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## Markov Chain Profit Modelling and Evaluation between Two Dissimilar Systems under Two Types of Failures

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## Markov Chain Profit Modelling and Evaluation between Two Dissimilar Systems under Two Types of Failures

### Cover Page Footnote

The present paper deals with profit modelling and comparison between two dissimilar systems under two types of failures based on Markovian Birth Death process. Type I failure is minor in the sense that the work is in a reduced capacity whereas type II failure is major because it causes the entire system failure. Both systems consist of four subsystems arranged in series parallel with three possible states: working with full capacity, reduced capacity and failed state. The systems are attended to by two repairmen in tandem. Through the transition diagrams, systems of differential difference equations are developed and solved recursively to obtain the steady state availability, busy period of repairmen, and profit function. Profit matrices for each subsystem have been developed for different combinations of failure and repair rates. Furthermore, we compare the profit for the two systems and find that system I is more profitable than system II.



## Markov Chain Profit Modelling and Evaluation between Two Dissimilar Systems under Two Types of Failures

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### Abstract

The present paper deals with profit modelling and comparison between two dissimilar systems under two types of failures based on Markovian Birth-Death process. Type I failure is minor in the sense that the work is in a reduced capacity whereas type II failure is major because it causes the entire system failure. Both systems consist of four subsystems arranged in series-parallel with three possible states: working with full capacity, reduced capacity and failed state. The systems are attended to by two repairmen in tandem. Through the transition diagrams, systems of differential difference equations are developed and solved recursively to obtain the steady-state availability, busy period of repair men, and profit function. Profit matrices for each subsystem have been developed for different combinations of failure and repair rates. Furthermore, we compare the profit for the two systems and find that system I is more profitable than system II.

**Keywords:** Profit; Redundant; availability; busy period; modelling; series-parallel

**MSC 2010 No.:** 60J20, 90B25

### 1. Introduction

The industrial and manufacturing systems comprise of large complex engineering systems arranged either in series, parallel, parallel-series or series-parallel. Examples of these systems are feeding, crushing, refining, steam generation, evaporation, crystallization, fertilizer plant, crystallization unit of a sugar plant, piston manufacturing plant, etc. Reliability, availability

and profit are vital in any successful industries and manufacturing settings. Profit may be enhancing using highly reliable system or subsystem. If the reliability and availability of a system is improved, the production and associated profit will also increase. This can be achieved by maintaining reliability and availability at the highest order through maintenance. Large volumes of literature exist on the issue of predicting performance evaluation of various industrial and manufacturing systems. Aggarwal *et al.* (2014) used Markov model for the analysis of urea synthesis of a fertilizer plant. Damcese and Helmy (2012) presented reliability study with mixed standby components. Gupta and Tewari (2011) analyzed the reliability and availability of thermal power plant. Gupta *et al.* (2007) presented the reliability and availability of serial processes of plastics pipe manufacturing plant. Gupta *et al.* (2005) discussed the mission reliability and availability of flexible polymer powder production system. Gupta *et al.* (2007) presented reliability parameters of a powder generating system. Kadiyan *et al.* (2012) presented the reliability and availability of uncaser system of brewery plant. Khanduja *et al.* (2012) presented the steady-state behaviour and maintenance planning of bleaching system of a paper plant. Kaur (2014) discussed the reliability, availability and maintainability of an industrial process. Kaur *et al.* (2013a) discussed the numerical solution of differential difference equations in reliability engineering. Kaur (2013c) discussed the use of corrective maintenance data for performance analysis of textile industry. Kaur (2013b) discussed the performance analysis of an industrial system under corrective and preventive maintenance. Kumar *et al.* (2014) discussed stochastic modelling of a concrete mixture plant with preventive maintenance. Kumar and Mudgil (2014) discussed the availability analysis of the ice cream making unit of a milk plant. Kumar and Tewari (2011) discussed the mathematical modelling and performance optimization of CO<sub>2</sub> cooling system of a fertilizer plant. Kumar *et al.* (2011) discussed the performance modelling of furnace draft air cycle in a thermal plant. Kumar and Lata (2012) presented the reliability evaluation of condensate system using fuzzy Markov model.

