



6-2014

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Shantha K. Kumari

*P.A. College of Engineering*

Vasudevan T. M. Nambisan

*College of Engineering Trikaripur*

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### Recommended Citation

Kumari, Shantha K. and M. Nambisan, Vasudevan T. (2014). On some Summation Formulae for the I-Function of Two Variables, *Applications and Applied Mathematics: An International Journal (AAM)*, Vol. 9, Iss. 1, Article 23.

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## On some Summation Formulae for the I-Function of Two Variables

**Shantha K. Kumari**

Department of Mathematics  
P.A. College of Engineering  
Mangalore, India  
[skk\\_abh@rediffmail.com](mailto:skk_abh@rediffmail.com)

**Vasudevan T. M. Nambisan**

Department of Mathematics  
College of Engineering  
Trikaripur, India  
[tmvasudevannambisan@yahoo.com](mailto:tmvasudevannambisan@yahoo.com)

Received: January 18, 2014; Accepted: May 5, 2014

### Abstract

In this research paper, we aim to establish three interesting summation formulae for the I-function of two variables recently introduced in the literature. The results are derived with the help of classical summation theorems due to Watson, Dixon and Whipple. A few known results are also obtained as special cases of our main findings. Since the I-function of two variables is the most generalized function of two variables and it includes as special cases many of the known functions appearing in the literature, the results derived in this paper will therefore serve as the key formulas from which a large number of summation formulas including elementary functions can be obtained by specializing the parameters therein.

**Keywords:** I-function; Mellin-Barnes Contour integral; H-function;  
Summation theorems

**AMS MSC 2010 No.:** 33C20, 33C60

### 1. Introduction

The I-function of two variables defined and studied by Shantha Kumari et al. (2014) is represented by means of the double Mellin - Barnes Contour integral in the following manner.

$$\begin{aligned}
 & I [z_1, z_2] \\
 &= I \begin{matrix} 0, n_1: m_2, n_2; m_3, n_3 \\ p_1, q_1: p_2, q_2; p_3, q_3 \end{matrix} \left[ \begin{matrix} z_1 \\ z_2 \end{matrix} \left| \begin{matrix} (a_j, \alpha_j, A_j; \xi_j)_{1, p_1} : (c_j, C_j; U_j)_{1, p_2}; (e_j, E_j; P_j)_{1, p_3} \\ (b_j, \beta_j, B_j; \eta_j)_{1, q_1} : (d_j, D_j; V_j)_{1, q_2}; (f_j, F_j; Q_j)_{1, q_3} \end{matrix} \right. \right] \\
 &= \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \phi(s, t) \theta_1(s) \theta_2(t) z_1^s z_2^t ds dt, \tag{1.1}
 \end{aligned}$$

where  $\phi(s, t)$ ,  $\theta_1(s)$  and  $\theta_2(t)$  are given by

$$\phi(s, t) = \frac{\prod_{j=1}^{n_1} \Gamma^{\xi_j}(1-a_j + \alpha_j s + A_j t)}{\prod_{j=n_1+1}^{p_1} \Gamma^{\xi_j}(a_j - \alpha_j s - A_j t)} \frac{1}{\prod_{j=1}^{q_1} \Gamma^{\eta_j}(1-b_j + \beta_j s + B_j t)}, \tag{1.2}$$

$$\theta_1(s) = \frac{\prod_{j=1}^{n_2} \Gamma^{U_j}(1-c_j + C_j s)}{\prod_{j=n_2+1}^{p_2} \Gamma^{U_j}(c_j - C_j s)} \frac{\prod_{j=1}^{m_2} \Gamma^{V_j}(d_j - D_j s)}{\prod_{j=m_2+1}^{q_2} \Gamma^{V_j}(1-d_j + D_j s)}, \tag{1.3}$$

and

$$\theta_2(t) = \frac{\prod_{j=1}^{n_3} \Gamma^{P_j}(1-e_j + E_j t)}{\prod_{j=n_3+1}^{p_3} \Gamma^{P_j}(e_j - E_j t)} \frac{\prod_{j=1}^{m_3} \Gamma^{Q_j}(f_j - F_j t)}{\prod_{j=m_3+1}^{q_3} \Gamma^{Q_j}(1-f_j + F_j t)}. \tag{1.4}$$

