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Profit Analysis of a Two Unit Cold Standby System Operating Under Different Weather Conditions Subject to Inspection

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Abstract

A system, or unit, is said to be working under normal weather conditions if the system is working under prescribed conditions as defined/stated by the definition of reliability of system/unit, otherwise the system is said to be working in abnormal weather conditions. For example, if a car with the capacity for five persons is carrying more than five persons, it will be said to be working under abnormal weather conditions. Another example, if a hydraulic machine having the capacity to lift a maximum weight of 500 tons is lifting a weight of 600 tons, then the machine is working under abnormal weather conditions. Hence, in this situation, work done by the machine is out of its capacity and the machine is working in abnormal weather conditions. If the machine is working within the capacity of the stated conditions, it is said to be working in normal weather conditions. The main purpose of this paper is to analyze the profit of a two-unit system called the standby system that is working under different weather conditions in an inspection facility. There is a single perfect server who visits the system immediately whenever required. A server inspects the unit before repair/replacement of the failed unit. All the mechanical activities done by the server are only possible during normal weather conditions. There are two possibilities after inspection of the unit; either repair of the unit is feasible or not feasible. If repair of the unit is not feasible, then the unit will be replaced immediately by a new unit. Otherwise, the repaired unit works as a new unit. The operative unit undergoes preventive maintenance after a specific (maximum) operation time. All random variables are statistically independent. The failure rate and the rate by which the system undergoes for preventive maintenance are constant whereas the inspection rate, repair rate, and maintenance rate follow negative exponential distributions. The expressions for several reliability measures are derived in steady state

conditions using the regenerative point technique and semi-Markov process. The graphical behavior of MTSF, availability and profit function, has been depicted with respect to preventive maintenance rate for arbitrary values of other parameters and costs.

Keywords: Profit Analysis; Cold Standby system; Preventive Maintenance; Repair inspection; Weather Conditions

MSC2010 No.: 60N05 and 90B25

1. Introduction

It is not possible, in general, that every product can be used only under the prescribed conditions by the manufacturers. Sometimes, the system works under partial prescribed conditions. In this situation, the system cannot be covered by the manufacturer warranty. Keeping such type of situation in mind the researchers including Goel et al. (1985) examined cost analysis of a two-unit cold standby system under different weather conditions. Gupta and Goel (1991) have obtained profit analysis of a two-unit cold standby system with abnormal weather conditions. Rander et al. (1994) have evaluated cost analysis of a two dissimilar cold standby system with preventive maintenance and replacement of standby. Gupta and Chaudhary (1994) analyzed profit of a system with two-units having guarantee periods and delayed operation of standby. Gopalan and Bhanu (1995) have discussed cost analysis of a two unit repairable system subject to on-line preventive maintenance and/or repair. Kumar et al. (1996) have obtained a probabilistic analysis of a two-unit cold standby system. Sridharan and Mohanavadivu (1997) analyzed cost benefit of a one server two dissimilar unit system subject to different repair strategies. Kadyan et al. (2004) have evaluated stochastic analysis of non-identical units reliability models with priority and different modes of failure.

Reliability models with priority for operation and repair with arrival time of server have been studied by Chander (2005). Malik and Barak (2007) obtained a probabilistic analysis of a single system operating under different weather conditions. Malik et al. (2008) examined a stochastic analysis of an operating system with two types of inspection subject to degradation. Malik and Barak (2009) obtained a reliability and economic analysis of a system operating under different weather conditions. Pawar et al. (2010) have discussed steady state analysis of an operating system with repair at different levels of damage subject to inspection and weather conditions. Malik and Barak (2013) analyzed reliability measures of a cold standby system with preventive maintenance and repair. Barak and Neeraj (2016) have obtained economic analysis of a system reliability model with priority to preventive maintenance over repair subject to weather conditions. Barak and Neeraj (2016) examined a system reliability model with priority to repair over preventive maintenance under different weather conditions. Ram and Nagiya (2016) discussed the performance evaluation of mobile communication systems with reliability measures. Ram and Manglik (2016) analyzed reliability measures of an industrial system under standby modes and catastrophic failure. Ram and Manglik (2016) have analyzed a multi-state manufacturing system with common cause failure and waiting repair strategy.

Recently, Barak et al. (2017) have discussed stochastic analysis of a two-unit system with standby and server failure subject to inspection. Keeping the above study in mind, they developed a model two-unit cold standby system working under different weather conditions (Figure1 state transition diagram). There is an inspection facility, unit is inspected before

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repair/replacement of the failed unit, and no requirement of inspection before preventive maintenance of the unit. There is a single perfect server who visits the system immediately, whenever required, and works only in normal weather. The server is not allowed to do any activity in abnormal weather. During inspection, if the server found that repair of the unit is not feasible, then the failed unit will be replaced by a new one; otherwise, a repaired unit will work as a new unit. The operative unit undergoes preventive maintenance after a specific (maximum) operation time. All random variables are statistically independent. The failure rate and the rate by which the system undergoes preventive maintenance are constant whereas the inspection rate, repair rate, and maintenance rate follow negative exponential distributions. The expressions for several reliability measures are derived in steady state using the regenerative point technique and semi-Markov process. The graphical behavior of MTSF, availability and profit function, has been depicted with respect to preventive maintenance rate for arbitrary values of other parameters and costs.

2. Notations

E:	The set of regenerative states $\{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$
O/Cs:	The unit is operative/cold stand by
α_0:	Maximum constant rate of operation time
λ:	Constant failure rate of the unit.
β / β_1:	Abnormal weather rate / Normal weather rate
$f(t) / F(t)$:	pdf /cdf of preventive maintenance time
$g(t) / G(t)$:	pdf /cdf of repair time of a failed unit
$h(t) / H(t)$:	pdf/cdf of inspection time of failed unit
a / b:	probability that unit goes for repair/replacement after inspection
P_m / WP_m:	The unit is under preventive maintenance/waiting for preventive maintenance
$\overline{P}_m / \overline{WP}_m$:	Preventive maintenance/waiting for preventive maintenance activities are stopped due to abnormal weather.
PM / FUR:	The unit is continuous under preventive maintenance/under repair from previous state.
$\overline{PM} / \overline{FUR}$:	The unit is continuous under preventive maintenance/under repair from previous state server activities are stopped due to abnormal weather conditions.
$FUr / WFUr$:	The failed unit under repair/waiting for repair due to server is busy with another unit.
$\overline{FUr} / \overline{WUR}_r$:	The failed unit under repair/waiting for repair is stopped due to abnormal weather.
$FUi / WFUi$:	The failed unit continuous under inspection/waiting for inspection from previous state.
$\overline{FUi} / \overline{FWUi}$:	The failed unit under inspection/waiting for inspection is stopped due to abnormal weather conditions.
$FUI / \overline{FUI}$:	The failed unit is continuous under inspection from previous state /inspection of the unit is stopped due to abnormal weather conditions.
Pmm / PMm:	The unit is under continuous preventive maintenance resumed from previous state which is halted in between due abnormal weather.

