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Fitting Skew Distributions to Iranian Auto Insurance Claim Data

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Abstract

In actuary, the derivation of loss distributions from insurance data is of great interest. Fitting an adequate distribution to real insurance data is not an easy task, mainly due to the nature of the data, which shows several features to be accounted for. Although, because of its stochastic and numerical simplicity, it is often assumed that the involved financial risk factors are normally distributed, but empirical studies indicate that most of financial risk factors have distributions with high peaks and heavy tails. Thus, it is important in the actuarial science to model insurance risks with skewed distributions. Claims size data in non-life insurance policies are very skewed and exhibit high kurtosis and extreme tails. Skew distributions are reasonable models for describing claims in property-liability insurance. We fit several well-known skew distributions (skew-normal, skew-Laplace, generalized logistic, generalized hyperbolic, variance gamma, normal inverse Gaussian, Marshal-Olkin Log-Logistic and Kumaraswamy Marshal-Olkin Log-Logistic distributions) to the amount of automobile accident claims for property damage to a third party. The data are from financial records of a state-owned major general insurance company in Iran. The fitted models are compared using AIC (Akaike information criterion), BIC (Bayesian information criterion) and Kolmogorov-Smirnov goodness-of-fit test statistics. We find that the Kumarasamy Marshal-Olkin Log-Logistic distribution is better than other considered distributions in describing the features of the observed data. This distribution is a very perfect distribution to describe the skew data. The value at risk and conditional tail expectation, as most common risk measures in insurance, are estimated for the data under consideration.

Keywords: Skew-normal distribution; skew-Laplace distribution; generalized logistic distribution; generalized hyperbolic distribution; value at risk; conditional tail expectation

MSC 2010 No.: 91B30, 62P05

1. Introduction

The derivation of loss distributions from insurance data is of great interest in actuary (Burnecki et al. (2005), Chen et al. (2008)). For example, an accurate estimate of the claims distribution makes it possible to obtain accurate predictions for pricing, accurate estimation of future company liabilities and better understanding of the implications of the claims to the solvency of the company (Frees and Valdez (2008)). However, fitting an adequate distribution to real insurance data is not an easy task, mainly due to the nature of the data, which shows several features to be accounted for (Eling (2012)).

Although, because of its stochastic and numerical simplicity, it is often assumed that the involved financial risk factors are normally distributed, empirical studies indicate that most of financial risk factors have distributions with high peaks and heavy tails (Chen et al. (2008)). Specifically, claims size data in non-life insurance policies are very skewed and exhibit high kurtosis and extreme tails (see Lane (2000); Embrechts et al. (2002); Vernic (2006); Frees and Valdez (2008)). Thus, it is important in the actuarial science to model insurance risks with skewed distributions. To this end, several skew distributions, such as skew-normal (Azzalini (1985)) and other distributions from the skew-elliptical class, generalized logistic distributions (Gupta and Kundu (2010)) and generalized hyperbolic distributions (Barndorff-Nielsen (1977)), are promising candidates for modelling claims distribution.

Along this line, Eling (2012) showed that the skew-normal and the skew-Student t distributions are reasonably competitive compared to some models when describing insurance data. Bolance et al. (2008) provided strong empirical evidence in favour of the use of the skew-normal, and log-skew-normal distributions to model bivariate claims data from the Spanish motor insurance industry (see also Vernic (2006)). Ahn et al. (2012) used the log-phase-type distribution as a parametric alternative in fitting heavy tailed data. In the study of Burnecki et al. (2005) usual claims distributions showed the presence of small, medium and large size claims, which are characteristics that are hardly compatible with the choice of fitting a single parametric analytical distribution. Chen et al. (2008) employed generalized hyperbolic distributions for modelling insurance data. Frees and Valdez (2008) used skew distributions for conditional distribution of claim sizes given the number and type of the claims.

In this paper, we consider an Iranian insurance company data set consisting of the amount of automobile accident claims for property damage to a third party. We fit skew-normal, skew-Laplace, generalized logistic, generalized hyperbolic, variance gamma, normal inverse Gaussian, Marshal-Olkin Log-Logistic and Kumaraswamy Marshal-Olkin Log-Logistic distributions to the data and compare the fitted models. The two last distributions are a couple of recently developed skew distributions which are more flexible and potentially more apt to a better fit. The rest of the paper is organized as follows: section 2 reviews two basic risk measures in actuary and section 3 provides a background on the considered skew distributions. The data are introduced in section 4. Results from fitting the model to the data are presented in section 5. The paper concludes in section 6 with some discussions.