Also:

- (i)  $z_1 \neq 0, z_2 \neq 0$ ;
- (ii)  $i = \sqrt{-1}$ ;
- (iii) an empty product is interpreted as unity;
- (iv) the parameters  $n_j, p_j, q_j (j = 1, 2, 3)$ ,  $m_j (j = 2, 3)$  are nonnegative integers such that  $0 \leq n_j \leq p_j (j = 1, 2, 3)$ ,  $q_1 \geq 0$ ,  $0 \leq m_j \leq q_j (j = 2, 3)$  (not all zero simultaneously);
- (v)  $\alpha_j, A_j (j = 1, \dots, p_1)$ ,  $\beta_j, B_j (j = 1, \dots, q_1)$ ,  $C_j (j = 1, \dots, p_2)$ ,  $D_j (j = 1, \dots, q_2)$ ,  $E_j (j = 1, \dots, p_3)$ ,  $F_j (j = 1, \dots, q_3)$  are assumed to be positive quantities for standardization purpose.
- (vi)  $a_j (j = 1, \dots, p_1)$ ,  $b_j (j = 1, \dots, q_1)$ ,  $c_j (j = 1, \dots, p_2)$ ,  $d_j (j = 1, \dots, q_2)$ ,  $e_j (j = 1, \dots, p_3)$  and  $f_j (j = 1, \dots, q_3)$  are complex numbers;
- (vii) The exponents  $\xi_j (j = 1, \dots, p)$ ,  $\eta_j (j = 1, \dots, q)$ ,  $U_j (j = 1, \dots, p_2)$ ,  $V_j (j = 1, \dots, q_2)$ ,  $P_j (j = 1, \dots, p_3)$ ,  $Q_j (j = 1, \dots, q_3)$  of various gamma functions involved in (1.2), (1.3) and (1.4) may take non-integer values; and

(viii)  $L_s$  and  $L_t$  are suitable contours of Mellin - Barnes type. Moreover, the contour  $L_s$  is in the complex s-plane and runs from  $\sigma_1 - i\infty$  to  $\sigma_1 + i\infty$ , ( $\sigma_1$  real) so that all the singularities of  $\Gamma^{V_j}(d_j - D_j s)(j = 1, \dots, m_2)$  lie to the right of  $L_s$  and all the singularities of  $\Gamma^{U_j}(1 - c_j + C_j s)(j = 1, \dots, n_2)$ ,  $\Gamma^{\xi_j}(1 - a_j + \alpha_j s + A_j t)(j = 1, \dots, n_1)$  lie to the left of  $L_s$ ; The other contour  $L_t$  follows similar conditions in the complex t-plane.

The function defined by (1.1) is an analytic function of  $z_1$  and  $z_2$  if

$$R = \sum_{j=1}^{p_1} \xi_j \alpha_j + \sum_{j=1}^{p_2} U_j C_j - \sum_{j=1}^{q_1} \eta_j \beta_j - \sum_{j=1}^{q_2} V_j D_j < 0, \tag{1.5}$$

$$S = \sum_{j=1}^{p_1} \xi_j A_j + \sum_{j=1}^{p_3} P_j E_j - \sum_{j=1}^{q_1} \eta_j B_j - \sum_{j=1}^{q_3} Q_j F_j < 0. \tag{1.6}$$

