Volume 7 | Issue 1 Article 25

6-2012

A Novel Algorithm to Forecast Enrollment Based on Fuzzy Time Series

Haneen T. Jasim
Mosul University

Abdul G. Jasim Salim
Mosul University

Kais I. Ibraheem
Mosul University

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 7, Issue 1 (June 2012), pp. 385 – 397

Applications and Applied Mathematics:
An International Journal
(AAM)

A Novel Algorithm to Forecast Enrollment Based on Fuzzy Time Series

Haneen Talal Jasim and Abdul Ghafoor Jasim Salim
Department of Mathematics
Mosul University
Mosul, Iraq

Kais Ismail Ibraheem
Department of Computers Sciences
Mosul University
Mosul, Iraq
kaisismail@yahoo.com

Received: January 2, 2012; Accepted: March 27, 2012

Abstract

In this paper we propose a new method to forecast enrollments based on fuzzy time series. The proposed method belongs to the first order and time-variant methods. Historical enrollments of the University of Alabama from year 1948 to 2009 are used in this study to illustrate the forecasting process. By comparing the proposed method with other methods we will show that the proposed method has a higher accuracy rate for forecasting enrollments than the existing methods.

Keywords: Fuzzy logical groups, fuzzified enrollments, fuzzy time series

MSC 2010: 20N25, 03B52, 37M10, 47S40

1. Introduction:

It is obvious that forecasting activities play an important role in our daily life. The classical time series methods can not deal with forecasting problems in which the values of time series are in linguistic terms, represented by fuzzy sets. Chena and Hsub (2004**),** proposed a new method to forecast enrollments using fuzzy time series. Şah and Degtiarev (2005) introduced a novel improvement of forecasting approach based on using time-invariant fuzzy time series. In contrast to traditional forecasting methods, fuzzy time series can also be applied to problems, in which historical data are linguistic values. It is shown that the proposed time-invariant method improves the performance of the forecasting process. Further, the effect of using different number of fuzzy sets is tested. Historical enrollment of the University of Alabama is used in this study to illustrate the forecasting process. Chen and Chung (2006) studied the forecasting enrollments of students by using fuzzy time series and genetic algorithms. They presented a new method to deal with forecasting problems based on fuzzy time series and genetic algorithms. Tahseen et al. (2007), studied multivariate high order fuzzy time series.

Lee et al. (2009), proposed the adoption of the weighted and the difference between actual data toward midpoint interval based on fuzzy time series, by using the enrollment of the University of Alabama and the University of Technology Malaysia (UTM). Chen and Chen (2009) discussed a new method to forecast the TAIEX based on fuzzy time series. The method is to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) based on fuzzy time series,. First, they fuzzify the historical data of the factor into fuzzy sets with a fixed length of intervals to form fuzzy logical relationships. Next, group the fuzzy logical relationships into fuzzy logical relationship groups. Then evaluate the leverage of fuzzy variations between the main factor and the secondary factor to forecast the TAIEX. Memmedli and Ozdemir (2010) studied an application of fuzzy time series to improve ISE forecasting. They achieved this by using different length of intervals with neural networks according to various performance measures.

2. Basic Concepts of Fuzzy Time Series:

This section briefly summarizes the basic fuzzy and fuzzy time series concepts. The main difference between the fuzzy time series and traditional time series is that the values of the fuzzy time series are represented by fuzzy sets rather than real values. Let U be the universe of discourse, where $U = \{u_1, u_2, ..., u_n\}$. A fuzzy set defined in the universe of discourse U can be represented as follows:

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + ... + f_A(u_n)/u_n$$

where f_A denotes the membership function of the fuzzy set A $f_A : U \rightarrow [0,1]$ and $f_A(u_i)$ denotes the degree of membership of u_i belonging to the fuzzy set A and $f_A(u_i) \in [0,1]$ and $1 \le i \le n$.

