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Yawen Guo
Florida International University

B. M. Golam Kibria
Florida International University

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On Some Statistics for Testing the Skewness in a Population: An Empirical Study

Yawen Guo and B.M. Golam Kibria

Department of Mathematics and Statistics
Florida International University
Miami, FL 33199
kibriag@fiu.edu

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Abstract

The purpose of this paper is to propose some test statistics for testing the skewness parameter of a distribution, not limited to a normal distribution. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the test statistics. We have compared both parametric methods (classical method with normality assumption) and non-parametric methods (bootstrap in Bias Corrected Standard Method, Efron's Percentile Method, Hall's Percentile Method and Bias Corrected Percentile Method). Our simulation results indicate that the power of the tests differ significantly across sample sizes, the choice of alternative hypotheses and methods one choose. When the data are generated from a normal distribution, both classical method and Efron's Percentile Method can attain a nominal size of 0.05, while other bootstrap methods cannot. However, for a skewed distribution, bootstrap methods show higher power with larger sample sizes whereas the classical method only performs well when the sample size is small.

Keywords: Bootstrap Methods; Hypothesis Testing; Power of the test; Skewness; Simulation Study

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MSC 2010: 62F03, 62F40, 62G10

1. Introduction

Shape parameters are useful in testing normality and robustness studies and widely used by researchers in many disciplines. Joanes and Gill (1998) proposed that skewness and kurtosis are popular as shape parameters and they could easily be estimated by using higher moments. Skewness is a measure of the symmetry of a distribution, and it could be either positive or negative. When the coefficient of skewness is equal to zero, it means that the distribution is symmetric. If the coefficient is positive, the tail on the right side is longer than the left side, and if the coefficient is negative, the tail on the left side is longer than the right side (Groeneveld and Meeden, 1984).

Perez-Meloand and Kibria (2016) considers several confidence intervals and proposed some bootstrap version of the existing interval estimators for estimating the skewness parameter of a distribution and compared them using a simulation study for a large sample size. In addition, Ankarali et al. (2009) mentioned that the shape of the distribution of the variable plays an important role in selecting appropriate test statistics among all criteria, in particular in small samples with a normal distribution.

Since there are only a handful of studies that have compared the confidence intervals of the skewness, the literature on the hypothesis testing of skewness is limited. In this paper, we will focus on the various hypothesis testing of skewness parameter and compare them in the sense of nominal size and empirical power of the test. The comparison will be made on the basis of following characteristics: different sample sizes, different proposed test statistics and different methods including parametric and non-parametric.

The organization of the paper is as follows. In Section 2, we review the previously proposed estimators and formulate the hypothesis testing for both a single parametric method and several non-parametric methods and their corresponding test statistics. A simulation study on the nominal size and power of the tests of skewness are discussed in Section 3. As an illustration, examples for skewness have been considered in Section 4. Some concluding remarks are presented in Section 5.

2. Statistical Methodology

In this section, we consider some parametric and non-parametric test statistics for testing the population skewness.

2.1. Parametric Methods

Skewness is viewed as a major shape parameter for a probability distribution. In probability theory and statistics, skewness is a measure of symmetry or asymmetry of the probability distribution. It could be represented by the third central moment and standard deviation as follows,

$$\gamma_1 = \frac{\mu_3}{\sigma^3} = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{E[(X-\mu)^3]}{(E[(X-\mu)^2])^{\frac{3}{2}}}, \quad (2.1)$$

where γ_1 is the population skewness parameter, μ_3 is the third central moment, μ is the mean, σ is the standard deviation and E is the expectation operator.

However, for different definitions of skewness, we have different ways to evaluate the performance. Let $X_1, X_2, \dots, X_n$ be an independently and identically distributed (*iid*) random sample from a population with mean μ and standard deviation σ. The traditional definition of skewness, proposed by Cramer (1946), has the form

$$g_1 = {m_3}/{m_2^{3/2}},$$

where the sample moments for variable X are defined as,

$$m_r = \frac{1}{n}\sum(x_i - \bar{x})^r. \quad (2.3)$$

Following the work of Joanes and Gill (1998), the three most commonly used parametric estimators for skewness from traditional measures, which has been developed by SAS and MINITAB are provided below:

$$g_1 = \frac{m_3}{{m_2}^{3/2}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{3/2}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{[\frac{1}{n}*(n-1)*s^2]^{3/2}} = (\frac{n}{n-1})^{3/2} * \frac{1}{n} * \frac{\sum_{i=1}^{n}(x_i-\bar{x})^3}{s^3},$$

$$G_1 = \frac{\sqrt{n(n-1)}}{n-2} g_1, \quad (2.4)$$

$$b_1 = (\frac{n-1}{n})^{3/2} g_1.$$

It is noted that for large sample sizes, the results do not deviate significantly. However, for small sample sizes, the results among three methods of estimators are sometimes significant at 0.05 level.

For normal distribution, Fisher (1930) stated that $E(g_1) = 0$ which is unbiased, and we could easily find that

$$E(G_1) = \frac{\sqrt{n(n-1)}}{n-2} E(g_1) = 0 \text{ and } E(b_1) = \left(\frac{n-1}{n}\right)^{\frac{3}{2}} E(g_1) = 0.$$

As given by Cramer (1946), in normal samples the variance of the Fisher-Pearson coefficient of skewness (g_1) is

$$Var(g_1) = \frac{6(n-2)}{(n+1)(n+3)}.$$

Then, the variance of G_1 and b_1 are obtained respectively as

$$Var(G_1) = \frac{n(n-1)}{(n-2)^2} Var(g_1) = \frac{6n(n-1)(n-2)}{(n+1)(n+3)(n-2)^2}$$

and

$$Var(b_1) = \left(\frac{n-1}{n}\right)^3 Var(g_1) = \left(\frac{n-1}{n}\right)^3 \frac{6(n-2)}{(n+1)(n+3)}.$$

Following Joanes and Gill (1998) and Perez-Meloand and Kibria (2016), we attempt to develop a Z-test statistic for testing the population skewness parameter. That means, we will test the following null and alternative hypotheses,

$$H_0\colon \gamma_1 = \gamma_s$$
$$H_1\colon \gamma_1 \neq \gamma_s \ , \tag{2.5}$$

and the test statistic for the three estimators $(g_1, G_1, and\ b_1)$ can be defined respectively as follows:

$$Z_{g1} = \frac{g_1 - \gamma_s}{\sqrt{\frac{6(n-2)}{(n+1)(n+3)}}} \ ,$$

$$Z_{G1} = \frac{G_1 - \gamma_s}{\sqrt{\frac{6n(n-1)}{(n+1)(n+3)(n-2)}}} \ ,$$

(2.6)
and

$$Z_{b1} = \frac{b_1 - \gamma_s}{\sqrt{\frac{6(n-2)}{(n+1)(n+3)}}\left(\frac{n-1}{n}\right)^{\frac{3}{2}}} \ ,$$

where g_1, G_1, b_1 are previously defined in equation (2.4), n is the sample size, γ_s is hypothesized value of skewness parameter. We will reject H_0 at α level of significance if the absolute values of the test statistics $(Z_{g_1}, Z_{G_1}, Z_{b_1})$ are greater than $Z\alpha_{/2}$, where $Z\alpha_{/2}$ is the upper $\frac{\alpha}{2}$ percentile of the standard normal distribution.

2.2. Bootstrap Approach.

In this section, we will discuss the bootstrap techniques for testing the skewness parameter. The bootstrap approach can be applied to any population as it does not require any assumption about the distribution, and if the sample size is large enough, the process of bootstrap could be very accurate (Efron, 1992). Following Perez-Meloand and Kibria (2016), the bootstrap methods for testing the skewness, can be summarized as follows: Let $X^{(*)} = X_1^{(*)}, X_2^{(*)}, \dots, X_n^{(*)}$, where the i^{th} sample is denoted $X^{(i)}$ for i=1,2,…,B, where B is the number of bootstrap samples. Parametric method requires normality assumption, however, in reality, most of the data do not follow a normal distribution. In this situation, the bootstrap is desired.

2.2.1. Bias-Corrected Standard Bootstrap Approach

Let $\hat{\theta}$ be a point estimator of θ (skewness parameter). Then, the bias-corrected standard bootstrap confidence interval for θ proposed by Perez-Meloand and Kibria (2016) takes the form,

$$\hat{\theta} - Bias(\hat{\theta}) \pm Z_{\alpha/2}\widehat{\sigma_B} \ ,$$

where $\widehat{\sigma_B} = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}(\theta_i^* - \bar{\theta})^2}$ is the bootstrap standard deviation, $\bar{\theta} = \frac{1}{B}\sum_{i=1}^{B}\theta_i^*$ is the bootstrap mean and $Bias(\hat{\theta}) = \bar{\theta} - \hat{\theta}$ is the estimated bias. Now we attempt to develop a Z-test statistic for testing the population skewness. In this regard, the null and alternative hypotheses are defined below:

$$H_0:\ \theta = \theta_0$$
$$H_0:\ \theta \neq \theta_0.$$

The test statistic for testing the alternative hypothesis can be written as follows:

$$Z_{\theta_0} = \frac{\hat{\theta} - Bias(\hat{\theta}) - \theta_0}{\widehat{\sigma_B}} \quad ,$$

where θ is the population skewness parameter. We will reject H_0 at α level of significance if the test statistic Z_{θ_0} is greater than $Z\alpha_{/2}$, where $Z\alpha_{/2}$ is the upper $\frac{\alpha}{2}$ percentile of the standard normal distribution.

2.2.2. Efron's Percentile Bootstrap Approach

Compared to bias-corrected standard bootstrap approach, Efron's Percentile method makes the computation of confidence intervals rather easy, since the confidence interval will depend on the value of upper $\alpha/2$ level of bootstrap samples and lower $\alpha/2$ level of bootstrap samples (Efron, 1987). First, we order the sample skewness of each bootstrap sample as follows:

$$\theta^*_{(1)} \leq \theta^*_{(2)} \leq \theta^*_{(3)} \leq \cdots \leq \theta^*_{(B)}.$$

Following Efron's (1987), the confidence interval will be given by

$$L = \theta^*_{[\left(\frac{\alpha}{2}\right)*B]} \text{ and } U = \theta^*_{[\left(1-\frac{\alpha}{2}\right)*B}$$

and we will reject the null hypothesis H_0: $\theta = \theta_0$ against alternative hypothesis H_a: $\theta \neq \theta_0$, if $L > \theta_0$ *or* $U < \theta_0$.

2.2.3. Hall's Percentile Bootstrap Approach

This is also a non-parametric approach proposed by Hall (1992), which does not require the standard deviation. In Hall's method, we order the errors of the estimator instead of estimator itself. The errors are ordered as follows:

$$\varepsilon^*_{(1)} \leq \varepsilon^*_{(2)} \leq \varepsilon^*_{(3)} \leq \cdots \leq \varepsilon^*_{(B)},$$

where $\varepsilon^*_i = \theta^*_i - \theta$. The confidence interval could be obtained in the similar manner as previous Efron's Percentile approach and it is presented below:

$$L = \theta - \varepsilon^*_{[\left(1-\frac{\alpha}{2}\right)*B]} \text{ and } U = \theta - \varepsilon^*_{[\left(\frac{\alpha}{2}\right)*B]}.$$

Following Hall (1992), the confidence interval could be simplified as:

$$L = 2\theta - \theta^*_{[\left(1-\frac{\alpha}{2}\right)*B]} \text{ and } U = 2\theta - \theta^*_{[\left(\frac{\alpha}{2}\right)*B]}$$

and we will reject the null hypothesis: H_0: $\theta = \theta_0$ against alternative hypothesis H_a: $\theta \neq \theta_0$, if $L > \theta_0$ *or* $U < \theta_0$.

2.2.4. Bias-Corrected Percentile Bootstrap Approach

This method was introduced by Efron (1987) and the first step is to find the proportion of times that θ_i^* is greater than θ, that is,

$$P = \frac{\#(\theta_i^* > \theta)}{B}$$

and then find Z_0 in order to make $\phi(Z_0) = 1 - P$, where ϕ is the cumulative distribution function of standard normal random variable. Z_0 will be used to construct the following confidence interval,

$$L = \theta^*_{[\phi(2Z_0 - Z_{1-\alpha/2})*B]} \text{ and } = \theta^*_{[\phi(2Z_0 + Z_{1-\alpha/2})*B]}$$

and we will reject the null hypothesis H_0: $\theta = \theta_0$ against alternative hypothesis H_a: $\theta \neq \theta_0$, if $L > \theta_0$ or $U < \theta_0$.