Further, the integral (1.1) is convergent if

$$\Delta_1 = \left[ \sum_{j=1}^{n_1} \xi_j \alpha_j - \sum_{j=n_1+1}^{p_1} \xi_j \alpha_j - \sum_{j=1}^{q_1} \eta_j \beta_j + \sum_{j=1}^{n_2} U_j C_j - \sum_{j=n_2+1}^{p_2} U_j C_j + \sum_{j=1}^{m_2} V_j D_j - \sum_{j=m_2+1}^{q_2} V_j D_j \right] > 0, \tag{1.7}$$

$$\Delta_2 = \left[ \sum_{j=1}^{n_1} \xi_j A_j - \sum_{j=n_1+1}^{p_1} \xi_j A_j - \sum_{j=1}^{q_1} \eta_j B_j + \sum_{j=1}^{n_3} P_j E_j - \sum_{j=n_3+1}^{p_3} P_j E_j + \sum_{j=1}^{m_3} Q_j F_j - \sum_{j=m_3+1}^{q_3} Q_j F_j \right] > 0, \tag{1.8}$$

$$|\arg(z_1)| < \frac{1}{2} \Delta_1 \pi, \quad |\arg(z_2)| < \frac{1}{2} \Delta_2 \pi. \tag{1.9}$$

In this paper, for the sake of brevity we shall use the following contracted notation for the I-function defined in (1.1):

$$I[z_1, z_2] = I \begin{matrix} 0, n_1 : m_2, n_2; m_3, n_3 \\ p_1, q_1 : p_2, q_2; p_3, q_3 \end{matrix} \begin{matrix} [z_1 | \mathcal{A}: \mathcal{C}; \mathcal{E}] \\ [z_2 | \mathcal{B}: \mathcal{D}; \mathcal{F}] \end{matrix}. \tag{1.10}$$

Further, if  $V_j = 1(j = 1, \dots, m_2)$ ,  $Q_j = 1(j = 1, \dots, m_3)$  in (1.1), then the function will be denoted by

$$\bar{I}[z_1, z_2] = I \begin{matrix} 0, n_1 : m_2, n_2; m_3, n_3 \\ p_1, q_1 : p_2, q_2; p_3, q_3 \end{matrix} \begin{matrix} [z_1 | \mathcal{A}: \mathcal{C}; \mathcal{E}] \\ [z_2 | \mathcal{B}: \mathcal{D}; \mathcal{F}] \end{matrix}. \tag{1.11}$$

and if  $V_j = 1(j = 1, \dots, m_2)$ ,  $Q_j = 1(j = 1, \dots, m_3)$ ,  $U_j = 1(j = 1, \dots, n_2)$  and  $P_j = 1(j = 1, \dots, n_3)$  and  $n_1 = 0$  in (1.1), then the corresponding function will be denoted by

$$\overline{I_1} [z_1, z_2] = I \begin{matrix} 0, 0: m_2, n_2; m_3, n_3 \\ p_1, q_1 : p_2, q_2; p_3, q_3 \end{matrix} \left[ \begin{matrix} z_1 | \mathcal{A}: \acute{C}; \acute{E} \\ z_2 | \mathcal{B}: \acute{D}; \acute{F} \end{matrix} \right] \quad (1.12)$$

