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Abstract

In this paper, the optimal homotopy asymptotic method is applied on the Cauchy reaction-diffusion problems to check the effectiveness and performance of the method. The obtained solutions show that the OHAM is more effective, simpler and easier than other methods. Moreover, this technique does not require any discretization or linearization and therefore it reduces significantly the numerical computations. The results reveal that the method is explicit.

Keywords: Optimal Homotopy Asymptotic Method ; Cauchy reaction-diffusion problem; Time-dependent partial differential equations

MSC 2010 No.: 35C10; 74H10

1. Introduction

Reaction-diffusion equations describe a wide variety of nonlinear systems in physics, chemistry, ecology, biology and engineering (Britton, 1998; Cantrell and Cosner, 2003). By a reaction-diffusion we mean an equation of the following form:

$$\frac{\partial u}{\partial t} = \Delta u + f(u, \Delta u; x, t),$$

The term u is diffusion term and $f(u, \Delta u; x, t)$ is the reaction term. More generally the diffusion term may be of type $A(u)$, where A is a second-order elliptic operator, which may be nonlinear and degenerate.
In this paper, we consider the one-dimensional reaction-diffusion equation

$$\frac{\partial u}{\partial t}(x,t) = D\frac{\partial^2 u}{\partial x^2}(x,t) + p(x,t)u(x,t), \tag{1}$$

where u is the concentration, p is the reaction parameter and $D > 0$ is the diffusion coefficient, subject to the initial or boundary conditions

$$u(x,0) = g(x), \quad x \in R, \tag{2}$$

$$u(0,t) = f_0(t), \quad \frac{\partial u}{\partial x}(0,t) = f_1(t), \qquad t \in R, \tag{3}$$

The problem given by equations (1) and (3) is called the characteristic Cauchy problem in the domain $\Omega = R \times R^+$ and the problem given by equations (1) and (3) is called the non-characteristic problem in the domain $\Omega = R^+ \times R$.
In (Dehdhan and Shakeri, 2008; Lesnic, 2007), this equation is solved by He's variational iteration method and the Adomian decomposition method,respectively.

Marinca et.al (Marinca and Herisanu, 2008; Marinca et al., 2008; Marinca et al., 2009; Marinca and Herisanu, 2010) developed a new technique which is called optimal homotopy asymptotic method (OHAM).This method was also successfully applied in solving many types of nonlinear problems for different equations (Ghoreishi et al., 2012; Gulzaman and Hussain, 2010; Idrees et al., 2010a; Idrees et al., 2010b; Idrees et al., 010b; Iqbal et al., 2010; Islam et al., 2010; Jafari and Gharbavy, 2012; Shah et al., 2010). The purpose of this paper is to extend the OHAM for the solution of Cauchy reaction-diffusion problem .

The paper is organized in the following fashion. Section 2 contains the basic mathematical theory of the optimal homotopy asymptotic method. Section 3 deals with applications of the OHAM to the Cauchy reaction-diffusion problem. Section 4 is reserved for conclusions.

2. Fundamentals of the OHAM

In this section we recall the basic idea of OHAM. Consider the following partial differential equation:

$$\begin{aligned} &L(u(x,t)) + g(x,t) + N(u(x,t)) = 0, \quad x \in \Omega, \\ &B(u, \frac{\partial u}{\partial t}) = 0, \quad x \in \Gamma, \end{aligned} \tag{4}$$

where L is a linear operator, x and t denote independent variable, $u(x,t)$ is an unknown function, $g(x,t)$ is a known function, $N(u(x,t))$ is a nonlinear operator, B is a boundary operator and Γ is the boundary of the domain Ω.
By means of OHAM, we first construct a family of equations

$$\begin{aligned} &(1-p)\left[L(\phi(x,t;p) + g(x,t))\right] = H(p)\left[L(\phi(x,t;p) + g(x,t)) + N(\phi(x,t;p))\right], \\ &B(\phi(x,t;p)\frac{\partial \phi(x,t;p)}{\partial t}) = 0, \end{aligned} \tag{5}$$