Pandey *et al.* (2011) discussed the reliability analysis of a series and parallel network using triangular intuitionistic fuzzy sets. Ram (2010) discussed the reliability measures of three-state complex system. Shakuntla (2012) presented reliability modelling and analysis of some process industrial systems. Sachdeva *et al.* (2008a) discussed availability modelling of screening system of a paper plant. Sachdeva *et al.* (2008b) studied the behaviour of a biscuit making plant using Markov regenerative modelling. Singh and Goyal (2013) presented a methodology to study the steady-state behaviour of repairable mechanical biscuit making plant. Tewari *et al.* (2012) computed the steady-state availability and performance optimization for the crystallization unit of sugar plant using genetic algorithm. Tuteja and Tuteja (1992a) studied the cost benefit analysis of a two server two unit system with different types of failure. Tuteja and Tuteja (1992b) presented profit evaluation of one server system with partial failure subject to random inspection. Tuteja and Malik (1992) presented reliability and profit analysis of two single unit models with three modes and different repair policies. Tuteja *et al.* (1991) analyzed two unit system with partial failure and three types of repairs. Tuteja *et al.* (1991) discussed the stochastic behaviour of a two unit system with two types of repairman and subject to random inspection.

The problem considered in this paper is different from those discussed by the authors above. The purpose of this paper is threefold. The first purpose is to develop the explicit expressions for the steady-state availability, busy period of repair men, and profit function. In this paper, we studied two dissimilar systems subject to two types of failures. The second purpose is to compare these systems in terms of their profit. The third is to capture the effect of both failure and repair rates on profit based on assumed numerical values given to the system parameters.

The organization of the paper is as follows. Section 2 presents the model's description and assumptions. Section 3 presents formulations of the models. Numerical examples are presented and discussed in Section 4. Finally, we make a concluding remark in Section 5.

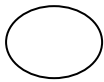
## Symbols



Indicates the system is in full working state



Indicates the system is in failed state



Indicates the system in reduced capacity state

A, B, C, D: Represent full working state of subsystem



a, b, c, d: Represent failed state of subsystem

$\beta_1, \beta_2, \beta_3, \beta_4$  Represent failure rates of subsystems A, B, C, D

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ : represent repair rates of subsystems A, B, C, D

$P_i(t)$ ,  $i = 0, 1, 2, \dots, 16$ : Probability that the system is in state  $S_i$  at time  $t$

$A_V^k(\infty)$ : Steady state availability of the system,  $k = 1, 2$

$B_{p1}^k(\infty)$ : Steady state busy period of repairman due to minor failure

$B_{p2}^k(\infty)$ : Steady state busy period of repairman due to major failure

$C_0$ : Total revenue generated from system using

$C_1$ : Cost incurred due to type I failure

$C_2$ : Cost incurred due to type II failure

$P_F^k(\infty)$ : Profit

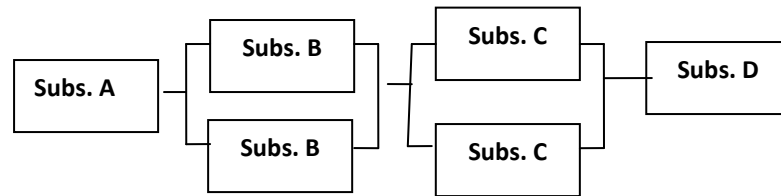
## 2. The Model's Description and Assumptions

### Model description of System I

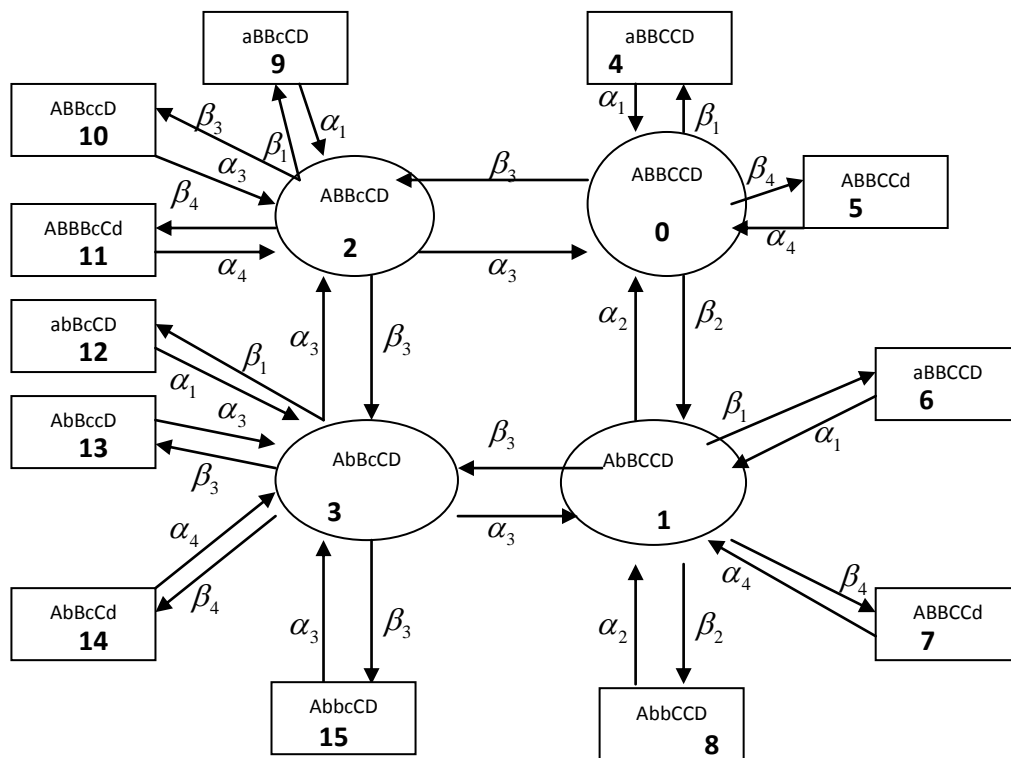
The system consists of four dissimilar subsystems arranged in series-parallel as follows:

1. Subsystem A: It is a single unit and has no standby unit. Its failure is catastrophic and causes complete failure of the system.

2. Subsystem B: Consists of two active parallel units. Failure of one unit causes the system to work in reduced capacity. Complete failure occurs when both units fail.
3. Subsystem C: Consists of two active parallel units. Failure of one unit causes the system to work in reduced capacity. Complete failure occurs when both units fail.
4. Subsystem D: It is a single unit and has no standby unit. Its failure is catastrophic and causes severe effect on the system performance; that is, a complete failure of the system.



**Figure 1.** Reliability block diagram of system I



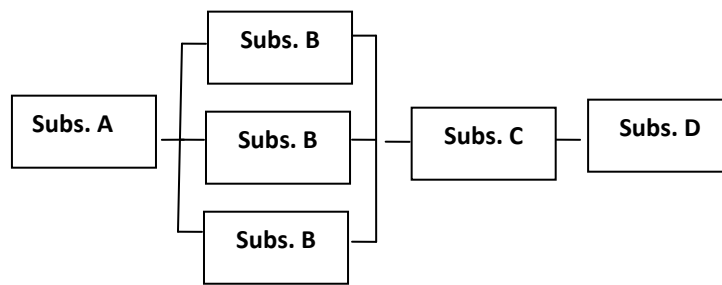
**Figure 2.** Transition diagram of system I

- State 0 indicate full working capacity
- States 1 – 3 indicate reduced capacity (type I failure)
- States 4 – 15 indicate failed states (type II failure)

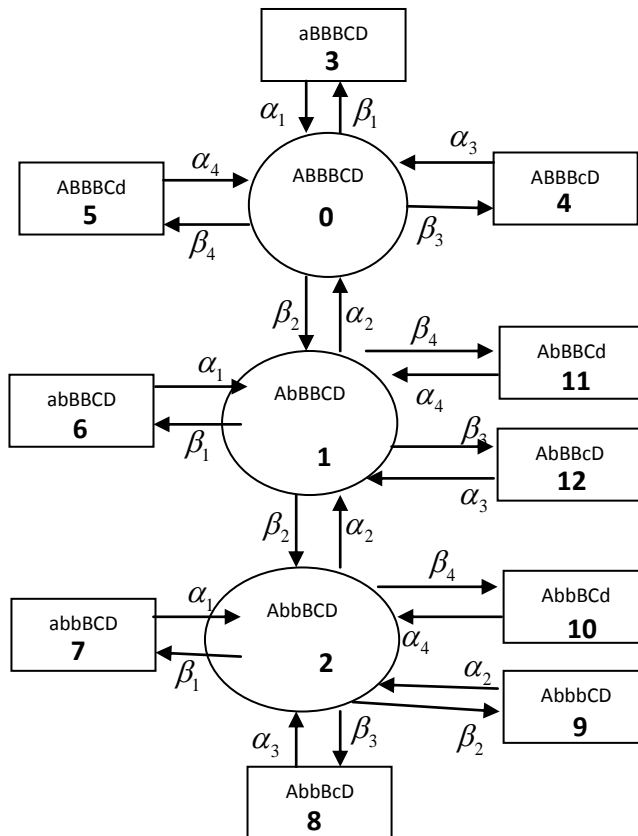
### Model description of System II

The system consists of four dissimilar subsystems arranged in series-parallel as follows:

1. Subsystem A: It is a single unit and has no standby unit. Its failure is complete failure of the system.
2. Subsystem B: Consists of three active parallel units. Failure of one unit, causes the system to work in reduced capacity. Complete failure occurs when both units fail.
3. Subsystem C: It is a single unit and has no standby unit. Its failure is catastrophic and causes severe effect on the system performance: that is, complete failure of the system.
4. Subsystem D: It is a single unit and has no standby unit. Its failure is catastrophic and causes severe effect on the system performance; that is, complete failure of the system.