2. Risk Measures

One of the most challenging tasks in the analysis of financial markets is to measure and manage risks properly (Chen et al. (2008)). Different risk measures and their properties have been widely studied in the literature (see Artzner et al. (1999); Dhaene et al. (2006); Jorion (2007); McNeil et al. (2010), and references therein). Most of the contributions and applications in risk management usually assume a parametric distribution for the loss random variable.

In collective risk theory, the aggregate claims process is defined as $S_t = \sum_{j=1}^{N_t} X_j, \quad t \geq 0$, where N_t is is the total number of claims in the time interval $[0, t]$ and $X_1, X_2, \ldots$ are claim sizes (severities). Typically, the claim arrival point process $\{N_t, t \geq 0\}$ is assumed to be a homogeneous Poisson process and the claim size sequence $\{X_1, X_2, \ldots\}$ is assumed to be an i.i.d. sequence of random variables with a distribution function $F_X(.)$. Usually, $F_X(.)$ is assumed to be absolutely continuous with probability density function $f_X(.)$. Moreover, it is assumed that the second moment of the claim size variable X_j is finite; i.e. $E[X_j^2] < \infty$. The standard choices for $F_X(.)$ are exponential, gamma, Weibull, Pareto, log-normal and mixture distributions. An insurance company needs to assess the claim size distribution $F_X(.)$ in order to appropriately charge a premium to take responsibility for the risk.

2.1. Value at Risk

Among different risk measures, Value at Risk (VaR) has become the standard measure of the market risk (Chen et al. (2008)) and it is widely used in applications (Artzner et al. (1999)). The VaR risk measure was actually in use by actuaries long before it was reinvented for investment banking. In actuarial context it is known as the quantile risk measure or quantile premium principle (Dhaene et al. (2006); Jorion (2007)). VaR is always specified with a given confidence level $0 \leq \gamma \leq 1$. In broad terms, the γ-VaR represents the loss that, with probability γ, will not be exceeded. More precisely, the γ-VaR of the claim size variable X, or the claim size distribution $F_X(.)$, is defined as (Jorion (2007))

$$VaR_\gamma(X) = \inf\{x \in R: F_X(x) \geq \gamma\} = F_X^{-1}(\gamma). \tag{1}$$

The γ-VaR assesses the extreme claims, where extreme is defined as the event with a $1-\gamma$ probability. The behaviour of the claim size distribution $f_X(.)$ above the γ-quantile does not affect the value of γ-VaR. In other words, the definition of VaR in (1) does not take into consideration what the claim will be if "the $1-\gamma$ extreme claim" actually occur.

2.2. Conditional Tail Expectation

The conditional tail expectation (CTE) of the claim size variable X was introduced to address some of the problems with the $VaR_\gamma(X)$. It is also called tail value at risk (TVaR), tail conditional expectation (TCE) and expected shortfall (Dickson et al. (2013)). Given the confidence level γ, $0 \leq \gamma \leq 1$, the CTE is the expected claim given that the claim falls in the $1-\gamma$ extreme part of the claim size distribution. The $1-\gamma$ extreme part of the claim size distribution is the part above the γ-VaR. Thus, given $VaR_\gamma(X)$ and assuming F_X is continuous at $VaR_\gamma(X)$, the CTE at confidence level γ is defined by

$$CTE_\gamma(X) = E\left[X \middle| X \geq VaR_\gamma(X)\right] = \frac{1}{1-\gamma} \int_{VaR_\gamma(X)}^{\infty} x f_X(x) dx.$$

The CTE is a popular actuarial risk measure and a useful tool in financial risk assessment. Since the distribution function F_X is unknown, statistical methods are required to make inference about the CTE_γ based on the observed claim size data $X_1, \dots, X_n$ (Bolance et al. (2008)). Considering parametric forms for the claim size distribution F_X provides parametric estimates for $VaR_\gamma(X)$ and $CTE_\gamma(X)$.

3. Some Skew Distributions

Skewed distributions have played an important role in the statistical literature since the pioneering work of Azzalini (1985). He has provided a methodology to introduce skewness in a normal distribution. Since then a number of papers appeared in this area. He showed that if $f(.)$ is a symmetric density function defined on R and $F(.)$ is its distribution function, then for any $\alpha \in R$,

$$g_\alpha(x) = 2f(x)F(\alpha x), \;\; x \in R,$$

defines a proper density function on R. If $\alpha = 0$, $g_0(x) = f(x)$ is symmetric but for $\alpha \neq 0$, $g_\alpha(.)$ is skewed. If $\alpha \to \pm\infty$, then $g_\alpha(.)$ tends to the density function of $\pm|X|$, where $X \sim f(.)$. This property has been studied extensively in the literature in connection with skew-t and skew-Cauchy distributions (Gupta and Kundu (2010)). In this section we review some skew distributions which are appropriate for the claim size data.

