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Abstract

Three different Bianchi type-I cosmological models as related to perfect fluid, with quintessence and with Chaplygin gas, in bimetric theory of gravitation have been deduced. The perfect fluid model has hyperbolic geometry and all its physical parameters are also hyperbolic in nature and therefore they have been studied from hyperbolic geometric view point. All these models are isotropize and shear-less. Other geometrical and physical behaviors of the models have also been studied. The hyperbolic geometric view point of the models will be helpful to the people who use observational data to search for such type of geometry.

Keywords: Bianchi type-I; bimetric gravitational theory; cosmology; hyperbolic geometric; isotropize; geometry

MSC 2010 No.: 83D05; 83F05

1. Introduction

General Relativity (GR) is modified by incorporating dark effects in two ways. The first category is that in which some exotic matter components are added in the energy-momentum tensor part of the action like the cosmological constant or quintessence field etc., while the second category is such that the whole gravitational action of GR is modified by including some dark energy source terms like extra order quantities of the Ricci scalar, such as $f(R), f(R,T), f(G)$ gravity etc. In $f(R)$ gravity, there have been several interesting dark energy as well as dark matter models that can be used to explore cosmological models. For example, quadratic Ricci corrections can be used to discuss dynamics of inflationary universe (Sharif and Yousaf (2014)), inverse Ricci scalar invariant corrections may be used to discuss late time cosmic acceleration; see, for instance (Sharif and Yousaf (2014)). There is a cubic three parametric $f(R)$ corrections (Sharif and Yousaf (2015)), whose results are viable with lambda CDM model. It is interesting to notice that polynomial $f(R)$ model can also be used to discuss inflationary universe (Sharif and Yousaf (2014)). The late time acceleration as well as inflationary universe can be well discussed through generalized CDTT model (Sharif and Yousaf (2014)).

Several theories of gravitation have been proposed as alternative to Einstein's theory of gravitation in order to explain the cosmic acceleration and existence of dark energy and dark matter in the universe. One among them is Rosen's bimetric theory of gravitation. The latest discovery of modern cosmology is that the current universe is expanding and accelerating. The data base like Cosmic Microwave Background Radiation (CMBR) and such as Type Ia supernova (SNe Ia) [Riess (2004), Eisenstein (2005), Astier (2006), Spergel (2007)] and Plank results shown in [Ade et al. (2014)] indicate that the universe contain 4.9% ordinary baryonic matter, 26.8% dark matter and 68.3% dark energy. The dark matter is an unknown form of matter which has clustering properties of ordinary matter and has not yet been detected. The dark energy is the term that represents an unknown form of energy which has not also been detected. Observations like the Wilkinson Microwave Anisotropic Probe (WMAP) [Bennet (2003), Spergel (2003)], Sloan Digital Sky survey (SDSS) [Tegmark (2004), Seljak (2005), Adelman-MaCarthy (2006)] have proved that our universe is undergoing in accelerating expansion. The various forms of dark energy such as quintessence [Ratra (1988), Wetterich (1988), Liddle (1998) and Zlatev (1999)] and phantom field (scalar field with negative sign of kinetic term [Caldwell (2002, 2003), Nojiri (2003), Onemli (2004) and Setare (2008)] and also the combination of quintessence have been studied in a unified model [Feng (2005), Guo (2005), Li (2005), Feng (2003), Zhao (2006), Setare (2008), Sadeghi (2008) and Setare (2008a, 2008b, 2009)].

The equation of state parameter $\omega(t)$ has an important role in dark energy models. The constant values of $\omega = -1, 0, +1/3$ and $+1$ represent vacuum fluid, dust, radiation and stiff dominated universe. The variable $\omega(t)$ of time or red shift is considered and the quintessence model, $\omega > -1$ (explanation of observations of accelerating universe) involving scalar field and phantom model $\omega < -1$ (expansion of universe increases to infinite degree in finite time) give rise to time dependent parameter $\omega(t)$ [Jimenez (2003), Das (2005) and Turner (1997)]. Various forms of parameter $\omega(t)$ have been used for dark energy models and a binary mixture of perfect fluid and dark energy have been considered by many researchers [Caldwell (1998), Liddle (1998), Steinhardt (1999), Rahaman (2006, 2009), Mukhopadhyay (2008), Ray (2011), Akarsu (201a, 2010b), Yadav (2011a, 2011b), Pradhan (2011a, 2011b, 20111c, 2011d, 2011e), Sharif (2010a, 2010b), Akarsu (2010c), Saha (2005, 2006) and Singh (2009)] to investigate the cosmological models of the universe and deduce the different geometrical and physical aspects of the models.

Rosen's (1973, 1975) bimetric theory of gravitation is one of the alternatives to general relativity and it is free from singularities appearing in the big-bang of cosmological models and it obeys the principle of covariance and equivalence of the general relativity. Therefore, the people are interested in investigating the cosmological models of the universe in bimetric theory of gravitation based on two matrices; one is Riemannian metric which described the geometry of curved space time, and the second is flat metric which refers to the geometry of the empty universe (no matter but gravitation is there) and described the initial forces.

The Rosen's field equations in bimetric theory of gravitation are

$$N_i^{\,j} - \frac{1}{2} N\, \delta_i^{\,j} = -T_i^{\,j}, \tag{1}$$

where

$$N_i^{\,j} = \frac{1}{2}\, \gamma^{\alpha\beta} \left(g^{sj}\, g_{si|\alpha}\right)_{|\beta},$$

$N = g^{ij} N_{ij}$ is the Rosen scalar. The vertical bar $(|)$ stands for $\gamma -$covariant differentiation where $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$. Many researchers have developed the theory and investigated many cosmological models of the universe in bimetric theory of gravitation and in general relativity, and studied their behavior geometrically and physically [Karade (1980), Israelit (1981), Reddy et al. (1989, 1998), Mohanty et al. (2002), Bali (2003a, 2003b, 2005, 2006, 2007), Katore (2006), Khadekar (2007), Borkar (2010a, 2013, 2014a, 2014b), Gaikwad (2011)]. Athough the non – existence of Bianchi types I, III, V and VI_0 cosmological models with perfect fluid in the Rosen's bimetric theory of gravitation have been shown by Reddy et al. (1989, 1998), Mohanty et al. (2002) and Borkar et al. (2010b) have deduced the existence of Bianchi type I magnetized cosmological model in Bimetric theory of gravitation and studied its geometrical and physical properties. We plan to extend the work with the combination of dark energy candidates namely quintessence and Chaplygin gas in our work in this article. It is realized that the model with perfect fluid with the combination of quintessence and Chaplygin gas dark energy exist and have the geometrical and physical significance due to the presence of dark energy candidates.

Three different Bianchi type I cosmological models corresponding to perfect fluid, with quintessence and with Chaplygin gas, in Bimetric theory of gravitation have been deduced. The perfect fluid model has hyperbolic geometry and all its physical parameters are also hyperbolic in nature and therefore they have been studied from a hyperbolic geometric view point. The dark energy quintessence model, for $\omega_q = -1/3$ is contracting in nature and for $\omega_q = -1$, the model is expanding. The Chaplygin gas model has volumetric power law expansion. All these models are isotropize and shear-less. Other geometrical and physical behaviors of the model have been studied. The hyperbolic geometric view point of the models will be helpful to the people of observational data to search such type of geometry.

2. Solutions of Rosen's field equations

We consider the Bianchi Type I metric [Borkar et al. (2010b)] in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \tag{2}$$

where A, B and C are functions of t only.