Definition 1. Assume $Y(t) \subset R$ (real line), t = ..., 0, 1, 2, ..., n to be a universe of discourse defined by the fuzzy set $f_i(t)$. $F(t)$ consisting of $f_i(t)$, i = 1, 2, ..., n is defined as a fuzzy time series on $Y(t)$. At that, $F(t)$ can be understood as a *linguistic variable*, whereas $f_i(t)$ i = 1, 2, ..., n are possible *linguistic values* of $F(t)$ [Şah and Degtiarev (2005); Chen and Chung (2006)].

Definition 2. If $F(t)$ is caused by $F(t - 1)$ denoted by $F(t - 1) \rightarrow F(t)$ then this relationship can be represented by $F(t) = F(t - 1) \circ R(t, t - 1)$ where the symbol " $\circ$ " is an operator; $R(t, t - 1)$ is a fuzzy relation between $F(t)$ and $F(t-1)$ and is called the first-order model of $F(t)$, [Chen and Hsu (2004); Nasser et al. (2008)].

Definition 3. Denoting $F(t-1)$ by A_i and $F(t)$ by A_j the relationship between $F(t-1)$ and $F(t)$ can be defined by a logical relationship $A_i \rightarrow A_j$ [Şah and Degtiarev (2005); Lotfi and Firozja (2007)].

Definition 4. Let $R(t; t-1)$ be a first-order model of $F(t)$. If for any t, $R(t; t-1) = R(t - 1; t - 2)$, then $F(t)$ is called a time-invariant fuzzy time series. Otherwise, it is called a time-variant fuzzy time series (Chen and Chung, 2006).

Definition 5. Fuzzy logical relationships, which have the same left- hand sides, can be grouped together into fuzzy logical relationship groups. For example, for the identical left-hand side A_i such grouping can be depicted as follows [Şah and Degtiarev (2005)]:

$$\left.\begin{array}{l} A_i \rightarrow A_{j1} \\ A_i \rightarrow A_{j2} \\ . \\ . \end{array}\right\} \Rightarrow A_i \rightarrow A_{j1}, A_{j2}, \ldots$$

3. A New Method for Forecasting Enrollments Based on Fuzzy Time Series

In this paper, we present a new method to forecast the enrollments of the University of Alabama based on fuzzy time series. The historical enrollments of the University of Alabama are shown in Table 1(The Board of Trustees The University of ALABAMA) The main steps for the fuzzy time series forecasting is shown in the following algorithm (Şah and Degtiarev, 2005) :

Step 1: Define the universe of discourse (universal set *U)* starting from variations of the historical enrollment data.

Step 2: Partition *U* into equally long subintervals.

Step 3: Define fuzzy sets *A*.

Step 4: Fuzzify variations of the historical enrollment data.

Step 5: Determine fuzzy logical relationships $A_i \rightarrow A_j$.

Step 6: Group fuzzy logical relationships (in Step 5) having the same left-hand side and calculate F_i for each *i*-th fuzzy logical relationship group.

Step 7: Forecast and deffuzify the forecasted outputs.

Step 8: Calculate the forecasted enrollments.

In the following we present the algorithm of the proposed method.

4. The Algorithm of the Proposed Method

In this section we proposed a novel algorithm to forecast enrollment of the University of Alabama based on fuzzy time series .The steps of it are given below:

Step 1: Collect the data.

Step 2: Determine the maximum and the minimum of the interval $\left[D_{\min} - D_1, D_{\max} + D_2\right]$, where D1 and D2 are constant to define the universe of discourse *U*.

Step 3: The determination of the interval I can be performed by the average based length method (Duru and Yoshida, 2009) as follows:

a: Find the difference of D_{vt}, D_{vt-1} then find the rest of the first differences, then find the average of the first differences.

$$av = \frac{\sum_{i=1}^{n}\left(D_i - D_{i-1}\right)}{n-1},$$ where n is the number of observation.

b: Take one half of the average.(i.e. B=av/2)

c: Determine the range where B is located.

d: According to the base, the length of the interval I is taken by rounding B, according to Table (6) (Duru and Yoshida,2009).