where $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$ and also $H(0)=0$, $\phi(x,t;p)$ is an unknown function, respectively.
Obviously, when $p=0$ and $p=1$ it holds

$$\phi(x,t;0)=u_0(x,t), \quad \phi(x,t;1)=u(x,t), \tag{6}$$

respectively. Therefore, when p increase from 0 to 1, the solution $\phi(x,t)$ varies from $u_0(x,t)$ to the solution $u(x,t)$. The zeroth-order problem is obtained from (5) when $p=0$,

$$L(u_0(x,t))+g(x,t)=0, \quad B\left(u_0,\frac{\partial u_0}{\partial t}\right)=0. \tag{7}$$

The auxiliary function $H(p)$ is chosen in the form

$$H(p)=pC_1+p^2C_2+p^3C_3+\ldots, \tag{8}$$

where $C_1,\, C_2,\, C_3, \cdots$ are constants which can be determined later. To get an approximate solution, $\phi(x,t;p,Ci)$ is expanded in a Taylors series about p as

$$\phi(x,t;p,Ci)=u_0(x,t)+\sum_{k\geq 1}u_k(x,t;C_i)p^k, \quad i=1,2,3,\cdots \tag{9}$$

Substituting equation (9) into equation (5) and equating the coefficients of like powers of p, the first and second-order problems are given as

$$L(u_1(x,t))=C_1N_0(u_0(x,t)), \quad B\left(u_1,\frac{\partial u_1}{\partial t}\right)=0, \tag{10}$$

$$L(u_2(x,t))-L(u_1(x,t))=C_2N_0(u_0(x,t))+C_1\left[L(u_1(x,t))+N_1(u_0(x,t),u_1(x,t))\right],$$
$$B\left(u_2,\frac{\partial u_2}{\partial t}\right)=0, \tag{11}$$

and the general governing equations for $u_k(x,t)$ are given as

$$\begin{aligned} L(u_k(x,t)) &- L(u_{k-1}(x,t))=C_kN_0(u_0(x,t)) \\ &+ \sum_{i=1}^{k-1}C_i\left[L(u_{k-i}(x,t))+N_{k-i}(u_0(x,t),u_1(x,t),\cdots,u_{k-1}(x,t))\right] \\ & k=2,3,\cdots, \quad B\left(u_k,\frac{\partial u_k}{\partial t}\right)=0 \end{aligned} \tag{12}$$

where$N_i; i\geqslant 0$, are the coefficients of p^i in the nonlinear operator N:

$$N(u(x,t))=N_0(u_0)+p\,N_1(u_0,u_1)+p^2\,N_2(u_0,u_1,u_2)+\cdots \tag{13}$$

It should be emphasized that the u_k for $k\geqslant 0$ are governed by the linear equations (7), (10), (11) and (12) with the linear boundary conditions that come from the original problem, which can be easily solved. The convergence of the series (9) depends upon the auxiliary constants $C_1,C_2,\cdots$. If it is convergent at $p=1$, one has

$$u(x,t;C_i)=\sum_{k=1}^{\infty}u_k(x,t;C_i). \tag{14}$$

Generally speaking, the solution of equation (4) can be determined approximately in the form

$$\widetilde{u}^{(m)} = u_0(x,t) + \sum_{k=1}^{m} u_k(x,t;C_i). \tag{15}$$

We note that the last coefficient C_k can be a function of x, t. Substituting equation (15) into equation (4) results in the following residual:

$$R(x,t;C_i) = L(\widetilde{u}^{(m)}(x,t;C_i)) + g(x,t) + N(\widetilde{u}^{(m)}(x,t;C_i)), \quad i = 1,2,\cdots \tag{16}$$