where

- $\mathcal{A}$  stands for  $(a_j; \alpha_j, A_j; \xi_j)_{1, p_1} \equiv (a_1; \alpha_1, A_1; \xi_1), \dots, (a_{p_1}; \alpha_{p_1}, A_{p_1}; \xi_{p_1})$ ;
- $\mathcal{B}$  stands for  $(b_j; \beta_j, B_j; \eta_j)_{1, q_1} \equiv (b_1; \beta_1, B_1; \eta_1), \dots, (b_{q_1}; \beta_{q_1}, B_{q_1}; \eta_{q_1})$ ;
- $\mathcal{C}$  stands for  $(c_j, C_j; U_j)_{1, p_2} \equiv (c_1, C_1; U_1), \dots, (c_{p_2}, C_{p_2}; U_{p_2})$ ;
- $\mathcal{D}$  stands for  $(d_j, D_j; V_j)_{1, q_2} \equiv (d_1, D_1; V_1), \dots, (d_{q_2}, D_{q_2}; V_{q_2})$ ;
- $\mathcal{E}$  stands for  $(e_j, E_j; P_j)_{1, p_3} \equiv (e_1, E_1; P_1), \dots, (e_{p_3}, E_{p_3}; P_{p_3})$ ;
- $\mathcal{F}$  stands for  $(f_j, F_j; Q_j)_{1, q_3} \equiv (f_1, F_1; Q_1), \dots, (f_{q_3}, F_{q_3}; Q_{q_3})$ ;
- $\acute{C}$  stands for  $(c_j, C_j; 1)_{1, n_2}, (c_j, C_j; U_j)_{n_2+1, p_2}$ ;
- $\acute{D}$  stands for  $(d_j, D_j; 1)_{1, m_2}, (d_j, D_j; V_j)_{m_2+1, q_2}$ ;
- $\acute{E}$  stands for  $(e_j, E_j; 1)_{1, n_3}, (e_j, E_j; P_j)_{n_3+1, p_3}$ ;
- $\acute{F}$  stands for  $(f_j, F_j; 1)_{1, m_3}, (f_j, F_j; Q_j)_{m_3+1, q_3}$ .

A detailed account of the I-function, its behavior and various special cases in one and two variables (including the generalized hypergeometric function  ${}_pF_q$ ) can be found in the paper by Shantha Kumari et al. (2014).

**Remark:**

It is not out of place to mention two interesting papers by Mishra et al. (2012, 2013).

**2. Results Required**

In our present investigation, we shall require the following classical summation theorems.

**Watson's Theorem** (Bailey, 1935)

$${}_3F_2 \left[ \begin{matrix} a, b, c \\ \frac{1}{2}(a+b+1), 2c \end{matrix} ; 1 \right] = \frac{\Gamma(\frac{1}{2}) \Gamma(c+\frac{1}{2}) \Gamma(\frac{1}{2}(a+b+1)) \Gamma(c+\frac{1}{2}(1-a-b))}{\Gamma(\frac{1}{2}(a+1)) \Gamma(\frac{1}{2}(b+1)) \Gamma(c+\frac{1}{2}(1-a)) \Gamma(c+\frac{1}{2}(1-b))}, \quad (2.1)$$

provided  $\Re(2c - a - b) > -1$  and the parameters are such that the series on the left is defined.

**Dixon's Theorem** (Bailey, 1935)

$${}_3F_2 \left[ \begin{matrix} a, b, c \\ (1+a-b), (1+a-c) \end{matrix} ; 1 \right] = \frac{\Gamma(1+\frac{1}{2}a) \Gamma(1+a-b) \Gamma(1+a-c) \Gamma(1-b-c+\frac{1}{2}a)}{\Gamma(1+a) \Gamma(1-b+\frac{1}{2}a) \Gamma(1-c+\frac{1}{2}a) \Gamma(1+a-b-c)}, \tag{2.2}$$

provided  $\Re(a - 2b - 2c) > -2$ .

**Whipple's Theorem** (Bailey, 1935)

$${}_3F_2 \left[ \begin{matrix} a, b, c \\ f, 2c+1-f \end{matrix} ; 1 \right] = \frac{\pi \Gamma(f) \Gamma(2c+1-f) 2^{1-2c}}{\Gamma(c+\frac{1}{2}(1+a-f)) \Gamma(\frac{1}{2}(a+f)) \Gamma(c+\frac{1}{2}(1+b-f)) \Gamma(\frac{1}{2}(b+f))}, \tag{2.3}$$

provided  $b = 1 - a$  and  $\Re(c) > 0$ .

**3. Main Summation Formulae**

In this section, the following three very general summation formulae will be established.

