Applications and Applied Mathematics: An International Journal (AAM)

Volume 7 | Issue 2 | Article 1

12-2012

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 7, Issue 2 (December 2012), pp. 488 - 499

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Received: August 26, 2012; Accepted: November 2, 2012

Abstract

In this paper, we implemented modification of truncated expansion method for the exact solutions of the Konopelchenko-Dubrovsky equation the (n+1)-dimensional combined sinh-cosh-Gordon equation and the Maccari system. Modification of truncated expansion method is a powerful solution method for obtaining exact solutions of nonlinear evolution equations. This method presents a wider applicability for handling nonlinear wave equations.

Keywords: Modification of truncated expansion method; Konopelchenko-Dubrovsky equation; (n+1)-dimensional combined sinh-cosh-Gordon equation; Maccari system

MSC 2000 No.: 35Q53; 35Q80; 35Q55; 35G25.

1. Introduction

The theory of nonlinear dispersive and dissipative wave motion has recently undergone much research. Phenomena in physics and other fields are often described by nonlinear evolution equations which play a crucial role in applied mathematics and physics. Furthermore, when an original nonlinear equation is directly solved, the solution preserves the actual physical characters of the equations. Explicit solutions of nonlinear equations are therefore of fundamental importance. Various methods for obtaining explicit solutions of nonlinear evolution equations are proposed. Many explicit exact methods are introduced in the literature. Among these methods, the tanh method [Ma (1993), Malfliet (1992)], the multiple exp-function method [Ma et al. (2010), Ma and Zhu (2012)], the Backlund transformation method [Miura (1978)], the Hirotas direct method [Hirota (1971), Hirota (2004)], the transformed rational function method [Ma and Lee (2009)], the first integral method [Feng (2002), Feng and Wang (2003), Feng and Knobel (2007), Feng (2002), Feng and Chen (2005)], the simplest equation method [Kudryashov (2005)], the automated tanh-function method [Parkes (1996)], modification of truncated expansion method [Kudryashov (2004), Kudryashov (1990), Ryabov (2010)] and the solitary wave ansatz method [Biswas et al. (2012), Triki et al. (2012), Ebadi et al. (2012), Johnpillai et al. (2012), Girgis et al. (2012), Crutcher et al. (2012)] are some of the methods used to develop nonlinear dispersive and dissipative problems.

Konopelchenko and Dubrovsky (1984) presented the Konopelchenko-Dubrovsky (KD) equation

$$\begin{cases} u_t - u_{xxx} - 6buu_x + \frac{3}{2}a^2u^2u_x - 3v_y + 3au_xv = 0, & (1a) \\ u_y = v_x, & (1b) \end{cases} \tag{1}$$

where a and b are real parameters. Equation (1) is a new nonlinear integrable evolution equation on two spatial dimensions and one temporal. This equation was investigated by the inverse scattering transform method. The F-expansion method is also used in Wang and Zhang (2005) to investigate the KD equation.

The aim of this paper is to find exact solutions of the KD equation and the (n+1)-dimensional combined sinh-cosh-Gordon equation and the Maccari system by using modification of truncated expansion method [Kudryashov (2004), Kudryashov (1990), Ryabov (2010)].

2. Modification of Truncated Expansion Method

We present the modification of the truncated expansion method [Kudryashov (2004), Kudryashov (1990), Ryabov (2010)]. We consider a general nonlinear partial differential equation (PDE) in the form

$$E\left(u, u_x, u_y, u_t, ...\right) = 0. \tag{2}$$

Using traveling wave $u(x,y,t) = y(z)$, $z = kx + ly - wt$ carries (2) into the following ordinary differential equation (ODE):

$$L\left(y, y_z, y_{zz}, ..., k, l, w\right) = 0. \tag{3}$$

The modification of the truncated expansion method involves the following steps [Kudryashov (2004), Kudryashov (1990), Ryabov (2010)].

***Step* 1.**

Determination of the dominant term with highest order of singularity. To find the dominant terms we substitute

$$y = z^{-p}, \tag{4}$$

into all terms of Equation (3). Then compare degrees, and choose two or more with the lowest degree. The maximum value of p is the pole of Equation (3) and is denoted by N. The method is applicable when N is integer. Otherwise the equation has to be transformed.