Step 4: The number of the fuzzy intervals is defined by:

$$m = \left(D_{\max} + D_1 - D_{\min} + D_2\right)/I$$

Step 5: Determine the fuzzy logical sets as follow: $A_i = \left(d_{i-1}, d_i, d_{i+1} d_{i+2}\right)$ Starting by $A_1 = \left(d_0, d_1, d_2, d_3\right)$ and ending $A_m = \left(d_{m-1}, d_m, d_{m+1}, d_{m+2}\right)$, where $d_0 = D_{\min} - I$ $d_{m+2} = D_{\max}$ and fuzzify the historical enrollments shown in Table 1, where fuzzy set A_i denotes a linguistic value of the enrollments represented by a fuzzy set and $1 \le i \le$ m.

Step 6: Determine the fuzzy logical relations as:

$$A_j \to A_i$$

Step 7: Find the fuzzy logical groups.

Step 8: Calculate the forecasted outputs. The forecasted value at time *t* is determined by the following rules:

A: If the fuzzy logical relationship group A_j is empty $A_j \to \phi$, then the value of F_{vt} is the middle of the fuzzy interval A_j which is

$$A_j = \left(d_{j-1}, d_j, d_{j+1}, d_{j+2}\right)$$

B: If the fuzzy logical relationship group A_j is one to one $A_j \to A_k$ then the intervals that contain the forecasted value is A_k and we will use the following rules to find the forecasting:

Assume that the fuzzy logical relationship is $A_i \to A_j$, where Ai denotes the fuzzified enrollment of year n-1 and Aj denotes the fuzzified enrollment of year n. Compute Y as follows:

Y = {[enrollment of year (*n*) - enrollment of year(*n*-1)] – [enrollment of year (*n*-1)- enrollment of year (*n*-2)]},

Then, compare the value of Y with zero and apply the following:

(1) If $j > i$, and $Y > 0$, then the trend of the forecasting will go up and we use the following **Rule 2** to forecast the enrollments.

(2) If $j > i$, and $Y < 0$, then the trend of the forecasting will go down and we use the following **Rule 3** to forecast the enrollments.

(3) If $j < i$, and $Y > 0$, then the trend of the forecasting will go up and we use the following **Rule 2** to forecast the enrollments.

(4) If $j < i$, and $Y < 0$, then the trend of the forecasting will go down and we use the following **Rule 3** to forecast the enrollments.

(5) If $j = i$, and $Y > 0$, then the trend of the forecasting will go up and we use the following **Rule 2** to forecast the enrollments.

(6) If $j = i$, and $Y < 0$, then the trend of the forecasting will go down and we use the following **Rule 3** to forecast the enrollments, where **Rule 1**, **Rule 2** and **Rule 3** are shown as follows:

Rule 1: When forecasting the enrollment of year 1950, there are no data before the enrollments of year 1947, therefore we are not able to calculate Z.

Z = {[enrollment (1949)- enrollment(1948)] – [enrollment (1948)- enrollment(1947)]}

Therefore,

if $-\left|Enrollment\ \ of\ year\ (1949) - Enrollment\ \ of\ year\ (1948)\right|/2 > \frac{A_j}{2}$, then, the trend of the forecasting of this interval will be upward and $F_n = 0.75\ of\ A_j$.

if $\left|Enrollment\ \ of\ year\ (1949) - Enrollment\ \ of\ year\ (1948)\right|/2 = \frac{A_j}{2}$, then the forecasting enrollment falls at the middle value of this interval.

if $\left|Enrollment\ \ of\ year\ (1949) - Enrollment\ \ of\ year\ (1948)\right|/2 < \frac{A_j}{2}$, then the trend of the forecasting of this interval will be downward, and $F_n = 0.25\ of\ A_j$.