**Figure 3.** Reliability block diagram of system II



**Figure 4:** Transition diagram of system II

- State 0 indicates full working capacity.
- States 1 – 2 indicate reduced capacity (type I failure).
- States 3 – 12 indicate failed states (type II failure).

## Assumptions

The assumptions used in the model's development are as follows:

1. At any given time the system is either in operating state, reduced capacity or in failed state.
2. Subsystems/units do not fail simultaneously.
3. The system is exposed to two types of failures minor and major failure.
4. Minor failure forces the system to work in reduced capacity states (there is no system failure) whereas major failure brings about system failure.
5. The system is attended to by two repairmen in tandem.
6. Standby units in the same subsystem are of the same nature and capacity as the active units.

## 3. Models Formulation

### Steady State availability, busy periods and profit of System I

The following differential difference equations associated with the transition diagram in Figure 2 of the system are formed using Markov birth-death process:

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i\right) P_0(t) = \alpha_2 P_1(t) + \alpha_3 P_2(t) + \alpha_1 P_4(t) + \alpha_4 P_5(t), \quad (1)$$

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i + \alpha_2\right) P_1(t) = \alpha_3 P_3(t) + \alpha_1 P_6(t) + \alpha_4 P_7(t) + \alpha_2 P_8(t) + \beta_2 P_0(t), \quad (2)$$

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i + \alpha_3\right) P_2(t) = \alpha_2 P_3(t) + \alpha_1 P_9(t) + \alpha_3 P_{10}(t) + \alpha_4 P_{11}(t) + \beta_3 P_0(t), \quad (3)$$

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i + \alpha_2 + \alpha_3\right) P_3(t) = \alpha_1 P_{12}(t) + \alpha_3 P_{13}(t) + \alpha_4 P_{14}(t) + \alpha_2 P_{15}(t) + \beta_3 P_1(t) + \beta_2 P_2(t), \quad (4)$$

$$\left(\frac{d}{dt} + \alpha_m\right) P_i(t) = \beta_m P_j(t), \quad m = 1, 2, 3, 4, \quad j = 0, 1, 2, 3, \quad i = 4, 5, 6, 7, \dots, 15, \quad (5)$$

with initial conditions

$$P_i(t) = \begin{cases} 1 & i = 0, \\ 0 & i > 0. \end{cases} \quad (6)$$



In the steady state, the derivatives of the state probabilities in Equations 1 – 5 are set to zero and solving the resulting equations recursively, we obtained the following steady state probabilities:

$$\begin{aligned}
 P_1(\infty) &= X_2 P_0(\infty), P_6(\infty) = X_1 X_2 P_0(\infty), P_{11}(\infty) = X_3 X_4 P_0(\infty), \\
 P_2(\infty) &= X_3 P_0(\infty), P_7(\infty) = X_2 X_4 P_0(\infty), P_{12}(\infty) = X_1 X_2 X_3 P_0(\infty), \\
 P_3(\infty) &= X_2 X_3 P_0(\infty), P_8(\infty) = X_2^2 P_0(\infty), P_{13}(\infty) = X_2 X_3^2 P_0(\infty), \\
 P_4(\infty) &= X_1 P_0(\infty), P_9(\infty) = X_1 X_3 P_0(\infty), P_{14}(\infty) = X_2 X_3 X_4 P_0(\infty), \\
 P_5(\infty) &= X_4 P_0(\infty), P_{10}(\infty) = X_3^2 P_0(\infty), P_{15}(\infty) = X_2^2 X_3 P_0(\infty).
 \end{aligned}$$

The probability of full working state  $P_0(\infty)$  is determined by using the normalizing condition below:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + \dots + P_{15}(\infty) = 1. \tag{7}$$

Substituting the values of  $P_1(\infty)$ -  $P_{15}(\infty)$  in terms of  $P_0(\infty)$  into the normalizing condition in (7) below

$$P_0(\infty) \left( 1 + X_2 + X_3 + X_2 X_3 + X_1 + X_4 + X_1 X_2 + \dots + X_2^2 X_3 \right) = 1. \tag{8}$$

$$P_0(\infty) = \frac{1}{d_0}, \tag{9}$$

the steady-state availability, busy period due to type I and II failure and profit function of system I are given below:

$$A_V^1(\infty) = \sum_{k=0}^3 P_k(\infty) = \frac{n_0}{d_0}, \tag{10}$$

$$B_1^1(\infty) = \sum_{k=1}^3 P_k(\infty) = \frac{n_1}{d_0}, \tag{11}$$

$$B_2^1(\infty) = \sum_{k=4}^{15} P_k(\infty) = \frac{n_2}{d_0}, \tag{12}$$

$$P_F^1(\infty) = C_0 A_V^1(\infty) - C_1 B_{P_1}^1(\infty) - C_2 B_{P_2}^1(\infty), \tag{13}$$