**Summation Formula 3.1.**

$$\sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{\left(\frac{1}{2}(a+b+1)\right)_k k!} \cdot I_{\begin{matrix} 0, n_1 + 1; m_2, n_2; m_3, n_3 \\ p_1 + 1, q_1 + 1; p_2, q_2; p_3, q_3 \end{matrix}} \left[ \begin{matrix} z_1 \\ z_2 \end{matrix} \middle| \begin{matrix} (1-k-c; \lambda_1, \lambda_2; 1), \mathcal{A}: \mathcal{C}; \mathcal{E} \\ \mathcal{B}, (1-k-2c; 2\lambda_1, 2\lambda_2; 1): \mathcal{D}; \mathcal{F} \end{matrix} \right]$$

$$= \frac{\pi 2^{1-2c} \Gamma(\frac{1}{2}(a+b+1))}{\Gamma(\frac{1}{2}(a+1)) \Gamma(\frac{1}{2}(b+1))} \cdot I_{\begin{matrix} 0, n_1 + 1; m_2, n_2; m_3, n_3 \\ p_1 + 1, q_1 + 2; p_2, q_2; p_3, q_3 \end{matrix}} \left[ \begin{matrix} \frac{z_1}{2^{2\lambda_1}} \\ \frac{z_2}{2^{2\lambda_2}} \end{matrix} \middle| \begin{matrix} (\frac{1}{2}(a+b+1) - c; \lambda_1, \lambda_2; 1), \\ \mathcal{B}, (\frac{1}{2}(a+1) - c; \lambda_1, \lambda_2; 1), \\ \mathcal{A}: \mathcal{C}; \mathcal{E} \\ (\frac{1}{2}(b+1) - c; \lambda_1, \lambda_2; 1): \mathcal{D}; \mathcal{F} \end{matrix} \right], \tag{3.1}$$

provided

- (i)  $\lambda_1 \geq 0, \lambda_2 \geq 0$  (both  $\lambda_1$  and  $\lambda_2$  are not simultaneously zero);
- (ii) The conditions given in (1.7), (1.8) and (1.9) are satisfied.
- (iii)  $\Re \left[ c + \lambda_1 \left( \frac{d_i}{D_i} \right) + \lambda_2 \left( \frac{f_j}{F_j} \right) \right] > 0, i = 1, \dots, m_2; j = 1, \dots, m_3.$

$$(iv) \Re \left[ c + \frac{1}{2}(1 - a - b) + \lambda_1 \left( \frac{d_i}{D_i} \right) + \lambda_2 \left( \frac{f_j}{F_j} \right) \right] > 0, \quad i = 1, \dots, m_2; \quad j = 1, \dots, m_3,$$

where  $(a)_k$  stands for the Pochhammer symbol defined by

$$(a)_k = a(a + 1) \cdots (a + k - 1) = \frac{\Gamma(a+k)}{\Gamma(a)}, \quad (a)_0 = 1 \quad \text{and} \quad k \text{ is an integer.}$$

**Summation Formula 3.2.**

$$\begin{aligned} & \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(1 + a - b)_k k!} \\ & \cdot I_{0,0: m_2, n_2 + 1; m_3, n_3} \left[ \begin{matrix} z_1 \\ z_2 \end{matrix} \middle| \begin{matrix} \mathcal{A}: (1 - k - c, \lambda; 1), \hat{C}, 1 + a + k - c, \lambda; 1; \hat{\mathcal{E}} \\ \mathcal{B}: \hat{\mathcal{D}}; \hat{\mathcal{F}} \end{matrix} \right] \\ & = \frac{\Gamma(1 + \frac{1}{2}a) \Gamma(1 + a - b)}{\Gamma(1 + a) \Gamma(1 + \frac{1}{2}a - b)} \cdot I_{0,0: m_2 + 1, n_2 + 1; m_3, n_3} \left[ \begin{matrix} z_1 \\ z_2 \end{matrix} \middle| \begin{matrix} \mathcal{A}: (1 - c, \lambda; 1), \hat{C} \\ \mathcal{B}: (1 + \frac{1}{2}a - c, \lambda; 1), (1 + a - b - c, \lambda; 1); \hat{\mathcal{E}} \\ (1 + \frac{1}{2}a - b - c, \lambda; 1), \hat{\mathcal{D}}; \hat{\mathcal{F}} \end{matrix} \right], \quad (3.2) \end{aligned}$$