3.1. Skew-normal Distribution

The random variable X has a skew-normal (SN) distribution with location parameter $\mu \in R$, scale parameter $\sigma > 0$ and shape parameter $\alpha \in R$ if its density function is given by

$$f_{SN}(x; \mu, \sigma, \alpha) = \frac{2}{\sigma} \phi\left(\frac{x-\mu}{\sigma}\right) \Phi\left(\alpha \frac{x-\mu}{\sigma}\right), \qquad x \in R,$$

where $\Phi(.)$ and $\phi(.)$ are the standard normal cumulative distribution function and the standard normal probability density function, respectively (Azzalini (1985)). We denote this by $X \sim SN(\mu, \sigma^2, \alpha)$. If $X \sim SN(\mu, \sigma^2, \alpha)$, then

$$E[X] = \mu + \sigma \sqrt{\frac{2}{\pi}}\, \delta \text{ and } Var(X) = \sigma^2 (1 - \frac{2}{\pi} \delta^2),$$

where $\delta = \frac{\alpha}{\sqrt{1+\alpha^2}}$. In addition, the coefficients of skewness and kurtosis of X are

$$S(X) = \frac{4-\pi}{2}\frac{sign(\alpha)\alpha^3}{\left[\frac{\pi}{2}+\left(\frac{\pi}{2}-1\right)\alpha^2\right]^{3/2}}, \quad K(X) = \frac{2(\pi-3)\alpha^4}{\left[\frac{\pi}{2}+\left(\frac{\pi}{2}-1\right)\alpha^2\right]^2},$$

and thus $S(X)$ varies in $(-0.9953, 0.9953)$ and $0 \leq K(X) \leq 0.8692$. The ranges of skewness and kurtosis show that the SN distribution is not appropriate for highly skewed data with extreme tail values. Further properties of the $SN(\mu, \sigma^2, \alpha)$ distribution are given in Azzalini (1985).

3.2. Skew-logistic and Generalized Logistic Distribution

Using the same basic principle of Azzalini (1985), the skewness can be easily introduced to the logistic distribution. The density function of a skew-logistic (SL) distribution with location parameter $\mu \in R$, scale parameter $\sigma > 0$ and shape parameter $\alpha \in R$, denoted $SL(\mu, \sigma, \alpha)$, is (Nadarajah (2009))

$$f_{SL}(x; \mu, \sigma, \alpha) = \frac{2e^{-\frac{x-\mu}{\sigma}}}{\beta\left(1 + e^{-\frac{x-\mu}{\sigma}}\right)^2 (1 + e^{-\alpha\frac{x-\mu}{\sigma}})}, \qquad x \in R.$$

Skew logistic distribution has some of the properties of the skew normal distribution. As such $f_{SN}(.)$, $f_{SL}(.)$ can have positive $(\alpha > 0)$ or negative $(\alpha < 0)$ skewness. However, the $SL(\mu, \sigma, \alpha)$ distribution is a more heavy tailed skewed distributions than the $SN(\mu, \sigma, \alpha)$ distribution. Also, for large values of α, the tail behaviors of the different members of the $SL(\mu, \sigma, \alpha)$ family are very similar.

Although $f_{SL}(.)$ is unimodal and log-concave, the distribution function, failure rate function, and different moments of $SL(\mu, \sigma, \alpha)$ do not have in explicit forms. Moreover, even when the location and scale parameters are known, the maximum likelihood estimator of the skewness parameter may not always exist (Gupta and Kundu (2010)). Thus, the SL distribution is difficult to use for data analysis purposes. As suggested by Gupta and Kundu (2010), instead of the SL distribution the type-I generalized logistic (GL) distribution, also known as proportional reversed hazard logistic (PRHL) distribution, can be employed for data analysis.

The generalized logistic distribution with location parameter $\mu \in R$, scale parameter $\sigma > 0$ and shape parameter $\alpha > 0$, denoted by $GL(\mu, \sigma, \alpha)$, has the density function

$$f_{GL}(x) = \frac{\alpha \exp(-\frac{x-\mu}{\sigma})}{\sigma\left[\,1 + \exp\left(-\frac{x-\mu}{\sigma}\right)\right]^{\alpha+1}}, \qquad x \in R.$$