FUrr / *FURr* :   The failed unit is under repair continuous resumed from previous state which is halted in between due abnormal weather/ unit is under repair continuous from previous state after halting due to abnormal weather.

FUii / *FUIi* :   The failed unit is under continuous inspection resumed from previous state which is halted in between due abnormal weather/ unit is under inspection continuously from previous state after halting due to abnormal weather.

M_{ij} :   The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in state S_j . Mathematically it can be written as

$$m_{ij} = \int_0^\infty t\, d[Q_{ij}(t)] = -q_{ij}^{*'}(O):$$

μ_i The mean Sojourn time in state S_i this is given by

$$\mu_i = E(t) = \int_0^\infty P(T > t)\,dt = \sum_j m_{ij} \text{ , where T denotes the time to system failure}$$

$q_{i,j}(t)/Q_{i,j}(t)$:   pdf/cdf of passage time from regenerative state S_i to a regenerative stage or to a failed state/weather affected state S_j visiting state once in$(0,t]$

$q_{i,j;k(r,s)}(t)$ $/Q_{i,j;k(r,s)}(t)$:   pdf/cdf of direct transition time from regenerative state S_i to a regenerative stage S_j or to a failed /weather affected state S_j visiting state S_k once and more times states S_r and S_s in $(0,t]$.

$P_{i,j}$:   Probability of transition from state S_i to S_j

$P_{ij,k(r,s)^n}(t)$:   Probability of transition time from state S_i to S_j visiting state S_j, S_k once and more times states S_r and S_s in $(0,t]$

$\sim/*$:   Symbol for Laplace Steltjes transform/ Laplace transform

$\otimes/\oplus$:   Symbol for Laplace Stieltjes convolution/Laplace convolution

$\overline{}$:   used to stopped all mechanical activity due to abnormal weather

$'$(desh):   Used to represent alternative result

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for non-zero elements in particular case: let $f(t) = \theta e^{-\theta t}$ and $g(t) = \phi e^{-\phi t}$

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt \tag{1}$$

$$p_{0,1} = \frac{\alpha_0}{\alpha_0 + \lambda} \, , \, p_{0,2} = \frac{\lambda}{\alpha_0 + \lambda} \, , \, p_{1,0} = \frac{\theta}{\lambda + \beta + \theta + \alpha_0} \, , \, p_{1,3} = \frac{\beta}{\lambda + \beta + \theta + \alpha_0} \, , \, p_{1,9} = \frac{\lambda}{\lambda + \beta + \theta + \alpha_0}$$

$$p_{1,10} = \frac{\alpha_0}{\lambda + \beta + \theta + \alpha_0} \, , \, p_{2,0} = \frac{b\eta}{\lambda + \beta + \eta + \alpha_0} \, , \, p_{2,4} = \frac{a\eta}{\lambda + \beta + \eta + \alpha_0} \, , \, p_{2,6} = \frac{\beta}{\lambda + \beta + \eta + \alpha_0}$$

$$p_{2,23} = \frac{\alpha_0}{\lambda + \beta + \eta + \alpha_0} \, , \, p_{2,31} = \frac{\lambda}{\lambda + \beta + \eta + \alpha_0} \, , \, p_{3,1} = p_{5,4} = p_{6,2} = \frac{\beta_1}{\lambda + \beta_1 + \alpha_0}$$

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$$p_{4,0} = \frac{\phi}{\phi + \alpha_0 + \lambda + \beta} , \; p_{3,13} = p_{5,37} = p_{6,27} = \frac{\alpha_0}{\lambda + \beta_1 + \alpha_0} , \; p_{3,15} = p_{5,36} = p_{6,29} = \frac{\lambda}{\lambda + \beta_1 + \alpha_0}$$

$$p_{4,5} = \frac{\beta}{\phi + \alpha_0 + \lambda + \beta} , \; p_{4,17} = \frac{\lambda}{\phi + \alpha_0 + \lambda + \beta} , \; p_{4,20} = \frac{\alpha_0}{\phi + \alpha_0 + \lambda + \beta} , \; p_{1,1,10} = \frac{\theta\alpha_0}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)}$$

$$p_{7,2} = p_{9,2} = p_{10,1} = p_{12,1} = p_{14,1} = p_{16,2} = \frac{\theta}{\theta + \beta} , \; p_{7,8} = p_{9,8} = p_{10,11} = p_{12,11} = p_{14,13} = p_{16,15} = \frac{\beta}{\theta + \beta}$$

$$p_{17,2} = p_{19,2} = p_{20,1} = p_{22,1} = p_{26,1} = p_{34,2} = p_{35,2} = p_{38,1} = \frac{\phi}{\phi + \beta}$$

$$p_{1,1,10(11,12)} = \frac{\beta\alpha_0}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)} , \; p_{1,2,9} = \frac{\theta\lambda}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)} , \; p_{2,1,23} = \frac{b\alpha_0\eta}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)}$$

$$p_{1,2,9(8,7)} = \frac{\beta\lambda}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)} , \; p_{2,2,31} = \frac{b\lambda\eta}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)} , \; p_{4,2,17} = \frac{\lambda\phi}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)}$$

$$p_{17,18} = p_{19,18} = p_{20,21} = p_{22,21} = p_{26,37} = p_{34,36} = p_{35,36} = p_{38,37} = \frac{\beta}{\phi + \beta}$$

$$p_{23,1} = p_{25,1} = p_{28,1} = p_{30,2} = p_{31,2} = p_{33,2} = \frac{b\eta}{\eta + \beta} , \; p_{6,2,(29,30)34} = \frac{a\lambda\phi}{(\beta + \phi)(\alpha_0 + \lambda + \beta_1)}$$

$$p_{23,24} = p_{25,24} = p_{28,27} = p_{30,29} = p_{31,32} = p_{33,32} = \frac{\beta}{\eta + \beta} , \; p_{4,2,17(18,19)} = \frac{\lambda\beta}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)}$$

$$p_{23,26} = p_{25,26} = p_{28,26} = p_{30,34} = p_{31,34} = p_{33,34} = \frac{a\eta}{\eta + \beta} , \; p_{2,1,23(24,25)} = \frac{b\beta\alpha_0}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)}$$