For more on bootstrap technique we refer our readers to DiCiccio & Romano (1988) among others.

3. Simulation Study

In this section, we will compare the performance of the proposed test statistics. We conducted a simulation study using R Version 3.2.1 to compare the performance of the test statistics in the sense of standard nominal size and high empirical power of the test.

3.1. Simulation Technique.

Even though the proposed test statistics are mainly developed for testing data from a normal (or symmetric) population, we will make an attempt to see the performance of these test statistics when the data are from a skewed distribution. The flow chart of our simulation study is pointed below:

1. Sample size, n=10, 20, 30, 50, 100 and 300.
2. 3000 simulation replications are used for each case, 1000 bootstrap samples for each simulation replication.
3. The normal and right skewed distributions are generated.
   (a) Normal distribution with mean 0 and SD 1
   (b) Gamma distribution with shape parameter 4, 7.5 and 10 and scale parameter 1.

For more on simulation technique, we refer our readers to Kibria and Banik (2013) and Banik and Kibria (2016) among others.

3.2. Performance for Normal distribution

It is well known that the normal distribution is symmetric and the skewness for normal distribution equals 0. Under this assumption and at α = 0.05 level of significance, we expect to get the power = 0.05 from the simulation dataset. Figure 3.1 shows the empirical size of the test when we are testing whether the skewness equals 0. It appears from Figure 3.1 that the classical method performs the best among all methods in the sense of attaining nominal size of 0.05 for different sample sizes. It differs only when sample size is small, that is when n =10. Among four types of bootstrap methods, only Efron's Percentile method attained the nominal size of 0.05. For the Bias Corrected Standard Method, Hall's Percentile Method and Bias Corrected Percentile Method, the empirical nominal size is beyond 0.1 when the sample size is less than 100. However, they attained nominal size 0.05 when the sample size is very large say, 300. In this case bootstrap methods do not provide better results than the classical method, despite the limit of sample size to test the skewness for normal distribution. It should be mentioned that for power test, we deleted the unqualified statistics using a 0.05 nominal size and all good test statistics are demonstrated in the graph.

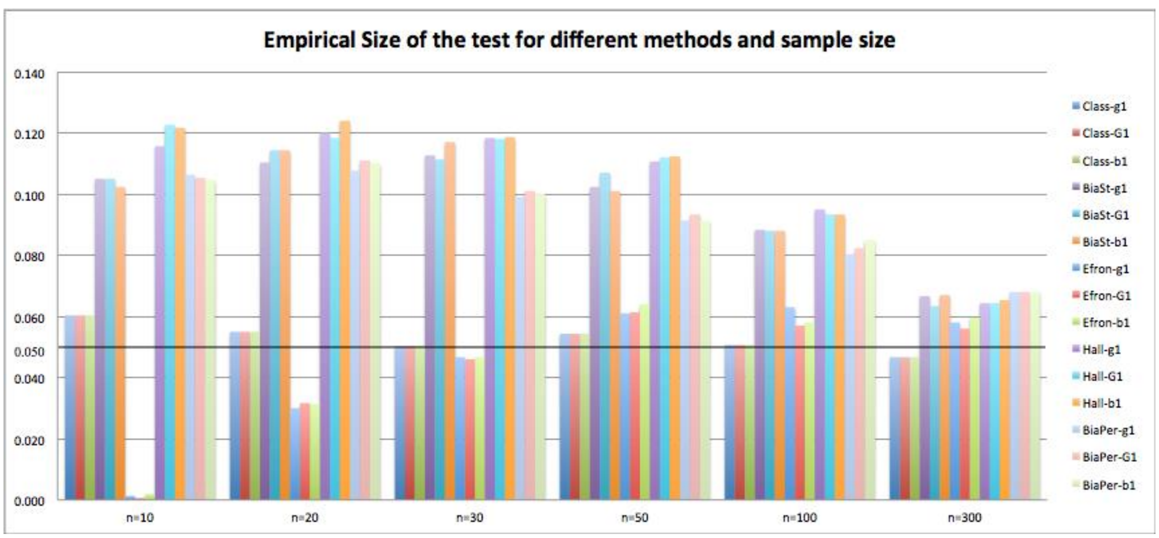

Figure 3.1. Empirical size of testing skewness=0 with different methods and sample size

Figures 3.2 to 3.7 show the empirical power against different hypothesized values for all proposed test statistics with different sample sizes: n = 10, 20, 30, 50, 100 and 300. The x-axis represents different hypothesized values and Y-axis is the empirical power. We would expect to have the empirical power close to 1 when increasing the hypothesized value from 0 to a larger value. From these six Figures, it appears that empirical powers are close to 1 when skewness equals to 2 or less than 2.

From Figures 3.2 to 3.7, we can see that for small sample sizes and near the null hypothesis or for large sample sizes and for high skewness, the power of the tests does not vary greatly. However, for small sample size with moderate departure from null hypothesis, the power of the tests varies among the test statistics. It appears that among all test statistics, the classical method is more powerful when the sample size is small (say 10) while for sample size greater than 10, Efron's Percentile Method shows absolute advantage other than classical method. Overall, the power approaches 1 when the alternative hypothesis is testing for skewness =2.

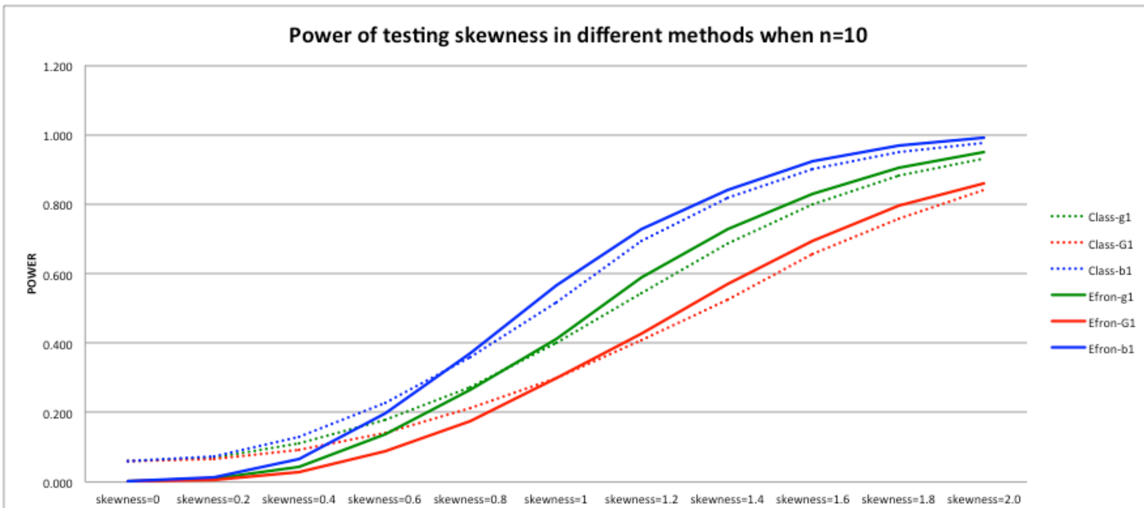

Figure 3.2. Power of testing skewness of *N* (0, 1) in different methods when *n* = 10

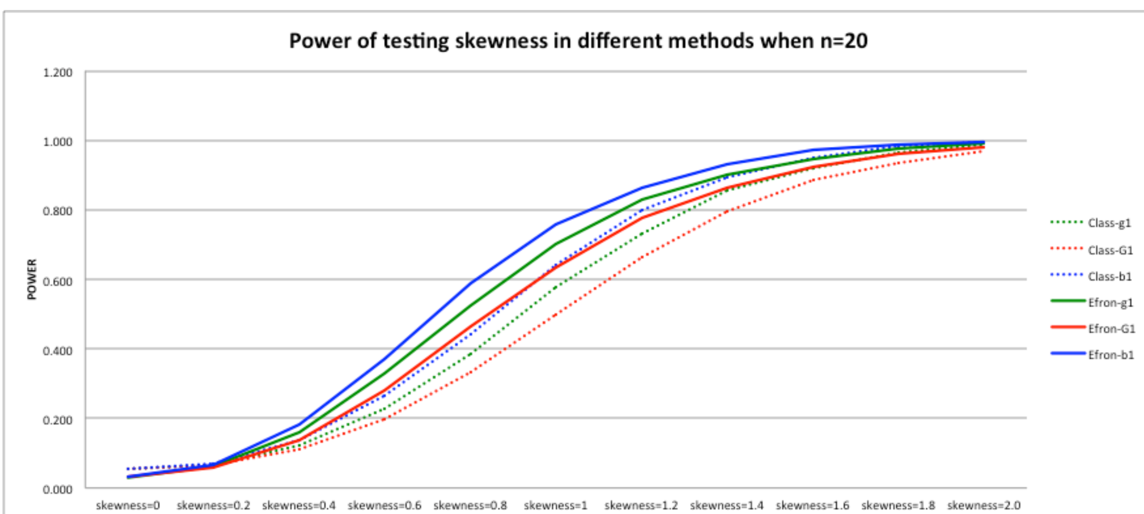

Figure 3.3. Power of testing skewness of *N* (0, 1) in different methods when *n* = 20

Both the classical and Efron's Percentile methods show acceptable results. By changing the alternative hypothesis, the Efron's Percentile is getting close to other bootstrap methods and apparently away from the classical method. The power approaches 1 when skewness is 1.6 and 1.2 respectively for n = 30 and 50.

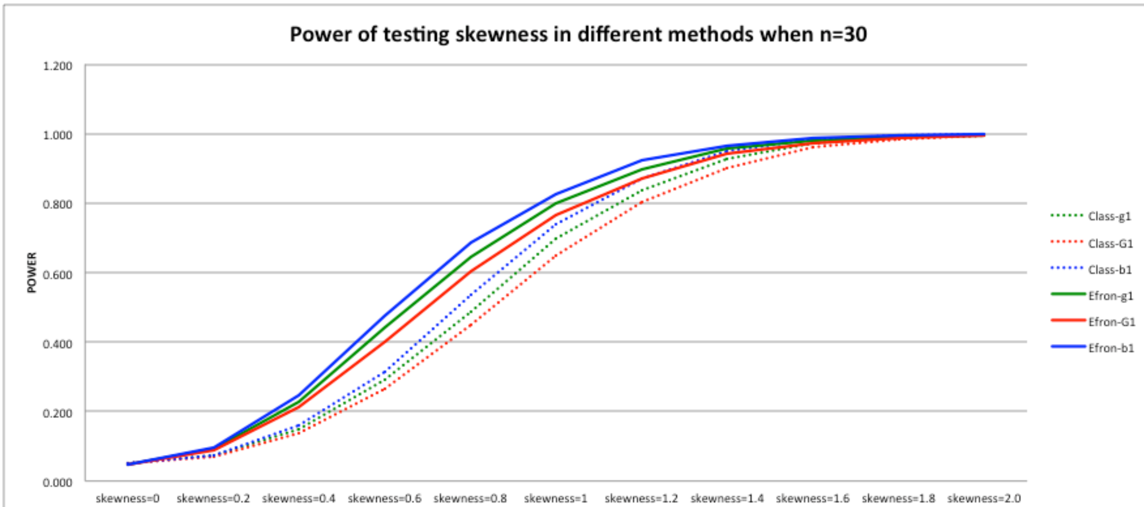

Figure 3.4. Power of testing skewness of N (0, 1) in different methods when n = 30