If $R(x,t;C_i) = 0$ then $\widetilde{u}^{(m)}(x,t;C_i)$ happens to be the exact solution. Generally such a case will not arise for nonlinear problems,but we can minimize the functional

$$J(C_i) = \int_0^t \int_\Omega R^2(x,t;C_i)dxdt, \tag{17}$$

where R is the residual. The unknown constants $C_i (i = 1,2,\cdots,m)$ can be optimally identified from the conditions

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \cdots = \frac{\partial J}{\partial C_m} = 0. \tag{18}$$

The disadvantage of the OHAM is the requirement to solve a set of coupled nonlinear algebraic equation for the unknown convergence-control parameters $C_1, C_2, C_3, \cdots, C_m$ which will be obtained from relation (18). It is clear that for the low order of m, the nonlinear algebraic system can be solved with some ease but if m is large it becomes more difficult to solve.

3. Application of the OHAM to the Cauchy reaction-diffusion problem

In this section, the OHAM is used in three special cases of the Cauchy reaction-diffusion problem (1).

Example 1. Consider equation (1) with $D = 1$, $p(x,t) = -1 + cos(x) - sin^2(x)$ *i.e*

$$\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) - (-1 + cos(x) - sin^2(x))u(x,t) = 0 \tag{19}$$

and the following initial conditions:

$$\begin{aligned} u(x,0) &= \frac{1}{10}e^{cos(x)-11}, \qquad x \in R, \\ u(0,t) &= \frac{1}{10}e^{-t-10}, \qquad t \in R, \\ \frac{\partial u}{\partial x}(0,t) &= 0, \qquad t \in R. \end{aligned} \tag{20}$$

The exact solution of this problem is $u(x,t) = \frac{1}{10}e^{cos(x)-t-11}$. Applying the method formulated in Section 2, leads to the following:

$$L[\phi(x,t;p)] = \frac{\partial \phi(x,t;p)}{\partial t}, \tag{21}$$

$$N[\phi(x,t;p)] = -\frac{\partial^2 \phi(x,t;p)}{\partial x^2}(x,t) - (-1 + cos(x) - sin^2(x))\phi(x,t;p), \tag{22}$$

$$g(x,t) = 0, \tag{23}$$

with initial condition:

$$\phi(x,0;p)=\frac{1}{10}e^{cos(x)-11}. \tag{24}$$

For the zeroth-order problem, we have

$$\frac{\partial u_0(x,t)}{\partial t}=0, \quad u_0(x,0)=\frac{1}{10}e^{cos(x)-11}$$

which has the solution

$$u_0(x,t)=\frac{1}{10}e^{cos(x)-11}. \tag{25}$$

The first-order problem can be defined as

$$\begin{aligned}&\frac{\partial u_1(x,t)}{\partial t}=(1+C_1)\frac{\partial u_0(x,t)}{\partial t}+C_1(-\frac{\partial^2 u_0(x,t)}{\partial x^2}-(-1+cos(x)-sin^2(x))u_0(x,t)).\\&u_1(x,0)=0\end{aligned}$$

It has the solution

$$u_1(x,t)=\frac{1}{10}C_1te^{cos(x)-11}. \tag{26}$$

The second-order problem can be defined as

$$\begin{aligned}&\frac{\partial u_2(x,t)}{\partial t}=(1+C_1)\frac{\partial u_1(x,t)}{\partial t}+C_2(-\frac{\partial^2 u_0(x,t)}{\partial x^2}-(-1+cos(x)-sin^2(x))u_0(x,t))\\&+C_1(-\frac{\partial^2 u_1(x,t)}{\partial x^2}-(-1+cos(x)-sin^2(x))u_1(x,t)),\\&u_2(x,0)=0,\end{aligned}$$

and it has the solution

$$u_2(x,t)=\frac{1}{20}te^{cos(x)-11}(tC_1^2+2C_1+2C_1^2+2C_2). \tag{27}$$

By repeating this process, we can obtain the solution of third-order problems as

$$\begin{aligned}u_3(x,t)&=\frac{1}{60}te^{cos(x)-11}(t^2C_1^3+6tC_1^2+6tC_1^3+6tC_1C_2\\&+\ 6C_1+12C_1^2+6C_2+6C_1^3+12C_1C_2+6C_3).\end{aligned} \tag{28}$$