where

$$d_0 = \left( (1 + X_2) \left( 1 + X_1 + X_3 + X_4 + X_3^2 + X_1 X_3 + X_3 X_4 \right) + X_2^2 (1 + X_3) \right),$$

$$n_0 = 1 + X_2 + X_3 + X_2 X_3,$$

$$n_1 = X_2 + X_3 + X_2 X_3,$$

$$n_2 = (X_1 + X_4) (1 + X_2 + X_3) + X_2 X_3 (X_1 + X_2 + X_3 + X_4) + X_2^2 + X_3^2.$$

## Steady State availability, busy periods and profit of System II

The following differential difference equations associated with the transition diagram in Figure 4 of the system are formed using Markov birth-death process:

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i\right) P_0(t) = \alpha_1 P_3(t) + \alpha_2 P_1(t) + \alpha_3 P_4(t) + \alpha_4 P_5(t), \quad (14)$$

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i + \alpha_2\right) P_1(t) = \alpha_2 P_2(t) + \alpha_1 P_6(t) + \alpha_4 P_{11}(t) + \alpha_3 P_{12}(t) + \beta_2 P_0(t), \quad (15)$$

$$\left(\frac{d}{dt} + \sum_{i=1}^4 \beta_i + \alpha_2\right) P_2(t) = \alpha_1 P_7(t) + \alpha_3 P_8(t) + \alpha_2 P_9(t) + \alpha_4 P_{10}(t) + \beta_2 P_1(t), \quad (16)$$

$$\left(\frac{d}{dt} + \alpha_m\right) P_i(t) = \beta_m P_j(t), \quad m = 1, 2, 3, 4, \quad j = 0, 1, 2, 3, \quad i = 3, 4, 5, 6, 7, \dots, 12. \quad (17)$$

$$P_i(t) = \begin{cases} 1 & i = 0, \\ 0 & i > 0. \end{cases} \quad (18)$$

In the steady state, the derivatives of the state probabilities in Equations 14 – 17 are set to zero and solving the resulting equations recursively we obtained the following steady state probabilities:

$$\begin{aligned} P_1(\infty) &= X_2 P_0(\infty), & P_4(\infty) &= X_3 P_0(\infty), & P_7(\infty) &= X_1 X_2^2 P_0(\infty), & P_{10}(\infty) &= X_4 X_2^2 P_0(\infty), \\ P_2(\infty) &= X_2^2 P_0(\infty), & P_5(\infty) &= X_4 P_0(\infty), & P_8(\infty) &= X_3 X_2^2 P_0(\infty), & P_{11}(\infty) &= X_2 X_4 P_0(\infty), \\ P_3(\infty) &= X_1 P_0(\infty), & P_6(\infty) &= X_1 X_2 P_0(\infty), & P_9(\infty) &= X_2^3 P_0(\infty), & P_{12}(\infty) &= X_2 X_3 P_0(\infty). \end{aligned}$$

The probability of full working state  $P_0(\infty)$  is determined by using the normalizing condition below:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + \dots + P_{15}(\infty) = 1. \quad (19)$$

Substituting the values of  $P_1(\infty) - P_{12}(\infty)$  in terms of  $P_0(\infty)$  into the normalizing condition in (19) below

$$P_0(\infty) \left(1 + X_2 + X_2^2 + X_1 + X_3 + X_4 + X_1 X_2 + \dots + X_2 X_3\right) = 1. \quad (20)$$

$$P_0(\infty) = \frac{1}{d_1}, \quad (21)$$

The steady-state availability, busy period due to type I failure, busy period due to type II failure and profit function of system II are given by

$$A_V^2(\infty) = \sum_{j=0}^2 P_j(\infty) = \frac{n_3}{d_1}, \quad (22)$$

$$B_1^2(\infty) = \sum_{j=1}^2 P_j(\infty) = \frac{n_4}{d_1}, \quad (23)$$

$$B_2^2(\infty) = \sum_{j=3}^{12} P_k(\infty) = \frac{n_5}{d_1}, \tag{24}$$

$$P_F^2(\infty) = C_0 A_V^2(\infty) - C_1 B_{P_1}^2(\infty) - C_2 B_{P_2}^2(\infty), \tag{25}$$

where

$$d_1 = \left( (1 + X_2 + X_2^2)(1 + X_1 + X_3 + X_4) + X_2^3 \right),$$

$$n_3 = 1 + X_2 + X_2^2, \quad n_4 = X_2 + X_2^2,$$

$$n_5 = \left( (X_1 + X_3 + X_4)(1 + X_2 + X_2^2) + X_2^3 \right),$$

$$X_1 = \frac{\beta_1}{\alpha_1}, \quad X_2 = \frac{\beta_2}{\alpha_2}, \quad X_3 = \frac{\beta_3}{\alpha_3}.$$

#### 4. Numerical Examples and Discussion

In this section, we numerically obtained the results for the profit for the systems using the failure and repair rates of Aggarwal *et al.* (2014). For each table the following set of parameter values were fixed:  $C_0 = 100,000$ ,  $C_1 = 10,000$ ,  $C_2 = 15,000$ .

