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Existence of Mild Solutions for Semilinear Impulsive Functional Mixed Integro-differential Equations with Nonlocal Conditions

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Abstract

In this paper, we prove the existence, uniqueness and continuous dependence of initial data on mild solutions of first order semilinear functional impulsive mixed integro-differential equations with nonlocal condition in general Banach spaces. The results are obtained by using the semigroup theory and Banach contraction theorem.

Keywords: Mild Solution; Nonlocal condition; Impulsive condition; Semigroup theory; Semilinear functional mixed integro-differential equation; Banach fixed point theorem

MSC 2010 No.: 45N05, 47H10, 47B38, 34A37

1. Introduction

Many evolution processes are characterized by the fact that at certain moments of time they experience an abrupt change of state. These processes are subject to short-term perturbations whose duration is negligible in comparison to the duration of the process. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulse. It is known, for example, that many biological phenomena involving thresholds, burning rhythm models in medicine and biological, optimal control models in economics, pharmacokinetics and frequency modulated systems, do exhibit impulsive effects. Thus, impulsive differential equations, that is, differential equations involving the impulsive effect, appear as a natural description of observed evolution phenomena of several real world problems. For more details on this theory and applications, see the monograph of Lakshmikantham et al. (1989), Perestyuk et al. (2011), Bainov and Simeonov (1989), and the papers of Akca et al. (1998), Ji et al. (2010), Liang et al. (2009), Belarbi et al. (2014).

Most of the practical systems in nature are generally integro-differential equations. So, the study of integro-differential equations is very important. Integro-differential equations with impulsive conditions have been studied by Balachandran et al. (2009), Ravichandran et al. (2013), and Yan (2011). The problems of existence, uniqueness and other qualitative problems of semilinear differential equations in Banach space has been studied extensively in the literature, see for instance, Akca, et al. (1998), Byszewski, et al. (1991, 1997, 1998), Pazy (1983). On the other hand, the nonlocal initial value problem was first studied by Byszewski (1992), where the existence, uniqueness and continuous dependence of a mild solution of a semilinear functional differential equation were discussed. Then it has been extensively studied by many authors, see for example, Balachandran et al. (1996), Lin et al. (1996).

Akca et al. (2002) established the existence, uniqueness and continuous dependence of a mild solution of an impulsive functional differential evolution nonlocal Cauchy problem of the form

$$\begin{aligned}
&\dot{u}(t) = Au(t) + f(t,u_t),\ t \in (0,a], && t \neq \tau_k,\\
&u(\tau_k + 0) = Q_k u(\tau_k) \equiv u(\tau_k) + I_k u(\tau_k), && k = 1,2,\ldots,\kappa,\\
&u(t) + \left(g\left(u_{t_1}, \ldots, u_{t_p}\right)\right)(t) = \phi(t), && t \in [-r,0],
\end{aligned}$$

where $0 < t_1 < \ldots < t_p \le a,\ p \in \mathbb{N},\ A$ and $I_k (k = 1,2,\ldots,\kappa)$ are linear operators acting in a Banach space E; f, g and ϕ are given functions satisfying some assumptions, $u_t(s) := u(t+s)$ for $t \in [0,a],\ s \in [-r,0],\ I_k u(\tau_k) = u(\tau_k + 0) - u(\tau_k - 0)$ and the impulsive moments τ_k are such that $0 < \tau_1 < \ldots < \tau_k < \ldots,\quad \tau_\kappa < a,\ \kappa \in \mathbb{N}.$

Recently, Machado et al. (2013) studied a class of abstract impulsive mixed-type functional integro-differential equations with finite delay in a Banach space of the form

$$\begin{aligned}
&x'(t) = A(t)x(t) + f\left(t, x_t, \int_0^t h(t,s,x_s)\,ds, \int_0^b k(t,s,x_s)\,ds\right) + (Bu)(t),\\
&t \in J = [0,b],\ t \neq t_i,\ i = 1,2,\ldots,s,\ \Delta x|_{t_i} = I_i\left(x(t_i)\right), \quad i = 1,2,3,\ldots,s,\\
&x(0) = \phi + g(x), \quad t \in [-r,0],
\end{aligned}$$

by using the Mönch fixed point theorem via measures of non-compactness and semigroup theory.