***Step* 2.**

We look for exact solution of Equation (3) in the form

$$y = \sum_{i=0}^{N} a_i Q^i(z), \tag{5}$$

where $a_i \left(i = 0,1,...,N\right)$ are constants to be determined later, such that $a_N \neq 0$, while $Q(z)$ has the form

$$Q(z) = \frac{1}{1 + c\exp\left(z\right)}, \qquad c = const, \tag{6}$$

a solution to the Riccati equation

$$Q_z = Q^2 - Q.$$

***Remark* 1.**

This Riccati equation also admits the following exact solutions [Ma and Fuchssteiner (1996)]:

$$Q_1(z) = \frac{1}{2}\left(1 - \tanh\left[\frac{z}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \qquad \xi_0 > 0,$$

$$Q_2(z) = \frac{1}{2}\left(1 - \coth\left[\frac{z}{2} - \frac{\varepsilon \ln\left(-\xi_0\right)}{2}\right]\right), \qquad \xi_0 < 0,$$

More general solutions are presented in the reference [Ma and Fuchssteiner (1996)].

***Remark* 2.**

Exponential functions are also applied to the construction of the (3+1)-dimensional and the three wave solutions to the bilinear equations [Ma and Zhu (2012)], and linear ordinary differential equations of arbitrary order are used to establish invariant subspaces of the solutions to the nonlinear equations [Ma (2012)].

***Step* 3.**

We calculate the necessary number of derivatives of function y. It is easy to do using Maple or Mathematica package. In case $N=1$ we have some derivatives of the function $y(z)$ in the form

$$\begin{aligned} &y = a_0 + a_1 Q, \\ &y_z = -a_1 Q + a_1 Q^2, \\ &y_{zz} = a_1 Q - 3a_1 Q^2 + 2a_1 Q^3, \\ &y_{zzz} = -a_1 Q + 7a_1 Q^2 - 12 a_1 Q^3 + 6a_1 Q^4. \end{aligned} \tag{7}$$

***Step* 4.**

We substitute expressions (5)-(7) to Equation (3). Then we collect all terms with the same powers of function $Q(z)$ and equate expressions to zero. As a result we obtain algebraic system of equations. Solving this system we get the values of unknown parameters.

3. Konopelchenko-Dubrovsky Equation

The wave variable $z = kx + ly - wt$ transforms the KD equation (1) into a system of ODEs:

$$\begin{cases} -wu_z - k^3 u_{zzz} - 6bkuu_z + \dfrac{3}{2}a^2 k u^2 u_z - 3lv_z + 3aku_z v = 0, & (8a) \\ lu_z = kv_z, & (8b) \end{cases} \tag{8}$$

Integrating Equation (8b) with respect to z and neglecting the constant of integration we obtain

$$v(z) = \frac{l}{k} u(z). \tag{9}$$

Substituting (9) into Equation (8a), we obtain the ordinary differential equation:

$$-\left(kw + 3l^2\right) u_z - k^4 u_{zzz} - 3k\left(2bk - al\right) uu_z + \frac{3}{2} a^2 k^2 u^2 u_z = 0. \tag{10}$$

Integrating Equation (10) with respect to z, we have

$$C_1-\left(kw+3l^2\right)u-k^4u_{zz}-\frac{3k}{2}\left(2bk-al\right)u^2+\frac{a^2k^2}{2}u^3=0, \tag{11}$$

where C_1 is integration constant.

The pole order of Equation (11) is $N=1$. So we look for solution of Equation (11) in the following form

$$u(z)=a_0+a_1Q(z). \tag{12}$$

Substituting (12) into Equation (11) and taking into account relations (7) we obtain the system of algebraic equations in the form

$$\begin{cases} -2k^4a_1+\frac{1}{2}a^2k^2a_1^3=0, \\ 3k^4a_1-\frac{3}{2}k\left(2bk-al\right)a_1^2+\frac{3}{2}a^2k^2a_0a_1^2=0, \\ -k^4a_1-\left(kw+3l^2\right)a_1-3k\left(2bk-al\right)a_0a_1+\frac{3}{2}a^2k^2a_0^2a_1=0, \\ C_1-\left(kw+3l^2\right)a_0-\frac{3}{2}k\left(2bk-al\right)a_0^2+\frac{1}{2}a^2k^2a_0^3=0. \end{cases} \tag{13}$$

From (13) we have following values of coefficients a_0, a_1 and parameters C_1, w

$$\begin{cases} a_0=-\frac{k^2a-2bk+al}{ka^2}, \quad a_1=\frac{2k}{a}, \quad w=\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}, \\ C_1=-\frac{1}{2}\frac{\left(k^2a-2bk+al\right)\left(-4b^2k^2+4bkal-al^2-2bk^3a+a^2lk^2\right)}{ka^4}. \end{cases} \tag{14}$$

$$\begin{cases} a_0=\frac{k^2a+2bk-al}{ka^2}, \quad a_1=-\frac{2k}{a}, \quad w=\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}, \\ C_1=-\frac{1}{2}\frac{\left(k^2a+2bk-al\right)\left(4b^2k^2-4bkal+al^2-2bk^3a+a^2lk^2\right)}{ka^4}. \end{cases} \tag{15}$$