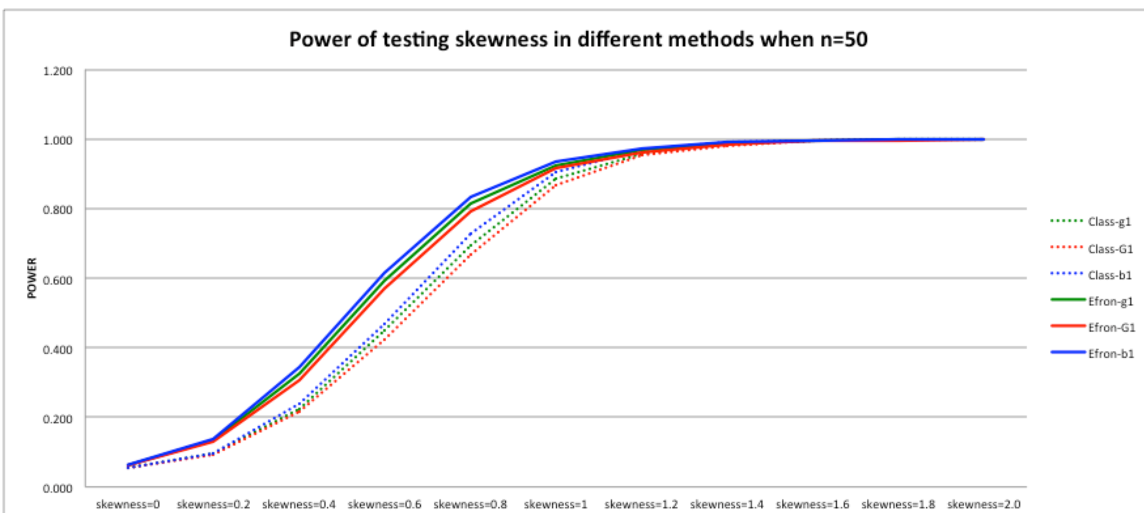

Figure 3.5. Power of testing skewness of N (0, 1) in different methods when n = 50

When we consider a larger sample size, say 100, and are testing skewness = 0.2, 0.4 or 0.6, then, the classical method is less powerful than the bootstrap methods. The power increases sharply to 0.9 for all methods when skewness = 0.8 and it goes up steadily to 1 from that point on. When the sample size goes up to 300, the power rises by an order of magnitude from 0.05 to 0.7 when the skewness shifts from 0 to 0.4, and thereafter, it increases gradually until 1 when skewness=0.6. Thus, it may be concluded that the classical method shows a little less power than Efron's Percentile method for moderate departure from null value, and when the sample size is large enough, there is no significant difference among bootstrap methods. However, it is noted that when the classical and Efron's Percentile methods attain a nominal size 0.05, other proposed bootstrap methods, from data in a normal population, are not useful.

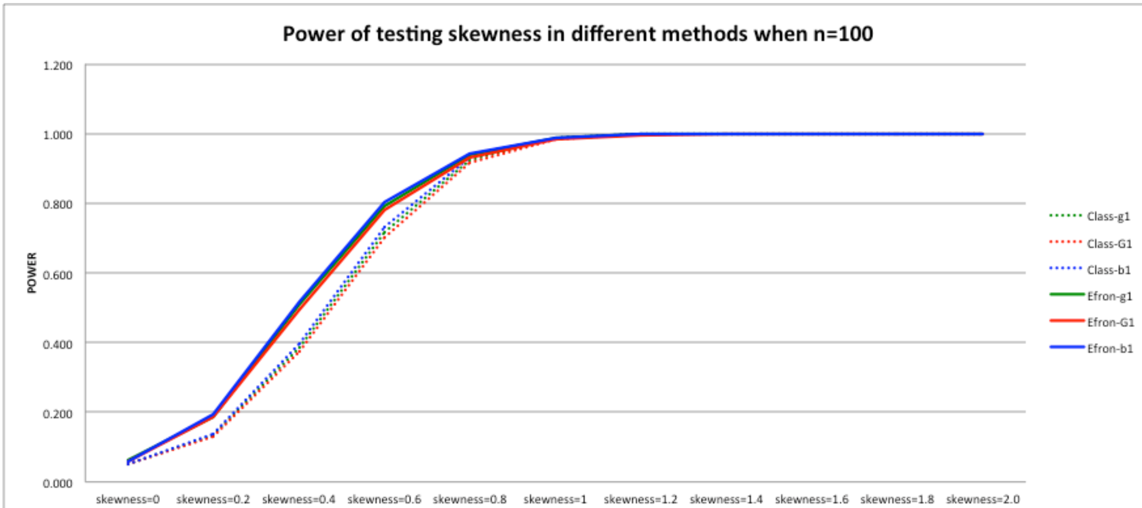

Figure 3.6. Power of testing skewness of $N(0, 1)$ in different methods when $n = 100$

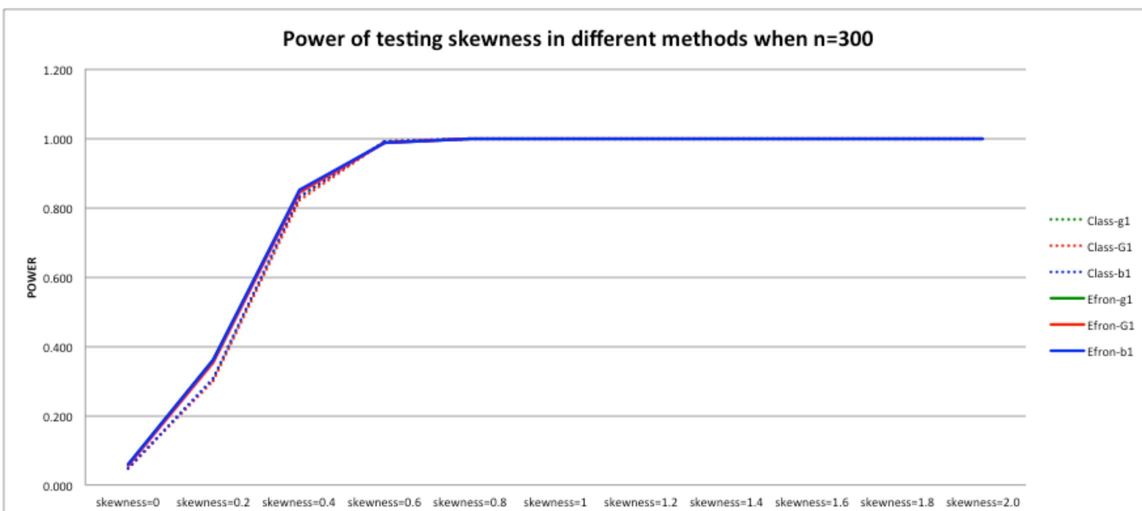

Figure 3.7. Power of testing skewness of $N(0, 1)$ in different methods when $n = 300$

We analyzed the performance of test statistics using sample size with different methods separately. Figures 3.8 and 3.9 illustrates the power of testing skewness in different sample size with classical method and Efron's Percentile Method only as other methods failed to attained the nominal level. These figures indicate that if the sample size is large enough, there seems to be no obvious difference among those three test statistics. The difference is only visible when the sample size is small, say n=10. Within each test statistic using those three estimators, increasing the sample size could improve the power of test for both classical and Efron's Percentile Method. Moreover, we find that the test statistic based on G_1 has the smallest power while the test statistic based on estimator b_1 has the highest power within each sample size.

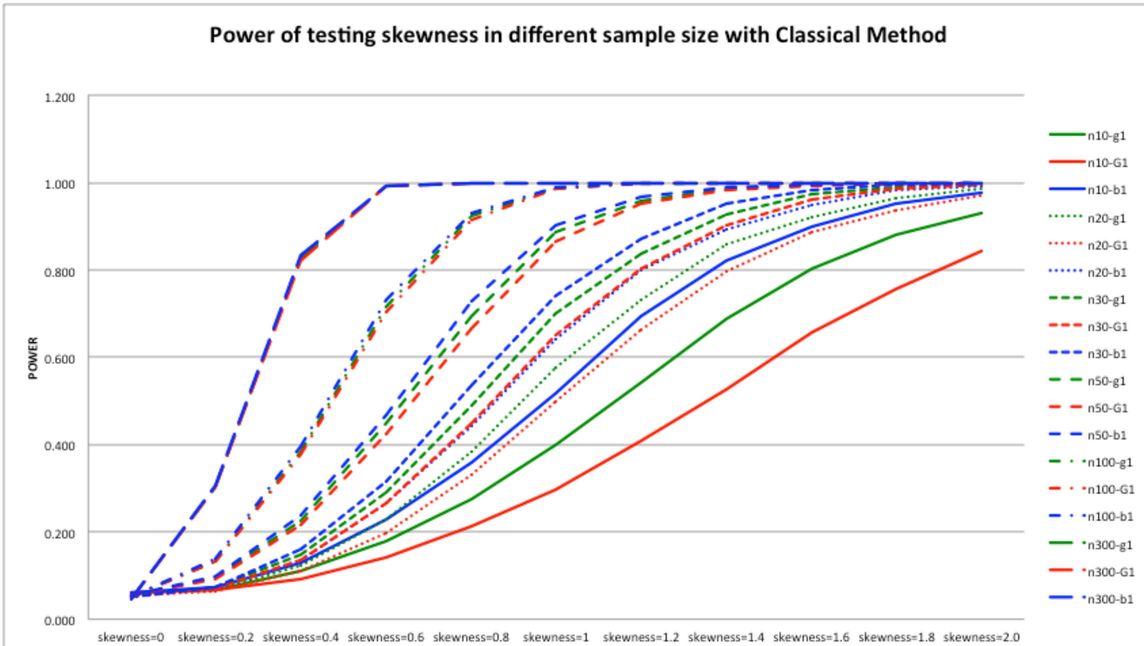

Figure 3.8. Power of testing skewness of N (0, 1) in different sample size with Classical Method

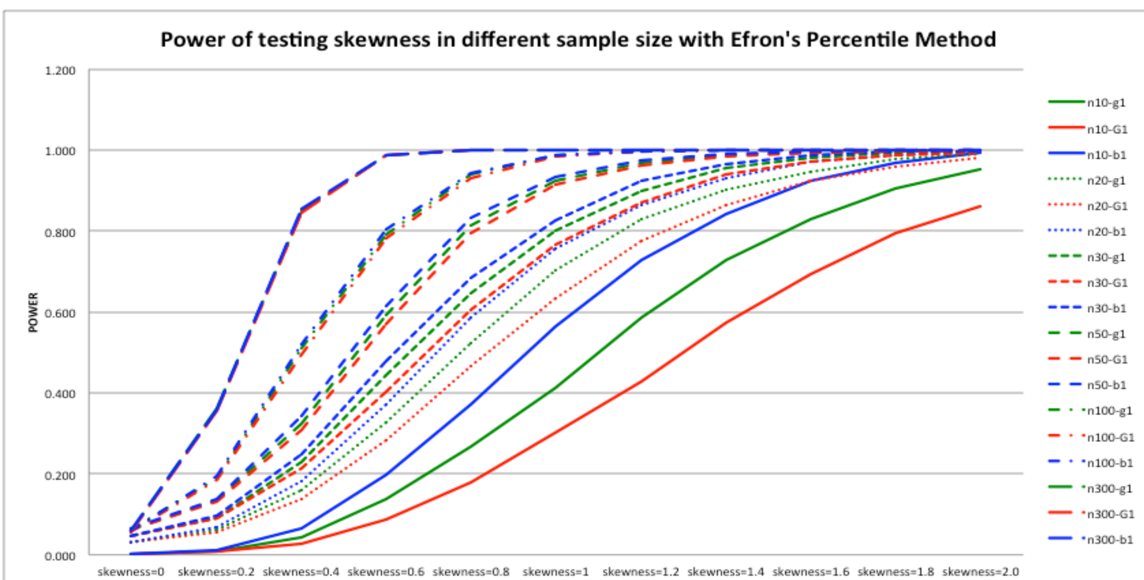

Figure 3.9. Power of testing skewness of N (0, 1) in different sample size with Efron's Percentile Method

3.3. Performance for Gamma distribution

Even though the parametric methods are developed for testing the skewness parameter of normal distribution, we made an attempt to apply this method along with bootstrap methods to other asymmetric distributions, which will be discussed in this section.