Ravichandran et al. (2011) proved the existence and uniqueness of mild solutions for a class of impulsive fractional integro-differential equations of the form

$$D_*^\alpha x(t) = Ax(t) + f\left(t, x(t), \int_0^t a(t,s,x(s))\,ds, \int_0^T b(t,s,x(s))\,ds\right),$$
$$t \in I = [0,T],\ t \neq t_k,\ k = 1,2,\ldots,m,\ x(0) = x_0 \in \mathbb{X},\ Delta\ x|_{t=t_k} = I_k\left(x\left(t_k^-\right)\right),\ k = 1,2,\ldots,m,$$

assuming that A is a sectorial operator on a Banach space $\mathbb{X}$ by means of Banach contraction principle and Leray-Schauder's alternative fixed point theorem. In the present paper, we study the nonlocal semilinear functional Volterra-Fredholm type of differential equations with impulses of the form

$$u'(t) = Au(t) + f\left(t, u_t, \int_0^t h(t,s,u_s)\,ds, \int_0^a k(t,s,u_s)\,ds\right), \tag{1}$$
$$t = (0,a],\ t \neq \tau_k,\ k = 1,2,3,\ ...,\ m,$$

$$u(t) + \left[g\left(u_{t_1}, \ldots, u_{t_p}\right)\right](t) = \phi(t), \qquad t \in [-r,0], \tag{2}$$

$$u(\tau_k + 0) = Q_k u(\tau_k) \equiv u(\tau_k) + I_k u(\tau_k), \qquad k = 1,2,3,\ \ldots,\ m, \tag{3}$$

where $0 < t_1 < \ldots < t_p \leq a$ $(p \in \mathbb{N})$, A is the infinitesimal generator of a C_0 semigroup of bounded linear operators $\{T(t), t \geq 0\}$ and $I_k\,(k = 1,2,\ldots,m)$ are the linear operators acting in a Banach space E; $\phi \in C([-r,0],E)$ and f,h,k,g are given functions satisfying some assumptions, $I_k u(\tau_k) = u(\tau_k + 0) - u(\tau_k - 0)$ and the impulsive moments τ_k are such that $0 \leq \tau_0 < \tau_1 < \ldots, \tau_m < \tau_{m+1} \leq a, m \in \mathbb{N}$.

Motivated by the above mentioned discussed and the work of Balachandran et al. (2001) and Ravichandran et al. (2013), we study the existence, uniqueness and continuous dependence of mild solution of nonlocal problem for an impulsive functional Volterra-Fredholm type of integro-differential equations. The results are obtained by using the semigroup theory and Banach contraction theorem. In this paper we generalize and extend the results of Akca et al. (2002, 2013) and Ji (2010). As usual, in the theory of impulsive differential equations, see for example, Lakshmikantham et al. (1989) and Samoilenko et al. (1995), at the points of discontinuity τ_i of the solution $t \mapsto u(t)$, we assume that $u(\tau_i) \equiv u(\tau_i - 0)$. It is clear that, in general, the derivatives $u'(\tau_i)$ do not exist. On the other hand, from (1), there exist the limits $u'(\tau_i \pm 0)$. According to the above convention, we assume $u'(\tau_i) = u'(\tau_i - 0)$.