Substitution equations (25), (26), (27) and (28) into equation (15) yields the third-order approximate solution ($m=3$) for equation (19):

$$\widetilde{u}^{(3)}=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t), \tag{29}$$

and substituting the approximate solution into equation (16) yields the residual R and the functional J:

$$R(x,t,C_1,C_2,C_3)=\frac{\partial\widetilde{u}^{(3)}}{\partial t}-\frac{\partial^2\widetilde{u}^{(3)}}{\partial x^2}-(-1+cos(x)-sin^2(x))\widetilde{u}^{(3)}, \tag{30}$$

$$J(C_1,C_2,C_3)=\int_{-1}^{1}\int_0^1 R^2(x,t,C_1,C_2,C_3)dtdx. \tag{31}$$

The constants C_1,C_2,C_3 result from the conditions (18) as

$$C_1=6.898033136\times10^{-9}, \quad C_2=-1.947679826\times10^{-8}, \quad C_3=-0.6428571245, \tag{32}$$

and the approximate solution of the third order is

$$\begin{aligned} u(x,t) &= 5e^{cos(x)-11}(2\times 10^{-2} - 1.285714285\times 10^{-2}t - 1.259546155\times 10^{-18}t^2 \\ &+ 1.094093843\times 10^{-27}t^3). \end{aligned} \tag{33}$$

In the Table 1 we compare the exact solution and the OHAM solution (33).

Table 1. Comparison of the exact and approximate values by OHAM at $t = 1$ in Example 1.

x	Exact value	Approximate value by OHAM	Absolute error
-4	$3.195894243542235\times 10^{-7}$	$3.102621877351807\times 10^{-7}$	$9.327236619042777\times 10^{-9}$
-3	$2.283063225857796\times 10^{-7}$	$2.216431887956551\times 10^{-7}$	$6.663133790124527\times 10^{-9}$
-2	$4.052620550657186\times 10^{-7}$	$3.934344575538314\times 10^{-7}$	$1.182759751188725\times 10^{-8}$
-1	$1.054669840797373\times 10^{-6}$	$1.023889237903636\times 10^{-6}$	$3.078060289373657\times 10^{-8}$
0	$1.670170079024566\times 10^{-6}$	$1.621426064567313\times 10^{-6}$	$4.874401445725321\times 10^{-8}$
1	$1.054669840797373\times 10^{-6}$	$1.023889237903636\times 10^{-6}$	$3.078060289373657\times 10^{-8}$
2	$4.052620550657186\times 10^{-7}$	$3.934344575538314\times 10^{-7}$	$1.182759751188725\times 10^{-8}$
3	$2.283063225857796\times 10^{-7}$	$2.216431887956551\times 10^{-7}$	$6.663133790124527\times 10^{-9}$
4	$3.195894243542235\times 10^{-7}$	$3.102621877351807\times 10^{-7}$	$9.327236619042777\times 10^{-9}$