$$p_{8,7} = p_{11,12} = p_{12,13} = p_{13,14} = p_{15,16} = p_{18,19} = p_{21,22} = p_{24,25} = p_{27,28} = p_{29,30} = p_{32,33} = p_{36,35} = p_{37,38} = 1$$

$$p_{2,1,23,26} = \frac{a\alpha_0\phi\eta}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)(\phi + \beta)} , \; p_{2,1,23(24,25)26} = \frac{a\alpha_0\beta\phi}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)(\phi + \beta)}$$

$$p_{2,1,23(24,25)26(37,38)} = \frac{a\alpha_0\beta^2}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)(\phi + \beta)} , \; p_{2,1,23,26(37,38)} = \frac{a\alpha_0\eta\beta}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)(\phi + \beta)}$$

$$p_{2,2,31(32,33)} = \frac{b\lambda\beta}{(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)} , \; p_{2,2,31,34} = \frac{a\lambda\eta\phi}{(\beta + \eta)(\beta + \phi)(\lambda + \eta + \beta + \alpha_0)}$$

$$p_{2,2,31(32,33)34} = \frac{a\lambda\beta\phi}{(\beta + \phi)(\beta + \eta)(\lambda + \eta + \beta + \alpha_0)} , \; p_{2,2,31,34(36,35)} = \frac{a\lambda\eta\beta}{(\beta + \phi)(\eta + \lambda + \beta + \alpha_0)(\eta + \beta)}$$

$$p_{2,2,31(32,33)34(36,35)} = \frac{a\lambda\beta^2}{(\beta + \phi)(\lambda + \eta + \beta + \alpha_0)(\beta + \eta)} , \; p_{3,1,(13,14)} = p_{5,1,(37,38)} = \frac{\alpha_0}{\lambda + \beta_1 + \alpha_0}$$

$$p_{3,2;(15,16)} = p_{5,2;(36,35)} = \frac{\lambda}{\lambda + \beta_1 + \alpha_0} , \; p_{6,2;(29,30)} = \frac{b\lambda}{(\lambda + \beta_1 + \alpha_0)} , \; p_{4,1;20} = \frac{\alpha_0\phi}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)}$$

$$, p_{4,1;20(21,22)} = \frac{\alpha_0\beta}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)} , \; p_{6,1;(27,28)} = \frac{b\alpha_0}{(\lambda + \beta_1 + \alpha_0)} \; p_{6,1,(27,28)26} = \frac{a\alpha_0\phi}{(\beta + \phi)(\alpha_0 + \lambda + \beta_1)} ,$$

$$p_{6,1,(27,28)26(37,38)} = \frac{a\alpha_0\beta}{(\beta + \phi)(\alpha_0 + \lambda + \beta_1)} \; p_{6,2,(29,30)34(36,35)} = \frac{a\lambda\beta}{(\beta + \phi)(\alpha_0 + \lambda + \beta_1)} \qquad (2)$$

The mean sojourn times $(\mu_i \text{ and } \mu_i')$ is the state S_i are

$$\mu_0 = \frac{1}{\alpha_0 + \lambda} \, , \mu_1 = \frac{1}{\alpha_0 + \lambda + \beta + \theta} \, , \ \mu_2 = \frac{1}{\alpha_0 + \lambda + \beta + \eta} \, , \mu_3 = \mu_5 = \mu_6 = \frac{1}{\alpha_0 + \lambda + \beta_1} \, , \mu_4 = \frac{1}{\alpha_0 + \lambda + \beta + \phi}$$

$$(3)$$

$$\mu_1' = \frac{\theta\beta_1 + (\lambda + \alpha_0)(\beta + \beta_1)}{\theta\beta_1(\lambda + \beta + \theta + \alpha_0)} \, , \mu_2' = \frac{\eta\phi\beta_1 + (\alpha_0 + \lambda)(\beta + \beta_1)(\phi + a\eta)}{\phi\eta\beta_1(\lambda + \beta + \eta + \alpha_0)} \, , \mu_3' = \frac{\theta\beta_1 + (\lambda + \alpha_0)(\beta + \beta_1 + \theta)}{\theta\beta_1(\lambda + \beta_1 + \alpha_0)}$$

$$\mu_4' = \frac{\beta_1\phi + (\alpha_0 + \lambda)(\beta + \beta_1)}{\phi\beta_1(\lambda + \beta + \alpha_0 + \phi)} \, , \mu_5' = \frac{\phi\beta_1 + (\lambda + \alpha_0)(\beta + \beta_1 + \phi)}{\phi\beta_1(\lambda + \beta_1 + \alpha_0)} \, ,$$

$$\mu_6' = \frac{\beta_1\eta\phi + (\alpha_0 + \lambda)\phi(\beta + \beta_1 + \eta) + a(\alpha_0 + \lambda)(\beta + \beta_1)\eta}{\phi\eta\beta_1(\lambda + \beta_1 + \alpha_0)}$$

$$(4)$$

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state takes as absorbing state. We have the following recursive relations for $\phi_i(t)$

$$\phi_0(t) = q_{0,1}(t) \otimes \phi_1(t) + q_{0,2}(t) \otimes \phi_2(t)$$

$$\phi_1(t) = q_{1,0}(t) \otimes \phi_0(t) + q_{1,3}(t) \otimes \phi_3(t) + q_{1,9}(t) + q_{1,10}(t)$$

$$\phi_2(t) = q_{2,0}(t) \otimes \phi_0(t) + q_{2,4}(t) \otimes \phi_4(t) + q_{2,6}(t) \otimes \phi_6(t) + q_{2,23}(t) + q_{2,31}(t)$$

$$\phi_3(t) = q_{3,1}(t) \otimes \phi_1(t) + q_{3,13}(t) + q_{3,15}(t)$$

$$\phi_4(t) = q_{4,0}(t) \otimes \phi_0(t) + q_{4,5}(t) \otimes \phi_5(t) + q_{4,17}(t) + q_{4,20}(t)$$

$$\phi_5(t) = q_{5,4}(t) \otimes \phi_4(t) + q_{5,36}(t) + q_{5,37}(t)$$

$$\phi_6(t) = q_{6,2}(t) \otimes \phi_2(t) + q_{6,27}(t) + q_{6,29}(t)$$

$$(5)$$

taking Laplace transform of above relation (5) and solving for $\tilde{\phi}_0(t)$. We have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \, ,$$

$$(6)$$

the reliability of the system model can be obtained by taking Laplace inverse transformation of (6). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{MN}{MD} \, ,$$

where

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$$MN = [\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}\{(\eta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}$$
$$\{(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}] + [\alpha_0\{(\eta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}(\lambda + \alpha_0 + \beta_1 + \beta)$$
$$\{(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}] + [\lambda\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}(\lambda + \alpha_0 + \beta_1 + \beta)\}$$
$$\{(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}] + [a\lambda\eta(\lambda + \alpha_0 + \beta_1)(\lambda + \alpha_0 + \beta + \beta_1)$$
$$\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}]$$

$$MD = [(\lambda + \alpha_0)\{(\eta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}$$
$$- \theta\alpha_0(\lambda + \alpha_0 + \beta_1)\{(\eta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\} - b\eta\lambda(\lambda + \alpha_0 + \beta_1)\{(\theta + \lambda + \alpha_0 + \beta)$$
$$(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}] + [(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1]$$
$$- [a\eta\lambda\phi(\lambda + \alpha_0 + \beta_1)^2\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)]$$