The $GL(\mu, \sigma, \alpha)$ distribution is positively skewed for $\alpha > 1$ and negatively skewed for $0 < \alpha < 1$. If $X \sim GL(\mu, \sigma, \alpha)$, then $E[X] = \mu + \sigma[\psi(\alpha) - \psi(1)]$ and $Var(X) = \sigma^2[\psi'(\alpha) + \psi'(1)]$, where $\psi(y) = \frac{d}{dy}\log(\Gamma(y))$ and $\psi'(y) = \frac{d}{dy}\psi(y)$ are known as digamma and polygamma functions, respectively (Gupta and Kundu (2010)). The skewness and kurtosis of X are

$$S(X) = \frac{\psi''(\alpha) - \psi''(1)}{(\psi'(\alpha) + \psi'(1))^{\frac{3}{2}}}, \qquad K(X) = \frac{\psi'''(\alpha) - \psi'''(1)}{(\psi'(\alpha) + \psi'(1))^{2}}.$$

Here, $S(X)$ varies in $(-2.0, 1.1396)$ and $-2.4 \leq K(X) \leq 6$.

3.3. Skew-Laplace Distribution

The density function of the skew-Laplace (SLap) or skew-double exponential distribution is given by

$$f_{SLap}(x) = \begin{cases} \frac{1}{\alpha+\beta} \exp\{\frac{x-\mu}{\alpha}\}, & x \leq \mu, \\ \frac{1}{\alpha+\beta} \exp\left\{\frac{x-\mu}{\beta}\right\}, & x > \mu, \end{cases}$$

where $\mu \in R$ is the location parameter and the mode of the distribution and $\alpha > 0$ and $\beta > 0$ are mixture parameters (Fieller et al. (1992)). If $\alpha \to 0$ or $\beta \to 0$, then the two-parameter exponential or negative-exponential distribution is obtained. The case $\alpha = \beta$ corresponds to the classical symmetric Laplace distribution. If $X \sim SLap(\mu, \alpha, \beta)$, then $E[X] = \mu + \beta - \alpha$ and $Var(X) = \alpha^2 + \beta^2$. Also, the skewness and kurtosis of X are

$$S(X) = \frac{2(\beta^3 - \alpha^3)}{(\alpha^2 + \beta^2)^{\frac{3}{2}}}, \qquad K(X) = 3 + \frac{6(\beta^4 + \alpha^4)}{(\alpha^2 + \beta^2)^2},$$

where $S(X)$ varies in $(-2, 2)$. Parameter estimation and further properties of the SLap distribution are discussed in Puig and Stephens (2007) and references therein.

3.4. Generalized Hyperbolic Distribution

The random variable X is said to have a generalized hyperbolic (GH) or normal mean-variance mixture distribution if $X = \mu + W\gamma + \sqrt{W}\sigma Z$ (Barndorff-Nielsen (1977)), where $Z \sim N(0,1)$ and W is independent of Z and has a generalized inverse Gaussian (GIG) distribution, $W \sim GIG(\lambda, \chi, \psi)$, with density function

$$f_W(w) = \left(\frac{\psi}{\chi}\right)^{\frac{\lambda}{2}} \frac{w^{\lambda-1}}{2K_\lambda(\sqrt{\chi\psi})} \exp\left\{-\frac{1}{2}\left(\frac{\chi}{w} + \psi w\right)\right\}, \quad w > 0,$$

where $K_\lambda(.)$ is the modified Bessel function of the third kind (see e.g. Abramowitz and Stegun (2012)) and $\chi > 0,\ \psi \geq 0,\ \lambda < 0$ or $\chi > 0,\ \psi > 0,\ \lambda = 0$ or $\chi \geq 0,\ \psi > 0,\ \lambda > 0$. Here, $\mu \in R$ is the location parameter, $\sigma > 0$ is the dispersion parameter and $\gamma \in R$ is the skewness parameter. If $\gamma = 0$, then the distribution is symmetric around μ. The GH density is given by

$$f_{GH}(x)=\frac{\left(\sqrt{\frac{\psi}{\chi}}\right)^{\lambda}\left(\psi+\frac{\gamma^2}{\sigma^2}\right)^{\frac{1}{2}-\lambda}K_{\lambda-\frac{1}{2}}\left(\sqrt{\left[\chi+\left(\frac{x-\mu}{\sigma}\right)^2\right]\left(\psi+\frac{\gamma^2}{\sigma^2}\right)}\right)}{\sqrt{2\pi\sigma^2}K_{\lambda}(\sqrt{\psi\chi})\left(\sqrt{\left[\psi^2+\left(\frac{x-\mu}{\sigma}\right)^2\right]\left(\psi+\frac{\gamma^2}{\sigma^2}\right)}\right)^{\frac{1}{2}-\lambda}}\exp\left(\frac{(x-\mu)\gamma}{\sigma^2}\right).$$

Many distributions are a special or limiting cases of the GH distribuion. For $\lambda=1$, the distribution is called hyperbolic distribution and for $\lambda=-\frac{1}{2}$ yields normal inverse Gaussian (NIG) distribution. In the case of $\chi=0$ and $\lambda>0$, the distribution is known as Variance Gamma (VG) distribution and in the case $\psi=0$ and $\lambda<0$, is called the generalized hyperbolic Student-t distribution.

