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Abstract

In this present work, the Kudryashov method is used to construct exact solutions of the (1+1)-dimensional and the (1+2)-dimensional form of the generalized Ito integro-differential equation. The Kudryashov method is a powerful method for obtaining exact solutions of nonlinear evolution equations. This method can be applied to non-integrable equations as well as integrable ones.

Keywords: Kudryashov Method; The (1+1)-dimensional Form of the Generalized Ito Integro-differential Equation; The (1+2)-dimensional Form of the Generalized Ito Integro-differential Equation

MSC 2010 No.: 34K30, 35R09, 35R10, 47G20

1. Introduction

The study of nonlinear evolution equations (NLEE) has been going on for the past few decades, see Ebadi et al. (2012), Ceaser and Gomez (2010), Li and Zeng (2007), Li and Zhao (2009), Liu (2000), and Wazwaz (2008). During this time, there has been a measurable progress that has been made. There are lots of equations that have been integrated. There are various methods of integrability that have been developed so far. In addition to NLEEs, there has been a growing interest in the nonlinear integro-differential evolution equations. Some of these commonly studied integro-differential evolution equations are the Ito equation, the generalized shallow water wave equation and many others. There are various analytical methods of solving these NLEEs that has also been developed in the past couple of decades. Some of these methods are the exp-function method (see He and Wu (2006), Aminikhah et al. (2009)), the F-expansion method (see Abdou (2007), Wang and Li (2005), Ren and Zhang (2006)), the Jacobi elliptic function expansion method (see Dai and Zhang (2006), Fan and Zhang (2002), Liu et al. (2001)), the modified simplest equation method (see Zayed (2011), Vitanov et al. (2010), Vitanov (2011), Jawad et al. (2010), Akbari (2013)), the first integral method (see Raslan (2008), Abbasbandy and Shirzadi (2010), Feng (2002), Feng and Wang (2003)), the functional variable method (see Zerarka et al. (2010), Zerarka and Ouamane (2010), Cevikel et al. (2012)), and many others. In this paper, we propose a Kudryashov method to construct exact travelling wave solutions for nonlinear evolution equations (see Kudryashov (2004), Kudryashov (1990), Ryabov (2010)). First, we reduce the nonlinear evolution equations to ODEs by travelling wave variable transformation. Secondly, we suppose the solution can be expressed in a polynomial in a variable, where it satisfies the Riccati equation. At the end, the degree of the polynomial can be determined by the homogeneous balance method, and the coefficients can be obtained by solving a set of algebraic equations.

In this work, by using the Kudryashov method, we aim to investigate the (1+1)-dimensional and the (1+2)-dimensional form of the generalized Ito integro-differential equation.

This paper is organized as follows: In Section 2, we describe briefly the Kudryashov method. In Sections 3 and 4, we apply the proposed method to solve the (1+1)-dimensional and the (1+2)-dimensional form of the generalized Ito integro-differential equation. In Section 5, the conclusion will be presented.

2. Modification of truncated expansion method

We consider a general nonlinear partial differential equation (PDE) in the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0. \tag{1}$$

Using traveling wave $u(x,t) = U(\xi), \xi = kx - \omega t$ carries equation (1) into the following ODE:

$$P(U, -\omega U', kU', k^2U'', \ldots) = 0. \tag{2}$$

The main steps of the modification of the truncated expansion method are the following:

Step 1. Determination of the dominant term with highest order of singularity. To find dominant terms, we substitute

$$U = \xi^{-p}, \tag{3}$$

to all terms of equation (2). Then we compare degrees of all terms of equation (2) and choose two or more with the lowest degree. The maximum value of p is the pole of equation (2) and we denote it as N. This method can be applied when N is integer. If the value N is non-integer, one can transform the equation studied.

Step 2. We look for exact solution of equation (2) in the form

$$U(\xi) = \sum_{i=0}^{N} a_i Q^i(\xi), \tag{4}$$

where $a_i (i = 0, 1, \ldots, N)$ are constants to be determined later, such that $a_N \neq 0$ while $Q(\xi)$ has the form

$$Q(\xi) = \frac{1}{1 + d \exp(\xi)}, \tag{5}$$

which is a solution to the Riccati equation

$$Q'(\xi) = Q^2(\xi) - Q(\xi),$$

where d is arbitrary constant.

Step 3. We can calculate the necessary number of derivatives of the function U. It is easy to do using a Maple or Mathematica package. Using the case $N = 1$ we have some derivatives of the function $U(\xi)$ in the form

$$\begin{aligned} U &= a_0 + a_1 Q, \\ U_\xi &= -a_1 Q + a_1 Q^2, \\ U_{\xi\xi} &= a_1 Q - 3a_1 Q^2 + 2a_1 Q^3, \\ U_{\xi\xi\xi} &= -a_1 Q + 7a_1 Q^2 - 12Q^3 + 6a_1 Q^4. \end{aligned} \tag{6}$$

Step 4. We substitute expressions given by equations (4)-(6) in equation (2). Then we collect all terms with the same powers of function $Q(\xi)$ and equate the expressions to zero. As a result we obtain algebraic system of equations. Solving this system we get the values of unknown parameters.