(7)

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are given as

$$A_0(t) = M_0(t) + q_{0,1}(t) \oplus A_1(t) + q_{0,2}(t) \oplus A_2(t)$$

$$A_1(t) = M_1(t) + q_{1,0}(t) \oplus A_0(t) + [q_{1,1;10}(t) + q_{1,1,10(11,12)}(t)] \oplus A_1(t) + [q_{1,2,9}(t) + q_{1,2,9(8,7)}(t)] \oplus A_2(t)$$
$$+ q_{1,3}(t) \oplus A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + [q_{2,1,23}(t) + q_{2,1,23,26}(t) + q_{2,1,23(24,25)}(t) + q_{2,1,23(24,25)26}(t)$$
$$+ q_{2,1,23,26(37,38)}(t) + q_{2,1,23(24,25)\ 26(37,38)}(t)] \oplus A_1(t) + [q_{2,2,31}(t) + q_{2,2,31,34}(t) + q_{2,2,31(32,33)}(t)$$
$$+ q_{2,2,31(32,33)34}(t) + q_{2,2,31,34(36,35)}(t) + q_{2,2,31(32,33)34(36,35)}] \oplus A_2(t)$$
$$+ q_{2,4}(t) \oplus A_4(t) + q_{2,6}(t) \oplus A_6(t)$$

$$A_3(t) = M_3(t) + [q_{3,1}(t) + q_{3,1,(13,14)}(t)] \oplus A_1(t) + q_{3,2,(15,16)}(t) \oplus A_2(t)$$

$$A_4(t) = M_4(t) + q_{4,0}(t) \oplus A_0(t) + [q_{4,1,20}(t) + q_{4,1,20(21,22)}(t)] \oplus A_1(t)$$
$$+ [q_{4,2,17}(t) + q_{4,2,17(18,19)}(t)] \oplus A_2(t) + q_{4,5}(t) \oplus A_5(t)$$

$$A_5(t) = M_5(t) + [q_{5,1,(37,38)}(t)] \oplus A_1(t) + [q_{5,2,(36,35)}(t)] \oplus A_2(t) + q_{5,4}(t) \oplus A_4(t)$$

$$A_6(t) = M_6(t) + [q_{6,1,(27,28)}(t) + q_{6,1,(27,28)26}(t) + q_{6,1,(27,28)26(37,38)}(t)] \oplus A_1(t)$$
$$+ [q_{6,2}(t) + q_{6,2,(29,30)}(t) + q_{6,2,(29,30)34}(t) + q_{6,2,(29,30)34(36,35)}] \oplus A_2(t)$$

(8)

where, $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time 't' without visiting to any other regenerative state. We have

$$M_0(t) = e^{-(\alpha_0 + \lambda)t}, M_1(t) = e^{-(\alpha_0 + \lambda + \beta)t}\overline{F(t)}, M_2(t) = e^{-(\alpha_0 + \beta + \lambda)t}\overline{H(t)}, M_4(t) = e^{-(\alpha_0 + \beta + \lambda)t}\overline{G(t)}$$

and

$$M_3(t) = M_5(t) = M_6(t) = e^{-(\alpha_0 + \beta_1 + \lambda)t}\ ,$$

(9)

taking Laplace transform of above relations (8) and (9) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{sN}{D} \text{, where} \tag{10}$$

$$N = \frac{\begin{bmatrix} \{(\phi + \lambda + \alpha_0 + \beta_1)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}\{(\theta + \lambda + \alpha_0 + \beta\ )(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\} \\ \{(\eta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\} \end{bmatrix}}{[(\lambda + \alpha_0)(\lambda + \alpha_0 + \beta_1)^3(\lambda + \alpha_0 + \theta + \beta)(\lambda + \alpha_0 + \phi + \beta)(\lambda + \alpha_0 + \eta + \beta)]}$$

$$D = X + T(W_1 + W_2 + W_3) ,$$

where

$$X = \frac{[a\eta\lambda\{(\theta + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta) - \beta\beta_1\}\{\alpha_0 + \lambda + \phi + \beta + \beta_1)(\beta + \beta_1) - \beta_1\phi\}]}{\phi\beta_1(\lambda + \alpha_0 + \beta_1)^2(\lambda + \alpha_0 + \theta + \beta)(\lambda + \alpha_0 + \phi + \beta)(\lambda + \alpha_0 + \eta + \beta)}$$

$$T = \frac{[(\phi + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta_1) - \beta_1\beta\}]}{[(\lambda + \alpha_0 + \beta_1)(\lambda + \alpha_0 + \phi + \beta)]}$$

$$W_1 = \frac{\begin{bmatrix} [\{(\theta + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta) - \beta\beta_1\} - \alpha_0(\alpha_0 + \lambda + \beta + \beta_1)\} \\ \{(\eta + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta) - \beta\beta_1\} - \lambda(\alpha_0 + \lambda + \beta + \beta_1)\}] - \alpha_0\lambda(\alpha_0 + \lambda + \beta + \beta_1)^2 \end{bmatrix}}{(\lambda + \alpha_0 + \beta_1)^2(\theta + \alpha_0 + \lambda + \beta)(\eta + \alpha_0 + \lambda + \beta)}$$

$$W_2 = \frac{[\alpha_0(\beta + \beta_1)\{(\eta + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}\{\alpha_0 + \lambda + \theta + \beta + \beta_1)(\alpha_0 + \lambda) + \beta_1\theta\}]}{[\theta\beta_1(\lambda + \alpha_0)(\lambda + \alpha_0 + \beta_1)^2(\lambda + \alpha_0 + \theta + \beta)(\lambda + \alpha_0 + \eta + \beta)]}$$

$$W_3 = \frac{\begin{bmatrix} [\lambda\{(\theta + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1\}[(\alpha_0 + \lambda + \beta + \beta_1)\{(\alpha_0 + \lambda)(\beta + \beta_1)(\phi + a\eta) \\ + \beta\eta\phi\} + (\alpha_0 + \lambda)\eta\beta\phi] \end{bmatrix}}{[\beta_1\eta\phi(\lambda + \alpha_0)(\lambda + \alpha_0 + \beta_1)^2(\lambda + \alpha_0 + \theta + \beta)(\lambda + \alpha_0 + \eta + \beta)]} .$$