The flat metric corresponding to metric (2) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{3}$$

The energy momentum tensor T_i^j of the source (Sharif et al. (2013)) is given by

$$T_i^j = (\rho + p)v_i v^j + p\delta_i^j , \tag{4}$$

with

$$v_i v^j = -1. \tag{5}$$

Here, ρ and p are the energy density and pressure of perfect fluid, respectively and v^i is the flow vector. The quantity θ is the scalar of expansion which is given by

$$\theta = v^i{}_{|i}. \tag{6}$$

We assume the coordinates to be co-moving, so that

$$v^1 = v^2 = v^3 = 0, \quad v^4 = -1.$$

Equation (4) of energy momentum tensor yield

$$T_1^1 = T_2^2 = T_3^3 = p \quad \text{and} \quad T_4^4 = -\rho. \tag{7}$$

The pressure p and the density ρ are related by an equation of state $p = \gamma\rho$, $0 \le \gamma \le 1$. The Rosen's field Equations (1) for the metric (2) and (3) with the help of (7) give the differential equations

$$-\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p, \tag{8}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p, \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = 16\pi ABC\, \rho. \tag{11}$$

From Equations (8) – (10), we write

$$A = B = C. \tag{12}$$

Also, from Equations (8) and (11), we get

$$\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} = -8\pi ABC(p+\rho). \tag{13}$$

The volume V is a function of t and it is

$$V = ABC. \tag{14}$$

From Equation (12), we write

$$V = A^3, \quad i.e., \quad A = B = C = V^{1/3}. \tag{15}$$

From Equations (13) - (15), we get

$$\frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 24\pi V(p+\rho). \tag{16}$$

We plan to study three different models related to perfect fluid, with quintessence and Chaplygin gas dark energy.

3. The perfect fluid model with physical significance

The conservation law for the energy-momentum tensor from Equation (4), we write

$$\dot{\rho} = -\frac{\dot{V}}{V}(p+\rho). \tag{17}$$

The perfect fluid obeys the equation of state

$$p_{PF} = \gamma\ \rho_{PF}, \quad 0 \le \gamma \le 1. \tag{18}$$

Here, $\gamma = 0$ (Dust Universe), $\gamma = 1/3$ (Radiation Universe), $\gamma = \left(\frac{1}{3}, 1\right)$ (Hard Universe) and $\gamma = 1$ (Zel'dovich Universe or stiff matter).

In a co-moving coordinate, the conservation law of energy momentum tensor (17), for the perfect fluid and dark energy is

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}\left(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}\right), \tag{19}$$

from which we have

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}(\rho_{DE} + p_{DE}) = 0, \tag{20}$$

and

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}(\rho_{PF} + p_{PF}) = 0. \tag{21}$$

From Equations (18) and (21), we write

$$\rho_{PF} = \frac{c_1}{V^{(1+\gamma)}}, \qquad p_{PF} = \frac{\gamma\, c_1}{V^{(1+\gamma)}}, \tag{22}$$

where c_1 is an integration constant.

Using Equation (22), Equation (16) infers

$$\dot{V} = \pm\sqrt{c_2 V^2 - 48\,\pi\,\frac{(1+\gamma)}{\gamma}c_1 V^{2-\gamma}}, \tag{23}$$

where c_2 is a constant of integration.

This Equation (23) has a solution

$$\int \frac{dV}{\sqrt{c_2 V^2 - 48\,\pi\left(1 + \frac{1}{\gamma}\right)c_1 V^{2-\gamma}}} = t + t_0, \tag{24}$$

where t_0 is constant of integration and is taken to be zero. So

$$\int \frac{dV}{\sqrt{c_2 V^2 - 48\,\pi\left(1 + \frac{1}{\gamma}\right)c_1 V^{2-\gamma}}} = t. \tag{25}$$

We study the model in view of the values of γ.

Case (i) For $\gamma = 0$, the integral Equation (25) has a singularity in dust regime. The model does not predict or does not allow a phase in which the universe is dominated by dust.

Case (ii) For $\gamma = 1/3$, the solution of integral Equation (25) is

$$V = \left[c_3/\sqrt{c_2}\cosh\left(\sqrt{c_2}/6\right)t\right]^6, \tag{26}$$

where c_2 and c_3 are positive constants. For proper choice $(c_3/\sqrt{c_2})=1$ and $(\sqrt{c_2}/6)=\alpha$ (constant), we have

$$V = (\cosh \alpha t)^6 , \tag{27}$$

and the scale factors *A*, *B* and *C*, (from Equation (15)) have the values

$$A = B = C = (\cosh (\alpha t))^2 . \tag{28}$$

The line element (2) becomes

$$ds^2 = -dt^2 + (\cosh (\alpha t))^4 (dx^2 + dy^2 + dz^2) . \tag{29}$$

This is Bianchi type I perfect fluid cosmological model in the absence of dark energy in Bimetric theory of gravitation.

It is to be noted that this model has hyperbolic geometry. At $t=0$, $V=A=B=C=1$ and they are hyperbolically increasing in nature with increasing time t and attain infinite values, as $t\to\infty$. This shows that the model starts with nonzero volume and the volume and scale factors are hyperbolically increasing with increase in time t and they go over to infinity at a later stage.

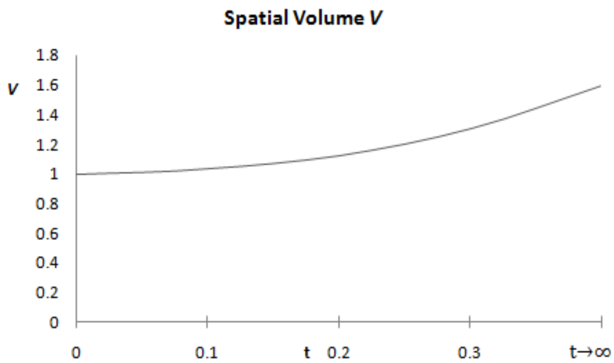

Figure 1. V vs t

This hyperbolic geometrical view point of the model will definitely be a benefit to the mathematical and physical community and the people of observational data to search such type of geometry. Recently, Ungar (2009) observed the hyperbolic geometry view point of Einstein's special relativity.

The energy density ρ_{PF} and the isotropic pressure p_{PF} of the perfect fluid model are

$$\rho_{PF} = 3p_{PF} \quad c_1/(\cosh(\alpha t))^{4/3} . \tag{30}$$

The physical quantities like scalar expansion θ, anisotropic parameter *A*, the shear scalar σ and the deceleration parameter *q* have been calculated as

$$\theta = 6\alpha \tanh(\alpha t), \tag{31}$$

$$A = \sigma = 0, \tag{32}$$

$$q = -\left(1/2(\sinh(\alpha t))^2\right) - 1. \tag{33}$$

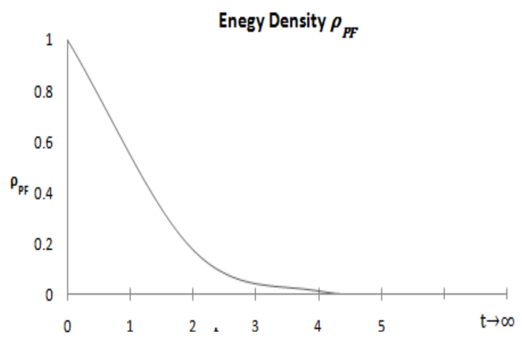

Figure 2. ρ_{PF} vs t

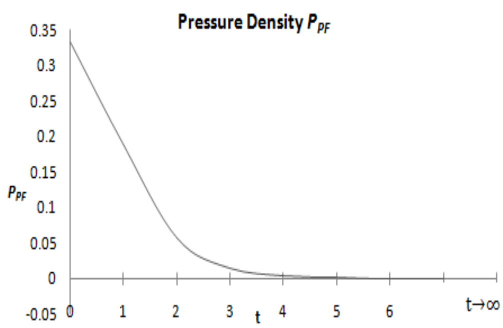

Figure 3. p_{PF} vs t

Figure 2 and Figure 3 show the nature of density and pressure (in similar nature) of the perfect fluid model. At $t = 0$, pressure and density attain the maximum value and they are decreasing very fast as t is increasing, and approaches zero value for $4.5(\text{appro.}) < t < \infty$. This shows that this radiating universe has very high density and pressure in the beginning and its density and pressure go on decreasing and attain zero value for $t \geq 4.5$ i.e., this radiating universe has the matter for some interval of time $0 \leq t \leq 4.5$ (appro.) and the model admit the vacuum case forever for $t \geq 4.5$.