There is a known identification issue with the parameters $(\lambda,\chi,\psi,\mu,\sigma^2,\gamma)$: for any $\nu>0$, the distribution $GH(\lambda,\chi,\psi,\mu,\sigma^2,\gamma)$ is identical with $GH(\lambda,\frac{\chi}{\nu},\nu\psi,\mu,\nu\sigma^2,\nu\gamma)$. This problem can be solved by introducing the constraint

$$E[W]=\sqrt{\frac{\chi}{\psi}}\frac{K_{1+\lambda}(\sqrt{\chi\psi})}{K_{\lambda}(\sqrt{\chi\psi})}=1.$$

Now, by setting $\bar{\alpha}=\sqrt{\chi\psi}$ we obtain

and $\chi=\bar{\alpha}\ \frac{K_{\lambda}(\bar{\alpha})}{K_{1+\lambda}(\bar{\alpha})},\psi=\bar{\alpha}\ \frac{K_{1+\lambda}(\bar{\alpha})}{K_{\lambda}(\bar{\alpha})}$,

and thus the distribution can be reparameterized as $GH(\lambda,\bar{\alpha},\mu,\sigma^2,\gamma)$. Using the parametrization $(\lambda,\bar{\alpha},\mu,\sigma^2,\gamma)$, the distribution does not exist in the case $\bar{\alpha}=0$ and $-1\leq\lambda\leq 0$, which corresponds to a generalized Student-t distribution with non-existing variance (Luethi and Breymann (2013)). If $X\sim GH(\lambda,\bar{\alpha},\mu,\sigma^2,\gamma)$, then $E[X]=\mu+\gamma$ and $Var(X)=\sigma^2+\gamma^2Var(W)$. The skewness and kurtosis of the GH distribution are not expressible in closed analytical forms but they can be approximated using numerical methods.

3.5. Marshal-Olkin Log-Logistic Distribution

Marshall and Olkin (1997) considered a new family of distribution for a given distribution with cdf $G(x)$, survival function $\bar{G}(x)$ and pdf $f(x)$. They defined the cdf and pdf of the Marshal-Olkin family of distributions respectively by

$$F(x)=1-\left[\frac{r\big(1-G(x)\big)}{1-(1-r)\bar{G}(x)}\right],$$

$$f(x)=\frac{rg(x)}{[1-(1-r)\bar{G}(x)]^2}.$$

If we consider the parent log-logistic distribution with positive parameters α and β and pdf and cdf given by $g(x)=\frac{\alpha\beta^{-\alpha}x^{\alpha-1}}{\left[\left(\frac{x}{\beta}\right)^{\alpha}+1\right]^2},x\in R,G(x)=\frac{1}{\left(\frac{x}{\beta}\right)^{-\alpha}+1}$, then the pdf of the Marshal-Olkin log-logistic (MO) distribution reduces to

$$f_{MO-LL}(x) = \frac{r\alpha\beta^{-\alpha}x^{\alpha-1}}{\left(r + \left(\frac{x}{\beta}\right)^{\alpha}\right)\left(1 + \left(\frac{x}{\beta}\right)^{\alpha}\right)}.$$

3.6. Kumaraswamy Marshal-Olkin Log-Logistic Distribution

Alizadeh et al. (2015) proposed a new extension of the Marshal-Olkin family for a given baseline distribution with cdf $G(x)$, survival function $\bar{G}(x)$ and pdf $f(x)$ depending on a parameter vector ξ. They defined the cdf and pdf of the new kumaraswamy Marshal-Olkin family of distributions with the three additional shape parameters $a, b\ , p > 0$, respectively by

$$F(x) = 1 - \left\{1 - \left[\frac{G(x)}{1 - p\bar{G}(x)}\right]\right\},$$

$$f(x) = \frac{ab(1-p)g(x)G(x)^{a-1}}{[1 - p\bar{G}(x)]^{a+1}}\left\{1 - \left[\frac{G(x)}{1 - p\bar{G}(x)}\right]^{a}\right\}^{b-1}.$$