**Table 1.** Profit matrix for subsystem A

$\alpha_1$	0.35		0.4		0.45		0.5		$\alpha_2 = 0.1$ $\beta_2 = 0.005$ $\alpha_3 = 0.5$ $\beta_3 = 0.001$ $\alpha_4 = 0.1$ $\beta_4 = 0.002$
	System I	System II	System I	System II	System I	System II	System I	System II	
$\beta_1$									
0.004	96876	95785	96888	95938	96898	96058	96907	96153	
0.005	96755	95479	96779	95670	96799	95819	96815	95938	
0.006	96634	95176	96670	95403	96700	95581	96725	95723	
0.007	96513	94873	96561	95138	96601	95344	96634	95510	

**Table 2.** Profit matrix for subsystem B

$\alpha_2$	0.05		0.1		0.15		0.2		$\alpha_1 = 0.4$ $\beta_1 = 0.005$ $\alpha_3 = 0.5$ $\beta_3 = 0.001$ $\alpha_4 = 0.1$ $\beta_4 = 0.002$
	System I	System II	System I	System II	System I	System II	System I	System II	
$\beta_2$									
0.004	96350	96054	96351	96068	96352	96079	96352	96087	
0.005	96338	96026	96341	96044	96342	96057	96344	96068	
0.006	96327	95999	96331	96019	96333	96036	96336	96049	
0.007	96315	95971	96320	95995	96324	96014	96327	96029	

**Table 3.** Profit matrix for subsystem C

$\alpha_2$	$\alpha_3$	0.45		0.5		0.55		0.6		$\alpha_2 = 0.1$
		System I	System II	System I	System II	System I	System II	System I	System II	
	$\beta_3$									$\alpha_1 = 0.4$
0.0005		95653	94670	95654	94821	95655	94938	95656	95032	$\beta_1 = 0.005$
0.001		95643	94371	95645	94558	95647	94704	95648	94821	
0.0015		95632	94074	95635	94297	95638	94471	95640	94611	$\alpha_4 = 0.1$
0.002		95621	93777	95625	94036	95629	94239	95632	94401	$\beta_4 = 0.002$

**Table 4.** Profit matrix for subsystem D

$\alpha_4$	$\beta_4$	0.05		0.1		0.15		0.2		$\alpha_2 = 0.1$
		System I	System II	System I	System II	System I	System II	System I	System II	
										$\alpha_3 = 0.5$
0.001		97702	96594	97714	96750	97724	96871	97733	96969	$\beta_3 = 0.001$
0.002		97579	96285	97603	96478	97624	96629	97640	96750	$\alpha_1 = 0.4$
0.003		97456	95976	97493	96209	97523	96388	97548	96532	$\beta_1 = 0.005$
0.004		97333	95670	97382	95938	97423	96147	97456	96315	

**Table 5.** Optimal values of Profit obtained

S/N	Subsystem	Maximum Profit	
		System I	System II
1	A	96907	96153
2	B	96352	96087
3	C	95656	95032
4	D	97733	96969

Table 1 and Figure 5 present the impact of failure and repair rates of subsystem A against the profit for different values of parameters  $\alpha_1$  and  $\beta_1$  for both system I and II. The failure and repair rates of other subsystems are kept constant as can be seen in the last column of Table 1. It is clear from Table 1 and Figure 3 that the profit shows increasing pattern with respect to repair rate  $\alpha_1$  and decreasing pattern with respect to failure rate  $\beta_1$  in both systems. However, system I tends to have more profit than system II.

Table 2 and Figure 6 present the impact of failure and repair rates of subsystem B against the profit for different values of parameters  $\alpha_2$  and  $\beta_2$  for both system I and II. The failure and repair rates of other subsystems are kept constant as can be seen in the last column of Table 2. It is clear from Table 2 and Figure 6 that the profit shows increasing pattern with respect to repair rate  $\alpha_2$  and decreasing pattern with respect to failure rate  $\beta_2$  in both systems. System I tend to have more profit than system II.

Table 3 and Figure 7 present the impact of failure and repair rates of subsystem C against the profit for different values of parameters  $\alpha_3$  and  $\beta_3$  for both system I and II. The failure and repair rates of other subsystems are kept constant as can be seen in the last column in the Table 3. It is clear from Table 3 and Figure 7 that the profit shows increasing pattern with respect to repair rate  $\alpha_3$  and decreasing pattern with respect to failure rate  $\beta_3$  in both systems. Here, again system I has more profit than system II.