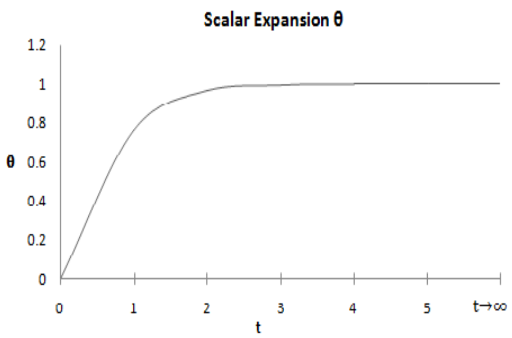

Figure 4. θ vs t

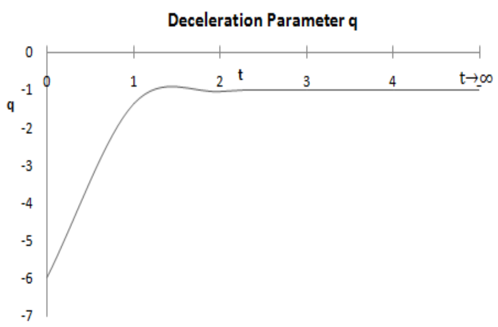

Figure 5. q vs t

The graph of scalar expansion θ is hyperbolic tangential. At $t = 0$, $\theta = 0$ and it is increasing hyperbolic tangentially and attains the finite value when $t \to \infty$. This shows that the model begins with zero expansion and the expansion goes on increasing hyperbolic tangentially and it is finite at a later stage (as shown in Figure 4).

It is well known that the universe underwent an accelerating expansion right after the big-bang (Inflation Era) and at high red shifts (Dark Energy Era). The universe must decelerate *(q>0)* when radiation dominates its dynamics. Figure 5 reflects quite unpleasant nature of deceleration parameter q which has more negative value in the beginning which shows that the model starts with highly accelerating phase and acceleration is slowing down continuously. We are putting this argument on the basis of our mathematical results and this argument may be unpleasant since nobody knows the secret of the nature and the wonder of the physics of the universe.

Case (iii) For $\gamma = 1$, Equation (25) has a solution

$$V = \beta \left(\cosh(\sqrt{c_2}\, t) + 1\right), \tag{34}$$

where β is positive constant.

From Equations (15), we write

$$A = B = C = (\beta(\cosh\sqrt{c_2}\, t + 1))^{1/3}. \tag{35}$$

The required line element (2) is

$$ds^2 = -dt^2 + (\beta(\cosh\sqrt{c_2}\, t + 1))^{2/3}(dx^2 + dy^2 + dz^2). \tag{36}$$

This is the Bianchi type I cosmological model with perfect fluid (stiff matter) in biometric theory of gravitation. This Zeldovich universe has volumetric hyperbolic expansion (as shown in Figure 6 similar to that of the radiating universe case (ii). The scale factors also have similar behavior as case (ii). The model starts with nonzero volume and volume increases in hyperbolic cosine nature and has infinite values at the final stage.

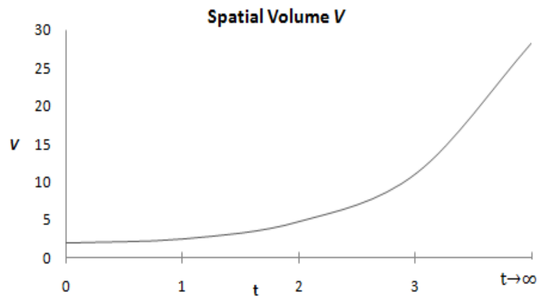

Figure 6. V vs t

The energy density ρ_{PF} and the pressure p_{PF} of the perfect fluid (stiff matter) are

$$\rho_{PF} = p_{PF} = c_1 / \beta^2 (\cosh(\sqrt{c_2}\, t) + 1)^2. \tag{37}$$

The physical parameters are

$$\theta \;=\; \frac{6\,\alpha\;\sinh(\sqrt{c_2}\,t)}{(\cosh(\sqrt{c_2}\,t)+1)}\,, \tag{38}$$

$$A = \sigma = 0\,, \tag{39}$$

$$q \;=\; -\frac{3}{\cos h\,(\sqrt{c_2}\;t)-1} \;-\; 1\,. \tag{40}$$

In this perfect fluid Zel'dovich universe, it is observed that all the physical parameters ρ_{PF}, p_{PF}, θ and decelerating parameter q behave in similar nature as that of case (ii) of radiating universe and having the nature of hyperbolic functions as explained earlier in case (ii) and there is no new contribution regarding the geometrical and physical behavior of these parameters, whose graphs are shown below.

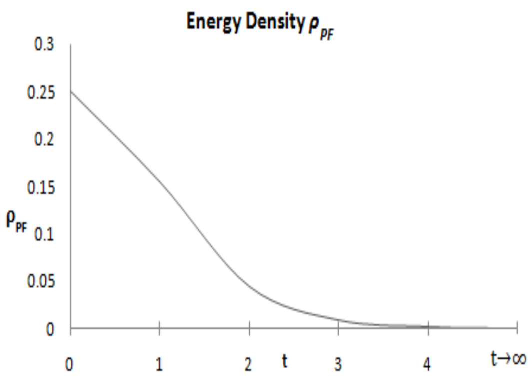

Figure 7. ρ_{PF} vs t

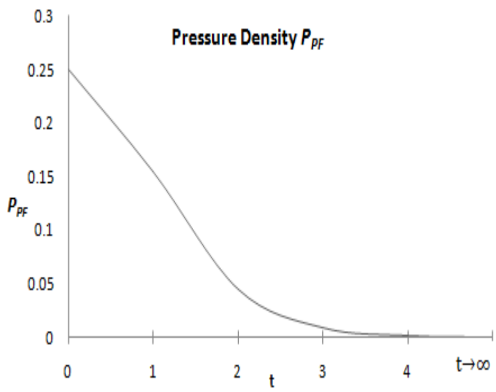

Figure 8. p_{PF} vs t

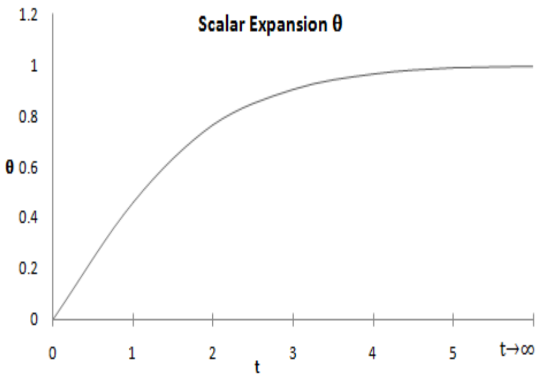

Figure 9. θ vs t

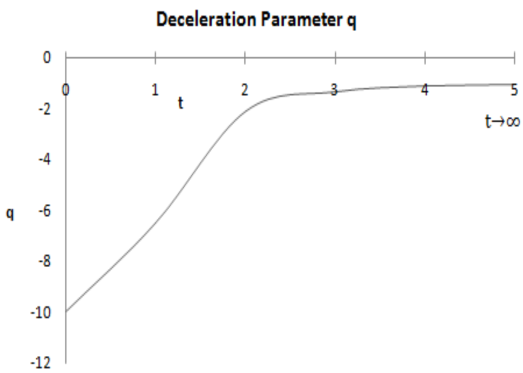

Figure 10. q vs t

It is important to say that in these perfect fluid models (29) and (36) corresponding to radiating universe and Zel'dovich universe, the geometry and all the physical parameters of the models behave hyperbolically in nature, since hyperbolic geometric functions are present in it. Therefore, the geometrical and physical properties of the models with physical parameters have been studied from hyperbolic geometric view point. This is the remarkable point observed in the geometry of the model that the model has hyperbolic geometry and it is helpful to the people of observational data to search such type of universe.