Using the values of parameters (14) we have the following solution of Equations. (12), (9)

$$\begin{cases} u(z) = -\dfrac{k^2a - 2bk + al}{ka^2} + \dfrac{2k}{a}Q(z), \\ v(z) = -\dfrac{lk^2a - 2lbk + al^2}{k^2a^2} + \dfrac{2l}{a}Q(z). \end{cases} \tag{16}$$

Combining (16) with (6), we obtain the exact solution to Equation (11) and the exact solution to the KD equation can be written as

$$\begin{cases} u(x,y,t) = -\dfrac{k^2a - 2bk + al}{ka^2} + \dfrac{2k}{a}\dfrac{1}{1+ce^{\left(kx+ly-\left(\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}\right)t\right)}}, \\ v(x,y,t) = -\dfrac{lk^2a - 2lbk + al^2}{k^2a^2} + \dfrac{2l}{a}\dfrac{1}{1+ce^{\left(kx+ly-\left(\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}\right)t\right)}}. \end{cases} \tag{17}$$

Using the values of parameters (15) we have following solution of Eqs. (12), (9)

$$\begin{cases} u(z) = \dfrac{k^2a + 2bk - al}{ka^2} - \dfrac{2k}{a}Q(z), \\ v(z) = \dfrac{lk^2a + 2lbk - al^2}{k^2a^2} - \dfrac{2l}{a}Q(z). \end{cases} \tag{18}$$

Combining (18) with (6), we obtain the exact solution to Equation (11) and the exact solution to the KD equation can be written as

$$\begin{cases} u(x,y,t) = \dfrac{k^2a + 2bk - al}{ka^2} - \dfrac{2k}{a}\dfrac{1}{1+ce^{\left(kx+ly-\left(\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}\right)t\right)}}, \\ v(x,y,t) = \dfrac{lk^2a + 2lbk - al^2}{k^2a^2} - \dfrac{2l}{a}\dfrac{1}{1+ce^{\left(kx+ly-\left(\frac{k^4a^2-6l^2a^2-12b^2k^2+12kal-3al^2}{2ka^2}\right)t\right)}}. \end{cases} \tag{19}$$

4. The (*N*+1)-Dimensional Combined Sinh-Cosh-Gordon Equation

Let us consider the (n+1)-dimensional combined sinh-cosh-Gordon equation in the form

$$u_{tt} - \sum_{i=1}^{n}\frac{\partial^2 u}{\partial x_i^2} + \alpha\sinh(u) + \beta\cosh(u) = 0. \tag{20}$$

By using the wave variable $z=\sum_{i=1}^{n} k_i x_i - wt$ we transform the (n+1)-dimensional combined sinh-cosh-Gordon equation (20) into the ODE:

$$\left(w^2-\sum_{i=1}^{n} k_i^2\right)u_{zz}+\alpha\sinh(u)+\beta\cosh(u)=0. \tag{21}$$

Engaging the Painleve property

$$v=\exp(u), \tag{22}$$

or equivalently

$$u=\ln(v),$$

we find

$$u_z=\frac{v'}{v},\quad u_{zz}=\frac{v''}{v}-\left(\frac{v'}{v}\right)^2.$$

The transformation (22) also gives

$$\sinh(u)=\frac{v-v^{-1}}{2},\quad \cosh(u)=\frac{v+v^{-1}}{2}, \tag{23}$$

Equivalently

$$u(z)=\cosh^{-1}\left(\frac{v+v^{-1}}{2}\right). \tag{24}$$

Substituting the transformations introduced above into Equation (21) yields the ODE

$$2\left(\sum_{i=1}^{n} k_i^2-w^2\right)(v')^2+2\left(\sum_{i=1}^{n} k_i^2-w^2\right)vv''+(\alpha+\beta)v^3-(\alpha+\beta)v=0. \tag{25}$$

The pole order of Equation (25) is $N=2$. So we look for solution of Equation (25) in the following form

$$v(z)=a_0+a_1Q(z)+a_2Q^2(z). \tag{26}$$

We substitute Equation (26) into Equation (25) and collect all terms with the same power in $Q_i\left(i=0,1,2,\ldots\right)$. Equating each coefficient of the polynomial to zero yields a set of simultaneous algebraic equations omitted here for the sake of brevity. Solving these algebraic equations by either Maple or Mathematica, we obtain

$$a_0=\pm\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\quad a_1=-4\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\quad a_2=4\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\quad w=\pm\sqrt{\left(\sum_{i=1}^{n}k_i\right)-\left(\sqrt{\alpha^2-\beta^2}\right)}. \tag{27}$$

Using values of parameters (27) we have the following solution of Equation (26)

$$v\left(z\right)=\sqrt{\frac{\alpha-\beta}{\alpha+\beta}}\left(4Q^2\left(z\right)-4Q\left(z\right)\pm1\right). \tag{28}$$