The skewness of the gamma distribution depends on the scale parameter only. For instance, the skewness of Gamma (k, p) is $\frac{2}{\sqrt{k}}$. At α = 0.05 level of significance, we are expecting the nominal size to be 0.05 from the simulation data when we are testing the skewness equal to $\frac{2}{\sqrt{k}}$. Figures 3.10 and 3.11 illustrate the empirical sizes for testing the skewness = 1 of Gamma (4,1) and skewness = 0.63 of Gamma (10,1) respectively. Unfortunately, the results are not acceptable for both parametric and bootstrap methods for Gamma (4,1), while the results are closer to 0.05 for Gamma (10,1) distribution. For small sample size n = 10, as Efron's Percentile method is under 0.05 limit, it can be chosen as a good test statistic. By increasing k, the shape of gamma distribution became closer to the bell-shaped "normal" distribution, which allowed us to find a nominal size closer to 0.05. We considered the following gamma distributions in simulations: Gamma (4, 1), Gamma (7.5, 1) and Gamma (10, 1) and the full results can be found in the Appendix A2 to A4. In the following Figures 3.10 and 3.11, we find that the nominal size is much closer to 0.05 for Gamma (10, 1) than for Gamma (4, 1). Because of the imperfect results, we can organize a graph to see the trend of changes of power as a reference but do not encourage using these results as conclusive. The classical method is selected from all five methods as the relatively best method, which shows the trend of power changes from above 0.05 to 1 in Gamma (10, 1). In Figure 3.12, we can find the test statistic based on estimator G_1 is less powerful for a small sample size, say n=10 or 20 when other conditions remain the same. When sample size increases to 100, we can easily find test statistic of G_1 has lower power while that of b_1 has higher power. By increasing the sample size to 300 two results were gathered: the power increases sharply to 1 at skewness=2 and stays at 1 thereafter, and there is no apparent difference among the test statistics based on these three estimators. In contrast, when the sample size is small, say n=10, the power rises gradually to 1 at skewness=3. In this paper, we will not discuss more about the results deeply but they are provided in Appendices A2 to A4 as a reference.

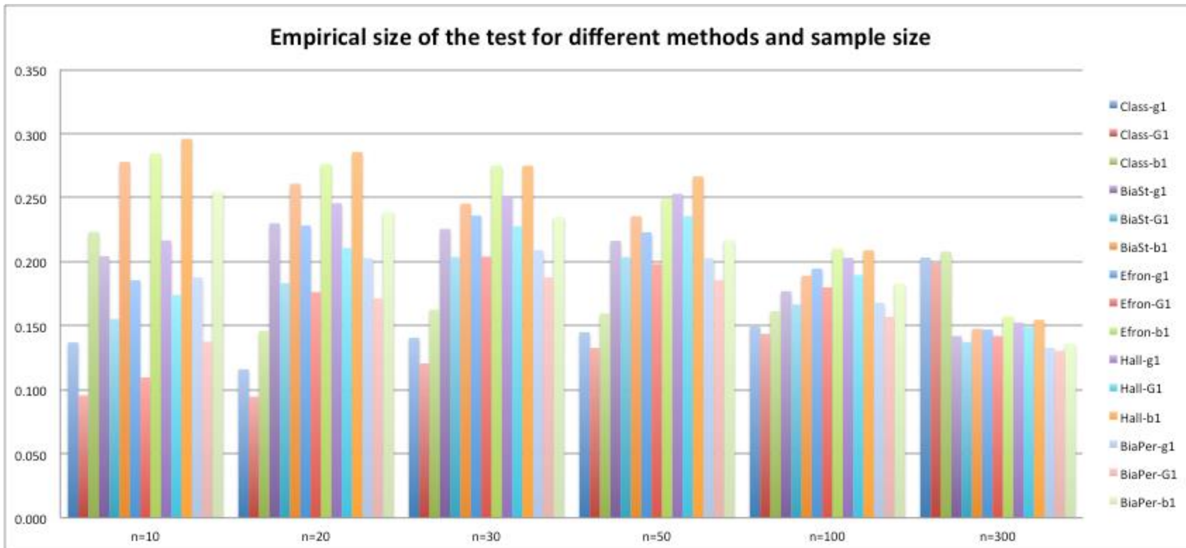

Figure 3.10. Empirical size of testing Gamma (4,1) skewness=1 with different methods and sample size

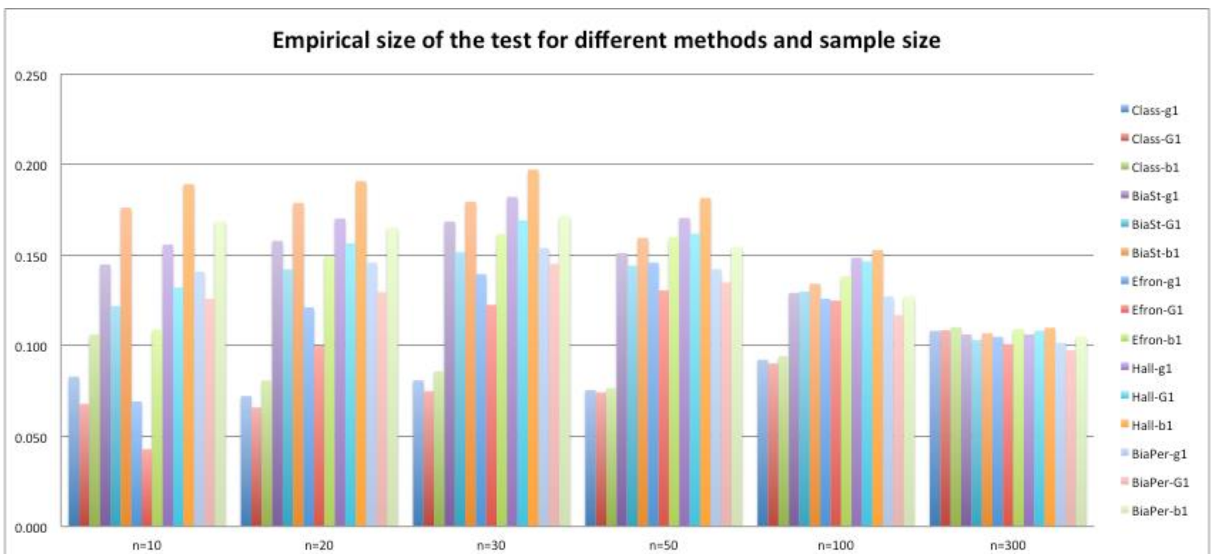

Figure 3.11. Empirical size of testing Gamma (10,1) skewness=0.63 with different methods and sample size

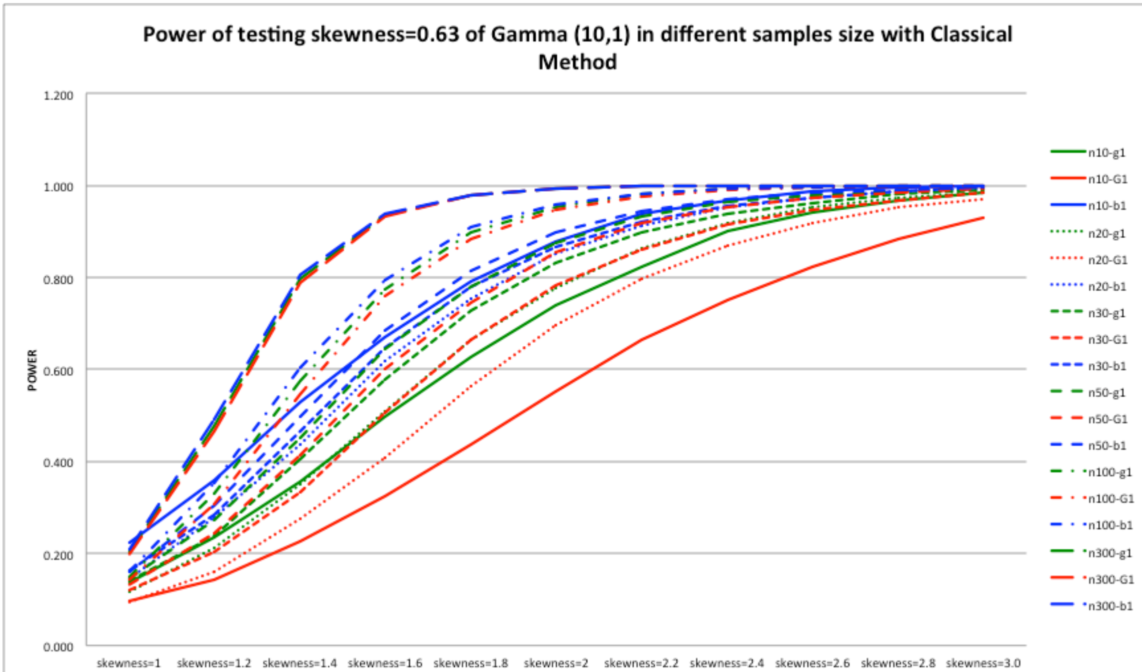

Figure 3.12. Power of testing skewness of Gamma (10,1) in different sample size with Classical Method

4. Applications

In this section, we will analyze two real life data sets to illustrate the performance of the test statistics based on the three estimators. We have a dataset in regards to 48 SIDS (Sudden Infant Death Syndrome) cases observed in King County, Washington during the years 1974 and 1975 (Belle at el., 2004). However, we used only one variable, birth weights (in grams) of these 48 cases in our study. Using this data the results of test statistics for testing the skewness for various alternative hypotheses are presented in Table 4.1. Before testing the hypotheses, we would like to confirm that whether the data follow a normal distribution or not. We have performed the Shapiro test (test statistic, W=0.9832, p-value=0.7168), which indicated that the data follow a normal distribution. We can easily find from Table 4.1, the classical method could correctly reject the null hypothesis when the skewness is departed from hypothesized value, say skewness=0.7. From that on, the classical method performs very well, however, the Bias Corrected Standard method shows unusual results which even reject the hypothesis when hypothesized value is close to null hypothesis. The Efron's Percentile method performs as well as the classical method.

Table 4.1. Testing skewness for n = 48 normal distribution data

			sk=0	sk=0.15	sk=0.4	sk=0.6	sk=0.7	sk=0.8	sk=1.0
Method	Estimator	Point Estimate	P-Value						
Classical	g1	0.135	0.657	0.482	0.212	0.081	0.044	0.023	0.005
	G1	0.139	0.657	0.487	0.224	0.090	0.051	0.027	0.006
	b1	0.131	0.657	0.476	0.201	0.072	0.038	0.019	0.003
Bias Corrected standard	g1	0.135	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	G1	0.139	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	b1	0.131	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		CI	Decision						
Efron's percentile bootstrap	g1	(−0.631, 0.676)	A	A	A	A	R	R	R
	G1	(−0.586, 0.726)	A	A	A	A	A	R	R
	b1	(−0.595, 0.648)	A	A	A	A	R	R	R
Hall's Percentile bootstrap	g1	(−0.445, 0.879)	A	A	A	A	A	A	R
	G1	(−0.444, 0.878)	A	A	A	A	A	A	R
	b1	(−0.424, 0.869)	A	A	A	A	A	A	R
Bias-corrected percentile	g1	(−0.517, 0.724)	R	A	A	A	A	R	R
	G1	(−0.539, 0.727)	R	A	A	A	A	R	R
	b1	(−0.574, 0.728)	R	A	A	A	A	R	R

Another example, which is used to test the skewness, is also related to SIDS. We obtained a dataset that consist of 78 cases of SIDS occurring in King County between 1976 and 1977 (Morris et al, 1993). They recorded the age at death (in Days) of 78 cases of SIDS and finally classify them into 11 different age intervals. For each age interval, the number of deaths was recorded and eventually the number of deaths was employed in this example study. The Shapiro test (test statistic, W = 0.82135, p-value = 0.0329), which cannot support normality assumption. By using classical method, the results of testing the statistics based on g_1 and b_1 could reject the null hypothesis when testing skewness=2.0 while Bias Corrected Standard method does not perform correctly in this test. For bootstrap method, only when the testing hypothesized value is large enough, say skewness = 1.9 and above, the results from the test statistics based on estimator b_1 from Efron's Percentile and Hall's Percentile method can provide a good solution to make a correct decision, otherwise the other methods can not.