4. Model with dark energy

We plan to consider two sources of dark energy viz. Quintessence and Chaplygin gas in our discussion in this literature. These two sources of dark energy have their own physical significance. Quintessence dark energy candidate, corresponding to equation of state ω_q, $-1 \le \omega_q \le 0$, causes the accelerating expansion of the universe, as observed by observation data and the Chaplygin gas can be introduced just to solve the cumbersome set of equations and not for a source of dark energy (Sharif and Yousaf (2014)).

4.1 Quintessence model

Let us consider the model of dark energy in which the dark energy is given by quintessence which obeys the equation of state

$$p_q = \omega_q \rho_q, \quad -1 \le \omega_q \le 0 \tag{41}$$

in which ω_q is equation of state parameter that corresponds to quintessence model.

From conservation law of energy momentum tensor Equation (19) and the above condition (41), we have

$$\rho_q = \frac{c_4}{V^{(1+\omega q)}}, \quad p_q = \frac{\omega_q c_4}{V^{(1+\omega q)}}, \tag{42}$$

where c_4 is the constant of integration.

In this model, the differential Equation (16) along with the density ρ_q and pressure p_q given by Equation (42) can be written in the form of integral equation as

$$\int \frac{dV}{\sqrt{c_5 V^2 - 48\pi c_4 (1 + 1/\omega_q) V^{2-\omega q}}} = t + c_6, \tag{43}$$

where c_5 and c_6 are constants of integration. For $c_6 = 0$, we write

$$\int \frac{dV}{\sqrt{c_5 V^2 - 48\pi c_4 (1 + 1/\omega_q) V^{2-\omega q}}} = t. \tag{44}$$

We discuss the solution with the cases of $\omega_q = 0, -1/3$ and -1.

Case (iv) For $\omega_q = 0$, the integral Equation (44) has no solution and hence the model is meaningless.

Case (v) For $\omega_q = -1/3$, the integral Equation (44) has a solution

$$V = \frac{1}{(\sinh(mt+n))^6}, \tag{45}$$

where m and n are nonzero positive constants.

From Equation (15) and (44), we have

$$A = B = C = V^{1/3} = \frac{1}{(\sinh(mt+n))^2}. \tag{46}$$

Thus, the required metric is

$$ds^2 = -dt^2 + \frac{1}{(\sinh(mt+n))^4}(dx^2 + dy^2 + dz^2). \tag{47}$$

This is Bianchi type I cosmological model with quintessence dark energy in Bimetric theory of gravitation for $\omega_q = -1/3$, which represents dark energy star.

This model has hyperbolic geometry and its volume V and the scale factor A, B and C are the inverse of hyperbolic sine functions. From the Figure 11, it is observed that, at $t = 0$, the volume of the model attains maximum value and it is gradually decreasing as t increases and approaches to zero value, when $t \to \infty$. This shows that the model starts with maximum volume and the volume slows down and approaches zero at a later stage.

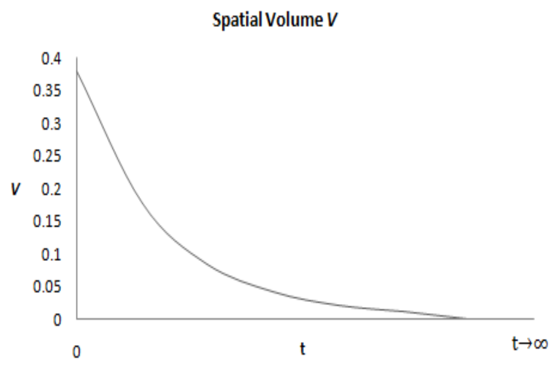

Figure 11. V vs t

The energy density ρ_q and pressure p_q, corresponding to quintessence model are

$$\rho_q = -3p_q = c_4(\sinh(mt+n))^4. \tag{48}$$

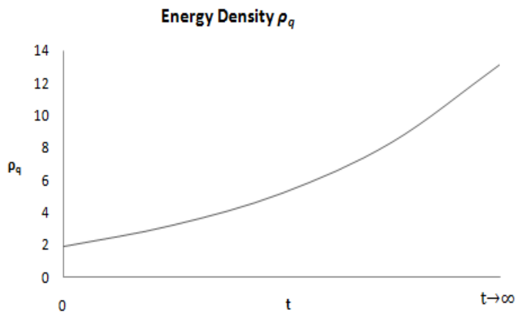

Figure 12. ρ_q vs t

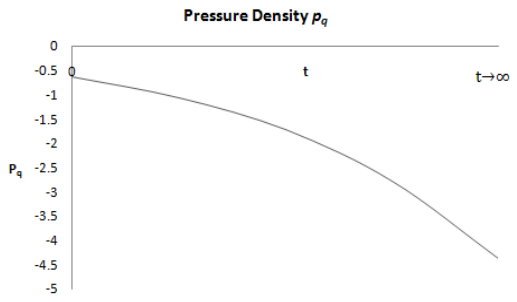

Figure 13. p_q vs t

The nature of density ρ_q follows the graph of hyperbolic sine function. At $t = 0$, the density attains the nonzero positive value and goes on increasing as time, t increases, and reaches infinity when $t \to \infty$ as shown in Figure 12. This shows that in the beginning the model has matter and the density of the matter goes on increasing with time, t and it is infinity at a later stage. It is shown in Figure 13 that the nature of the pressure p_q in the model is negative and decreasing in nature and tends to minus infinity at a later stage.

The physical parameters θ, A, σ and q are given by

$$\theta = -6m\coth(mt+n), \tag{49}$$

$$A = \sigma = 0, \tag{50}$$

$$q = -\frac{1}{2}\sec h^2(mt+n) - 1. \tag{51}$$

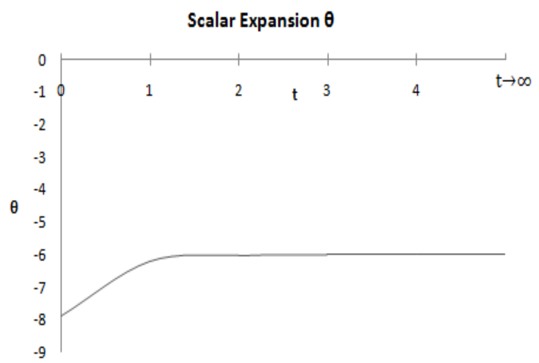

Figure 14. θ vs t

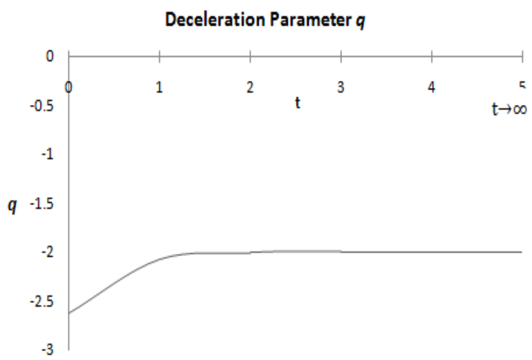

Figure 15. q vs t

The deceleration parameter q is always negative and its graph is shown in Figure 15. Thus, this quintessence dark energy model represents dark energy star and it is contracting starting with a maximum volume.