6. Busy Period Analysis for Server

(a) Let $B_i^p(t)$ be the probability that the server is busy in preventive maintenance of the unit at an instant 't' given that system entered state S_i at t=0. The recursive relations for $B_i^p(t)$ are as follows

$$B_0^P(t) = q_{0,1}(t) \oplus B_1^P(t) + q_{0,2}(t) \oplus B_2^P(t)$$

$$B_2^P(t) = q_{20}(t) \oplus B_0^P(t) + [q_{2,1,23}(t) + q_{2,1,23,26}(t) + q_{2,1,23(24,25)}(t) + q_{2,1,23(24,25)26}(t) + q_{2,1,23,26(37,38)}(t)$$
$$+ q_{2,1,23(24,25)26(37,38)}(t)] \oplus B_1^P(t) + [q_{2,2,31}(t) + q_{2,2,31,34}(t) + q_{2,2,31(32,33)}(t) + q_{2,2,31(32,33)34}(t)$$
$$+ q_{2,2,31,34(36,35)}(t) + q_{2,2,31(32,33)34(36,35)}] \oplus B_2^P(t) + q_{2,4}(t) \oplus B_4^P(t) + q_{2,6}(t) \oplus B_6^P(t)$$

$$B_3^P(t) = U_3^P(t) + [q_{3,1}(t) + q_{3,1,(13,14)}(t)] \oplus B_1^P(t) + q_{3,2,(15,16)}(t) \oplus B_2^P(t)$$

$$B_4^P(t) = q_{4,0}(t) \oplus B_0^P(t) + [q_{4,1,20}(t) + q_{4,1,20(21,22)}(t)] \oplus B_1^P(t) + [q_{4,2,17}(t) + q_{4,2,17(18,19)}(t)] \oplus B_2^P(t)$$
$$+ q_{4,5}(t)] \oplus B_5^P(t)$$

$$B_5^P(t) = q_{5,1,(37,38)}(t) \oplus B_1^P(t) + q_{5,2,(36,35)}(t) \oplus B_2^P(t) + q_{5,4}(t) \oplus B_4^P(t)$$

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$$B_6^P(t) = [q_{6,1,(27,28)}(t) + q_{6,1,(27,28)26}(t) + q_{6,1,(27,28)26(37,38)}(t)] \oplus B_1^P(t) + [q_{6,2}(t) + q_{6,2,(29,30)}(t)$$
$$+ q_{6,2,(29,30)34}(t) + q_{6,2,(29,30)34(36,35)}] \oplus B_2^P(t) \quad , \quad (11)$$

where, $U_i^P(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states.

$$U_1^P(t) = \frac{\theta + \alpha_0 + \lambda}{\theta(\lambda + \beta + \theta + \alpha)} \ , \ U_3^P(t) = \frac{\alpha_0 + \lambda}{\theta(\lambda + \beta_1 + \alpha_0)} \ .$$

(b) Let $B_i^R(t)$ be the probability that the server is busy in repair of the unit at an instant 't' given that system entered state S_i at t=0. The recursive relations for $B_i^R(t)$ are as follows

$$B_0^R(t) = q_{0,1}(t) \oplus B_1^R(t) + q_{0,2}(t) \oplus B_2^R(t)$$

$$B_1^R(t) = q_{1,0}(t) \oplus B_0^R(t) + [q_{1,1,10}(t) + q_{1,1,10(11,12)}(t)] \oplus B_1^R(t) + [q_{1,2,9}(t) + q_{1,2,9(8,7)}(t)] \oplus B_2^R(t)$$
$$+ q_{1,3}(t) \oplus B_3^R(t)$$

$$B_2^R(t) = U_2^R(t) + q_{20}(t) \oplus B_0^R(t) + [q_{2,1,23}(t) + q_{2,1,23,26}(t) + q_{2,1,23(24,25)}(t) + q_{2,1,23(24,25)26}(t)$$
$$+ q_{2,1,23,26(37,38)}(t) + q_{2,1,23(24,25)26(37,38)}(t)] \oplus B_1^R(t) + [q_{2,2,31}(t) + q_{2,2,31,34}(t)$$
$$+ q_{2,2,31(32,33)}(t) + q_{2,2,31(32,33)34}(t) + q_{2,2,31,34(36,35)}(t) + q_{2,2,31(32,33)34(36,35)}] \oplus B_2^R(t)$$
$$+ q_{2,4}(t) \oplus B_4^R(t) + q_{2,6}(t) \oplus B_6^R(t)$$

$$B_3^R(t) = [q_{3,1}(t) + q_{3,1,(13,14)}(t)] \oplus B_1^R(t) + q_{3,2,(15,16)}(t) \oplus B_2^R(t)$$

$$B_4^R(t) = U_4^R(t) + q_{4,0}(t) \oplus B_0^R(t) + [q_{4,1,20}(t) + q_{4,1,20(21,22)}(t)] \oplus B_1^R(t) + [q_{4,2,17}(t)$$
$$+ q_{4,2,17(18,19)}(t)] \oplus B_2^R(t) + q_{4,5}(t)] \oplus B_5^R(t)$$

$$B_5^R(t) = U_5^R(t) + q_{5,1,(37,38)}(t) \oplus B_1^R(t) + q_{5,2,(36,35)}(t) \oplus B_2^R(t) + q_{5,4}(t) \oplus B_4^R(t)$$

$$B_6^R(t) = U_6^R(t) + [q_{6,1,(27,28)}(t) + q_{6,1,(27,28)26}(t) + q_{6,1,(27,28)26(37,38)}(t)] \oplus B_1^R(t)$$
$$+ [q_{6,2}(t) + q_{6,2,(29,30)}(t) + q_{6,2,(29,30)34}(t) + q_{6,2,(29,30)34(36,35)}(t)] \oplus B_2^R(t) \quad . \quad (12)$$

$U_i^R(t)$ be the probability that the server is busy in state S_i due to repair up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states.