provided

- (i)  $\lambda \geq 0$ ;
- (ii) The conditions given in (1.7), (1.8) and (1.9) are satisfied.
- (iii)  $\Re \left[ c + \lambda \left( \frac{d_i}{D_i} \right) \right] > 0; \quad i = 1, \dots, m_2.$
- (iv)  $\Re \left[ c - \frac{1}{2}a + b + \lambda \left( \frac{c_i - 1}{c_i} \right) \right] < 1; \quad i = 1, \dots, n_2.$

**Summation Formula 3.3.**

$$\begin{aligned} & \sum_{k=0}^{\infty} \frac{(a)_k (1 - a)_k}{(b)_k k!} \\ & \cdot I_{0, n_1 + 1; m_2, n_2; m_3, n_3} \left[ \begin{matrix} z_1 \\ z_2 \end{matrix} \middle| \begin{matrix} (1 - c - k; \lambda_1, \lambda_2; 1), \mathcal{A}: \mathcal{C}; \mathcal{E} \\ \mathcal{B}, (b - k - 2c; 2\lambda_1, 2\lambda_2; 1); \hat{\mathcal{D}}; \hat{\mathcal{F}} \end{matrix} \right] \\ & = \frac{\pi 2^{1-2c} \Gamma(b)}{\Gamma\left(\frac{1}{2}(a + b)\right) \Gamma\left(\frac{1}{2}(b - a + 1)\right)} \end{aligned}$$

$$\cdot I_{p_1+1, q_1+2; p_2, q_2; p_3, q_3} \left[ \begin{matrix} 0, n_1+1; m_2, n_2; m_3, n_3 \\ \frac{z_1}{2^{2\lambda_1}} \end{matrix} \middle| (1-c; \lambda_1, \lambda_2; 1), \right. \\
 \left. \frac{z_2}{2^{2\lambda_2}} \middle| \mathcal{B}, \left( \frac{1}{2}(b-a+1) - c; \lambda_1, \lambda_2; 1 \right), \right. \\
 \left. \mathcal{A}: \mathcal{C}; \mathcal{E} \right. \\
 \left. \left( \frac{1}{2}(a+b) - c; \lambda_1, \lambda_2; 1 \right): \mathcal{D}; \mathcal{F} \right], \quad (3.3)$$

provided

- (i)  $\lambda_1 \geq 0, \lambda_2 \geq 0$  (both  $\lambda_1$  and  $\lambda_2$  are not simultaneously zero);
- (ii) The conditions given in (1.7), (1.8) and (1.9) are satisfied.
- (iii)  $\Re \left[ c + \lambda_1 \left( \frac{d_i}{D_i} \right) + \lambda_2 \left( \frac{f_j}{F_j} \right) \right] > 0; i = 1, \dots, m_2; j = 1, \dots, m_3.$

**Proof:**

In order to establish our first general summation formula (3.1) we proceed as follows. Denoting the left-hand side of (3.1) by  $S$ , using the definition of the I-function of two variables with the help of (1.1), we have

$$S = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{\left( \frac{1}{2}(a+b+1) \right)_k k!} \\
 \cdot \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \phi(s, t) \theta_1(s) \theta_2(t) z_1^s z_2^t \frac{\Gamma(k+c+\lambda_1 s+\lambda_2 t)}{\Gamma(k+2c+2\lambda_1 s+2\lambda_2 t)} ds dt. \quad (3.4)$$