Case (vi) For $\omega_q = -1$, the integral Equation (44) has a solution

$$V = e^{\sqrt{c_5}\,t}, \tag{52}$$

and Equation (15) gives

$$A = B = C = e^{a\,t}, \tag{53}$$

where, a is a positive constant. Thus, the required metric (2) is

$$ds^2 = -dt^2 + e^{2at}(dx^2 + dy^2 + dz^2). \tag{54}$$

This is the Bianchi type I cosmological model with dark energy (quintessence) in Bimetric theory of gravitation for $\omega_q = -1$ which has volumetric exponential expansion. The volume V and scale factors A, B and C obey exponential laws. At $t = 0$, they admit value one and increasing exponentially as t increases and goes to infinity, when $t \to \infty$. This shows that model starts with nonzero volume and nonzero scale factors and the volume and the scale factors are exponentially increasing as t increases and admit infinite values at later stages.

The Hubble parameter H and its directional's H_1, H_2, H_3 are

$$H = H_1 = H_2 = H_3 = a. \tag{55}$$

The energy density ρ_q and the pressure p_q corresponds in this model (55) are

$$\rho_q = -p_q = c_4 \quad \text{(constant)}. \tag{56}$$

The physical parameters θ, A, σ and q admits the values

$$\theta = 3a, \quad A = \sigma = 0, \quad q = -1. \tag{57}$$

It is seen that the model has a constant rate of expansion forever. The density ρ_q and pressure p_q of the matter for $\omega_q = -1$ are constant right from the beginning. The scalar expansion θ admits a constant value which supports the uniform expansion. Anisotropic parameter A and shear σ admit zero values show that the model is isotropize in all directions without shear. The deceleration parameter q is $q = -1$ shows the model has accelerating phase forever and there is no decelerating phase. It is noted that although we consider the model with matter of perfect fluid and dark energy (quintessence), for $\omega_q = -1$ there is no role of perfect fluid, and the geometry of the model is fully based on quintessence.

4.2. Chaplygin gas model

Let us now consider the case when the dark energy is represented by Chaplygin gas. In this model, we have

$$p_C = -\frac{\mu}{\rho_C}, \tag{58}$$

in which μ is constant.

From Equation (20) of conservation of energy momentum and the above condition (58), we write the values of density ρ_c and pressure p_c as

$$\rho_C = \sqrt{\frac{\mu_1}{V^2} + \mu}, \tag{59}$$

$$p_C = -\frac{\mu}{\sqrt{\frac{\mu_1}{V^2} + \mu}}, \tag{60}$$

where μ_1 is the constant of integration.

The differential Equation (16) in view of density ρ_C (Equation (59)) and pressure p_c (Equation (60)), we have

$$\frac{d}{dt}\left(\frac{\dot{V}^2}{V^2}\right) = 48\pi\left\{\frac{2c_1}{V^2} + \frac{\mu_1}{V\sqrt{\mu_1 + \mu V^2}}\right\}\dot{V}. \tag{61}$$

For the sake of simplicity, choose $\mu_1 = 0$. So the above Equation (61) has the form

$$\left(\frac{dV}{dt}\right)^2 + lV = 0, \tag{62}$$

which has a solution

$$V = 4l\,t^2, \tag{63}$$

where $l = 96\pi\, c_1$ (constant).

From Equation (15) and (63), we get

$$A = B = C = (4lt^2)^{1/3}. \tag{64}$$

The line element (2) has the form

$$ds^2 = -dt^2 + (4lt^2)^{2/3}(dx^2 + dy^2 + dz^2). \tag{65}$$

This is the Bianchi type I cosmological model with Chaplygin gas in bimetric theory of gravitation. The volume and scale factors of the model are in power law expansion. At $t = 0$, the volume V and scale factors A, B, C are zero and they increase with increases in time t and tend to infinity, as $t = 0$. This shows that the model has volumetric power law expansion starting with zero volume and zero scale factors and volume and scale factors increasing with time t and approaches infinity at later stage of time t.

The Hubble parameter H and its directional's are

$$H = H_1 = H_2 = H_3 = 2/3t. \tag{66}$$

The energy density ρ_C and pressure p_C are given by

$$\rho_C = -p_c = \sqrt{\mu}\ , \text{(constant)} \tag{67}$$

and

$$\theta = 2/t,\ A = \sigma = 0, q = 1/2. \tag{68}$$

The Hubble parameter H and its direction H_1, H_2 and H_3 are inversely proportional to time t. At $t = 0$, they admit infinite values and when $t \to \infty$, they admit zero values. This shows that the model has infinite rate of expansion in the beginning and the rate of expansion is zero at later a stage. In this Chaplygin gas model, the energy density ρ_c and pressure p_c admit the constant values forever which shows that this dark energy chaplygin gas model has uniform density and pressure. The scalar expansion θ is inversely proportional to time t, and has similar features as that of the Hubble parameter H. Anisotropic parameter A and the shear σ attain the zero values which shows the model is isotropize without shear. The isotropize nature of the model is also supported by the zero value of $\overset{\text{ratio}}{t \to \infty}\ \sigma/\theta$. The deceleration parameter, q appears with positive value 1/2. This shows that the model has decelerating expansion always and there is no chance of accelerating expansion in this Chaplygin gas model.

5. Summary

1. As related to perfect fluid, quintessence and Chaplygin gas dark energy, the three different "Bianchi type I cosmological model in bimetric theory of gravitation" have been deduced.

2. The perfect fluid models corresponding to $\gamma = 1/3$ (radiating) and $\gamma = 1$ (stiff matter) are hyperbolic geometric in nature. The perfect fluid model correspondence to $\gamma = 0$ (dust) does not exist.

3. All the physical matters in these perfect fluid models obey the graph of hyperbolic geometric functions and therefore their natures have been studied from hyperbolic geometric view point.

4. These perfect fluid models have volumetric hyperbolic expansion.

5. These perfect fluid models are isotropize in nature without shear.

6. These perfect fluid models are highly accelerating and acceleration of the model goes on increasing as time t is increasing and has the constant acceleration at later stages of time t.

7. The quintessence model does not exist for $\omega_q = 0$.

8. The dark energy quintessence model for $\omega_q = -1/3$, represents dark energy star. In the beginning, the model attains the maximum volume and the volume slows down and approaches zero at a later stage.

9. For $\omega_q = -1/3$, in this dark energy quintessence model, the density ρ_q is an increasing function of time t and the pressure p_q is a decreasing function of time t and model starts with matter with nonzero density.

10. In this model with $\omega_q = -1$, a negative rate of expansion shows this dark energy quintessence is contracting. The contraction of the model is also supported by the nature of the scalar expansion θ which is negative, and the behavior of its volume which is decreasing continuously.

11. For $\omega_q = -1$, the dark energy quintessence model has volumetric exponential expansion which starts with nonzero volume and nonzero scale factors and they are increasing exponentially with increasing time and admits zero value at a later stage.

12. This dark energy quintessence model for $\omega_q = -1$ has constant rate of expansion and it has uniform density and pressure of the matter.