2. Preliminaries

Throughout this work, $(E, \|.\|)$ is a Banach space, A is the infinitesimal generator of a C_o semigroup $\{T(t), t \geq 0\}$ on E, $D(A)$ is the domain of A, and $M = \operatorname{Sup}_{t\in[0,a]} \|T(t)\|_{B(E)}$. In this consequence the operator norm $\|.\|_{B(E)}$ will be denoted by $\|.\|$. Consider

$$J_o = [-r, 0],\ J = [0, a]$$

and

$$X = C([-r, 0] : E), 0 < r < \infty,\ Y = C([-r, a] : E),\ Z = C([0, a] : E).$$

For a continuous function $u : [-r, a] \to E$, we denote u_t a function belonging to X and defined by

$$u_t = u(t + s) \text{ for } t \in J,\ s \in J_0.$$

Let

$$f : J \times X \times X \times X \to E, h, k : J \times J \times X \to X \text{ and } \phi \in X.$$

To proceed, we need the following assumptions:

(A_1): For every $u, v, w \in Y$ and $t \in J$, $f(., u_t, v_t, w_t) \in Z$.

(A_2): There exists a constant $L > 0$ such that

$$\|f(t, x_t, y_t, z_t) - f(t, u_t, v_t, w_t)\| \leq L\left(\|x - u\|_{C([-r,t],E)} + \|y - v\|_{C([-r,t],E)} + \|z - w\|_{C([-r,t],E)}\right);$$
$$x, y, z, u, v, w \in Y, t \in J.$$

(A_3): There exists a constant $H > 0$ such that

$$\|h(t, s, u_s) - h(t, s, v_s)\| \leq H \|u - v\|_{C([-r,s],E)};\ u, v \in Y, s \in J.$$

(A_4): There exists a constant $K > 0$ such that

$$\left\| k\left(t,s,u_s\right) - k\left(t,s,v_s\right) \right\| \le K \left\| u - v \right\|_{C\left(\left[-r,s\right],E\right)}; \; u,v \in Y, s \in J .$$

(A_5): Let $g : X^p \to X$ such that there exists a constant $G \ge 0$ satisfying

$$\left\| \left(g\left(u_{t_1}, \ldots, u_{t_p}\right)\right)(t) - \left(g\left(v_{t_1}, \ldots, v_{t_p}\right)\right)(t) \right\| \le G \left\| u - v \right\|_{C\left(\left[-r,a\right],E\right)}; \; u,v \in Y, t \in J_0 .$$

(A_6): There exists a constant $L_k > 0$ such that

$$\left\| I_k\left(v\right)\right\| \le L_k \left\| v \right\|, v \in E, k = 1, 2, \ldots, m ,$$

(A_7): $MG + MLa\left(1 + Ha + Ka\right) + M \sum_{0<\tau_k<t} L_k < 1$.

A function $u \in C\left(\left[-r,a\right],E\right)$ satisfying the equations

$$u(t) = T(t)\phi(0) - T(t)\left[\left(g\left(u_{t_1}, \ldots, u_{t_p}\right)\right)(0)\right] + \int_0^t T(t-s)\, f\left(s, u_s, \int_0^s h\left(s,\xi,u_\xi\right) d\xi, \int_0^a k\left(s,\xi,u_\xi\right) d\xi\right) ds + \sum_{0<\tau_k<t} T\left(t-\tau_k\right) I_k u\left(\tau_k\right); \quad t \in [0,a], \tag{4}$$

and

$$u(t) + \left[g\left(u_{t_1}, \ldots, u_{t_p}\right)\right](t) = \phi(t), t \in [-r,0], \tag{5}$$

is said to be the mild solution of problem $(1)-(3)$.

The following inequality will be useful while proving our result:

Lemma 1. (Perestyuk et al. (2011), p.11)

Suppose that a nonnegative piecewise-continuous function $u(t)$ satisfies the following inequality for $t \ge t_0$:

$$u(t) \le C + \int_{t_0}^t \gamma u(s)\, ds + \sum_{t_0<\tau_i<t} \beta u\left(\tau_i\right),$$

where $C \geq 0, \beta \geq 0, \gamma > 0$, and τ_i are the point of discontinuity of the first kind of the function $u(t)$. Then the following estimates hold for the function $u(t)$:

$$u(t) \leq C(1+\beta)^{i(t_0,t)} e^{\gamma(t-t_0)},$$

where $i(t_0,t)$ is the number of points τ_i on the interval $[t_0,t)$.

