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Mathematical Modelling of Blood Flow through Catheterized Artery under the Influence of Body Acceleration with Slip Velocity

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Abstract

The flow of blood through stenosed catheterized artery with the effect of external body acceleration has been considered. The pulsatile flow behaviour of blood in an artery subjected to the pulsatile pressure gradient and slip velocity has been studied considering blood as a Newtonian fluid. The non-linear differential equations governing the fluid flow are solved using the perturbation method. The analytic expressions are derived for the velocity profile, flow rate, wall shear stress and effective viscosity. The computer codes are developed for the analysis of physiological parameters. The effects of various parameters on blood flow are discussed through graphs. It is observed that insertion of catheter increases the wall shear stress enormously depending upon the size of the catheter. Body acceleration enhances the axial velocity and flow rate. However it is found that with the application of slip velocity, the wall shear stress is significantly decreases.

Keywords: Pulsatile flow; Body acceleration; Slip velocity; Newtonian fluid; Stenosis

AMS-MSC 2010 No.: 76Z05, 74G10

1. Introduction

In recent years, the interest in hemodynamical studies has grown appreciably due to the fact that many cardiovascular diseases are closely related to the flow conditions in the blood vessels Liepsich (2002). The development of various fatal disorders such as Atherosclerosis, Diabetes, Thrombosis and other cardiovascular disorders, which continue to be leading cause of deaths, is increasing public concern about their treatments. Stenosis is one of them.

Externally imposed body acceleration also has major influence on the flow through stenosed artery. Activities such as driving, cycling and jogging cause the human body to be exposed to accelerations. Body acceleration changes the blood velocity significantly. So the velocity of blood under the pumping action of heart is increased to the high amplitude of body acceleration.

When the human body experiences a sudden velocity change, the blood flow is disturbed. The human body has a remarkable adaptability to changes and a prolonged exposure of the body to such vibrations leads to many health problems like headache, abdominal pain, venous pooling of blood in the extremities and increased pulse rate on account of disturbances in blood flow. If the response of the human system to such acceleration is understood properly, the controlled accelerations can be used for therapeutic treatments, development of new diagnostic tools and for better designing of protective pads.

A model for abnormal growth in the lumen of the artery is called stenosis, which is developed due to the accumulation of cholesterol on the vessel wall is represented by Rodbard (1966), Shukla (1980), Sankar (2009). May (1963), Bugliarellow and Sevilla (1970), Young (1979), Chaturani (1986), Sapna (2009), Ponnalagasamy (2012) have studied the flow characteristic of blood in an artery with stenosis by introducing various non-Newtonian fluid models to represent the blood. Although blood is a suspension of cells it behaves like a non-Newtonian fluid at low shear rates in smaller arteries. Yeleswarupu (1996) has presented a generalized Newtonian model used mainly for blood, at a high rate of shear when blood behaves like a Newtonian fluid. Many researchers assumed stenosis to be symmetric and human blood to be Newtonian, to simplify the problem an assumption that remains largely valid for a large vessel as well [Ookawara (2000), Biswas (2010)].

Under normal physiological conditions transport of blood in the human circulatory system depends entirely on the pumping action of the heart producing a pressure gradient throughout the arterial system. Ishikawa (2000) has analysed periodic blood flow through stenosed and locally expanded tubes. They found that the vortex downstream of stenosis or expansion becomes strongest at a certain frequency of pulsation. Kumar (2010), Haghighi (2012) have studied the pulsatile flow of blood under the influence of externally imposed periodic body acceleration through a time dependent stenosed artery. The flow characteristic of blood through a multi stenosed artery when it is subjected to whole body acceleration has been studied by Chakravarty (1999), where they considered a flexible cylindrical tube containing a homogeneous Newtonian fluid with variable viscosity depicting blood. Nagarani (2008), Verma (2011) have considered the pulsatile flow of Casson fluid through a stenosed artery with the effect of body acceleration. It is observed that the yield stress of the fluid and body acceleration highly influenced the velocity of the fluid, shear stress, flow rate in a stenosed artery.