13. This model for $\omega_q = -1$ has uniform expansion and has accelerating phase forever and there is no chance of a decelerating phase in the model.

14. In both quintessence models (48) and (55) corresponding to $\omega_q = -1/3$ and $\omega_q = -1$, there is no shear and the models are isotropize in nature. It is also seen that there is no role of perfect fluid and the geometry of these two models is fully based on quintessence.

15. The Chaplygin gas models have been investigated and it is given by Equation (66) and it has volumetric power law expansion starting with zero volume and zero scale factors.

16. This Chaplygin gas model has infinite rate of expansion in the beginning and rate of expansion is zero at a later stage.

17. The Chaplygin gas model has uniform density and pressure.

18. The model is isotropize and shear-less in the Chaplygin gas case. The isotropize of this model also supports the zero value of ratio σ/θ. This Chaplygin gas model has decelerating expansion and there is no accelerating expansion in the model.

6. Conclusion

Three different Bianchi type-I cosmological models corresponding to perfect fluid, with quintessence and with Chaplygin gas, in Bimetric theory of gravitation have been deduced. The perfect fluid model has hyperbolic geometry and all its physical parameters are also hyperbolic in nature and therefore they have been studied from a hyperbolic geometric view point. The dark energy quintessence model, for $\omega_q = -1/3$ is contracting in nature and for $\omega_q = -1$, the model is expanding. The chaplygin gas model has volumetric power law expansion. All these models are isotropize and shear-less. Other geometrical and physical behaviors of the models have been studied. The hyperbolic geometric view point of the models will be helpful to the people of observational data to search such type of geometry.

It is interesting to extend this work for spherical symmetric, cylindrical symmetric, planer symmetric as well as restricted class of a axial stellar system, in our future plan.

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REFERENCES

Ade P. A. R., Aghanim, N., Armitage-Caplan, C., Arnaud M., (2014). Planck 2013 results. XVI. Cosmological parameters, Astronomy and Astrophysics, Vol. 571, pp. A16

Adelman-McCarthy, J. K. Agüeros, M. A., Allam, S. S., Anderson, K. S. J., Anderson, S. F., Annis, J., Bahcall, N. A., Baldry, I. K., Barentine, J. C., Berlind, A., Bernardi, M., Blanton, M. R., Boroski, W. N., Brewington, H. J., Brinchmann, J., Brinkmann, J., Brunner, R.J., Budavári, T., Carey, L.N., Carr, M. A., Castander, F.J., Connolly, A. J., Csabai, I., Czarapata, P.C., Dalcanton, J.J., Doi, M., Dong, F., Eisenstein, D.J., Evans, M.L.,Fan, X., Finkbeiner, D.P., Friedman, S.D., Frieman, J.A., Fukugita, M., Gillespie, B., Glazebrook, K., Gray, J., Grebel, E.K., Gunn, J.E., Gurbani, V.K., de Haas, E., Hall, P. B., Harvanek, M., Hawley, S.L., Hayes, J., Hendry, John S., Hennessy, G.S., Hindsley, R.B., Hirata, C.M., Hogan, C. J., Hogg, D.W., Holmgren, D. J., Holtzman, J.A., Ichikawa, S., Ivezić, Ž., Jester, S., Johnston, D.E., Jorgensen, A.M., Jurić, M., Kent, S.M., Kleinman, S. J., Knapp, G. R., Kniazev, A. Y., Kron, R.G., Krzesinski, J., Kuropatkin, N., Lamb, D.Q., Lampeitl, H., Lee, B.C., Leger, R. F., Lin, H., Long, D.C., Loveday, J., Lupton, R.H., Margon, B., Vanden, B., Daniel, E., Vandenberg, Jan, Vogeley, M.S., Vogt, N. P., Walkowicz, L. M., Weinberg, D.H., West, A.A., White, S.D.M., Xu, Y., Yanny, B., Yocum, D. R., York, D.G., Zehavi, I., Zibetti, S., Zucker, D.B. (2006). The fourth data release of the Sloan Digital Sky Survey, The Astrophysical Journal Supplement Series, Vol. 162, pp. 38.

Astier, P., Guy, J. Regnault, N., Pain, R., Aubourg, E., Balam, D., Basa, S., Carlberg, R. G., Fabbro, S., Fouchez, D., Hook, I. M., Howell, D. A. , Lafoux, H. , Neill, J. D. , Palanque-Delabrouille, N. , Perrett, K. ,Pritchet, C. J. , Rich, J. ,Sullivan, M. , Taillet, R. , Aldering, G. , Antilogus, P.; Arsenijevic, V.; Balland, C.; Baumont, S.; Bronder, J.;

Courtois, H. , Ellis, R. S. , Filiol, M., Gonçalves, A. C. , Goobar, A. , Guide, D. , Hardin, D. , Lusset, V., Lidman, C., McMahon, R. , Mouchet, M., Mourao, A. , Perlmutter, S., Ripoche, P., Tao, C., Walton, N. (2006). The supernovae legacy survey: measurement of Ω_M, Ω_Λ and ω from the first year data set, Astronomy and Astrophysics, Vol. 447, pp. 31.

Akarsu, O. and Kilinc, C. B. (2010). LRS bianchi type I models with anisotropic dark energy and constant deceleration parameter, General Relativity and Gravitation, Vol. 42, No.1, pp. 119.

Akarsu, O. and Kilinc, C. B. (2010). Bianchi type III models with anisotropic dark energy, General Relativity and Gravitation, Vol. 42, No. 4, pp. 763.

Akarsu, O. and Kilinc, C.B. (2010). De-sitter expansion with anisotropic fluid in bianchi type I space time, Astrophysics and Space Science, Vol. 326, No. 2, pp. 315.

Bali, R. and Dave, S. (2003). Bianchi type-IX string cosmological models with bulk viscous fluid in general relativity, Astrophysics and Space Science, Vol. 288, pp. 503.

Bali, R. and Upadhaya, R.D. (2003). LRS bianchi type I string dust magnetized cosmological models, Astrophysics and Space Science, Vol. 283, pp. 97.

Bali, R. and Singh, D. (2005). Bianchi type-V bulk viscous fluid string dust cosmological model in general relativity, Astrophysics and Space Science, Vol. 300, pp. 387.

Bali, R. and Anjali (2006). Bianchi type I magnetized string cosmological model in general relativity, Astrophysics and Space Science, Vol. 302, pp. 201.

Bali, R. and Pareek, U. K. (2007). Bianchi type I string dust cosmological model with magnetic field in general relativity, Astrophysics and Space Science, Vol. 312, pp. 305.

Bennett, C. L., Hill R. S., Hinshaw G., Nolta M. R., Odegard N., Page L., Spergel D. N., Weiland J. L., Wright E. L., Halpern M., Jarosik N., Kogut A., Limon M., Meyer S. S., Tucker G. S. and Wollack E., (2003). First year Wilkinson anisotropy probe observations: Foreground Emission, The Astrophysical Journal Supplement Series, Vol. 148, pp. 97.

Borkar, M. S. and Charjan, S. S. (2010). Bianchi type I magnetized cosmological model in bimetric theory of gravitation, Application and Applied Mathematics (AAM), Vol. 5, No. 2, pp. 563.

Borkar, M. S. and Charjan, S. S. (2010). Bianchi type I bulk viscous fluid string dust cosmological model with magnetic field in bimetric theory of gravitation, Applications and Applied Mathematics (AAM), Vol. 5, No. 9, pp. 96.