Table 4 and Figure 8 present the impact of failure and repair rates of subsystem D against the profit for different values of parameters  $\alpha_4$  and  $\beta_4$  for both system I and II. The failure and repair rates of other subsystems are kept constant as can be seen in the last column of Table 4. It is clear from Table 4 and Figure 6 that the profit shows increasing pattern with respect to repair rate  $\alpha_4$  and decreasing pattern with respect to failure rate  $\beta_4$  in both systems. However, system I tends to have more profit than system II.

Table 5 helps in determining the subsystem with maximum profit. It is observed that subsystem D has maximum profits of 97733 and 96969 for system I and II, respectively. From Table 5, it is observed that the most critical subsystem as far as maintenance is concerned and required immediate attention is subsystem C.

## 5. Conclusion

In this paper, we analyzed two dissimilar systems, each consisting of subsystems A, B, C and D. Explicit expressions for steady-state availability, busy period and profit function for the two systems were derived and comparison between the systems was performed numerically. It is evident from Tables 1 - 4 and Figures 5 - 8 that the optimal system is system I. Models presented in this paper are important to engineers, maintenance managers and plant management for proper maintenance analysis, decision and safety of the system as a whole. The models will also assist engineers, decision makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision making, which may lead to unnecessary expenditures, incorrect maintenance scheduling and reduction of safety standards.

Overall, based on numerical results in the Tables and Figures, it is evident that

- The revenue obtained decreases with increase in failure rates.
- The revenue is affected by the number of operating units.
- The system availability as well as revenue of the system can be increased by adding more redundant units/subsystems, taking more units in the system in cold standby, and by increasing the repair rate.

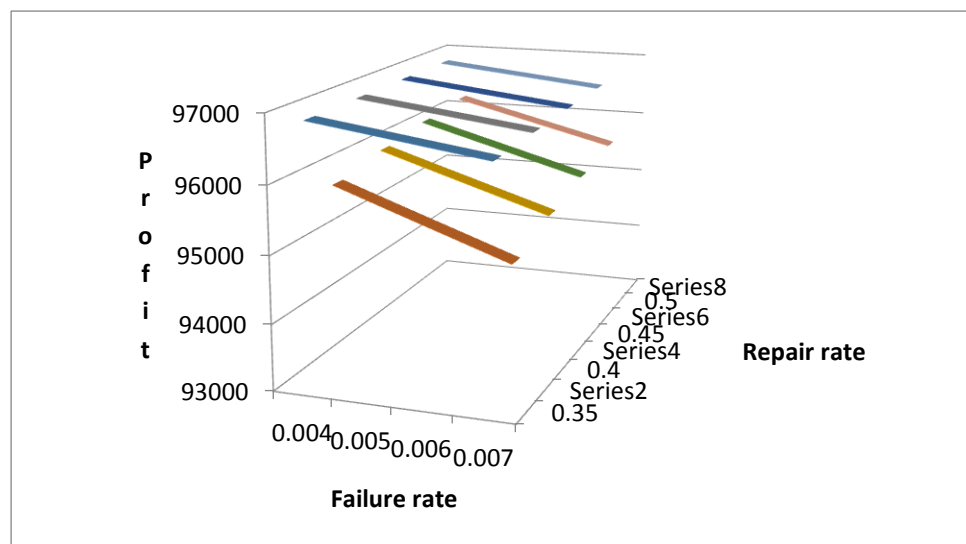
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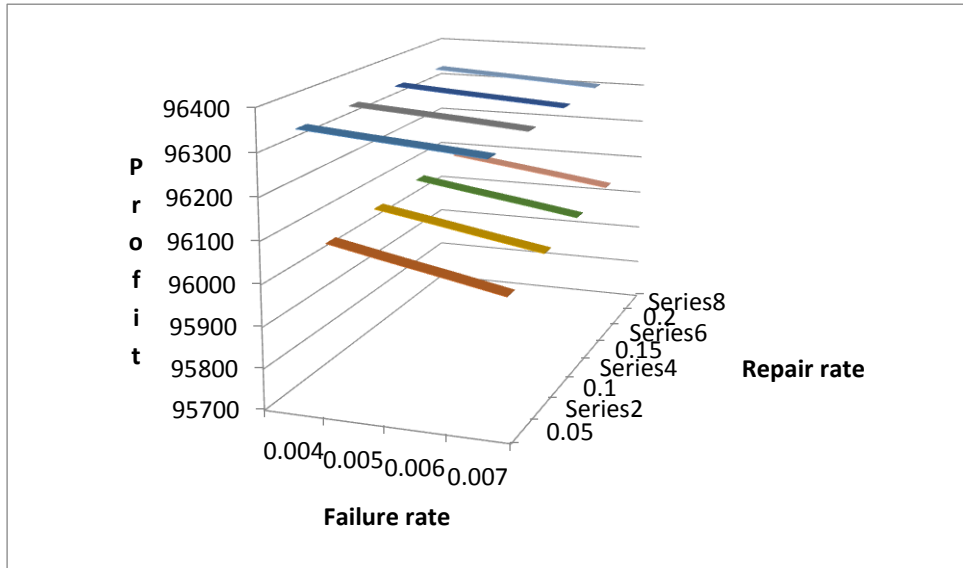
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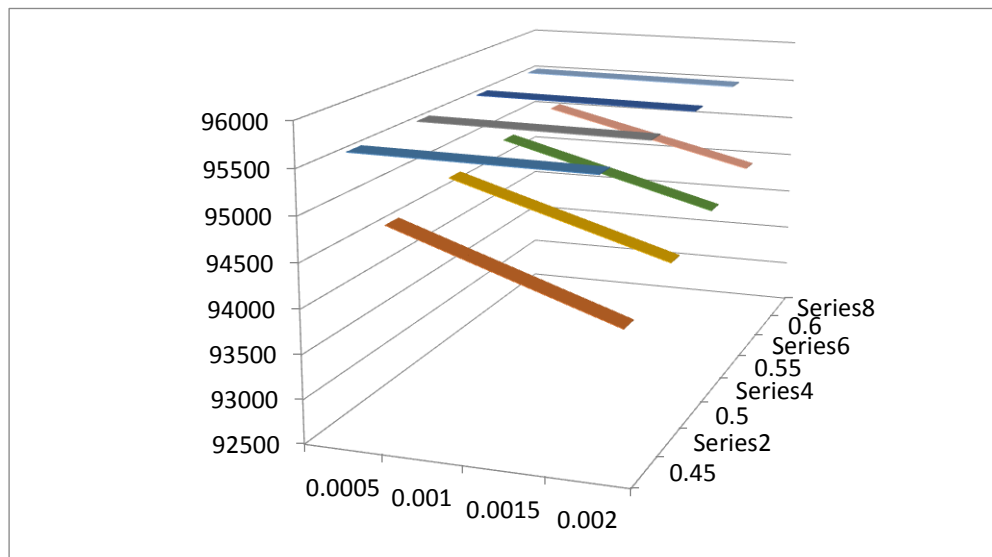
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**Figure 5.** Impact of failure and repair of subsystem A on profit