Table 4.2. Testing skewness for n=11 non-normal distribution data

			sk=0	sk=0.5	sk=1	sk=1.5	sk=1.9	sk=2.0
Method	Estimator	Point Estimate	P-Value					
Classical	g1	1.020	0.964	0.821	0.514	0.199	0.060	0.042
	G1	1.189	0.964	0.851	0.612	0.319	0.141	0.110
	b1	0.884	0.964	0.783	0.407	0.105	0.019	0.012
Bias Corrected standard	g1	1.020	0.256	0.235	0.216	0.197	0.183	0.180
	G1	1.189	0.270	0.249	0.229	0.209	0.195	0.191
	b1	0.884	0.244	0.224	0.205	0.187	0.191	0.170
		CI	Decision					
Efron's percentile bootstrap	g1	(−0.131, 2.202)	A	A	A	A	A	A
	G1	(−0.065, 2.504)	A	A	A	A	A	A
	b1	(−0.043, 1.887)	A	A	A	A	R	R
Hall's Percentile bootstrap	g1	(−0.189, 2.078)	A	A	A	A	A	A
	G1	(−0.210, 2.558)	A	A	A	A	A	A
	b1	(−0.142, 1.833)	A	A	A	A	R	R
Bias-corrected percentile	g1	(0.185, 2.370)	R	A	A	A	A	A
	G1	(0.179, 2.674)	R	A	A	A	A	A
	b1	(0.114, 2.036)	R	A	A	A	A	A

5. Conclusion

This paper proposed several test statistics for testing the skewness parameter of a distribution. Since a theoretical comparison is not possible, a simulation study has been conducted to compare the performance of the test statistics. We have compared both parametric method (Classical method with normality assumption) and non-parametric methods (bootstrap in Bias Corrected Standard Method, Efron's Percentile Method, Hall's Percentile Method and Bias Corrected Percentile Method) in the hypothesis testing of skewness, where the data are generated from normal and gamma distributions. Table 5.1 illustrates the performance of the tests and our simulation results indicate that the power of the tests differ significantly across sample sizes, the choice of alternative hypotheses and methods we choose. When the data are generated from normal distribution, both classical method and Efron's Percentile Method can attain a nominal size 0.05, while other bootstrap methods cannot provide good results in this situation. However, for skewed distribution, bootstrap methods show higher power for increased sample sizes whereas the classical method only performs well with small sample sizes. The results of Bias Corrected Percentile Method are approaching those of other bootstrap methods, which are obviously away from the classical method. Moreover, for testing different hypotheses among all distributions, as usual, a larger sample size always provide with higher empirical power.

Table 5.1. Performance of hypothesis test of skewness

		Performance of Hypothesis Test of Skewness					
Distribution	Method	n=10	n=20	n=30	n=50	n=100	n=300
N (0,1)	Classical	Good	Good	Good	Good	Good	Good
	Bias Corrected Standard Bootstrap	Weak	Weak	Weak	Weak	Weak	Fair
	Efron's Percentile Bootstrap	Good	Good	Good	Good	Fair	Good
	Hall's Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Fair
	Bias Corrected Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Fair
Gamma (4,1)	Classical	Weak	Weak	Weak	Weak	Weak	Weak
	Bias Corrected Standard Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Efron's Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Hall's Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Bias Corrected Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
Gamma (7.5,1)	Classical	Weak	Fair	Fair	Fair	Fair	Weak
	Bias Corrected Standard Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Efron's Percentile Bootstrap	Fair	Weak	Weak	Weak	Weak	Weak
	Hall's Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Bias Corrected Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
Gamma (10,1)	Classical	Fair	Fair	Fair	Fair	Fair	Weak
	Bias Corrected Standard Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Efron's Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
	Hall's Percentile Bootstrap	Good	Weak	Weak	Weak	Weak	Weak
	Bias Corrected Percentile Bootstrap	Weak	Weak	Weak	Weak	Weak	Weak
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The test statistics used in this paper are based on the assumption of normal distribution, however, the simulated results suggest that these statistics can be used for some non-normal distributions as well. It is noted that the performance of gamma distribution needs further investigation since the bootstrap methods do not work for the data coming from this distribution. We would suggest continuing to explore the test of skewness of gamma distribution and some other distributions with specific skewness features.

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APPENDICES

APPENDIX A

Table A1: Power for N(0,1) with skewness= 0 against with other value for different sample sizes

		n=10										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
Classical	g1	0.060	0.069	0.110	0.179	0.275	0.399	0.543	0.688	0.802	0.881	0.930
	G1	0.060	0.066	0.092	0.143	0.214	0.298	0.410	0.525	0.658	0.758	0.842
	b1	0.060	0.073	0.129	0.228	0.360	0.518	0.693	0.820	0.901	0.951	0.977
Bias Corrected standard	g1	0.105	0.126	0.174	0.256	0.359	0.473	0.596	0.694	0.774	0.843	0.884
	G1	0.105	0.115	0.158	0.211	0.294	0.382	0.482	0.584	0.675	0.745	0.809
	b1	0.102	0.130	0.198	0.303	0.433	0.580	0.698	0.792	0.854	0.901	0.932
Efron's percentile bootstrap	g1	0.001	0.008	0.043	0.137	0.266	0.414	0.588	0.728	0.829	0.905	0.951
	G1	0.000	0.008	0.028	0.087	0.177	0.301	0.429	0.572	0.695	0.795	0.861
	b1	0.002	0.012	0.067	0.198	0.372	0.565	0.729	0.843	0.923	0.969	0.993
Hall's Percentile bootstrap	g1	0.116	0.128	0.177	0.247	0.349	0.469	0.591	0.692	0.775	0.837	0.874
	G1	0.123	0.129	0.160	0.210	0.283	0.375	0.474	0.579	0.676	0.745	0.805
	b1	0.122	0.136	0.200	0.297	0.431	0.575	0.698	0.790	0.851	0.890	0.924
Bias Corrected percentile	g1	0.106	0.126	0.180	0.255	0.354	0.458	0.565	0.664	0.742	0.810	0.870
	G1	0.105	0.118	0.158	0.218	0.290	0.375	0.465	0.554	0.639	0.716	0.778
	b1	0.105	0.128	0.203	0.301	0.420	0.548	0.666	0.760	0.836	0.900	0.952
		n=20										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
Classical	g1	0.055	0.067	0.123	0.229	0.384	0.578	0.731	0.858	0.921	0.965	0.987
	G1	0.055	0.065	0.111	0.198	0.333	0.500	0.663	0.798	0.886	0.936	0.970
	b1	0.055	0.071	0.136	0.266	0.442	0.641	0.799	0.895	0.950	0.983	0.994
Bias Corrected standard	g1	0.110	0.140	0.231	0.390	0.551	0.686	0.791	0.862	0.910	0.938	0.959
	G1	0.114	0.133	0.215	0.355	0.504	0.641	0.753	0.824	0.880	0.918	0.943
	b1	0.114	0.142	0.257	0.431	0.597	0.730	0.830	0.890	0.929	0.955	0.973
Efron's percentile bootstrap	g1	0.030	0.062	0.161	0.328	0.524	0.702	0.830	0.901	0.947	0.976	0.990
	G1	0.032	0.057	0.139	0.282	0.465	0.635	0.777	0.865	0.923	0.960	0.980
	b1	0.031	0.068	0.183	0.373	0.587	0.757	0.865	0.930	0.971	0.987	0.996
Hall's Percentile bootstrap	g1	0.120	0.146	0.240	0.395	0.555	0.688	0.791	0.855	0.905	0.933	0.954
	G1	0.118	0.142	0.221	0.357	0.513	0.638	0.750	0.826	0.881	0.915	0.939
	b1	0.124	0.147	0.266	0.437	0.598	0.735	0.825	0.886	0.924	0.948	0.967
Bias Corrected percentile	g1	0.108	0.132	0.233	0.373	0.533	0.678	0.785	0.864	0.907	0.943	0.967
	G1	0.111	0.128	0.207	0.336	0.487	0.627	0.740	0.827	0.881	0.921	0.951
	b1	0.110	0.139	0.248	0.416	0.581	0.725	0.831	0.889	0.931	0.963	0.981
		n=30										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
Classical	g1	0.032	0.050	0.128	0.291	0.506	0.726	0.884	0.957	0.986	0.996	0.998
	G1	0.032	0.048	0.118	0.260	0.460	0.676	0.844	0.940	0.977	0.992	0.998
	b1	0.032	0.053	0.143	0.322	0.555	0.770	0.914	0.971	0.991	0.998	0.999
Bias Corrected standard	g1	0.103	0.166	0.328	0.529	0.712	0.832	0.914	0.954	0.974	0.987	0.992
	G1	0.099	0.160	0.307	0.497	0.680	0.814	0.893	0.942	0.967	0.981	0.989
	b1	0.103	0.173	0.342	0.564	0.743	0.859	0.930	0.963	0.978	0.989	0.994
Efron's percentile bootstrap	g1	0.045	0.099	0.273	0.504	0.731	0.878	0.952	0.982	0.993	0.999	0.999
	G1	0.046	0.096	0.253	0.469	0.691	0.850	0.934	0.975	0.991	0.997	0.999
	b1	0.051	0.109	0.297	0.544	0.767	0.903	0.968	0.987	0.995	0.999	0.999
Hall's Percentile bootstrap	g1	0.111	0.172	0.337	0.542	0.714	0.838	0.908	0.950	0.971	0.985	0.994
	G1	0.112	0.169	0.319	0.508	0.678	0.812	0.890	0.937	0.965	0.978	0.989
	b1	0.110	0.178	0.354	0.572	0.742	0.857	0.925	0.957	0.979	0.990	0.994
Bias Corrected percentile	g1	0.091	0.151	0.306	0.516	0.711	0.848	0.924	0.971	0.983	0.993	0.997
	G1	0.090	0.150	0.289	0.486	0.673	0.815	0.906	0.958	0.979	0.990	0.996
	b1	0.091	0.158	0.333	0.546	0.743	0.876	0.942	0.977	0.990	0.995	0.999

Table A1 (Continued)

		n=50										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
	g1	0.034	0.075	0.203	0.442	0.707	0.887	0.968	0.992	0.998	0.999	1.000
Classical	G1	0.034	0.073	0.193	0.419	0.680	0.868	0.960	0.989	0.997	0.999	1.000
	b1	0.034	0.078	0.215	0.467	0.734	0.900	0.976	0.995	0.999	0.999	1.000
Bias	g1	0.100	0.182	0.392	0.630	0.807	0.910	0.960	0.982	0.990	0.995	0.998
Corrected	G1	0.102	0.182	0.376	0.614	0.788	0.896	0.952	0.981	0.989	0.995	0.998
standard	b1	0.104	0.190	0.406	0.656	0.823	0.920	0.966	0.985	0.993	0.996	0.998
Efron's	g1	0.059	0.140	0.367	0.638	0.837	0.941	0.981	0.996	0.999	1.000	1.000
percentile	G1	0.058	0.137	0.349	0.613	0.820	0.930	0.975	0.992	0.998	0.999	1.000
bootstrap	b1	0.058	0.145	0.385	0.663	0.855	0.949	0.985	0.996	0.999	1.000	1.000
Hall's	g1	0.114	0.197	0.399	0.635	0.804	0.904	0.960	0.984	0.990	0.996	0.998
Percentile	G1	0.109	0.189	0.385	0.617	0.792	0.893	0.955	0.980	0.990	0.994	0.997
bootstrap	b1	0.111	0.203	0.418	0.652	0.821	0.918	0.966	0.986	0.992	0.996	0.999
Bias	g1	0.093	0.173	0.381	0.636	0.815	0.924	0.973	0.991	0.997	0.999	1.000
Corrected	G1	0.091	0.170	0.371	0.611	0.798	0.914	0.967	0.988	0.996	0.999	0.999
percentile	b1	0.093	0.181	0.392	0.651	0.833	0.931	0.978	0.992	0.997	0.999	1.000
		n=100										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
	g1	0.048	0.123	0.379	0.733	0.923	0.988	0.998	0.999	1.000	1.000	1.000
Classical	G1	0.048	0.121	0.371	0.717	0.917	0.987	0.998	0.999	1.000	1.000	1.000
	b1	0.048	0.125	0.391	0.746	0.929	0.989	0.998	0.999	1.000	1.000	1.000
Bias	g1	0.096	0.215	0.529	0.803	0.928	0.978	0.993	0.997	0.998	0.999	0.999
Corrected	G1	0.096	0.210	0.514	0.796	0.924	0.975	0.992	0.997	0.998	0.999	0.999
standard	b1	0.097	0.223	0.536	0.808	0.930	0.978	0.995	0.998	0.998	0.999	1.000
Efron's	g1	0.071	0.191	0.528	0.818	0.943	0.988	0.996	0.998	0.999	1.000	1.000
percentile	G1	0.069	0.191	0.512	0.811	0.939	0.986	0.996	0.998	0.999	1.000	1.000
bootstrap	b1	0.067	0.195	0.534	0.830	0.946	0.988	0.997	0.999	0.999	1.000	1.000
Hall's	g1	0.100	0.226	0.533	0.806	0.928	0.976	0.993	0.997	0.999	0.999	1.000
Percentile	G1	0.102	0.220	0.524	0.794	0.924	0.973	0.992	0.997	0.998	0.999	1.000
bootstrap	b1	0.098	0.223	0.544	0.813	0.932	0.977	0.994	0.998	0.999	0.999	1.000
Bias	g1	0.090	0.217	0.530	0.807	0.931	0.982	0.996	0.998	0.999	1.000	1.000
Corrected	G1	0.092	0.210	0.518	0.798	0.927	0.980	0.995	0.998	0.999	1.000	1.000
percentile	b1	0.092	0.218	0.536	0.815	0.936	0.984	0.996	0.998	0.999	1.000	1.000
		n=300										
Method	Est	γ1=0	γ1=0.2	γ1=0.4	γ1=0.6	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0
	g1	0.044	0.296	0.821	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Classical	G1	0.044	0.292	0.819	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	b1	0.044	0.298	0.825	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Bias	g1	0.064	0.365	0.843	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Corrected	G1	0.064	0.362	0.837	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000
standard	b1	0.066	0.362	0.842	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Efron's	g1	0.060	0.366	0.850	0.991	0.999	1.000	1.000	1.000	1.000	1.000	1.000
percentile	G1	0.059	0.363	0.844	0.990	0.999	1.000	1.000	1.000	1.000	1.000	1.000
bootstrap	b1	0.059	0.367	0.854	0.991	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Hall's	g1	0.065	0.369	0.844	0.990	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Percentile	G1	0.067	0.366	0.842	0.990	0.999	1.000	1.000	1.000	1.000	1.000	1.000
bootstrap	b1	0.068	0.368	0.846	0.990	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Bias	g1	0.065	0.375	0.839	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Corrected	G1	0.064	0.369	0.841	0.988	0.999	1.000	1.000	1.000	1.000	1.000	1.000
percentile	b1	0.065	0.372	0.843	0.987	0.999	1.000	1.000	1.000	1.000	1.000	1.000