An arterial catheter is a thin, hollow, tube which is placed into the artery to measure blood pressure more accurately than is possible with a blood pressure cuff. The catheter can also be used to get repeated blood samples when it is necessary to frequently measure the levels of oxygen and/or carbon dioxide in the bloodstream. Many researchers discussed the flow of blood in an artery, in presence of a catheter by modelling the catheter and artery as rigid co-axial cylinders and blood as either Newtonian or non-Newtonian fluid. During catheterization, small

tubes (catheters) are inserted into the circulatory system under x-ray guidance in order to obtain information about blood flow and pressures within the heart and to determine if there are obstructions within the blood vessels feeding the heart muscle (coronary arteries). Sarkar and Jayaraman (1998) studied the pulsatile flow in a catheterized stenosed artery and estimated the correction in mean pressure drop along the stenosis due to catheterization. The effect of catheterization on various flow characteristics in a curved artery with or without stenosis was studied by Jayaraman (1995) and Dash (1999). Sankar (2009) has studied a two-fluid model for the pulsatile flow of blood in a catheterized artery, by considering the core layer as a Casson fluid and the peripheral layer as a Newtonian fluid.

To understand the existence of slip at the tube wall Nubar (1967), Brunn (1975) have reviewed the several treatments of slip at the walls of the capillary tubes. In view of theoretical and experimental observations implying the existence of slip at the wall, it is improper to ignore the slip in blood flow. It is also noted that in the literature, there is no direct formula to calculate the slip velocity. It is therefore worthwhile to find a formula to calculate the slip velocity at the wall. Pulsatile flow of blood through a catheterized artery in presence of different geometry of stenosis with a velocity slip at a stenotic wall has been investigated by Biswas and Chakraborty (2010), Verma (2011). It is found that the wall shear stress and effective viscosity decreases while axial velocity increases with velocity slip at wall.

The aim of present analysis to study the characteristic of blood flow through a artery with axially symmetric stenosis considering blood as a Newtonian fluid. Effect of slip velocity, body acceleration and presence of catheter on the flow variables such as velocity profile wall shear stress flow rate has been studied for pulsatile blood flow in constricted artery.

2. Mathematical Formulation

Consider a pulsatile flow of blood in the presence of an external body acceleration along with the effect of slip velocity at the stenotic wall. We consider the axially symmetric, laminar, one dimensional and fully developed flow of blood through a catheterized artery with mild stenosis. It is assumed that the wall of the tube is rigid and the body fluid blood is represented by a Newtonian fluid. The geometry of the stenosis is shown in the (Figure 1) and is given by



Figure 1. Geometry of stenosed artery

$$\overline{R}(\overline{z}) = \begin{cases} \overline{R_0} - (\overline{\delta_s}/2)(1 + \cos(\pi \overline{z}/\overline{z_0})) &, \text{ for } |\overline{z}| \le \overline{z_0}, \\ \overline{R_0} &, & \text{ for } |\overline{z}| > \overline{z_0}, \end{cases}$$

$$(1)$$

where $\overline{R}(\overline{z})$ is the radius of the artery in the stenotic region, R_0 is the constant radius of the artery in the non-stenotic region, $\overline{z_0}$ is the half length of the stenosis and $\overline{\delta_s}$ is the maximum height of the stenosis such that $\overline{\delta_s} / \overline{R_0} < 1$.

It is found that the radial velocity being very small can be neglected for low Reynolds number in case of mild stenosis. The Navier-Stokes equations governing the fluid flow is given by Schlichting and Gersten (2004).