$$U_2^R(t) = \frac{\left[\phi\eta(\beta+\eta)(\beta+\phi) + (\alpha_0+\lambda)\{a\eta^2(\beta+\phi) + \beta\phi(\beta+\phi) + a\eta\beta^2\}\right]}{\phi\eta(\beta+\eta)(\beta+\phi)(\lambda+\beta+\eta+\alpha_0)}, U_4^R(t) = \frac{\alpha_0 + \lambda + \phi}{\phi(\lambda+\beta+\phi+\alpha_0)}$$

$$U_5^R(t) = \frac{\lambda + \alpha_0}{\phi(\lambda+\beta_1+\alpha_0)} \ , \ U_6^R(t) = \frac{(\alpha_0+\lambda)(\phi+a\eta)}{\phi\eta(\lambda+\beta_1+\alpha_0)} \ . \quad (13)$$

Taking Laplace transform of above relations (11) and (12) and solving for $B_0^{*p}(t)$ and $B_0^{*R}(t)$ the time for which server is busy due to preventive maintenance and repair respectively is given by

$$B_0^p(t) = \lim_{s \to 0} s B_0^{*p}(s) = \frac{M_1^p}{D} , \quad B_0^R(t) = \lim_{s \to 0} s B_0^{*R}(s) = \frac{M_2^R}{D} , \tag{14}$$

where

$$M_1^P(t) = \frac{\begin{bmatrix} \alpha_0\{(\theta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0) - \beta\beta_1\}\{(\eta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0) - \beta\beta_1\} \\ \{(\phi + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0) - \beta\beta_1\} \end{bmatrix}}{[\theta(\lambda + \alpha_0)(\theta + \lambda + \beta + \alpha_0)(\phi + \lambda + \beta + \alpha_0)(\eta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0)^3]} . \tag{15}$$

$$M_1^R(t) = G(HT + I), \quad G = \frac{[\lambda[\{(\theta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0) - \beta\beta_1\}]]}{[(\theta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0)(\lambda + \alpha_0)]} .$$

$$H = \frac{\begin{bmatrix} (\lambda + \beta_1 + \alpha_0)[\phi\eta(\beta + \eta)(\beta + \phi) + (\alpha_0 + \lambda)\{(a\eta^2 + \beta\phi)(\beta + \phi) + a\eta\beta^2\}] \\ + \beta(\lambda + \alpha_0)(\beta + \phi)(\beta + \eta)(\phi + a\eta) \end{bmatrix}}{\phi\eta(\beta + \eta)(\beta + \phi)(\lambda + \beta + \eta + \alpha_0)(\lambda + \beta_1 + \alpha_0)} .$$

$$T = \frac{[(\phi + \alpha_0 + \lambda + \beta)(\lambda + \alpha_0 + \beta_1) - \beta_1\beta\}]}{[(\lambda + \alpha_0 + \beta_1)(\lambda + \alpha_0 + \phi + \beta)]} , \quad I = \frac{[a\eta[\{(\phi + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0) - \beta\beta_1\}]]}{[\phi(\phi + \lambda + \beta + \alpha_0)(\eta + \lambda + \beta + \alpha_0)(\lambda + \beta_1 + \alpha_0)]} . \tag{16}$$

7. Expected Number of Repair and Preventive Maintenance of the Units

Let $N_i^P(t)$ and $N_i^R(t)$ be the expected number of preventive maintenance and repair of unit by the server in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $N_i^P(t)$ and $N_i^R(t)$ are given as

$$N_0^K(t) = q_{0,1}(t) \otimes (N_1^K(t) + \delta_{PK}) + q_{0,2}(t) \otimes (N_2^K(t) + \delta_{RK})$$

$$N_1^K(t) = q_{1,0}(t) \oplus N_0^K(t) + [q_{1,1,10}(t) + q_{1,1,10(11,12)}(t)] \oplus N_1^K(t) + [q_{1,2,9}(t) + q_{1,2,9(8,7)}(t)] \oplus N_2^K(t)$$
$$+ q_{1,3}(t) \oplus N_3^K(t)$$

$$N_2^K(t) = q_{20}(t) \oplus N_0^K(t) + [q_{2,1,23}(t) + q_{2,1,23,26}(t) + q_{2,1,23(24,25)}(t) + q_{2,1,23(24,25)26}(t) + q_{2,1,23,26(37,38)}(t)$$
$$+ q_{2,1,23(24,25)26(37,38)}(t)] \oplus N_1^K(t) + [q_{2,2,31}(t) + q_{2,2,31,34}(t) + q_{2,2,31(32,33)}(t) + + q_{2,2,31(32,33)34}(t)$$
$$+ q_{2,2,31,34(36,35)}(t) + q_{2,2,31(32,33)34(36,35)}] \oplus N_2^K(t) + q_{2,4}(t) \oplus N_4^K(t) + q_{2,6}(t) \oplus N_6^K(t)$$

$$N_3^K(t) = [q_{3,1}(t) + q_{3,1,(13,14)}(t)] \oplus B_1^K(t) + q_{3,2,(15,16)}(t) \oplus B_2^K(t)$$

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$$N_4^K(t) = q_{4,0}(t) \oplus N_0^K(t) + [q_{4,1,20}(t) + q_{4,1,20(21,22)}(t)] \oplus N_1^K(t) + [q_{4,2,17}(t) + q_{4,2,17(18,19)}(t)] \oplus N_2^K(t)$$
$$+ q_{4,5}(t)] \oplus N_5^K(t)$$

$$N_5^K(t) = q_{5,1,(37,38)}(t) \oplus N_1^K(t) + q_{5,2,(36,35)}(t) \oplus N_2^K(t) + q_{5,4}(t) \oplus N_4^K(t)$$

$$N_6^K(t) = [q_{6,1,(27,28)}(t) + q_{6,1,(27,28)26}(t) + q_{6,1,(27,28)26(37,38)}(t)] \oplus N_1^K(t) + [q_{6,2}(t) + q_{6,2,(29,30)}(t)$$
$$+ q_{6,2,(29,30)34}(t) + q_{6,2,(29,30)34(36,35)}(t)] \oplus N_2^K(t)$$

(K=P, for preventive maintenance; K=R, for repair of the units) (17)