Table A2: Power for Gamma(4,1) with skewness=1 against with other value for different sample size

		n=10										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.137	0.236	0.355	0.497	0.628	0.741	0.823	0.901	0.941	0.968	0.984
	G1	0.096	0.142	0.227	0.325	0.437	0.552	0.664	0.752	0.822	0.884	0.931
	b1	0.223	0.360	0.530	0.671	0.790	0.879	0.938	0.968	0.987	0.997	0.999
Bias Corrected standard bootstrap	g1	0.204	0.293	0.390	0.493	0.587	0.663	0.730	0.779	0.829	0.868	0.898
	G1	0.155	0.211	0.288	0.365	0.455	0.536	0.613	0.671	0.722	0.767	0.810
	b1	0.278	0.392	0.517	0.620	0.702	0.767	0.823	0.870	0.904	0.925	0.944
Efron's percentile bootstrap	g1	0.186	0.302	0.446	0.585	0.701	0.810	0.907	0.973	0.999	1.000	1.000
	G1	0.110	0.190	0.292	0.405	0.531	0.637	0.730	0.824	0.903	0.961	0.996
	b1	0.285	0.447	0.608	0.749	0.864	0.958	0.998	1.000	1.000	1.000	1.000
Hall's Percentile bootstrap	g1	0.217	0.310	0.407	0.499	0.590	0.660	0.720	0.769	0.815	0.854	0.886
	G1	0.174	0.224	0.302	0.377	0.467	0.541	0.614	0.669	0.720	0.760	0.801
	b1	0.296	0.417	0.519	0.619	0.692	0.755	0.809	0.857	0.894	0.916	0.938
Bias-corrected percentile bootstrap	g1	0.188	0.261	0.351	0.445	0.541	0.631	0.736	0.843	0.970	1.000	1.000
	G1	0.138	0.191	0.256	0.325	0.403	0.485	0.562	0.646	0.723	0.819	0.924
	b1	0.255	0.359	0.472	0.575	0.681	0.809	0.960	1.000	1.000	1.000	1.000
		n=20										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.116	0.214	0.351	0.510	0.664	0.778	0.865	0.917	0.954	0.973	0.985
	G1	0.095	0.160	0.276	0.409	0.563	0.696	0.797	0.869	0.917	0.952	0.971
	b1	0.146	0.279	0.437	0.618	0.753	0.853	0.913	0.952	0.974	0.986	0.994
Bias Corrected standard bootstrap	g1	0.230	0.321	0.433	0.545	0.637	0.714	0.776	0.823	0.865	0.894	0.917
	G1	0.183	0.272	0.369	0.480	0.575	0.654	0.723	0.780	0.825	0.863	0.892
	b1	0.261	0.375	0.496	0.606	0.694	0.763	0.816	0.862	0.898	0.919	0.938
Efron's percentile bootstrap	g1	0.228	0.355	0.500	0.641	0.748	0.836	0.897	0.939	0.962	0.979	0.992
	G1	0.176	0.298	0.426	0.554	0.678	0.770	0.843	0.901	0.938	0.960	0.979
	b1	0.276	0.426	0.577	0.715	0.815	0.885	0.934	0.961	0.982	0.993	0.998
Hall's Percentile bootstrap	g1	0.246	0.340	0.449	0.557	0.639	0.714	0.772	0.822	0.859	0.891	0.917
	G1	0.211	0.293	0.396	0.499	0.589	0.659	0.722	0.775	0.818	0.858	0.887
	b1	0.286	0.395	0.514	0.612	0.695	0.762	0.813	0.860	0.894	0.917	0.936
Bias-corrected percentile bootstrap	g1	0.203	0.299	0.412	0.523	0.626	0.705	0.773	0.832	0.877	0.916	0.945
	G1	0.171	0.254	0.347	0.455	0.555	0.639	0.719	0.778	0.837	0.876	0.908
	b1	0.239	0.352	0.475	0.587	0.685	0.763	0.827	0.880	0.916	0.949	0.972
		n=30										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.141	0.239	0.404	0.579	0.728	0.832	0.899	0.937	0.962	0.981	0.989
	G1	0.121	0.204	0.332	0.505	0.665	0.781	0.861	0.916	0.946	0.968	0.984
	b1	0.163	0.284	0.465	0.646	0.779	0.868	0.921	0.955	0.974	0.987	0.995
Bias Corrected standard bootstrap	g1	0.226	0.340	0.469	0.592	0.688	0.758	0.808	0.851	0.881	0.908	0.927
	G1	0.204	0.301	0.425	0.545	0.646	0.724	0.783	0.826	0.862	0.888	0.914
	b1	0.245	0.383	0.513	0.639	0.718	0.784	0.836	0.868	0.897	0.924	0.943
Efron's percentile bootstrap	g1	0.236	0.395	0.555	0.692	0.789	0.864	0.914	0.941	0.964	0.982	0.989
	G1	0.204	0.340	0.499	0.637	0.751	0.830	0.890	0.924	0.951	0.971	0.984
	b1	0.275	0.439	0.604	0.739	0.826	0.892	0.929	0.955	0.978	0.987	0.995
Hall's Percentile bootstrap	g1	0.252	0.365	0.490	0.602	0.687	0.750	0.808	0.844	0.877	0.906	0.929
	G1	0.228	0.323	0.445	0.553	0.652	0.724	0.776	0.825	0.857	0.888	0.913
	b1	0.275	0.402	0.525	0.640	0.720	0.785	0.830	0.864	0.897	0.923	0.940
Bias-corrected percentile bootstrap	g1	0.209	0.324	0.458	0.588	0.698	0.776	0.836	0.882	0.916	0.934	0.951
	G1	0.188	0.284	0.419	0.547	0.650	0.739	0.801	0.856	0.892	0.919	0.939
	b1	0.235	0.364	0.506	0.635	0.740	0.807	0.865	0.905	0.930	0.948	0.967

Table A2 (Continued)

		n=50										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.145	0.274	0.452	0.645	0.780	0.874	0.931	0.964	0.978	0.987	0.993
	G1	0.133	0.244	0.415	0.600	0.747	0.855	0.919	0.954	0.974	0.985	0.992
	b1	0.160	0.305	0.497	0.686	0.815	0.899	0.944	0.969	0.983	0.991	0.995
Bias Corrected standard bootstrap	g1	0.216	0.345	0.478	0.604	0.701	0.782	0.833	0.872	0.906	0.928	0.944
	G1	0.204	0.321	0.445	0.571	0.678	0.760	0.817	0.862	0.894	0.917	0.939
	b1	0.236	0.367	0.503	0.631	0.725	0.797	0.852	0.888	0.913	0.938	0.951
Efron's percentile bootstrap	g1	0.223	0.392	0.561	0.704	0.812	0.881	0.927	0.959	0.974	0.985	0.990
	G1	0.198	0.359	0.527	0.671	0.785	0.865	0.914	0.950	0.968	0.981	0.990
	b1	0.250	0.420	0.593	0.735	0.833	0.896	0.940	0.965	0.979	0.989	0.993
Hall's Percentile bootstrap	g1	0.253	0.364	0.483	0.608	0.701	0.779	0.831	0.870	0.903	0.930	0.943
	G1	0.236	0.346	0.458	0.580	0.680	0.755	0.812	0.855	0.891	0.920	0.939
	b1	0.267	0.388	0.509	0.637	0.727	0.795	0.848	0.885	0.914	0.936	0.951
Bias-corrected percentile bootstrap	g1	0.203	0.331	0.483	0.619	0.738	0.825	0.884	0.925	0.953	0.970	0.980
	G1	0.186	0.312	0.452	0.593	0.714	0.804	0.868	0.911	0.939	0.962	0.974
	b1	0.217	0.362	0.518	0.656	0.763	0.846	0.899	0.937	0.963	0.974	0.983
		n=100										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.150	0.331	0.576	0.775	0.899	0.953	0.981	0.992	0.996	0.999	0.999
	G1	0.144	0.307	0.547	0.759	0.885	0.946	0.976	0.992	0.995	0.998	0.999
	b1	0.161	0.353	0.603	0.794	0.908	0.958	0.983	0.993	0.996	0.999	0.999
Bias Corrected standard bootstrap	g1	0.177	0.353	0.530	0.677	0.791	0.859	0.903	0.933	0.952	0.969	0.979
	G1	0.167	0.334	0.516	0.662	0.779	0.853	0.896	0.930	0.950	0.966	0.976
	b1	0.189	0.366	0.547	0.695	0.800	0.867	0.911	0.936	0.958	0.973	0.979
Efron's percentile bootstrap	g1	0.195	0.404	0.609	0.772	0.870	0.924	0.959	0.979	0.990	0.994	0.997
	G1	0.180	0.387	0.588	0.750	0.859	0.916	0.955	0.977	0.986	0.994	0.996
	b1	0.210	0.428	0.630	0.787	0.879	0.930	0.965	0.982	0.990	0.994	0.998
Hall's Percentile bootstrap	g1	0.203	0.362	0.531	0.683	0.789	0.859	0.906	0.935	0.956	0.972	0.981
	G1	0.190	0.347	0.514	0.663	0.775	0.851	0.899	0.931	0.951	0.970	0.979
	b1	0.209	0.382	0.556	0.699	0.802	0.867	0.909	0.939	0.960	0.974	0.982
Bias-corrected percentile bootstrap	g1	0.168	0.357	0.547	0.708	0.833	0.897	0.942	0.970	0.984	0.992	0.995
	G1	0.157	0.334	0.530	0.692	0.822	0.892	0.936	0.966	0.981	0.990	0.995
	b1	0.183	0.370	0.566	0.731	0.845	0.910	0.949	0.973	0.985	0.992	0.996
		n=300										
Method	Est	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8	γ1=3.0
Classical	g1	0.203	0.478	0.797	0.936	0.979	0.994	0.998	1.000	1.000	1.000	1.000
	G1	0.199	0.467	0.789	0.933	0.977	0.993	0.998	1.000	1.000	1.000	1.000
	b1	0.208	0.490	0.806	0.939	0.979	0.994	0.999	1.000	1.000	1.000	1.000
Bias Corrected standard bootstrap	g1	0.142	0.376	0.641	0.808	0.900	0.944	0.968	0.983	0.990	0.993	0.996
	G1	0.137	0.369	0.631	0.805	0.896	0.941	0.966	0.982	0.989	0.992	0.996
	b1	0.148	0.384	0.651	0.813	0.901	0.944	0.969	0.982	0.990	0.992	0.996
Efron's percentile bootstrap	g1	0.147	0.422	0.696	0.856	0.925	0.964	0.983	0.991	0.995	0.998	0.999
	G1	0.142	0.411	0.687	0.847	0.922	0.962	0.983	0.991	0.995	0.997	0.999
	b1	0.157	0.430	0.703	0.855	0.927	0.965	0.984	0.992	0.996	0.997	0.999
Hall's Percentile bootstrap	g1	0.152	0.380	0.654	0.817	0.903	0.950	0.973	0.985	0.990	0.995	0.998
	G1	0.149	0.369	0.643	0.815	0.899	0.947	0.971	0.985	0.990	0.995	0.997
	b1	0.155	0.387	0.660	0.824	0.907	0.951	0.975	0.985	0.991	0.996	0.998
Bias-corrected percentile bootstrap	g1	0.133	0.369	0.650	0.823	0.909	0.953	0.979	0.989	0.993	0.997	0.998
	G1	0.130	0.362	0.639	0.818	0.907	0.951	0.976	0.989	0.993	0.997	0.999
	b1	0.136	0.380	0.659	0.825	0.910	0.954	0.978	0.989	0.993	0.997	0.999