3. New exact travelling wave solution of the (1+1)-dimensional form of the generalized Ito integro-differential equation

The (1+1)-dimensional form of the generalized Ito integro-differential equation that is going to be studied in this section is given by

$$q_{tt} + q_{xxxt} + a(2q_x q_t + q q_{xt}) + a q_{xx} \int_{-\infty}^{x} q_t dx' = 0, \tag{7}$$

Here, in (7), q is the dependent variable while x and t are the independent variables. The coefficient a is constant. Equation (7) can reduced to

$$v_{ttx} + v_{xxxxt} + a(2v_{xx}v_{xt} + v_x v_{xxt} + v_{xxx}v_t) = 0, \tag{8}$$

using the potential $q = v_x$. Equation (8) is converted to the ODE

$$c^2 e u''' - c e^4 u^{(v)} + a(-2ce^3 u'' u'' - ce^3 u' u''' - ce^3 u''' u') = 0. \tag{9}$$

Equivalently,

$$cu''' - e^3u^{(v)} - ae^2((u')^2)'' = 0, \tag{10}$$

by the wave variables $v = u(\xi),\ \xi = ex - ct$, where primes denote the derivatives with respect to ξ, and e, c are real constants to be determined later. Equation (10) is then integrated twice. This converts it to

$$cu' - e^3u''' - ae^2(u')^2 = 0. \tag{11}$$

The pole order of equation (11) is $N = 1$. So we look for the solution of equation (11) in the following form

$$u(\xi) = a_0 + a_1Q. \tag{12}$$

Substituting equation (12) into equation (11), we obtain the system of algebraic equations in the following form

$$\begin{aligned}
Q^1 &: \ -ca_1 + e^3a_1 = 0,\\
Q^2 &: \ ca_1 - 7e^3a_1 - ae^2a_1^2 = 0,\\
Q^3 &: \ 12e^3a_1 + 2ae^2a_1^2 = 0,\\
Q^4 &: \ -6e^3a_1 - ae^2a_1^2 = 0.
\end{aligned}$$

Solving the algebraic equations above, this yields:

$$a_1 = \frac{-6e}{a}, \qquad c = e^3. \tag{13}$$

From (12) and (13), we obtain the following travelling wave solution of equation (11),

$$u(\xi) = a_0 - \frac{6e}{a}\left(\frac{1}{1 + d\exp(\xi)}\right), \tag{14}$$

where a_0 and d are arbitrary constants.

Then the exact solution to equation (7) is written as

$$q(x,t) = \frac{6e^2d}{a}\left(\frac{\exp(ex - e^3t)}{(1 + d\exp(ex - e^3t))^2}\right).$$

4. New exact travelling wave solution of the (1+2)-dimensional form of the generalized Ito integro-differential equation

The (1+2)-dimensional form of the generalized Ito integro-differential equation to be studied in this section is given by

$$q_{tt} + q_{xxxt} + a(2q_xq_t + qq_{xt}) + aq_{xx}\int_{-\infty}^{x} q_t dx' + bq_{yt} + dq_{xt} = 0, \tag{15}$$

Here, in (15), q is the dependent variable while x, y, and t are the independent variables. The coefficient a, b, and d are constants. Equation (15) can reduced to

$$v_{ttx} + v_{xxxxt} + a(2v_{xx}v_{xt} + v_xv_{xxt} + v_{xxx}v_t) + bv_{xyt} + dv_{xxt} = 0, \tag{16}$$

by using the potential $q = v_x$. Equation (16) is converted to the ODE

$$c^2 e u''' - c e^4 u^{(v)} + a(-2ce^3 u'' u'' - ce^3 u' u''' - ce^3 u''' u') - cbefu''' - cde^2 u''' = 0. \tag{17}$$

Equivalently,

$$(c - bf - de)u''' - e^3 u^{(v)} - ae^2((u')^2)'' = 0, \tag{18}$$

by the wave variables $v = u(\xi)$, $\xi = ex + fy - ct$, where primes denote the derivatives with respect to ξ, and e, f, and c are real constants to be determined later. The equation (18) is then integrated twice. This converts it to

$$(c - bf - de)u' - e^3 u''' - ae^2 (u')^2 = 0. \tag{19}$$

The pole order of equation (19) is $N = 1$. So we look for solution of equation (19) in the following form

$$u(\xi) = a_0 + a_1 Q. \tag{20}$$

Substituting equation (20) into equation (19), we obtain the system of algebraic equations in the following form

$$\begin{aligned}
Q^1 &: \ -(c - bf - de)a_1 + e^3 a_1 = 0,\\
Q^2 &: \ (c - bf - de)a_1 - 7e^3 a_1 - ae^2 a_1^2 = 0,\\
Q^3 &: \ 12e^3 a_1 + 2ae^2 a_1^2 = 0,\\
Q^4 &: \ -6e^3 a_1 - ae^2 a_1^2 = 0.
\end{aligned}$$

Solving the algebraic equations above, this yields

$$a_1 = \frac{-6e}{a}, \qquad c = e^3. \tag{21}$$

From (20) and (21), we obtain the following travelling wave solution of equation (19)

$$u(\xi) = a_0 - \frac{6e}{a}\left(\frac{1}{1 + d\exp(\xi)}\right), \tag{22}$$

where a_0 and d are arbitrary constants.

Then the exact solution to equation (15) is written as

$$q(x, y, t) = \frac{6e^2 d}{a}\left(\frac{\exp(ex - (bf + de + e^3 t)}{(1 + d\exp(ex - (bf + de + e^3 t))^2}\right).$$

5. Conclusion

Modification of the truncated expansion method is applied successfully for solving the Ito equation, which is a nonlinear integro-differential evolution equation. Compared to the methods used before, one can see that this method is direct, concise and effective. Moreover, the method can also be applied to many other nonlinear evolution equations.

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