Taking Laplace Stieltjes transform of relations (16) and solving for $\tilde{R}_0^R(s)$ and $\tilde{R}_0^P(s)$. The expected no of repair and preventive maintenance per unit time are respectively of given by

$$R_0^P(\infty) = \lim_{s \to 0} s\tilde{R}_0^P(s) = \frac{N_3^P}{D} \text{ and } R_0^R(\infty) = \lim_{s \to 0} s\tilde{R}_0^R(s) = \frac{N_4^R}{D} \quad , \tag{18}$$

where

$$N_3^P = \frac{\begin{bmatrix} [\alpha_0\{(\phi+\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}[\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ -\{\alpha_0(\alpha_0+\lambda+\beta+\beta_1)\}\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}-\lambda(\alpha_0+\lambda+\beta+\beta_1)\} \\ -\alpha_0\lambda(\alpha_0+\lambda+\beta+\beta_1)^2]] - [a\eta\lambda\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ (\alpha_0+\lambda+\beta+\beta_1)(\alpha_0+\lambda+\beta_1)] \end{bmatrix}}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^3(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)(\eta+\alpha_0+\phi+\beta)}$$

$$N_4^R = \frac{\begin{bmatrix} [\lambda\{(\phi+\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}[\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ -\{\alpha_0(\alpha_0+\lambda+\beta+\beta_1)\}\{(\eta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\}-\lambda(\alpha_0+\lambda+\beta+\beta_1)\} \\ -\alpha_0\lambda(\alpha_0+\lambda+\beta+\beta_1)^2]] - [a\eta\lambda\{(\theta+\lambda+\alpha_0+\beta)(\lambda+\alpha_0+\beta_1)-\beta\beta_1\} \\ (\alpha_0+\lambda+\beta+\beta_1)(\alpha_0+\lambda+\beta_1)] \end{bmatrix}}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^3(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)(\eta+\alpha_0+\phi+\beta)}$$

and D has already defined.

8. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0 A_0 - K_1 B_0^R - K_2 B_0^P - K_3 R_0^R - K_4 R_0^P \quad , \tag{19}$$

K_0= (5,000): Revenue per unit up-time of the system
K_1= (400): Cost per unit time for which server is busy due preventive maintenance
K_2= (500): Cost per unit time for which server is busy due to repair and inspection
K_3= (350): Cost per visit per unit time repair and inspection
K_4= (300): Cost per visit per unit time preventive maintenance.

9. Result Discussion

The model is a case study of water supply system with particular values to the parameters like (α, β, β_1, λ, ϕ and θ) having facility of inspection before repair or replacement of the unit. The graphs for mean time to system failure, availability and profit function have been drawn with respect to preventive maintenance rate as shown in the figures 2-4 respectively.

Figure 2: indicates that the mean time to system failure goes on increasing with the increase of preventive maintenance rate (θ) and declines with the rate by which unit undergoes for preventive maintenance (or maximum operation time α_0). Therefore the mean time to system failure is highly affected by the preventive maintenance rate rather than other parameters. The trend of the mean time to system failure cannot affect so much by stopping all mechanical activities in abnormal conditions.

Figures 3: highlighted the trend of availability, which follow increasing pattern with the increase of preventive maintenance rate (θ) as well as normal weather rate (β_1) and the decline when maximum operation time α_0 is increasing. Availability of the system clearly can be seen that all parameters like (α, β, β_1, λ, ϕ and η) having their affect but pattern could not be changed after changing the values of the parameters expect the parameters η and ϕ. Hence, inspection of the failed unit before repair or replacement of the unit increases the availability of the system for use, whenever, the server allowed any activity only in the normal weather for avoiding unnecessary damage of the system.

Figures 4: clearly shows the trend of profit of the system, which follow increasing pattern with the increase of preventive maintenance rate (θ) as well as normal weather rate (β_1) and the decline when maximum operation time α_0 is increasing. The profit of the system clearly can be seen that all parameters like (α, β, β_1, λ, ϕ and η) having their affect but pattern could not be changed after changing the values of the parameters expect the parameters η and ϕ. Hence, inspection of the failed unit before repair or replacement of the unit increases the profit of the system, whenever, the server allowed any activity only in the normal weather for avoiding unnecessary damage of the system.

10. Conclusion

Hence, the study reveals that a cold standby system of two identical units working under different weather conditions and server works only in normal weather conditions and idea of inspection of the unit before repair/replacement of the unit, can be made more reliable and profitable to use by conducting preventive maintenance after a pre-specific period of operation as well as increasing normal weather rate rather than to increase repair rate of the system.

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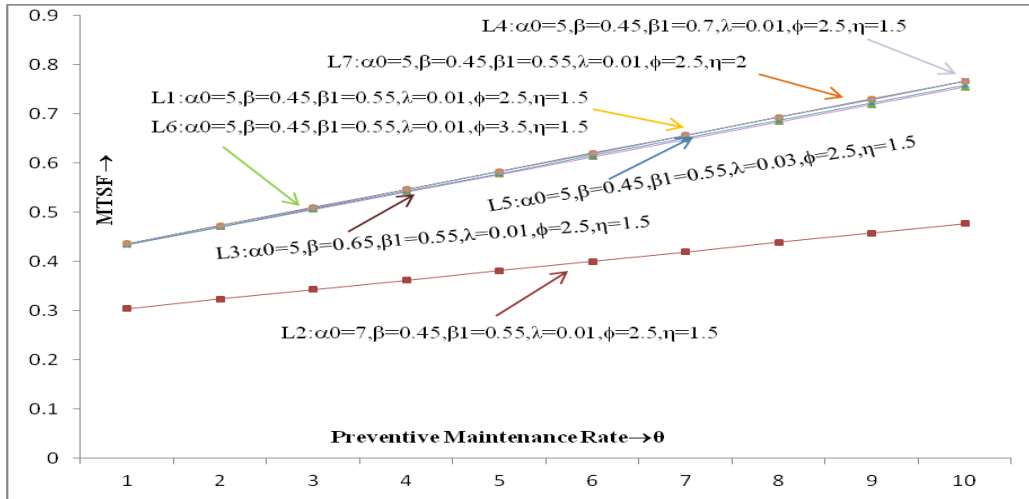
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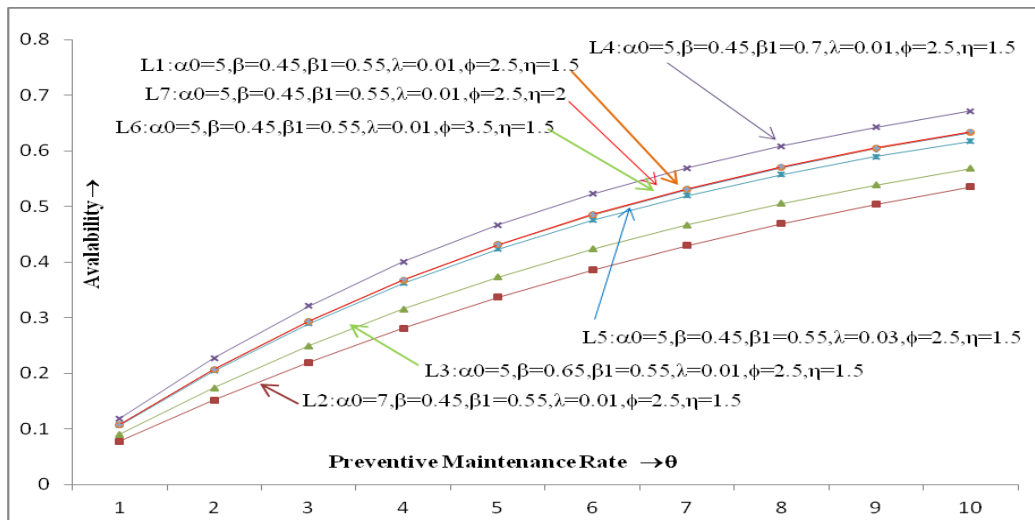**Figure 2.** Graph of MTSF Vs Prventive Maintenance Rate