Combining (28) with (6), we obtain the exact solution to Equation (25) in the form

$$v\left(z\right)=\sqrt{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{4}{\left(1+ce^z\right)^2}-\frac{4}{\left(1+ce^z\right)}\pm1\right). \tag{29}$$

By using (24), we have the exact solution of the (n+1)-dimensional combined sinh-cosh-Gordon equation in the form

$$u\left(x,t\right)=\cosh^{-1}\left[\frac{\sqrt{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{4}{\left(1+ce^z\right)^2}-\frac{4}{\left(1+ce^z\right)}\pm1\right)+\left(\sqrt{\frac{\alpha-\beta}{\alpha+\beta}}\left(\frac{4}{\left(1+ce^z\right)^2}-\frac{4}{\left(1+ce^z\right)}\pm1\right)\right)^{-1}}{2}\right], \tag{30}$$

where $x=\left(x_1,x_2,\ldots,x_n\right)$, $z=\sum_{i=1}^{n}k_ix_i\mp\left(\sqrt{\left(\sum_{i=1}^{n}k_i\right)-\left(\sqrt{\alpha^2-\beta^2}\right)}\right)t$.

5. The Maccari System

For the system [Bekir (2009)]

$$\begin{cases} iu_t+u_{xx}+uv=0, \\ v_t+v_y+\left(|u|^2\right)_x=0. \end{cases} \tag{31}$$

In order to find traveling wave solutions of Equation (31), we set

$$u(x,y,t)=e^{i(px+qy+rt)}u(z), \quad v(x,y,t)=v(z),$$
$$z=x+\alpha y-2pt, \tag{32}$$

where p,q,r and α are constants.

Substituting (32) into (31), which is then reduced to the following nonlinear ordinary differential equation:

$$\begin{cases} -\left(r+p^2\right)u+u_{zz}+uv=0, & (33a) \\ \left(\alpha-2p\right)v_z+2uu_z=0. & (33b) \end{cases} \tag{33}$$

Integrating Equation (33b) with respect to z and neglecting the constant of integration we obtain

$$v(z)=-\frac{1}{(\alpha-2p)}u^2(z). \tag{34}$$

Substituting (34) into Equation (33a), we obtain ordinary differential equation:

$$-(\alpha-2p)\left(r+p^2\right)u+(\alpha-2p)u_{zz}-u^3=0. \tag{35}$$

The pole order of Equation (35) is $N=1$. So we look for solution of Equation (35) in the following
form

$$u(z)=a_0+a_1Q(z). \tag{36}$$

We substitute Equation (36) into Equation (35) and collect all terms with the same power in $Q_i\ (i=0,1,2,...)$. Equating each coefficient of the polynomial to zero yields a set of simultaneous algebraic equations omitted here for the sake of brevity. Solving these algebraic equations by either Maple or Mathematica, we obtain

$$r=-\left(p^2+\frac{1}{2}\right), \quad \alpha=2p+\frac{a_1^2}{2}, \quad a_0=-\frac{a_1}{2}, \tag{37}$$

where a_1 is arbitrary constant.

Using the values of parameters (37) we have following solution of Eqs. (36), (34)

$$\begin{cases} u(z) = a_1\left(Q(z) - \dfrac{1}{2}\right), \\ v(z) = 2Q(z) - 2Q^2(z) - \dfrac{1}{2}. \end{cases} \tag{38}$$

Combining (38) with (6), we obtain the exact solution to Equation (35) and the exact solution to the Maccari system can be written as

$$\begin{cases} u(x,y,t) = a_1 e^{i\left(px + qy - \left(p^2 + \frac{1}{2}\right)t\right)}\left(-\dfrac{1}{2} + \dfrac{1}{1 + ce^{\left(x + \left(2p + \frac{a_1^2}{2}\right)y - 2pt\right)}}\right), \\ v(x,y,t) = -\dfrac{2}{\left(1 + ce^{\left(x + \left(2p + \frac{a_1^2}{2}\right)y - 2pt\right)}\right)^2} + \dfrac{2}{1 + ce^{\left(x + \left(2p + \frac{a_1^2}{2}\right)y - 2pt\right)}} - \dfrac{1}{2}. \end{cases} \tag{39}$$

6. Conclusion

We have thus obtained exact solutions of Konopelchenko-Dubrovsky equation and (n+1)-dimensional combined sinh-cosh-Gordon equation and the Maccari system by using the modification of truncated expansion method. The efficiency of this method was aptly demonstrated. The solutions obtained are potentially significant and important for the explanation of some practical physical problems. The method may also be applied to other nonlinear partial differential equations.

Acknowledgment

The authors are highly grateful to the referees for their constructive comments.

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