3. Existence of Mild Solution

Theorem 1.

Suppose that the assumptions $(A_1)-(A_7)$ are satisfied and $q<1$, where

$$q = MG + MLa(1+Ha+Ka) + M\sum_{0<\tau_k<t} L_k .$$

Then, the impulsive nonlocal Cauchy problem $(1)-(3)$ has a unique mild solution.

***Proof**:*

Define an operator F on the Banach space Y by the formula

$$(Fu)(t) = \begin{cases} \phi(t) - \left[g\left(u_{t_1},\ldots,u_{t_p}\right)\right](t),\ t \in [-r,0], \\ T(t)\phi(0) - T(t)\left[g\left(u_{t_1},\ldots,u_{t_p}\right)\right](0) \\ \qquad + \int_0^t T(t-s) f\left(s, u_s, \int_0^s h(s,\xi,u_\xi)d\xi, \int_0^a k(s,\xi,u_\xi)d\xi\right)ds \\ \qquad + \sum_{0<\tau_k<t} T(t-\tau_k) I_k u(\tau_k),\ t \in [0,a], \end{cases} \tag{6}$$

where $u \in Y$. It is easy to see that F maps Y into itself. Now, we will show that F is contraction on Y. Consider

$$(Fu)(t) - (Fv)(t) = \left[g\left(v_{t_1},\ldots,v_{t_p}\right)\right](t) - \left[g\left(u_{t_1},\ldots,u_{t_p}\right)\right](t);\ \ u,v \in Y; t \in [-r,0) \tag{7}$$

and

$$(Fu)(t) - (Fv)(t) = T(t)\left[\left(g\left(v_{t_1},\ldots,v_{t_p}\right)\right)(0) - \left(g\left(u_{t_1},\ldots,u_{t_p}\right)\right)(0)\right]$$

$$+\int_0^t T(t-s)\Bigg[f\left(s,u_s,\int_0^s h\left(s,\xi,u_\xi\right)d\xi,\int_0^a k\left(s,\xi,u_\xi\right)d\xi\right)$$
$$-f\left(s,v_s,\int_0^s h\left(s,\xi,v_\xi\right)d\xi,\int_0^a k\left(s,\xi,v_\xi\right)d\xi\right)\Bigg]ds$$
$$+\sum_{0<\tau_k<t} T(t-\tau_k)\left[I_k u(\tau_k)-I_k v(\tau_k)\right]; \qquad u,v\in Y, t\in J. \tag{8}$$

From (7) and (A_5), we have

$$\left\|(Fu)(t)-(Fv)(t)\right\| \le G\left\|u-v\right\|_Y;\ \ u,v\in Y, t\in J_0. \tag{9}$$

Moreover, by (8), and $(A_2)-(A_7)$,

$$\left\|(Fu)(t)-(Fv)(t)\right\| \le \Bigg\|T(t)\left[\left(g\left(v_{t_1},\ldots,v_{t_p}\right)\right)(0)-\left(g\left(u_{t_1},\ldots,u_{t_p}\right)\right)(0)\right]$$
$$+\int_0^t T(t-s)\Bigg[f\left(s,u_s,\int_0^s h\left(s,\xi,u_\xi\right)d\xi,\int_0^a k\left(s,\xi,u_\xi\right)d\xi\right)$$
$$-f\left(s,v_s,\int_0^s h\left(s,\xi,v_\xi\right)d\xi,\int_0^a k\left(s,\xi,v_\xi\right)d\xi\right)\Bigg]ds$$
$$+\sum_{0<\tau_k<t} T(t-\tau_k)\left[I_k u(\tau_k)-I_k v(\tau_k)\right]\Bigg\|, \text{ for } u,v\in Y, t\in J,$$