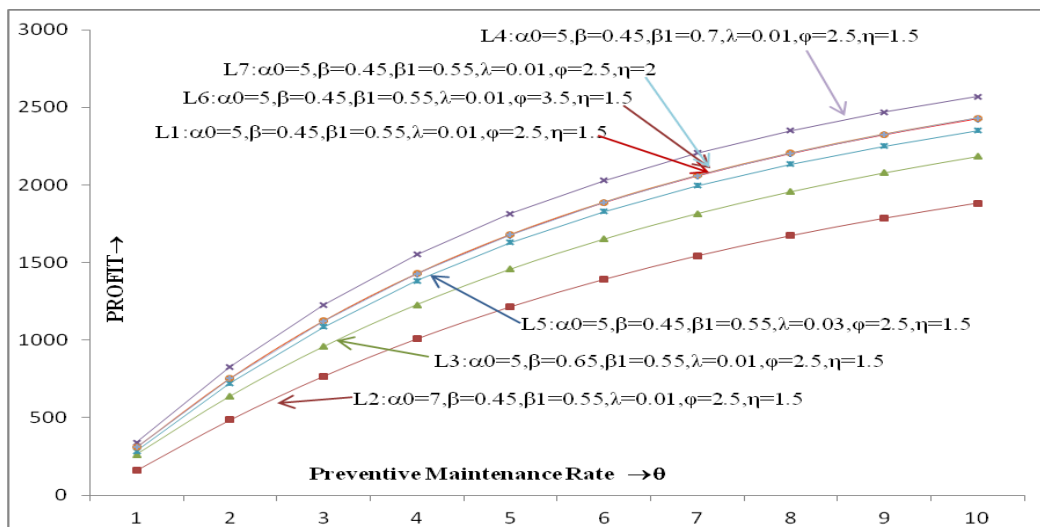**Figure 3.** Graph of Availability Vs Prventive Maintenance Rate

Figure 4. Graph of Profit Vs Prventive Maintenance Rate

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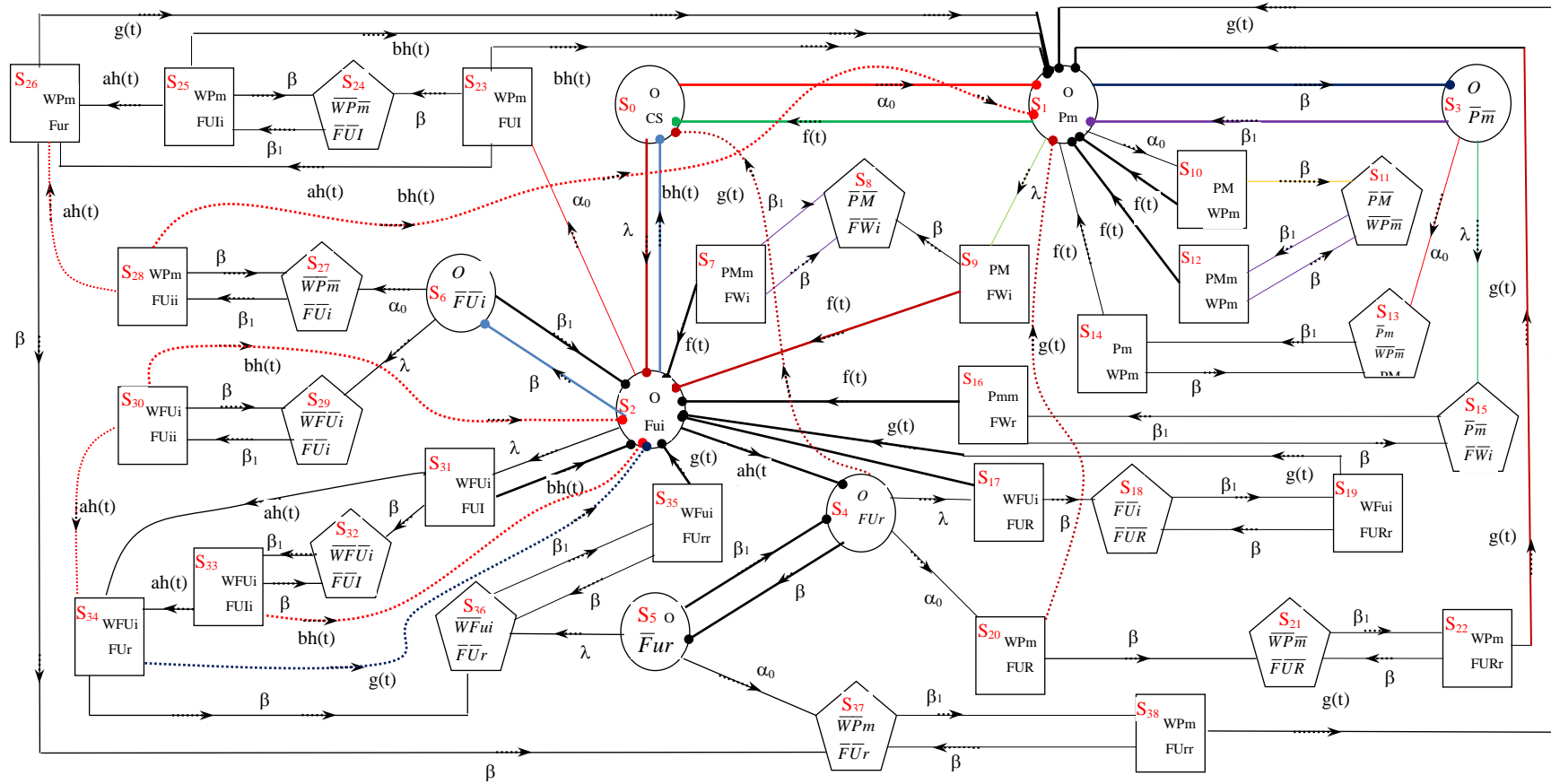AAM: Intern. J., Vol. 13, Issue 1 (June 2018)   83

Figure 1. State Transitions Diagram of Model