If we consider the parent log-logistic distribution with positive parameters α and β and pdf and cdf given by $g(x) = \frac{\alpha\beta^{-\alpha}x^{\alpha-1}}{\left[\left(\frac{x}{\beta}\right)^{\alpha}+1\right]^{2}}, x \in R, G(x) = \frac{1}{\left(\frac{x}{\beta}\right)^{-\alpha}+1}$, then the pdf of the Kumaraswamy Marshal-Olkin log-logistic (KMO) distribution reduces to

$$f_{KwMO-LL}(x) = \frac{ab(1-p)\alpha\beta^{-\alpha}x^{\alpha-1}\left(\frac{\left(\frac{x}{\beta}\right)^{-\alpha}+1}{\left(\frac{x}{\beta}\right)^{\alpha}+1}\right)^{2}}{\left[1 - p\left(\left(\frac{x}{\beta}\right)^{-\alpha} - 1\right)\right]^{a+1}}\left\{1 - \left[\frac{1}{1 - p\left(\left(\frac{x}{\beta}\right)^{-\alpha} - 1\right)}\right]^{a}\right\}^{b-1}.$$

4. Data

As in many countries, owners of automobiles in Iran are obliged to have minimum coverage for property damage and personal injury to third parties (parties other than the insured). The data in the present study are gathered from vehicle insurance portfolios from a state-owned major general insurance company in Iran, Alborz Insurance Company. The observations are from financial records of the amount of automobile accident claims for property damage to a third party over a period of one year, March 2011-March 2012. Only the dates on which claims for payment were submitted have been used and the effect of placing an upper limit on the amount reimbursed to a policyholder in the event of a claim, known as a coverage limit, is ignored.

Figure 1 shows the claim sizes and aggregate claim sizes over the time period along with the histogram and log-histogram of the claim size data. The plot of claim sizes over the time shows the homogeneity of the claim arrival process and presence of extreme values among claim sizes. Histogram and log-histogram of the claim sizes indicate the long-tailed nature of the distribution of claims. Figure 1 also presents descriptive statistics for the data. In addition to the number of observations, indicators for the first four moments (mean, standard deviation, skewness, excess kurtosis), and minimum and maximum, we also present the 99% quantile and the mean loss, if the loss is above 99%. The 99% quantile is an empirical estimate of $VaR_{0.99}(X)$ and the mean loss exceeding the 99% quantile is an empirical estimate of the $CTE_{0.99}(X)$. The histogram, log-histogram and descriptive statistics show that the third party car property damage claims distribution has a high level of skewness and kurtosis and any candidate parametric model for the data is required to mimic these features.

5. Results

In this section, we fit the skew-normal (SN), skew-Laplace (SLap), generalized logistic (GL), generalized hyperbolic (GH), variance gamma (VG), normal inverse gamma (NIG), Marshal-Olkin Log-Logistic (MO) and Kumaraswamy Marshal-Olkin Log-Logistic (KMO) distributions to the data. Parameters of all models are estimated using maximum likelihood estimation. All the calculations are implemented in the statistical programming language R (R Core Team (2013)), using packages sn (Azzalini (2014)), glogis (Zeileis and Windberger (2014)) and ghyp (Luethi and Breymann (2013)).