(10)

$$\le \left\|T(t)\right\|\left\|\left(g\left(v_{t_1},\ldots,v_{t_p}\right)\right)(0)-\left(g\left(u_{t_1},\ldots,u_{t_p}\right)\right)(0)\right\|$$
$$+\int_0^t \left\|T(t-s)\right\|\left\|\begin{array}{l} f\left(s,u_s,\int_0^s h\left(s,\xi,u_\xi\right)d\xi,\int_0^a k\left(s,\xi,u_\xi\right)d\xi\right) \\ \qquad -f\left(s,v_s,\int_0^s h\left(s,\xi,v_\xi\right)d\xi,\int_0^a k\left(s,\xi,v_\xi\right)d\xi\right)\end{array}\right\|ds$$
$$+\sum_{0<\tau_k<t}\left\|T(t-\tau_k)\right\|\left\|I_k u(\tau_k)-I_k v(\tau_k)\right\|,$$

$$\leq MG\|u-v\|_Y + ML\int_0^t \left[\begin{array}{l} \|u-v\|_{C([-r,s],E)} \\ \qquad + \int_0^s \left\| h\left(s,\xi,u_\xi\right) - h\left(s,\xi,v_\xi\right) \right\| d\xi \end{array} \right.$$

$$\left. + \int_0^a \left\| k\left(s,\xi,u_\xi\right) - k\left(s,\xi,v_\xi\right) \right\| d\xi \right] ds + \sum_{0<\tau_k<t} M \left\| I_k u\left(\tau_k\right) - I_k v\left(\tau_k\right) \right\|,$$

$$\leq MG\|u-v\|_Y$$

$$+ ML\int_0^t \left[\begin{array}{l} \|u-v\|_{C([-r,s],E)} + H\int_0^s \|u-v\|_{C([-r,\xi],E)}\, d\xi \\ \qquad + K\int_0^a \|u-v\|_{C([-r,\xi],E)}\, d\xi \end{array} \right] ds$$

$$+ \sum_{0<\tau_k<t} ML_k \left\| u\left(\tau_k\right) - v\left(\tau_k\right) \right\|,$$

$$\leq MG\|u-v\|_Y$$

$$+ ML\int_0^t \left[\|u-v\|_Y + H\|u-v\|_Y\, a + K\|u-v\|_Y\, a \right] ds$$

$$+ \sum_{0<\tau_k<t} ML_k \|u-v\|_Y,$$

$$\leq MG\|u-v\|_Y + ML\int_0^t \left(1+Ha+Ka\right)\|u-v\|_Y\, ds + \sum_{0<\tau_k<t} ML_k \|u-v\|_Y,$$

$$\leq MG\|u-v\|_Y + ML\left(1+Ha+Ka\right)a\|u-v\|_Y + M\sum_{0<\tau_k<t} L_k \|u-v\|_Y,$$

$$\leq \left[MG + ML\left(1+Ha+Ka\right)a + M\sum_{0<\tau_k<t} L_k \right] \|u-v\|_Y . \tag{11}$$

From equation $(9)-(11)$, we get

$$\left\|(Fu)-(Fv)\right\|_Y \leq q\|u-v\|_Y ; \quad u,v \in Y, \tag{12}$$

where

$$q = MG + ML\left(1+Ha+Ka\right)a + M\sum_{0<\tau_k<t} L_k .$$

Since, $q<1$, equation (12) shows that F is a contraction on Y. Consequently, the operator F satisfies all the assumptions of the Banach contraction theorem and therefore, in space Y there is only one fixed point of F and this is the mild solution of the nonlocal Cauchy problem with impulse effect. This completes the proof of the theorem.

4. Continuous Dependence of a Mild Solution

Theorem 2.