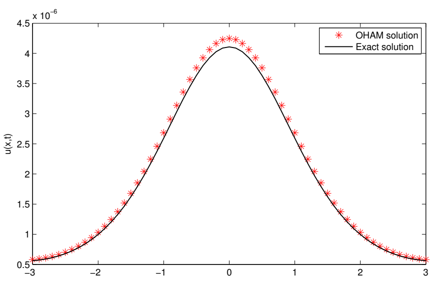

Fig. 1: OHAM, Exact solution at $t = 0.1$

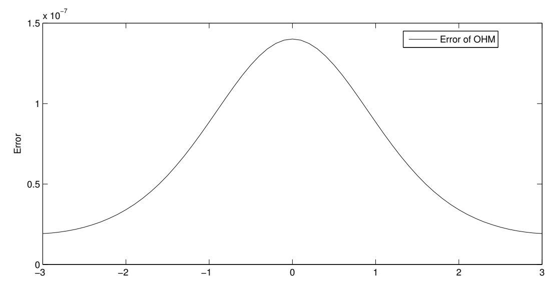

Fig. 2: Absolute error at $t = 0.1$

Example 2. In this example we solve equation (1) with $p(x,t) = -16t$ and $D = 1$ *i.e.*

$$\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) + 16tu(x,t) = 0, \tag{34}$$

with the following initial conditions:

$$\begin{aligned} &u(x,0) = e^{-x-4}, \qquad x\in R, \\ &u(0,t) = e^{-t(8t-1)-4}, \qquad t\in R, \\ &\frac{\partial u}{\partial x}(0,t) = -e^{-t(8t-1)-4}, \qquad t\in R, \end{aligned} \tag{35}$$

The exact solution of this problem is $u(x,t) = e^{-x-t(8t-1)-4}$. According to the OHAM formulation described in Section 2, we start with

$$L = \frac{\partial \phi(x,t;p)}{\partial t}, \tag{36}$$

$$N = -\frac{\partial^2 \phi(x,t;p)}{\partial x^2} + 16t\phi(x,t;p), \tag{37}$$

$$g(x,t) = 0, \tag{38}$$

with initial condition $\phi(x, 0; p) = e^{-x-4}$ The zeroth-order problem is

$$\frac{\partial u_0(x,t)}{\partial t} = 0, \qquad u_0(x,0) = e^{-x-4},$$

and it has the solution

$$u_0(x,t) = e^{-x-4}. \tag{39}$$

The first-order problem is defined as

$$\frac{\partial u_1(x,t)}{\partial t} = (1 + C_1)\frac{\partial u_0(x,t)}{\partial t} + C_1(-\frac{\partial^2 u_0(x,t)}{\partial x^2} + 16t\ u_0(x,t)), \quad u_1(x,0) = 0,$$

and the solution is given by

$$u_1(x,t) = C_1 e^{-x-4} t(8t - 1). \tag{40}$$

The second-order problem can be defined as

$$\begin{aligned} \frac{\partial u_2(x,t)}{\partial t} &= (1 + C_1)\frac{\partial u_1(x,t)}{\partial t} + C_2(-\frac{\partial^2 u_0(x,t)}{\partial x^2} + 16t\ u_0(x,t)) \\ &\quad + C_1(-\frac{\partial^2 u_1(x,t)}{\partial x^2} + 16t\ u_1(x,t)), \quad u_2(x,0) = 0, \end{aligned}$$

and the solution is given by

$$u_2(x,t) = \frac{1}{2}e^{-x-4}t(64C_1^2t^3 - 16C_1^2t^2 + 16tC_1 + 17tC_1^2 + 16C_2t - 2C_1 - 2C_1^2 - 2C_2). \tag{41}$$

By repeating this process, we can obtain the solution of third-order problems as

$$\begin{aligned} u_3(x,t) &= \frac{1}{6}(512C_1^3t^5 - 192C_1^3t^4 + 384t^3C_2C_1 + 384C_1^2t^3 + 408C_1^3t^3 - 96t^2C_2C_1 \\ &- 96C_1^2t_9^27t^2C_1^3 + 48C_2t + 48tC_1 + 102C_1^2 + 102C_1C_2t + 54t + 48C_3t \\ &- 6C_1 - 12C_1^2 - 6C_2 - 6C_1^3 - 12C_1C_2 - 6C_3). \end{aligned} \tag{42}$$

Adding equations (39), (40), (41) and (42), we obtain:

$$\widetilde{u}^{(3)} = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t). \tag{43}$$