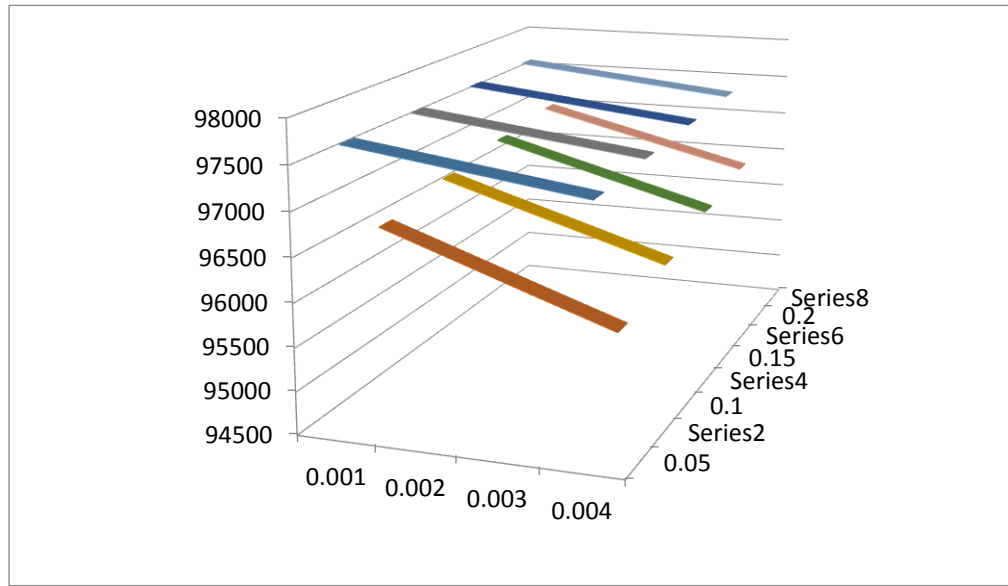


**Figure 6.** Impact of failure and repair of subsystem B on profit



**Figure 7.** Impact of failure and repair of subsystem C on profit





**Figure 8.** Impact of failure and repair of subsystem D on profit