Assume that the functions f,g,h,k and $I_k(u), k=1,2,\ldots,m,$ satisfy the assumptions $(A_1)-(A_6)$ and $q<1$. Then, for each $\phi_1,\phi_2 \in Y$ and for the corresponding mild solutions u_1,u_2 of the problems,

$$u'(t) = Au(t) + f\left(t, u_t, \int_0^t h(t,s,u_s)\,ds, \int_0^a k(t,s,u_s)\,ds\right),\ t=(0,a], t \neq \tau_k, \tag{13}$$

$$u(\tau_k+0) = Q_k u(\tau_k) \equiv u(\tau_k) + I_k u(\tau_k), k=1,2,3,\ldots,m, \tag{14}$$

$$u(t) + \left[g\left(u_{t_1},\ldots,u_{t_p}\right)\right](t) = \phi_i(t), (i=1,2), t \in [-r,0], \tag{15}$$

the following inequality holds:

$$\|u_1-u_2\|_Y \leq \left[M\|\phi_1-\phi_2\|_X + M\left(G+LKa^2\right)\|u_1-u_2\|_Y\right] e^{aML(1+Ha)}\left(1+ML_k\right)^k. \tag{16}$$

Additionally, if

$$G+LKa^2 < \frac{e^{-aML(1+Ha)}\left(1+ML_k\right)^{-k}}{M}, \tag{17}$$

then

$$\|u_1-u_2\|_Y \leq \frac{Me^{aML(1+Ha)}\left(1+ML_k\right)^k}{\left[1-M\left(G+LKa^2\right)e^{aML(1+Ha)}\left(1+ML_k\right)^k\right]}\|\phi_1-\phi_2\|_X. \tag{18}$$

Proof:

Assume that $\phi_i \in X\ (i=1,2)$ are arbitrary functions and let $u_i\ (i=1,2)$ be the mild solution of problem (13) – (15). Then,

$$u_1(t)-u_2(t) = T(t)\left[\phi_1(0)-\phi_2(0)\right]$$

$$-T(t)\Big[\Big(g\big((u_1)_{t_1},\ldots,(u_1)_{t_p}\big)\Big)(0)-\Big(g\big((u_2)_{t_1},\ldots,(u_2)_{t_p}\big)\Big)(0)\Big]$$
$$+\int_0^t T(t-s)\Bigg[f\Bigg(s,(u_1)_s,\int_0^s h\big(s,\xi,(u_1)_\xi\big)d\xi,\int_0^a k\big(s,\xi,(u_1)_\xi\big)d\xi\Bigg)$$
$$-f\Bigg(s,(u_2)_s,\int_0^s h\big(s,\xi,(u_2)_\xi\big)d\xi,\int_0^a k\big(s,\xi,(u_2)_\xi\big)d\xi\Bigg)\Bigg]ds$$
$$+\sum_{0<\tau_k<t}T(t-\tau_k)\big(I_k u_1(\tau_k)-I_k u_2(\tau_k)\big);\qquad t\in J, \tag{19}$$

and

$$u_1(t)-u_2(t)=\big[\phi_1(t)-\phi_2(t)\big]$$
$$-\Big[\Big(g\big((u_2)_{t_1},\ldots,(u_2)_{t_p}\big)\Big)(t)-\Big(g\big((u_1)_{t_1},\ldots,(u_1)_{t_p}\big)\Big)(t)\Big];\quad t\in J_0. \tag{20}$$