Substituting equation (43) into equation (16) and calculating the constants C_1, C_2 and C_3 using the least squares method in (17) and (18) we obtain:

$$C_1 = -.3792732735, \quad C_2 = 0.06574604733, \quad C_3 = 0.02702046464, \tag{44}$$

and the approximate solution of third order in the form

$$\begin{aligned} u(x,t) &= e^{-x-4}(1 - 5.081255854t^2 + .6521918342t + 8.503613449t^4 - 2.171368183t^3 \\ &- 4.655597548t^6 + 1.745849080t^5) \end{aligned} \tag{45}$$

In the Table 2 we compare the exact solution and OHAM solution (45).

Table 2. Comparison of the exact and approximate values by OHAM at $t = 1$ in Example 2.

x	Exact value	Approximate value by OHAM	Absolute error
-2	$1.234098040866795 \times 10^{-4}$	$-8.887768223807506 \times 10^{-4}$	$1.012186626467430 \times 10^{-3}$
-1.5	$7.485182988770060 \times 10^{-6}$	$-5.390703924158947 \times 10^{-4}$	$6.139222223035953 \times 10^{-4}$
-1	$4.539992976248485 \times 10^{-6}$	$-3.269627207435608 \times 10^{-4}$	$3.723626505060456 \times 10^{-4}$
-0.5	$2.753644934974716 \times 10^{-6}$	$-1.983129147140295 \times 10^{-4}$	$2.258493640637766 \times 10^{-4}$
0	$1.670170079024566 \times 10^{-6}$	$-1.983129147140295 \times 10^{-4}$	$1.369845637812812 \times 10^{-4}$
0.5	$1.013009359863071 \times 10^{-6}$	$-7.295524424207707 \times 10^{-5}$	$8.308533784070777 \times 10^{-5}$
1	$6.144212353328210 \times 10^{-5}$	$-4.424959241964330 \times 10^{-5}$	$5.039380477297151 \times 10^{-5}$
1.5	$3.726653172078671 \times 10^{-5}$	$-2.683873448230139 \times 10^{-5}$	$3.056538765438006 \times 10^{-5}$
2	$2.260329406981054 \times 10^{-5}$	$-1.627851533140247 \times 10^{-5}$	$1.853884473838352 \times 10^{-5}$

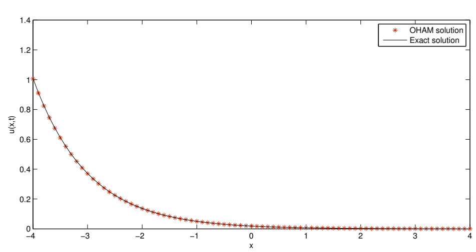

Fig. 3: OHAM, Exact solution at $t = 0.01$.

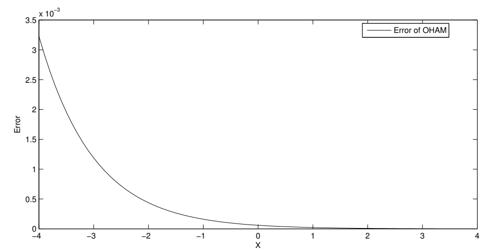

Fig. 4: Absolute error at $t = 0.01$.

Example 3. Consider Eq. (1) with$p(x,t) = -\frac{1}{4}$ and $D = 1$ *i.e.*

$$\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) + \frac{1}{4}u(x,t) = 0 \tag{46}$$

with the following initial conditions:

$$\begin{aligned} &u(x,0) = \frac{1}{2}x + e^{-x/2}, \qquad x \in R, \\ &u(0,t) = 1, \qquad t \in R, \\ &\frac{\partial u}{\partial x}(0,t) = \frac{1}{2}e^{-t/4} - \frac{1}{2}, \qquad t \in R, \end{aligned} \tag{47}$$

The exact solution of this problem is $u(x,t) = \frac{1}{2}xe^{-t/4} + e^{-x/2}$.