$$\overline{\rho}\left(\frac{\partial \overline{u}}{\partial \overline{t}}\right) = -\left(\frac{\partial \overline{p}}{\partial \overline{z}}\right) - \left(1/r\right)\frac{\partial}{\partial \overline{r}}\left(\overline{r\tau}\right) + \overline{F}\left(\overline{t}\right),\tag{2}$$

$$\frac{\partial \overline{p}}{\partial r} = 0,$$
 (3)

where \overline{u} represent the axial velocity along z-direction, \overline{p} is the pressure, $\overline{\rho}$ is the density, \overline{t} the time, $\overline{\tau}$ the shear stress and $\overline{F}(\overline{t})$ the body acceleration. Mathematically, $\overline{F}(\overline{t})$ is described in equation (8). Newtonian fluid (blood) can be given by the equation

$$\overline{\tau} = -\overline{\mu} \left(\frac{\partial \overline{u}}{\partial r} \right), \tag{4}$$

where $\overline{\mu}$ is the shear viscosity of blood.

Boundary conditions

The boundary conditions are

(i)
$$\overline{u} = \overline{u_s} \ at \ \overline{r} = \overline{R}(\overline{z}),$$
 (5)

(ii)
$$\overline{u} = 0 \ at \ \overline{r} = \overline{R_1},$$
 (6)

where $\overline{u_s}$ is the slip velocity at the stenotic wall, $\overline{R_1}$ is the radius of the catheter.

Since the pressure gradient is a function of \overline{z} and \overline{t} , we take

$$\frac{-\partial \overline{p}}{\partial \overline{z}}(\overline{z},\overline{t}) = A_0 + A_1 \cos\cos\left(\overline{\omega_p t}\right), t \ge 0,$$
(7)

where A_0 is the steady state pressure gradient, A_1 is the amplitude of the fluctuating component, $\overline{\omega_p} = 2\pi \overline{f_p}$, where $\overline{f_p}$ is the pulse rate frequency.

The periodic body acceleration $\overline{F}(\overline{t})$ in the axial direction is given by

$$\overline{F}(\overline{t}) = a_0 \cos \cos \left(\overline{\omega_b t} + \varphi \right), \tag{8}$$

where a_0 is the amplitude of body acceleration, $\overline{\omega_b} = 2\pi \overline{f_b}$; $\overline{f_b}$ is it's frequency in Hz. The frequency of the body acceleration $\overline{f_b}$ is assumed to be small so that wave effect can be neglected.

By introducing the following non-dimensional variables

$$u = \overline{u} / \frac{A_0 R_0^2}{4\overline{\mu}}, z = \overline{z} / \overline{R_0}, R(z) = \overline{R(z)} / \overline{R_0}, r = \overline{r} / \overline{R_0}, t = \overline{t\omega_p}, \omega = \overline{\omega_b} / \overline{\omega_p},$$

$$\delta_s = \overline{\delta_s} / \overline{R_0}, u_s = \overline{u_s} / \frac{A_0 \overline{R_0^2}}{4\overline{\mu}}, \tau = \overline{\tau} / \frac{A_0 \overline{R_0}}{2}, \alpha^2 = \overline{R_0^2 \omega_p \rho} / \overline{\mu}, e = A_1 / A_0,$$

$$B = a_0 / A_0, R_1 = \overline{R_1} / \overline{R_0},$$
(9)

where A_0 is steady component of pressure gradient, $\overline{R_0}$ is the radius of the normal artery, $\overline{\omega_p}$ is the frequency of oscillation of the pulsatile flow and then $A_0 \overline{R_0}^2 / 4\overline{\mu}$, represents the central line velocity in a poiseuille flow.

The non-dimensional momentum equation (2) can be written as

$$\alpha^{2} \frac{\partial u}{\partial t} = 4 (1 + e \cos t) + 4B \cos(\omega t + \phi) - (2/r) \frac{\partial}{\partial r} (r\tau), \qquad (10)$$

where $\alpha^2 = \omega_p R_o^2 / (\mu / \rho)$, is called Womersley frequency parameter.