Gaikwad, N. P. and Borkar, M. S. (2011). Bianchi type I massive string magnetized baratropic perfect fluid cosmological model in bimetric theory of gravitation, Chinese Physics Letter, Vol. 28, N. 8, pp. 089803.

Borkar, M. S., Charjan, S. S. and Lepse, V. V. (2013). Evaluation of bianchi type VI_0 cosmological models with a binary mixture of perfect fluid and dark energy in bimetric theory of gravitation, International Journal of Applied Mathematics, Vol. 28, No. 2, pp. 2051.

Borkar, M. S. and Gayakwad, P. V. (2014). Evaluation of Schwarz child's exterior and interior solutions in bimetric theory of gravitation, Applications and Applied Mathematics (AAM), Vol. 9, No. 1, pp. 260.

Borkar, M. S. and Gayakwad, P. V. (2014). The measurements of Ω_M, Ω_Λ and ω in bianchi type I dark energy cosmological model with constant deceleration parameter in bimetric theory of gravitation, International Organization for Scientific Research-Journal of Mathematics (IOSR-JM), Vol. 10, No. 3, pp. 28.

Caldwell, R. R, Dave, R. and Steinhardt P. J. (1998). Cosmological imprint of an energy component with general equation of state, Physical Review Letters, Vol. 80, pp. 1582.

Caldwell, R. R. (2002). A Phantom menace cosmological consequences of a dark energy component with super negative equation of state, Physics Letter B, Vol. 545, No. 1-2, pp. 23.

Caldwell, R. R., Kamionkowski, M. and Weinberg, N. N. (2003). Phantom energy and doomsday, Physical Review Letters, Vol. 91, pp. 071301.

Das, A., Gupta, S., Saini, T. D. and Kar S. (2005). Cosmology with decaying tachyon matter, Physical Review D, Vol. 72, pp. 043528.

Eisenstein, D. J. Zehavi, I., Hogg, D. W., Scoccimarro, R. (2005). Detection of acoustic peak in the large scale correlation function of SDSS luminous red galaxies, The Astrophysical Journal, Vol. 633, pp. 560.

Feng, B., Li, M., Piao, Y.-S. and Zhang, X. (2003). Oscillating quintom and the recurrent universe, Physics Letter B, Vol. 634, No. 2-3, pp. 101.

Feng, B., Wang, X. L. and Zhang, X. M. (2005). Dark energy constraints from age and supernova, Physics Letter B, Vol. 607, No. 1-2, pp. 35.

Guo, Z. K., Piao, Y-S., Zhang, X. and Zhang, Y-Z. (2005). Cosmological evolution of a quintom model of dark energy, Physics Letter B, Vol. 608, No. 3-4, pp. 177.

Israelit, M. (1981). Spherically symmetric fields in Rosen's bimetric theories of gravitation, General Relativity and Gravitation, Vol. 13, No.7, pp. 681.

Jimenez, R. (2003). The value of the equation of state of dark energy, New Astronomy Reviews, Vol. 47, pp. 761.

Katore, S. D. and Rane, R. S. (2006). Magnetized cosmological models in bimetric theory of gravitation, Pramana Journal of Physics, Vol. 67, No. 2, pp. 227.

Katore, S. D., Kapse, D. V. and Tayade, G. B. (2011). Bianchi type vi_0 cosmological models with perfect fluid and dark energy, International Journal of Theoretical Physics, Vol. 50, No. 11, pp. 3299.

Khadekar, G. S. and Tade, S. D. (2007). String cosmological model in five dimensional in bimetric theory of gravitation, Astrophysics and Space Science, Vol. 310, pp. 47.

Kamenshchik, A., Moschella, U. and Pasquier, V. (2001). An alternative to quintessence, Physics Letters B, Vol. 511, No. 2-4, pp. 265.

Karade, T. M. (1980). Spherically symmetric space-times in bimetric theory of gravitation, Indian Journal Pure Applied Mathematics, Vol. 11, No. 9, pp. 1202.

Liddle, A. R. and Scherrer, R. J. (1998). Classification of scalar field potentials with cosmological scaling solutions, Physical Review D, Vol. 59, pp. 023509.

Li, M.-Z., Feng, B. and Zhang, X.-M. (2005). A single field model of dark energy with equation of state crossing-1, Journal of Cosmology and Astroparticle Physics, Vol. 12, pp. 002.

Mohanty, G. and Sahoo P. K. (2002). Some Bianchi type cosmological models in bimetric theory, Czechoslovak Journal of Physics, Vol. 52, pp. 1041.

Mukhopadhyay, U., Ghosh, P. P. and Choudhury, S. B. D. (2008). $\Lambda -$CDM Universe: A phenomenological approach with many possibilities, International Journal of Modern Physics D, Vol. 17, pp. 301.

Nojiri, S. and Odintsov, S. D. (2003). Quantum de-sitter cosmology and phantom matter, Physics Letter B, Vol. 562, No. 3, pp. 147.

Onemli, V. K. and Woodard, R. P. (2004). Quantum effects can render $w < -1$ on cosmological scales, Physical Review D, Vol. 70, pp. 107301.

Pradhan, A., Amirhashchi, H. and Jaiswal, R. (2011). A new class of LRS bianchi type-II dark energy models with variable EoS parameter, Astrophysics and Space Science, Vol. 334, No. 2, pp. 249.

Pradhan, A., Amirhashchi, H. and Saha, B. (2011). Bianchi type-I anisotropic dark energy model with constant deceleration parameter, International Journal of Theoretical Physics, Vol. 50, No. 9, pp. 2923.

Pradhan, A. and Amirhashchi, H. (2011). Dark energy model in anisotropic bianchi type-III space-time with variable EOS parameter, Astrophysics and Space Science, Vol. 332, pp. 441.

Pradhan, A. and Singh, A. K. (2011). Anisotropic bianchi type-I string cosmological models in normal gauge for Lyra's manifold with constant deceleration parameter, International Journal of Theoretical Physics, Vol. 50, No. 3, pp. 916.

Ratra, B. and Peebles, J. (1988). Cosmological consequences of a rolling homogeneous scalar field, Physical Review D, Vol. 37, pp. 3406.

Rahaman, F., Bhui, B. and Bhui, B.C. (2006). Cosmological model with a viscous fluid in a Kaluza-Klein metric, Astrophysics and Space Science, Vol. 301, No. 1-4, pp. 47.

Rahaman, F., Kalam, M., Sarker, M. and Chakraborty, S. (2009). Wormholes with varying equation of state parameter, Acta Physica Polonica B, Vol. 40, No. 1, pp. 25.

Ray, S., Rahaman, F., Mukhopadhyay, U., Sarkar, R. (2011). Variable equation of state for generalized dark energy model, International Journal of theoretical Physics, Vol. 50, No.1-2, pp. 2687.

Reddy, D. R. K. and Venkateswarlu R. (1989). Non-existence of Bianchi type-I perfect fluid cosmological models in a bimetric theory of gravitation, Astrophysics and Space Science,Vol. 158, pp. 169.

Reddy, D.R.K. and Venkateshwara Rao, N. (1998). On some Bianchi type cosmological models in a bimetric theory of gravitation, Astrophysics and Space Science, Vol. 257, pp. 293.

Riess, A. G., Strolger, L-G., Tonry, J., et al. (16 more) (2004). Type I supernovae discoveries at z >1 from the Hubble space telescope: Evidence for past deceleration parameter and constraints on dark energy evolution, The Astrophysical Journal Supplement Series, Vol. 607, pp. 665.

Rosen, N. (1973). A bi-metric theory of gravitation, General Relativity and Gravitation, Vol. 4, No. 6, pp. 435.

Rosen, N. (1975). A bi-metric theory of gravitation II, General Relativity and Gravitation, Vol. 6, No. 3, pp. 259.

