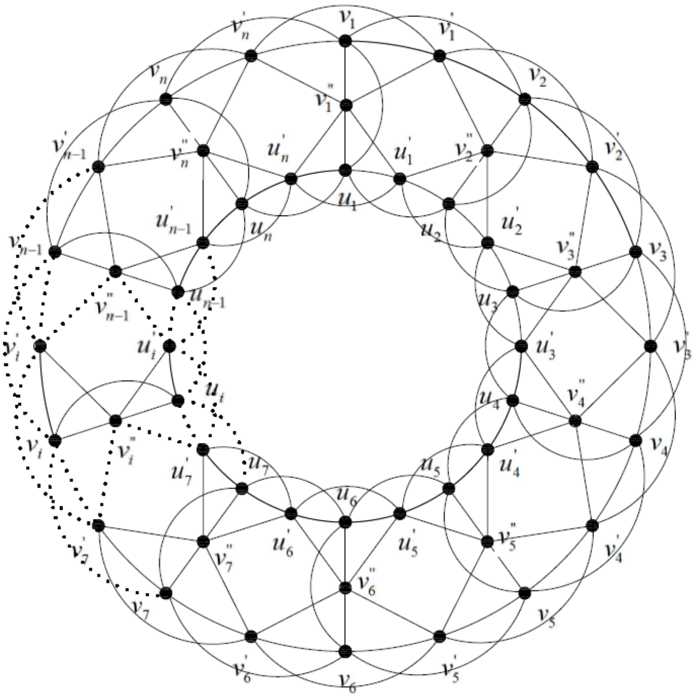

Figure 6. Total Graph of Prism Graph $T(Y_n)$

4. Conclusion

The study of b-coloring is important due to its applications in many real life problems like clustering, automatic recognition of documents, web service etc. In this paper, we investigated the b-chromatic number of prism graph and its central graph, middle graph and total graph. The investigation of analogous results for different graphs and different operation of above families of graph are still open.

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