From our assumptions, we get

$$\|u_1(\delta)-u_2(\delta)\|\le M\|\phi_1-\phi_2\|_X+MG\|u_1-u_2\|_Y$$
$$+ML\int_0^\delta\Bigg[\|u_1-u_2\|_{C([-r,s],E)}+\int_0^s\Big\|h\big(s,\xi,(u_1)_\xi\big)d\xi-h\big(s,\xi,(u_2)_\xi\big)\Big\|d\xi$$
$$+\int_0^a\Big\|k\big(s,\xi,(u_1)_\xi\big)d\xi-k\big(s,\xi,(u_2)_\xi\big)\Big\|d\xi\Bigg]ds$$
$$\le M\|\phi_1-\phi_2\|_X+MG\|u_1-u_2\|_Y$$
$$+ML\int_0^\delta\Bigg[\|u_1-u_2\|_{C([-r,s],E)}+H\int_0^s\|u_1-u_2\|_{C([-r,\xi],E)}\,d\xi$$
$$+K\int_0^a\|u_1-u_2\|_{C([-r,\xi],E)}\,d\xi\Bigg]ds+M\sum_{0<\tau_k<\xi}L_k\|u_1(\tau_k)-u_2(\tau_k)\|_E,$$
$$\le M\|\phi_1-\phi_2\|_X+MG\|u_1-u_2\|_Y+MLKa^2\|u_1-u_2\|_Y$$
$$+ML\int_0^\delta\Big[\|u_1-u_2\|_{C([-r,s],E)}+Ha\|u_1-u_2\|_{C([-r,s],E)}\Big]ds$$
$$+ML_k\sum_{0<\tau_k<\xi}\|u_1(\tau_k)-u_2(\tau_k)\|_E,$$
$$\le M\|\phi_1-\phi_2\|_X+M\left(G+LKa^2\right)\|u_1-u_2\|_Y$$

$$+ML\left(1+Ha\right)\int_0^t \left\|u_1-u_2\right\|_{C([-r,s],E)}\,ds$$
$$+ML_k\sum_{0<\tau_k<t}\left\|u_1\left(\tau_k\right)-u_2\left(\tau_k\right)\right\|_E;\ 0\le\xi\le s\le\delta\le t\le a\,.$$

With this result, and by virtue of (A_5) it follows that

$$\sup_{\delta\in[0,t]}\left\|u_1\left(\delta\right)-u_2\left(\delta\right)\right\|\le M\left\|\phi_1-\phi_2\right\|_X+M\left(G+LKa^2\right)\left\|u_1-u_2\right\|_Y$$
$$+ML\left(1+Ha\right)\int_0^t \left\|u_1-u_2\right\|_{C([-r,s],E)}\,ds$$
$$+ML_k\sum_{0<\tau_k<t}\left\|u_1\left(\tau_k\right)-u_2\left(\tau_k\right)\right\|_E;t\in J\,. \tag{21}$$

By hypothesis (A_5) and $\left(20\right)$ we have

$$\left\|u_1\left(t\right)-u_2\left(t\right)\right\|\le M\left\|\phi_1-\phi_2\right\|_X+MG\left\|u_1-u_2\right\|_Y;\ t\in\left[-r,0\right)\,. \tag{22}$$

Formula $\left(21\right)$ and $\left(22\right)$ imply that

$$\left\|u_1\left(t\right)-u_2\left(t\right)\right\|\le M\left\|\phi_1-\phi_2\right\|_X+M\left(G+LKa^2\right)\left\|u_1-u_2\right\|_Y$$
$$+ML\left(1+Ha\right)\int_0^t \left\|u_1-u_2\right\|_{C([-r,s],E)}\,ds$$
$$+ML_k\sum_{0<\tau_k<t}\left\|u_1\left(\tau_k\right)-u_2\left(\tau_k\right)\right\|_E,\qquad t\in J\,. \tag{23}$$

Applying Gronwall's inequality for discontinuous function (see Perestyuk et al. (2011)), from $\left(23\right)$ it follows that

$$\left\|u_1\left(t\right)-u_2\left(t\right)\right\|_Y\le\left[M\left\|\phi_1-\phi_2\right\|_X+M\left(G+LKa^2\right)\left\|u_1-u_2\right\|_Y\right]e^{aML(1+Ha)}\left(1+ML_k\right)^k, \tag{24}$$

and therefore, $\left(16\right)$ holds. Inequality $\left(18\right)$ is a consequence of $\left(16\right)$. This completes the proof of the theorem.

5. Conclusion

In this article, the existence, uniqueness and continuous dependence of initial data on a mild solution of semilinear functional impulsive mixed integro-differential equations with nonlocal condition in general Banach spaces are discussed. We apply the concepts of semigroup theory together with Banach contraction theorem to establish the results.

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