Equation (4) can be written as

$$\tau = -\frac{1}{2}\frac{\partial u}{\partial r},\tag{11}$$

On substituting the value of τ in Equation (10) we have

$$\alpha^{2} \left(\frac{\partial u}{\partial t} \right) = 4 \left(1 + e \cos t \right) + 4B \cos \left(\omega t + \phi \right) + \left(1/r \right) \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right).$$
(12)

The boundary conditions (5) and (6) reduces to

(i)
$$u = u_s \text{ at } r = R(z),$$

(ii) $u = 0 \text{ at } r = R_1.$ (13)

The geometry of an arterial stenosis in dimensionless form is given by

$$R(z) = \begin{cases} 1 - (\delta_s/2) \left(1 + \cos\left(\frac{\pi z}{z_0}\right) \right), & for |z| \le z_0, \\ 1, & for |z| > z_0. \end{cases}$$
(14)

The non-dimensional volumetric flow rate is defined by

$$Q(z,t) = 4 \int_{0}^{R(z)} r u(z,r,t) dr,$$
(15)

where

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$$Q(z,t) = \frac{\overline{Q}(\overline{z},\overline{t})}{\pi A_0 (\overline{R_0})^4 / 8\overline{\mu}}; \overline{Q}(\overline{z},\overline{t})$$
is the volumetric flow rate.

Effective viscosity $\overline{\mu_e}$ defined as

$$\overline{\mu_e} = \pi \left(-\frac{\partial \overline{p}}{\partial \overline{z}} \right) (\overline{R}(\overline{z}))^4 / \overline{Q}(\overline{z}, \overline{t}),$$
(16)

can be expressed in the dimension less form as

$$\mu_{e} = R^{4} \left(1 + e \cos t \right) / Q(z, t).$$
(17)

3. Method of solution:

Let the velocity u can be expressed in the following form

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$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \dots$$
(18)

Substituting Equation (18) in Equation (12) & (13) respectively and equating constant term and α^2 term we get

$$\frac{\partial}{\partial r} \left(r \frac{\partial u_0}{\partial r} \right) = -4r [(1 + e \cos t) + B \cos(\omega t + \phi)], \tag{19}$$

$$\frac{\partial u_0}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(\frac{\partial u_1}{\partial r} \right) \right), \tag{20}$$

(i)
$$u_0 = u_s \text{ and } u_1 = 0 \text{ at } r = R(z),$$
 (21)

(*ii*)
$$u_0 = 0$$
 and $u_1 = 0$ at $r = R_1$. (22)

To determine u_0 and u_1 , we integrate equation (19) and (20) twice with respect to r, using the boundary conditions (21) & (22)

$$u_{0} = f(t) \left(R_{1}^{2} - r^{2} \right) + \frac{\ln(r/R_{1})}{\ln(R/R_{1})} \left(u_{s} + f(t) \left(R^{2} - R_{1}^{2} \right) \right),$$
(23)

where $f(t) = (1 + e \cos t) + B \cos(\omega t + \phi)$.

$$u_{1} = D \begin{bmatrix} \left(\left(R_{1}^{2}r^{2}/4 \right) - \left(r^{4}/16 \right) - 3\left(R_{1}^{4}/16 \right) \right) + \frac{\left(R^{2} - R_{1}^{2} \right)}{4\ln\left(R/R_{1} \right)} \left(r^{2}\ln\left(r/R_{1} \right) - r^{2} + R_{1}^{2} \right) \\ + \frac{\ln\left(R_{1}/r \right)}{\ln\left(R/R_{1} \right)} \begin{bmatrix} \left(\left(R_{1}^{2}R^{2}/4 \right) - \left(R^{4}/16 \right) - 3\left(R_{1}^{4}/16 \right) \right) \\ + \frac{\left(R^{2} - R_{1}^{2} \right)}{4\ln\left(R/R_{1} \right)} \left(\ln\left(R/R_{1} \right) R^{2} - R^{2} + R_{1}^{2} \right) \end{bmatrix},$$
(24)

where D = f'(t).