According to the OHAM formulation described in Section 2, we start with

$$L = \frac{\partial \phi(x,t;p)}{\partial t}, \tag{48}$$

$$N = -\frac{\partial^2 \phi(x,t;p)}{\partial x^2} + \frac{1}{4}\phi(x,t;p), \tag{49}$$

$$g(x,t) = 0, \tag{50}$$

with initial condition $\phi(x,0;p) = \frac{1}{2}x + e^{-x/2}$. The zeroth-order problem is

$$\frac{\partial u_0(x,t)}{\partial t} = 0, \qquad u_0(x,0) = \frac{1}{2}x + e^{-x/2}.$$

It has the solution

$$u_0(x,t) = \frac{1}{2}x + e^{-x/2}. \tag{51}$$

By repeating the process described in Section 2 and previous examples, we obtain $u_1(x,t)$, $u_2(x,t)$ and $u_3(x,t)$. Therefore, we can obtain the solution of third-order approximation solution by using OHAM is

$$\widetilde{u}^{(3)} = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t). \tag{52}$$

The following values of C_1, C_2, C_3 are obtained:

$$C_1 = -0.9594124146, \quad C_2 = 0.0004828136284, \quad C_3 = 0.00001301250074. \tag{53}$$

Hence, our approximate solution becomes

$$u(x,t) = 0.5x + e^{-0.5x} - 0.1249851166xt + 0.01553539427xt^2 - 0.001149885987xt^3 \tag{54}$$

In the Table 3 we compare the exact solution and the OHAM solution (54).

Table 3. Comparison of the exact and approximate values by OHAM at $t = 1$ in Example 3

x	Exact value	Appr. value by OHAM	Absolute error
-3	3.313487895730958	3.313487895289065	$4.418927446181442 \times 10^{-10}$
-2	1.939481045387640	1.939481045093045	$2.945950150490262 \times 10^{-10}$
-1	1.259320879164426	1.259320879017128	$1.472975075245131 \times 10^{-10}$
0	1	1	0
1	0.9959310512483359	0.9959310513956334	$1.472975075245131 \times 10^{-10}$
2	1.146680224242847	1.146680224537442	$2.945952370936311 \times 10^{-10}$
3	1.391331334755537	1.391331335197430	$4.418925225735393 \times 10^{-10}$

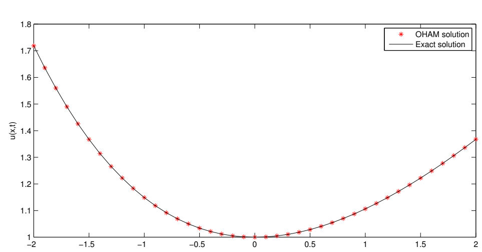

Fig. 5: OHAM, Exact solution at $t = 0.001$.

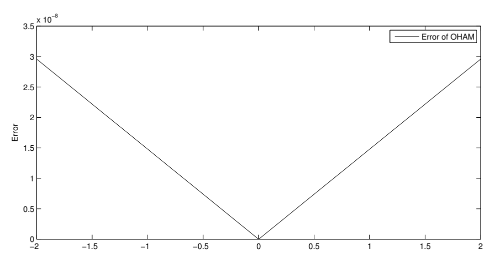

Fig. 6: Absolute error at $t = 0.001$.

4. Conclusions

In this paper, the OHAM has been used for solving the Cauchy reaction-diffusion problem. It is obvious from the solutions that there is no need for computing further higher order terms of $u(x,\, t)$. The OHAM also provides us with a very simple way to control and adjust the convergence of the series solution using the auxiliary constants C_i's which are optimally determined. Therefore, the method shows its validity and great potential for the solution of time dependent problems in science and engineering. *Mathematica* has been used for all computations in this paper.

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