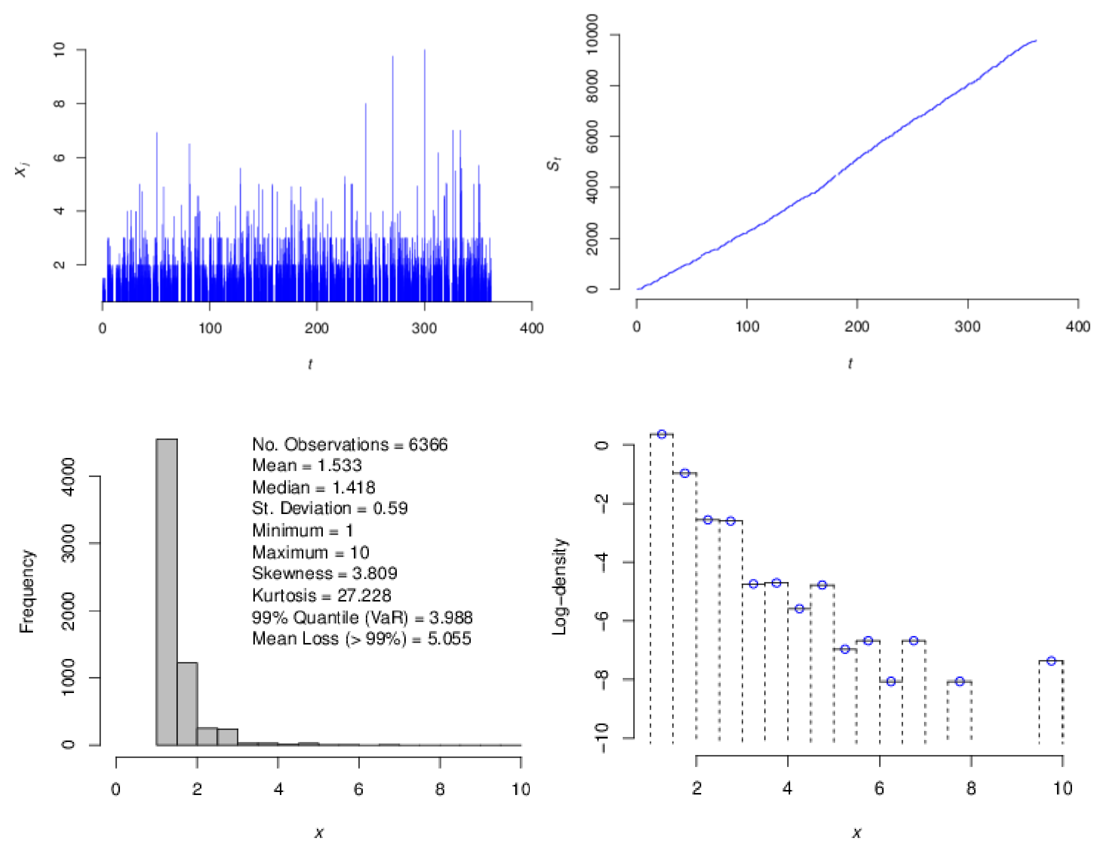

Figure 1. Claim size (top left), aggregate claims (top right), histogram (bottom left) and log-Histogram (bottom right) of the amount of automobile accident claims for property damage to a third party.

The results are presented in Table 1. The log-likelihood, Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) of each model are also reported in Table 1. Based on AIC and BIC criteria, the GH and its special case, the VG model are better, respectively. The AIC and BIC of the GH, VG, SLap models are fairly close and since the models are not nested, the likelihood ratio test cannot be employed to see if these models are significantly different from each other.

To examine which model is more in agreement with the data, the Kolmogorov-Smirnov (KS) goodness-of-fit test statistic is computed for each model. All of these values are above the critical value of the KS test at 5% level with $n = 6366$, which is 0.01702, but the two smallest values belong to the NIG and KMO model. This indicates that the two distribution functions of the fitted NIG and KMO models are closer to the empirical distribution function of the data than other models. This can also be seen from Figure 2, where the logarithm of the density functions of all fitted models are compared with the log-histogram of the data. This figure shows the KMOL is superior to other distributions in covering the long tail of the data distribution. Perhaps one reason it is so is that it is a special and more flexible distribution to descibe the skew data. In addition, the KS test statistic and Figure 2 reveal that the fitted SN distribution is the worst model among the fitted models. This is a consequence of its narrow range of skewness and kurtosis which is inadequate for the present data with high levels of skewness and kurtosis.

Finally, Table 2 compares the empirical mean, standard deviation, skewness, kurtosis, VaR and CET (at 95% and 99% confidence levels) of the data with their parametric counterparts under the fitted models. From the expected value, standard deviation, skewness and kurtosis, it can be seen that the KMO model has the closest characteristics to their corresponding empirical values. Also, the fitted KMO model provides very close VaR and CET values to their corresponding empirical values at both 95% and 99% confidence levels. Notice that the characteristics of the fitted SN model are very far from their corresponding empirical values.

Table 1. Estimated parameters, log-likelihood, AIC, BIC and Kolmogorov-Smirnov test statistic of the fitted skew-normal (SN), skew-Laplace (SLap), generalized logistic (GL), generalized hyperbolic (GH), variance-gamma (VG), normal inverse Gaussian (NIG) distributions to the amount of automobile accident claims for property damage to a third party.

Model	Estimates of model parameters	Log-likelihood	AIC	BIC	KS statistics
SN	$\hat{\mu} = 1.0014, \hat{\sigma} = 0.7931, \hat{\alpha} = 183.4461$	-3168.93	6343.9	6364.1	0.2443
SLap	$\hat{\mu} = 1.0080, \hat{\alpha} = 0.00355, \hat{\beta} = 0.5288$	-2352.85	4711.7	4732.0	0.1065
GL	$\hat{\mu} = -2.7137, \hat{\sigma} = 0.3161, \hat{\alpha} = 351023.4108$	-3290.26	6586.5	6606.8	0.1488
GH	$\hat{\lambda} = 1.024, \hat{\bar{\alpha}} = 0.147, \hat{\mu} = 0.999, \hat{\sigma} = 0.0198, \hat{\gamma} = 0.535$	-2342.67	4695.4	4729.1	0.1092
VG	$\hat{\lambda} = 1.0665, \hat{\mu} = 1.0040, \hat{\sigma} = 0.0433, \hat{\gamma} = 0.5293$	-2346.33	4700.7	4727.7	0.1114
NIG	$\hat{\bar{\alpha}} = 1.1473, \hat{\mu} = 0.9210, \hat{\sigma} = 0.0118, \hat{\gamma} = 0.6122$	-2416.16	4840.3	4867.4	0.0793
MO	$\hat{r} = 1.6576, \hat{\alpha} = 6.4999, \hat{\beta} = 1.3073$	-3293.25	6592.5	6612.8	0.1189
KMO	$\hat{p} = 0.991, \hat{\alpha} = 16.700, \hat{\beta} = 0.276, \hat{a} = 12.030, \hat{b} = 1.292$	-2503.25	5016.5	5050.3	0.0821