Neglecting terms of $o(\alpha^4)$ and other power of α , the expression for the axial velocity u(r, z, t) is obtained from equations (18), (23) and (24) as

$$u(z,r,t) = f(t)(R_1^2 - r^2) + \frac{\ln(r/R_1)}{\ln(R/R_1)}(u_s + f(t)(R^2 - R_1^2)) + \alpha^2 D(C_1 + C_2 + C_3 + C_4), \quad (25)$$

where $C_1 = \left(\frac{R_1^2 r^2}{4} - \left(\frac{r^4}{16}\right) - 3\left(\frac{R_1^4}{16}\right)\right),$ $C_2 = \frac{\left(\frac{R^2 - R_1^2}{4\ln(R/R_1)}\right) \left(r^2 \ln\ln(r/R_1) - r^2 + R_1^2\right),$ $C_3 = \frac{\ln(R_1/R)}{\ln(R/R_1)} \left(\left(\frac{R_1^2 R^2}{4}\right) - \left(\frac{R^4}{16}\right) - 3\left(\frac{R_1^4}{16}\right)\right),$ $C_4 = \frac{\left(\frac{R^2 - R_1^2}{4\ln(R/R_1)}\right) \left(\ln(R/R_1)R^2 - R^2 + R_1^2\right).$

The wall shear stress τ_w can be obtained by substituting the velocity expression from equation (23) and (24).

$$\tau_{w} = -\frac{1}{2} \left(\left(\frac{\partial u_0}{\partial r} \right) + \alpha^2 \left(\frac{\partial u_1}{\partial r} \right) \right)_{r=R(z)},$$
(26)

$$\tau_{w} = \begin{bmatrix} Rf(t) - \frac{1}{2Rln(R/R_{1})} \left(u_{s} + f(t)(R^{2} - R_{1}^{2}) \right) - \\ \left[\frac{\alpha^{2}D}{2} \left(\left(\left(R_{1}^{2}R/2 \right) - \left(R^{3}/4 \right) \right) + \frac{\left(R^{2} - R_{1}^{2} \right)}{4\ln(R/R_{1})} \left(2R \ln(R/R_{1}) - R \right) \\ - \frac{1}{R\ln(R/R_{1})} \left(\left(\left(R_{1}^{2}R^{2}/4 \right) - R^{4}/16 - \left(3R_{1}^{4}/16 \right) \right) \\ + \frac{\left(R^{2} - R_{1}^{2} \right)}{4\ln(R/R_{1})} \left(\ln\ln(R/R_{1})R^{2} - R^{2} + R_{1}^{2} \right) \right) \end{bmatrix} \end{bmatrix}$$

$$(27)$$

From equations (15) and (25) the volumetric flow rate Q(z,t) can be obtained as

$$Q(z,t) = f(t) \left(2R_1^2 R^2 - R^4 - R_1^4 \right) + \frac{1}{\ln(R/R_1)} \left(u_s + f(t) \left(R^2 - R_1^2 \right) \right) \\ \times \left(2\ln(R/R_1) R^2 - R^2 + R_1^2 \right) + \alpha^2 D \left(B_1 + \frac{\left(R^2 - R_1^2 \right)}{\ln(R/R_1)} B_2 B_3 B_4 \right),$$
(28)

where $B_1 = ((R_1^2 R^4 / 4) - (R^6 / 24) - (3R_1^4 R^2 / 8) - (5R_1^6 / 24) + (3R_1^6 / 8)),$

$$B_{2} = \left(\ln \left(\frac{R}{R_{1}} \right) \left(\frac{R^{4}}{4} \right) - \left(\frac{5R^{4}}{16} \right) + \left(\frac{R_{1}^{2}R^{2}}{2} \right) - \left(\frac{3R_{1}^{4}}{16} \right) \right),$$

$$B_{3} = \left(\frac{1}{\ln(R/R_{1})}(R_{1}^{2}R^{2} - (R^{4}/4) - (3R_{1}^{4}/4))\right) + \frac{(R^{2} - R_{1}^{2})}{\ln(R/R_{1})}(\ln(R/R_{1})R^{2} - R^{2} + R_{1}^{2})),$$

$$B_{4} = \ln(R_{1}/R)(R^{2}/2) + (R^{2}/4) - (R_{1}^{2}/4).$$

The expression for the effective viscosity μ_e can be obtained from equation (17) and (28).