Now, changing the order of integration and summation, which is easily seen to be justified due to the uniform convergence of the series involved in the process, we have after some simplification

$$S = \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \phi(s, t) \theta_1(s) \theta_2(t) z_1^s z_2^t \frac{\Gamma(c+\lambda_1 s+\lambda_2 t)}{\Gamma(2c+2\lambda_1 s+2\lambda_2 t)} \\
 \cdot \sum_{k=0}^{\infty} \frac{(a)_k (b)_k (c+\lambda_1 s+\lambda_2 t)_k}{\left( \frac{1}{2}(a+b+1) \right)_k (2c+2\lambda_1 s+2\lambda_2 t)_k k!} ds dt. \quad (3.5)$$

Summing up the series, we have

$$S = \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \phi(s, t) \theta_1(s) \theta_2(t) z_1^s z_2^t \frac{\Gamma(c+\lambda_1 s+\lambda_2 t)}{\Gamma(2c+2\lambda_1 s+2\lambda_2 t)} \\
 \cdot {}_3F_2 \left[ \begin{matrix} a, b, c+\lambda_1 s+\lambda_2 t \\ \frac{1}{2}(a+b+1), 2(c+\lambda_1 s+\lambda_2 t) \end{matrix} ; 1 \right] ds dt. \quad (3.6)$$



Now we observe that the  ${}_3F_2$  appearing in the inner side, can be evaluated with the help of classical summation theorem (2.1) and after a little simplification, interpreting the result with the help of the definition of the I-function of two variables (1.1) we easily arrive at the right hand side of (3.1). This completes the proof of our first general summation formula.

In exactly the same manner, the summation formulae (3.2) and (3.3) can be established with the help of the known results (2.2) and (2.3) respectively.

#### 4. Special Cases

On account of the most general nature of the I-function of two variables, it includes as special cases many of the known functions of one and two variables appearing in the literature and hence the results derived in this paper will serve as the key formulas from which we can obtain a large number of known and unknown results. However here we shall mention some of the known results.

- (i) If we take the exponents  $\xi_j (j = 1, \dots, p) = \eta_j (j = 1, \dots, q) = U_j (j = 1, \dots, p_2)$ ,  $V_j (j = 1, \dots, q_2) = P_j (j = 1, \dots, p_3) = Q_j (j = 1, \dots, q_3) = 1$  in various gamma functions involved in (1.2), (1.3) and (1.4), the I-function of two variables can be reduced to H-function of two variables defined by Mittal and Gupta, (1972) and hence we get the corresponding summation formulae recorded in (Srivastava et al. (1982)).
- (ii) When all the exponents  $\xi_j (j = 1, \dots, p)$ ,  $\eta_j (j = 1, \dots, q)$ ,  $U_j (j = 1, \dots, p_2)$ ,  $V_j (j = 1, \dots, q_2)$ ,  $P_j (j = 1, \dots, p_3)$ ,  $Q_j (j = 1, \dots, q_3)$  of various gamma functions involved in (1.2), (1.3) and (1.4) are equal to unity, and if we take  $p_1 = q_1 = 0$ ;  $m_3 = 1$ ,  $n_3 = p_3$ ,  $E_j = F_j = 1$ ;  $f_1 = 0$  with  $z_2 \rightarrow 0$ ;  $c = 0$ ,  $\lambda_1 = 1$  and specializing the parameters, (3.2) and (3.3) reduces to known results by Nair, V.C. (1968, p.256, Equation (8.62), pp. 254-255, Equation (8.59)).

Similarly other results can also be obtained.

#### 5. Conclusion

In this research paper we have established three summation formulae involving the I-function of two variables recently introduced by Shantha Kumari et al. (2014), by using some classical summation theorems. As we have seen the generalized function of two variables introduced by Agarwal(1965) and Sharma(1965) have found interesting applications in wireless communication [Xia et al. (2012)], and the results evaluated in this paper may be potentially useful.

#### Acknowledgements

*The authors are grateful to the referees for giving the valuable suggestions for the improvement of this paper.*

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