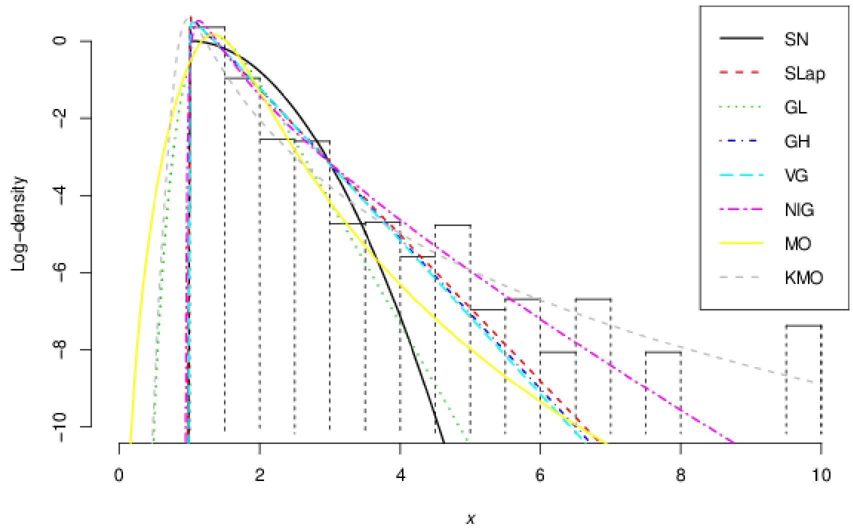

Figure 2. log-Histogram of the third-party car insurance claims and logarithm of density functions of the fitted models.

6. Conclusion

The aim of this work is to fit several distributions to 6366 third party car property damage claims submitted to an Iranian insurance company during one year. Because the empirical results show that the data are right-skewed, the skew-distributions to analyze of data perform very well. The results showed that the conventional skew-normal distribution is not an appropriate model for the data. On the other hand, the Kumaraswamy Marshal-Olkin Log-Logistic distribution has the ability of describing the features of the observed data better than other competing distributions.
The value at risk and conditional tail expectation of the claims are estimated both parametrically and empirically. In many fitted distributions, the Kumaraswamy Marsha-Olkin Log-Logistic model provided very close parametric estimates of expected value, standard deviation, skewness, kurtosis, VaR's and CTE's to the their corresponding empirical estimates. Thus, the fitted Kumaraswamy Marshal-olkin Log-Logistic model can be regarded as an appropriate model for the data which provides much more accurate estimate for the claim distribution than the skew-normal model.

Table 2. Empirical and estimated values of mean, standard deviation, skewness, kurtosis and 0.95 and 0.99 VaR and CTE from the fitted models to the amount of automobile accident claims for property damage to a third party.

Model	$E[X]$	$\sqrt{Var(X)}$	$S(X)$	$K(X)$	$VaR_{0.95}(X)$	$CET_{0.95}(X)$	$VaR_{0.99}(X)$	$CET_{0.99}(X)$
Empirical	1.5333	0.5897	3.8087	27.2280	2.6507	3.4451	3.9878	5.0221
SN	1.6342	0.4781	0.9951	0.8690	2.5559	2.8555	3.0443	0.0329
SLap	1.5333	0.5288	1.9999	3.9999	2.5887	3.1157	3.4398	3.9686
GL	1.5044	0.4054	1.1395	-2.3999	2.2607	2.5808	2.7759	3.0927
GH	1.5333	0.2687	1.9680	8.8172	2.5664	3.0802	3.3934	3.9065
VG	1.5333	0.2646	1.9433	8.5928	2.5577	3.0627	3.3707	3.8739
NIG	1.5332	0.3267	2.8008	16.0739	2.6382	3.3485	3.7730	4.5524
MO	1.4606	0.4285	1.5759	10.6896	2.2105	2.6244	2.8507	3.3728
KMO	1.5781	0.6931	3.8649	26.9143	2.7959	3.8094	4.5890	5.4794

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