4. Results and Discussions

In the present model our objective is to study the combined effect of body acceleration and the slip velocity in a catheterized stenosed artery assuming blood is a Newtonian fluid. The expressions for velocity, flow rate, wall shear stress and effective viscosity are obtained by solving the governing equation of flow using perturbation method with a very small Womersley frequency parameter. Computer codes are developed for numerical evaluation of the analytical results obtained in equations (25)-(29). The various parameter values are selected from Young (1979), Biswas (2009) and Srivastava (2010).

It is very important to study the pattern of velocity profile since the detailed description of the flow field is given by the velocity profile. The variation of axial velocity u with radial distance rdifferent values body acceleration for of В and for fixed values of $e = 1, \omega = 1, z = 0, \phi = 0.2, \alpha = 0.5, u_s = 0.05$ at the peak of the stenosis (z = 0) is presented in figure 2. From the graph it is predicted that the velocity is more in the presence of body acceleration and increases rapidly with the increase of body acceleration. As the body acceleration increases the plug flow region shrivel therefore more flow takes place.



Figure 2. Variation of axial velocity with radial distance

Figure 3 shows the effects of various parameters on the axial velocity u for the different values of stenosis height $\delta_s = 0, 0.2$ slip velocity u_s and for fixed values of $R_1 = 0.1, \alpha = 0.5, z = 0, t = 1$. It is observed that the magnitude of the axial velocity is more in the non-stenosed artery than that in stenosed artery. It is clearly indicated that the velocity profile is almost plugged profile for $\delta_s > 0$. Slip velocity shows a significant change in velocity profile. Axial velocity increases in normal as well as stenosed vessel with the increase in slip velocity.

Figure 4 depicts the variation of volumetric flow rate with pressure gradient *e* for different values of body acceleration parameter *B* and stenosis height δ_s and for fixed values of $R_1 = 0.2$, $\alpha = 0.1$, t = 1, $u_s = 0.1$. It is found that as the height of the stenosis increases flow rate decreases i.e. flow rate is more in normal artery than in stenosed artery.

The lowest value of flow rate Q is obtained at e = 0.Graph shows that the highest and lowest trends in the flow rate Q are obtained with $B = 1, u_s = 0.1, \delta_s = 0$ and $B = 0, u_s = 0.1, \delta_s = 0.1$ respectively. Further it is observed that the body acceleration as well as the slip velocity enhances the flow rate.

Figure 5 reveals the variation of the blood flow characteristic τ_w with catheter radius for different values of slip velocity u_s in a stenosed artery. It can be easily seen that the increase of catheter radius, increases the wall shear stress. On the other hand, as the slip velocity increases the wall shear stress reduces in an artery.



Radial distance r Figure 3. Variation of axial velocity with radial distance

Figure 6 shows the variation of wall shear stress with catheter radius \mathbf{R}_1 for different values of the body acceleration parameter B and for fixed values of $\delta_s = 0.2$, $\alpha = 0.1$, $u_s = 0.5$. It is found that the wall shear stress increases with the increase of body acceleration in an artery with inserted catheter.



Figure 4. Variation of flow rate Q(t) with pressure gradient e



Figure 5. Variation of wall shear stress with catheter radius



Figure 6. Variation of wall shear stress with Catheter radius

5. Conclusion

In the present mathematical model, pulsatile blood flow through stenosed catheterized artery with periodic body acceleration and axial slip velocity at the constricted wall has been considered. The variation of the flow variables with different flow parameters are presented graphically. It is seen that axial velocity and flow rate increase with the wall slip. Further it has been observed that the wall shear stress decreases with slip velocity. It can be seen that there is a great impact of both slip velocity and body acceleration on the flow variables. On the basis of the above analysis slip at the stenosed artery could play a very important role in blood-flow modeling by reducing the wall shear stress. Further this model can be used to estimate the influence of the various parameters mentioned above on different flow characteristic under severe stenosis.

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