Table A3: Power for Gamma(7.5,1) with skewness=0.73 against with other value for different sample size

		n=10										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.090	0.182	0.312	0.450	0.578	0.703	0.803	0.873	0.921	0.954	0.977
	G1	0.071	0.122	0.191	0.296	0.416	0.529	0.641	0.730	0.814	0.871	0.913
	b1	0.127	0.290	0.454	0.609	0.738	0.847	0.910	0.951	0.978	0.993	0.998
Bias Corrected standard	g1	0.151	0.264	0.367	0.476	0.575	0.656	0.725	0.785	0.831	0.865	0.893
	G1	0.132	0.194	0.269	0.356	0.447	0.532	0.612	0.676	0.732	0.785	0.826
	b1	0.194	0.355	0.478	0.593	0.686	0.759	0.823	0.861	0.895	0.916	0.937
Efron's percentile bootstrap	g1	0.092	0.237	0.384	0.518	0.653	0.765	0.863	0.935	0.980	0.999	1.000
	G1	0.054	0.141	0.243	0.368	0.487	0.599	0.695	0.784	0.866	0.931	0.975
	b1	0.146	0.368	0.526	0.673	0.797	0.904	0.969	0.998	1.000	1.000	1.000
Hall's Percentile bootstrap	g1	0.164	0.272	0.379	0.481	0.579	0.658	0.721	0.781	0.819	0.853	0.882
	G1	0.144	0.199	0.276	0.370	0.457	0.541	0.618	0.677	0.727	0.779	0.813
	b1	0.203	0.367	0.483	0.596	0.684	0.754	0.810	0.849	0.881	0.912	0.930
Bias Corrected percentile	g1	0.141	0.249	0.340	0.442	0.535	0.617	0.700	0.785	0.884	0.985	1.000
	G1	0.115	0.174	0.248	0.333	0.416	0.497	0.570	0.644	0.706	0.785	0.863
	b1	0.182	0.331	0.447	0.542	0.642	0.748	0.854	0.972	1.000	1.000	1.000
		n=20										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.082	0.188	0.337	0.495	0.662	0.783	0.870	0.929	0.962	0.979	0.987
	G1	0.076	0.151	0.264	0.409	0.568	0.704	0.803	0.882	0.931	0.962	0.976
	b1	0.096	0.246	0.413	0.595	0.742	0.846	0.918	0.959	0.977	0.988	0.993
Bias Corrected standard	g1	0.178	0.319	0.446	0.564	0.667	0.743	0.803	0.845	0.882	0.911	0.932
	G1	0.157	0.277	0.386	0.500	0.611	0.694	0.759	0.810	0.847	0.878	0.911
	b1	0.202	0.368	0.503	0.623	0.722	0.787	0.834	0.876	0.910	0.933	0.948
Efron's percentile bootstrap	g1	0.153	0.340	0.497	0.650	0.762	0.846	0.911	0.948	0.968	0.983	0.992
	G1	0.125	0.284	0.419	0.572	0.698	0.789	0.864	0.921	0.950	0.970	0.982
	b1	0.190	0.397	0.576	0.720	0.815	0.898	0.944	0.965	0.983	0.992	0.996
Hall's Percentile bootstrap	g1	0.191	0.336	0.459	0.569	0.669	0.741	0.794	0.837	0.875	0.908	0.929
	G1	0.172	0.292	0.401	0.506	0.614	0.695	0.755	0.804	0.841	0.876	0.904
	b1	0.220	0.383	0.515	0.634	0.718	0.780	0.830	0.873	0.908	0.930	0.947
Bias Corrected percentile	g1	0.167	0.311	0.423	0.548	0.661	0.744	0.810	0.863	0.905	0.933	0.956
	G1	0.144	0.263	0.366	0.478	0.595	0.686	0.764	0.818	0.867	0.904	0.931
	b1	0.195	0.351	0.489	0.614	0.719	0.796	0.849	0.898	0.931	0.956	0.973
		n=30										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.082	0.219	0.408	0.624	0.781	0.882	0.937	0.968	0.984	0.992	0.997
	G1	0.072	0.187	0.344	0.550	0.725	0.843	0.913	0.954	0.976	0.988	0.993
	b1	0.095	0.259	0.475	0.691	0.831	0.910	0.957	0.978	0.989	0.996	0.999
Bias Corrected standard	g1	0.224	0.412	0.554	0.682	0.773	0.837	0.880	0.911	0.932	0.950	0.966
	G1	0.205	0.369	0.515	0.641	0.743	0.810	0.862	0.895	0.919	0.938	0.955
	b1	0.240	0.452	0.603	0.721	0.803	0.859	0.896	0.921	0.945	0.961	0.971
Efron's percentile bootstrap	g1	0.216	0.461	0.655	0.782	0.871	0.925	0.962	0.978	0.989	0.995	0.998
	G1	0.194	0.414	0.598	0.740	0.842	0.903	0.943	0.969	0.982	0.990	0.995
	b1	0.255	0.517	0.699	0.825	0.895	0.942	0.970	0.985	0.992	0.997	1.000
Hall's Percentile bootstrap	g1	0.238	0.422	0.561	0.683	0.766	0.830	0.874	0.908	0.930	0.950	0.965
	G1	0.222	0.384	0.524	0.643	0.732	0.808	0.854	0.889	0.917	0.937	0.956
	b1	0.255	0.465	0.603	0.717	0.798	0.851	0.891	0.921	0.943	0.960	0.970
Bias Corrected percentile	g1	0.212	0.403	0.568	0.704	0.802	0.864	0.915	0.945	0.965	0.976	0.986
	G1	0.193	0.364	0.516	0.661	0.764	0.841	0.892	0.925	0.951	0.969	0.980
	b1	0.234	0.446	0.615	0.743	0.828	0.891	0.928	0.959	0.974	0.983	0.991

Table A3 (Continued)

		n=50										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.081	0.276	0.518	0.720	0.855	0.936	0.975	0.988	0.994	0.997	0.999
	G1	0.075	0.246	0.476	0.682	0.832	0.917	0.967	0.985	0.992	0.996	0.999
	b1	0.093	0.316	0.559	0.759	0.880	0.950	0.980	0.990	0.996	0.998	0.999
Bias Corrected standard	g1	0.197	0.428	0.594	0.733	0.808	0.874	0.912	0.942	0.961	0.973	0.979
	G1	0.184	0.402	0.566	0.699	0.794	0.856	0.897	0.932	0.954	0.968	0.976
	b1	0.212	0.450	0.624	0.745	0.823	0.881	0.922	0.950	0.965	0.977	0.983
Efron's percentile bootstrap	g1	0.207	0.483	0.678	0.811	0.896	0.946	0.976	0.987	0.993	0.997	0.999
	G1	0.188	0.456	0.646	0.786	0.878	0.938	0.970	0.982	0.991	0.996	0.998
	b1	0.224	0.516	0.707	0.835	0.914	0.957	0.980	0.989	0.995	0.998	0.999
Hall's Percentile bootstrap	g1	0.217	0.437	0.601	0.724	0.806	0.863	0.907	0.940	0.961	0.971	0.978
	G1	0.203	0.413	0.567	0.695	0.789	0.852	0.894	0.929	0.954	0.969	0.976
	b1	0.229	0.461	0.627	0.746	0.820	0.877	0.917	0.946	0.964	0.975	0.984
Bias Corrected percentile	g1	0.192	0.432	0.613	0.752	0.848	0.915	0.956	0.978	0.987	0.992	0.995
	G1	0.175	0.406	0.580	0.726	0.833	0.898	0.943	0.971	0.984	0.991	0.994
	b1	0.206	0.456	0.647	0.780	0.868	0.927	0.961	0.981	0.989	0.992	0.997
		n=100										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.097	0.353	0.648	0.848	0.944	0.979	0.991	0.997	0.998	0.999	1.000
	G1	0.093	0.325	0.623	0.830	0.935	0.975	0.990	0.996	0.998	0.999	1.000
	b1	0.099	0.374	0.668	0.859	0.949	0.983	0.992	0.998	0.999	0.999	1.000
Bias Corrected standard	g1	0.153	0.421	0.624	0.765	0.868	0.923	0.954	0.971	0.980	0.986	0.989
	G1	0.145	0.400	0.609	0.758	0.857	0.918	0.951	0.969	0.978	0.984	0.990
	b1	0.162	0.440	0.637	0.778	0.879	0.930	0.957	0.971	0.982	0.987	0.991
Efron's percentile bootstrap	g1	0.157	0.475	0.704	0.845	0.928	0.966	0.982	0.991	0.995	0.998	0.999
	G1	0.148	0.461	0.684	0.836	0.924	0.964	0.980	0.991	0.995	0.998	0.999
	b1	0.168	0.501	0.716	0.860	0.936	0.968	0.985	0.991	0.996	0.998	0.999
Hall's Percentile bootstrap	g1	0.172	0.428	0.627	0.767	0.866	0.924	0.953	0.971	0.981	0.987	0.991
	G1	0.164	0.410	0.611	0.755	0.856	0.919	0.949	0.969	0.980	0.986	0.991
	b1	0.175	0.444	0.641	0.776	0.879	0.930	0.957	0.972	0.982	0.989	0.992
Bias Corrected percentile	g1	0.140	0.431	0.648	0.804	0.908	0.951	0.977	0.988	0.993	0.997	0.999
	G1	0.134	0.411	0.633	0.786	0.900	0.951	0.973	0.988	0.992	0.996	0.998
	b1	0.155	0.442	0.662	0.815	0.912	0.957	0.978	0.990	0.995	0.998	0.999
		n=300										
Method	Est	γ1=0.73	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6	γ1=2.8
Classical	g1	0.133	0.597	0.894	0.982	0.996	0.999	1.000	1.000	1.000	1.000	1.000
	G1	0.132	0.586	0.889	0.981	0.995	0.999	1.000	1.000	1.000	1.000	1.000
	b1	0.136	0.606	0.898	0.982	0.996	0.999	1.000	1.000	1.000	1.000	1.000
Bias Corrected standard	g1	0.128	0.532	0.789	0.915	0.964	0.986	0.992	0.996	0.998	0.999	0.999
	G1	0.123	0.524	0.791	0.909	0.961	0.983	0.992	0.996	0.998	0.999	0.999
	b1	0.132	0.538	0.798	0.917	0.964	0.985	0.992	0.996	0.998	0.999	0.999
Efron's percentile bootstrap	g1	0.134	0.573	0.831	0.935	0.978	0.991	0.996	0.998	0.999	1.000	1.000
	G1	0.128	0.571	0.828	0.933	0.978	0.990	0.996	0.998	0.999	1.000	1.000
	b1	0.138	0.585	0.837	0.936	0.979	0.992	0.996	0.999	0.999	1.000	1.000
Hall's Percentile bootstrap	g1	0.133	0.541	0.803	0.916	0.970	0.989	0.993	0.997	0.999	0.999	1.000
	G1	0.130	0.535	0.795	0.917	0.968	0.989	0.993	0.997	0.999	0.999	1.000
	b1	0.138	0.550	0.808	0.922	0.971	0.989	0.993	0.997	0.999	1.000	1.000
Bias Corrected percentile	g1	0.123	0.532	0.797	0.917	0.968	0.988	0.995	0.998	0.999	0.999	1.000
	G1	0.119	0.524	0.790	0.916	0.968	0.987	0.994	0.998	0.999	0.999	1.000
	b1	0.125	0.541	0.803	0.920	0.971	0.988	0.995	0.998	0.999	1.000	1.000