Saha, B. (2005). Anisotropic cosmological models with perfect fluid and dark energy, Chinese Journal of Physics, Vol. 43, No. 6, pp. 1035.

Saha, B. (2006). Anisotropic cosmological models with a perfect fluid and a Λ term, Astrophysics and Space Science, Vol. 302, No. 1-4, pp. 83.

Sadeghi, J., Setare, M. R., Banijamali, A. and Milani, F. (2008). Non-minimally coupled quintom model inspired by string theory, Physics Letters B, Vol. 662, No. 2, pp. 92.

Setare, M. R. (2007). Interacting generalized chaplygin gas model in non-flat universe, The European Physical Journal, Vol. C52, pp. 689.

Setare, M. R., Sadeghi, J. and Amani, A. R. (2008). Shape invariance method for quintom model in the bent brane background, Physics Letters B, Vol. 660, No. 4, pp. 299.

Setare, M. R. and Saridakis, E. N. (2008). Coupled oscillators as models of quintom dark energy, Physics Letters B, Vol. 668, No. 3, pp. 177.

Setare, M. R. and Saridakis, E. N. (2008). The quintom model with O(N) symmetry, Journal of Cosmology and Astroparticle Physics, Vol. 09, pp. 026.

Setare, M. R., Sadeghi, J. and Amani, A. R. (2008). From inflation to acceleration, with phantom and canonical scalar fields in non-flat universe, Physics Letters B, Vol. 666, No. 4, pp. 288.

Setare, M. R. and Saridakis, E. N. (2009). Quintom cosmology with general potentials, International Journal of modern Physics D, Vol. 18, pp. 549.

Setare, M. R. (2009). Holographic Chaplygin DGP cosmologies, International Journal of Modern Physics D, Vol. 18, No. 3, pp. 419.

Seljak, U., Makarov A., Mcdonald P, Anderson S., Bahcall N. A., Brinkmann J. (2005). Cosmological parameter analysis including SDSS Ly α forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass, and dark energy, Physical Review D, Vol. 71, P. 103515.

Sharif, M. and Zubair, M. (2010). Effects of electromagnetic field on the dynamics of bianchi type VI_0 universe with anisotropic dark energy, International Journal of Modern Physics D, Vol. 19, pp. 1957.

Sharif, M. and Zubair, M. (2010). Dynamics of bianchi I universe with magnetized anisotropic dark energy, Astrophysics and Space Science, Vol. 330, No. 2, pp. 399.

Sharif, M. and Bhutti, M. Z. U. H. (2013). Effects of Electromagnetic Field on Shearfree Spherical Collapse, Astrophys. Space Sci., Vol 347, pp. 337.

Sharif, M. and Yousaf, Z. (2014). Cylindrical Thin-shell Wormholes in f (R) gravity, arXiv:1410.4757.

Sharif, M. and Yousaf, Z (2014) Instability of Dissipative Restricted Non-Static Axial Collapse with Shear Viscosity in f(R) Gravity, J. Cosmol. Astropart. Phys., Vol. 06, pp. 019.

Sharif, M. and Yousaf, Z (2014). Electromagnetic Field and Dynamical Instability Of Collapse with CDTT Model, Astropart. Phys., Vol 56, pp. 19.

Sharif, M. and Yousaf, Z (2014). Energy density inhomogeneities with polynomial f(R) cosmology, Astrophys.Space Sci., Vol 352 pp. 321.

Sharif, M. and Yousaf, Z (2014). Energy Density Inhomogeneities in Charged Radiating Stars with Generalized CDTT Model, Astrophysics and Space Science, vol. 354, pp. 431.

Sharif, M. and Yousaf, Z (2015). Stability of regular energy density in Palatini f(R) gravity, Eur. Phys. J. C, Vol. 75, pp. 58.

Singh, T. and Chaubey, R. (2009). Bianchi type-V cosmological models with perfect fluid and dark energy, Astrophysics and Space Science, Vol. 319, No. 2-4, pp. 149.

Spergel, D.N., Verde L. and Peiris H. V. (2003). First year Wilkinson microwave anisotropy probe (WMAP) Observations: Determination of Cosmological Parameters, The Astrophysical Supplement Series, Vol. 148, pp. 175.

Spergel, D. N., Bean, R., Doré, O., Nolta, M. R., Bennett, C. L., Dunkley, J., Hinshaw, G., Jarosik, N. , Komatsu, E. , Page, L. , Peiris, H. V., Verde, L., Halpern, M., Hill, R. S., Kogut, A., Limon, M., Meyer, S. S., Odegard, N., Tucker, G. S., Weiland, J. L., Wollack, E., Wright, E. L., (2007), The three year Wilkinson microwave anisotropy probe observations: implications for cosmology, The Astrophysical Journal Supplement Series, Vol. 170, pp. 377.

Steinhardt, P.J., Wang, L. and Zlatev, I. (1999). Cosmological Tracking Solutions, Physical Review D,Vol. 59, pp. 123504.

Tegmark, M, Strauss, Michael A. , Blanton, M R. , Abazajian, K, Dodelson, S, Sandvik, H, Wang, X, Weinberg, D H. , Zehavi, I, Bahcall, N A., Hoyle, F, Schlegel, D, Scoccimarro, R, Vogeley, M S. , Berlind, A, Budavari, T, Connolly, A, Eisenstein, D. J., Finkbeiner, D, Frieman, J A., Hui, L, Jain, B, Johnston, D, Kent, Stephen, L, Huan, Nakajima, R, Nichol, R. C., Ostriker, J. P., Pope, A, Scranton, R. Seljak,U, Sheth, R, Stebbins, A, Szalay, A.S., Szapudi, I., Xu, Y., Annis, J., Brinkmann, J., Burles, S., Castander, F.J., Csabai, I., Loveday, J., Doi, M., Fukugita, M., Gillespie, B., Hennessy, G., Hogg, D. W., Ivezić, Ž., Knapp, G., Lamb, D.Q., Lee, B.C., Lupton, R. H., McKay, T.A., Kunszt, P., Munn, J.A., O'Connell, L.P., John, Pier, J.R., Richmond, M., Rockosi, C., Schneider, D.P., Stoughton, C., Tucker, D.L., Vanden, B., Daniel E., Yanny, B., York, D.G., (2004), Cosmological Parameters from SDSS and WMAP, Physical Review D, Vol. 69, P. 103501.

Turner, M. S. and White, M. (1997). CDM models with a smooth component, Physical Review D, Vol. 56, pp. R4439.

Ungar, A. A. (2009). Einstein's Special Relativity: The hyperbolic Geometric view point, conference on mathematics, Physics and Philosophy on the Interpretations of Relativity,II, Budapest.

Wetterich, C. (1988). Cosmology and Fate of Dilation Symmetry, Nuclear Physics B, Vol. 302, No. 4, pp. 668.

Yadav, A. K., Rahaman, F. and Ray, S. (2011). Dark energy models with variable equation of state parameter, International Journal of Theoretical Physics, Vol. 50, No. 3, pp. 871.

Yadav, A. K. and Yadav, L. (2011). Bianchi type III anisotropic dark energy models with constant deceleration parameter, International Journal of Theoretical Physics, Vol. 50, No. 1, pp. 218.

Zhao, W. and Zhang, Y. (2006). Quintom models with an equation of state crossing -1, Physical Review D, Vol. 73, pp. 123509.

Zlatev, I., Wang, L. M. and Steinhardt, P.J. (1999). Quintessence, cosmic coincidence, and the cosmological constant, Physics Review Letters, Vol. 82, pp. 896.