Table A4: Power for Gamma(10,1) with skewness=0.63 against with other value for different sample size

		n=10										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.083	0.127	0.215	0.342	0.476	0.610	0.739	0.836	0.894	0.937	0.967
	G1	0.068	0.091	0.144	0.219	0.330	0.446	0.555	0.676	0.766	0.844	0.892
	b1	0.106	0.184	0.323	0.480	0.640	0.779	0.869	0.927	0.963	0.981	0.992
Bias	g1	0.145	0.200	0.293	0.401	0.500	0.602	0.689	0.755	0.812	0.853	0.888
Corrected	G1	0.122	0.158	0.219	0.302	0.392	0.479	0.561	0.643	0.709	0.765	0.807
standard	b1	0.176	0.259	0.383	0.509	0.623	0.714	0.784	0.844	0.884	0.915	0.936
Efron's	g1	0.069	0.140	0.259	0.397	0.553	0.682	0.794	0.877	0.943	0.986	1.000
percentile	G1	0.043	0.085	0.162	0.267	0.387	0.517	0.634	0.738	0.819	0.889	0.942
bootstrap	b1	0.109	0.223	0.380	0.557	0.705	0.825	0.913	0.978	0.999	1.000	1.000
Hall's	g1	0.156	0.208	0.297	0.410	0.509	0.610	0.685	0.751	0.795	0.840	0.874
Percentile	G1	0.132	0.170	0.219	0.305	0.400	0.486	0.569	0.647	0.706	0.754	0.797
bootstrap	b1	0.189	0.263	0.400	0.512	0.628	0.711	0.776	0.827	0.873	0.906	0.928
Bias	g1	0.141	0.186	0.278	0.369	0.471	0.558	0.649	0.736	0.824	0.904	0.979
Corrected	G1	0.126	0.155	0.202	0.283	0.361	0.450	0.524	0.595	0.671	0.747	0.813
percentile	b1	0.169	0.246	0.361	0.474	0.576	0.677	0.780	0.877	0.972	1.000	1.000
		n=20										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.072	0.117	0.223	0.358	0.538	0.701	0.822	0.893	0.937	0.966	0.983
	G1	0.066	0.096	0.174	0.290	0.441	0.603	0.742	0.843	0.905	0.940	0.966
	b1	0.081	0.141	0.273	0.444	0.633	0.780	0.877	0.932	0.964	0.982	0.991
Bias	g1	0.158	0.233	0.355	0.481	0.603	0.697	0.774	0.827	0.868	0.900	0.923
Corrected	G1	0.142	0.209	0.307	0.424	0.536	0.644	0.728	0.791	0.838	0.873	0.899
standard	b1	0.179	0.272	0.407	0.539	0.662	0.755	0.814	0.862	0.897	0.922	0.940
Efron's	g1	0.121	0.219	0.369	0.536	0.684	0.791	0.873	0.921	0.955	0.975	0.987
percentile	G1	0.100	0.183	0.309	0.462	0.610	0.729	0.819	0.888	0.930	0.958	0.974
bootstrap	b1	0.149	0.264	0.440	0.611	0.752	0.850	0.911	0.950	0.972	0.987	0.994
Hall's	g1	0.170	0.251	0.361	0.488	0.601	0.705	0.776	0.826	0.867	0.896	0.917
Percentile	G1	0.156	0.223	0.322	0.433	0.545	0.644	0.731	0.785	0.832	0.869	0.897
bootstrap	b1	0.191	0.285	0.415	0.546	0.659	0.755	0.814	0.857	0.891	0.917	0.939
Bias	g1	0.146	0.223	0.340	0.467	0.593	0.691	0.777	0.838	0.886	0.922	0.950
Corrected	G1	0.129	0.188	0.294	0.404	0.524	0.634	0.726	0.791	0.850	0.893	0.922
percentile	b1	0.165	0.260	0.384	0.524	0.651	0.748	0.823	0.883	0.919	0.948	0.966
		n=30										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.081	0.133	0.260	0.455	0.637	0.774	0.877	0.934	0.966	0.983	0.991
	G1	0.075	0.117	0.224	0.390	0.575	0.724	0.843	0.909	0.951	0.974	0.985
	b1	0.086	0.154	0.304	0.516	0.691	0.831	0.908	0.954	0.977	0.988	0.995
Bias	g1	0.168	0.255	0.398	0.543	0.666	0.752	0.822	0.867	0.906	0.929	0.947
Corrected	G1	0.151	0.229	0.363	0.500	0.626	0.717	0.795	0.847	0.888	0.917	0.936
standard	b1	0.179	0.286	0.433	0.585	0.702	0.785	0.847	0.889	0.920	0.941	0.956
Efron's	g1	0.139	0.269	0.444	0.622	0.751	0.845	0.907	0.951	0.971	0.985	0.993
percentile	G1	0.122	0.230	0.399	0.566	0.705	0.807	0.883	0.929	0.962	0.978	0.988
bootstrap	b1	0.161	0.306	0.491	0.668	0.786	0.877	0.929	0.963	0.981	0.992	0.995
Hall's	g1	0.182	0.269	0.408	0.544	0.667	0.752	0.814	0.867	0.897	0.925	0.944
Percentile	G1	0.169	0.252	0.374	0.510	0.628	0.717	0.791	0.842	0.883	0.910	0.932
bootstrap	b1	0.197	0.296	0.447	0.586	0.697	0.784	0.840	0.883	0.914	0.938	0.958
Bias	g1	0.154	0.248	0.394	0.549	0.674	0.773	0.850	0.901	0.935	0.961	0.973
Corrected	G1	0.145	0.221	0.362	0.502	0.633	0.735	0.813	0.879	0.917	0.948	0.966
percentile	b1	0.171	0.278	0.434	0.595	0.715	0.808	0.875	0.916	0.951	0.971	0.980

Table A4 (Continued)

		n=50										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.075	0.147	0.329	0.561	0.758	0.881	0.946	0.981	0.991	0.997	0.999
	G1	0.074	0.131	0.301	0.516	0.724	0.858	0.935	0.976	0.989	0.995	0.998
	b1	0.076	0.164	0.363	0.602	0.790	0.907	0.961	0.986	0.994	0.998	0.999
Bias Corrected standard	g1	0.151	0.270	0.442	0.607	0.729	0.819	0.883	0.919	0.949	0.966	0.979
	G1	0.144	0.251	0.416	0.582	0.710	0.801	0.869	0.910	0.939	0.959	0.974
	b1	0.159	0.287	0.475	0.632	0.756	0.837	0.894	0.928	0.954	0.971	0.983
Efron's percentile bootstrap	g1	0.146	0.291	0.501	0.691	0.821	0.906	0.950	0.980	0.991	0.996	0.998
	G1	0.130	0.266	0.467	0.661	0.798	0.889	0.944	0.973	0.987	0.994	0.997
	b1	0.160	0.313	0.536	0.716	0.844	0.922	0.959	0.984	0.992	0.997	0.999
Hall's Percentile bootstrap	g1	0.170	0.285	0.448	0.604	0.728	0.815	0.876	0.916	0.945	0.966	0.980
	G1	0.162	0.264	0.421	0.579	0.707	0.795	0.860	0.908	0.935	0.959	0.974
	b1	0.181	0.296	0.476	0.627	0.750	0.831	0.891	0.926	0.952	0.973	0.984
Bias Corrected percentile	g1	0.142	0.267	0.446	0.625	0.764	0.862	0.924	0.959	0.982	0.990	0.996
	G1	0.135	0.244	0.420	0.596	0.735	0.846	0.910	0.949	0.977	0.988	0.993
	b1	0.154	0.286	0.477	0.657	0.790	0.882	0.937	0.969	0.984	0.992	0.996
		n=100										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.092	0.189	0.464	0.728	0.886	0.959	0.986	0.995	0.999	0.999	1.000
	G1	0.090	0.178	0.437	0.707	0.876	0.954	0.984	0.995	0.998	0.999	1.000
	b1	0.094	0.205	0.486	0.747	0.897	0.964	0.989	0.996	0.999	0.999	1.000
Bias Corrected standard	g1	0.129	0.281	0.503	0.692	0.814	0.885	0.937	0.959	0.977	0.987	0.991
	G1	0.130	0.264	0.485	0.677	0.807	0.883	0.931	0.958	0.975	0.986	0.990
	b1	0.134	0.293	0.518	0.703	0.828	0.893	0.943	0.962	0.979	0.988	0.992
Efron's percentile bootstrap	g1	0.126	0.312	0.561	0.760	0.876	0.940	0.972	0.989	0.994	0.998	0.999
	G1	0.125	0.293	0.545	0.744	0.870	0.933	0.969	0.988	0.993	0.997	0.999
	b1	0.138	0.325	0.580	0.778	0.882	0.946	0.977	0.990	0.995	0.998	1.000
Hall's Percentile bootstrap	g1	0.148	0.288	0.506	0.693	0.816	0.885	0.937	0.961	0.980	0.988	0.993
	G1	0.146	0.277	0.486	0.675	0.809	0.879	0.933	0.958	0.977	0.987	0.992
	b1	0.153	0.295	0.523	0.705	0.825	0.891	0.943	0.966	0.982	0.989	0.993
Bias Corrected percentile	g1	0.127	0.275	0.510	0.717	0.845	0.919	0.961	0.983	0.991	0.996	0.998
	G1	0.117	0.264	0.500	0.699	0.840	0.911	0.959	0.981	0.991	0.996	0.998
	b1	0.127	0.297	0.534	0.731	0.856	0.926	0.964	0.985	0.992	0.997	0.999
		n=300										
Method	Est	γ1=0.63	γ1=0.8	γ1=1.0	γ1=1.2	γ1=1.4	γ1=1.6	γ1=1.8	γ1=2.0	γ1=2.2	γ1=2.4	γ1=2.6
Classical	g1	0.108	0.329	0.768	0.950	0.991	0.999	1.000	1.000	1.000	1.000	1.000
	G1	0.108	0.320	0.759	0.947	0.990	0.999	1.000	1.000	1.000	1.000	1.000
	b1	0.110	0.338	0.776	0.952	0.992	0.999	1.000	1.000	1.000	1.000	1.000
Bias Corrected standard	g1	0.106	0.328	0.677	0.868	0.949	0.979	0.990	0.995	0.997	0.999	0.999
	G1	0.103	0.318	0.671	0.864	0.947	0.978	0.989	0.994	0.997	0.999	0.999
	b1	0.107	0.338	0.680	0.870	0.951	0.979	0.991	0.994	0.997	0.999	0.999
Efron's percentile bootstrap	g1	0.105	0.365	0.718	0.895	0.963	0.987	0.994	0.997	0.999	0.999	1.000
	G1	0.101	0.353	0.711	0.893	0.961	0.985	0.993	0.998	0.999	0.999	1.000
	b1	0.109	0.373	0.726	0.899	0.966	0.987	0.994	0.998	0.999	1.000	1.000
Hall's Percentile bootstrap	g1	0.106	0.340	0.686	0.877	0.953	0.981	0.993	0.996	0.999	0.999	1.000
	G1	0.108	0.328	0.674	0.874	0.951	0.981	0.993	0.996	0.998	0.999	1.000
	b1	0.110	0.347	0.690	0.880	0.957	0.982	0.994	0.996	0.998	0.999	1.000
Bias Corrected percentile	g1	0.101	0.327	0.675	0.870	0.952	0.981	0.992	0.996	0.998	0.999	1.000
	G1	0.097	0.319	0.673	0.865	0.950	0.981	0.992	0.996	0.998	0.999	1.000
	b1	0.105	0.334	0.683	0.876	0.954	0.983	0.992	0.997	0.999	0.999	1.000