<u>Rule 2</u>: If $x = |Y| * 2 + Enrollment\ of\ year\ (n-1) \in A_j$ $or\ x = Enrollment\ of\ year\ (n-1) - |Y| * 2 \in A_j$, then the trend of the forecasting of this interval will be upward and $F_n = 0.75\ of\ A_j$.

If $x = \frac{|Y|}{2} + Enrollment\ \ of\ year\ (n-1) \in A_j$ $or\ x = Enrollment\ of\ year\ (n-1) - \frac{|Y|}{2} \in A_j$, then the trend of the forecasting of this interval will be downward and $F_n = 0.25\ of\ A_j$.

If neither is the case, then the forecasting enrollment will be the middle value of the interval A_j

Rule 3: If $x = \frac{|Y|}{2} + Enrollment \;\; of \;\; year \; (n-1) \in A_j$

or $x = Enrollment\, of\, year\,(n-1) - \frac{|Y|}{2} \in A_j$, then the trend of the forecasting of this interval will be downward and $F_n = 0.25 \; of \; A_j$.

If $x = |Y| * 2 + Enrollment \;\; of \;\; year \; (n-1) \in A_j$

or $x = Enrollment \; of \; year \; (n-1) - |Y| * 2 \in A_j$, then the trend of the forecasting of this interval will be upward and $F_n = 0.75 \; of \; A_j$.

If neither is the case, then we let the forecasting enrollment will be the middle value of the interval A_j [Chen and Hsu (2004)].

C: If the fuzzy logical relationship group A_j is one too many $A_j \rightarrow A_{k1}, A_{k2}, \ldots, A_{kp}$ then the intervals that contain the forecasted value is can be determined as follows:

a: If the difference between any two of $k1, k2, \ldots\ldots, kp \leq 2$ then the interval that contain the forecasted value is

$$IF_{vt} = \frac{A_{\mathrm{k1}} + A_{\mathrm{k2}} + \ldots.. + A_{\mathrm{kp}}}{\mathrm{p}}$$

$$= \left(\frac{d_{k1-1} + d_{k2-1} + \ldots.. + d_{kp-1}}{p}, \frac{d_{k1} + d_{k2} + \ldots.. + d_{kp}}{p}, \frac{d_{k1+1} + d_{k2+1} + \ldots.. + d_{kp+1}}{p}, \frac{d_{k1+2} + d_{k2+2} + \ldots.. + d_{kp+2}}{p} \right)$$

and the forecasting is the middle of this interval.

b: If the difference between any two of $k1, k2, \ldots\ldots, kp > 2$, then the forecasted value is the middle of the following interval :

$$IF_{vt} = \frac{A_{k1} + A_{k2} + \ldots.. + A_{ki-1} + A_{ki+1} + \ldots\ldots + A_{kp}}{p}$$
$$= \left(\frac{d_{k1-1} + \ldots + d_{(ki-1)-1} + d_{(ki+1)-1} + \ldots + d_{kp-1}}{p}, \frac{d_{k1} + \ldots + d_{(ki-1)} + d_{(ki+1)} + \ldots + d_{kp}}{p}, \frac{d_{k1+1} + \ldots + d_{(ki-1)+1} + d_{(ki+1)+1} + \ldots + d_{kp+1}}{p}, \frac{d_{k1+2} + \ldots + d_{(ki-1)+2} + d_{(ki+1)+2}. + \ldots + d_{kp+2}}{p} \right),$$

where A_{ki} is the intervals that have deference > 2, i = 1,2,…, p and the forecasting of the fuzzy sets A_{ki} is calculated as one to one fuzzy logical relationship by applying **Step 8**.

5. The Results

We will apply the proposed algorithm to forecast the historical enrollments of the University of Alabama. The universe set is U = [5172-172, 27052+948], i.e., U = [5000, 28000]. The determination of the interval I (which represent the length of the interval) can be performed by the average based length method [Duru and Yoshida (2009)] as follows:

Find the difference of D_{vt}, D_{vt-1} then find the rest of the first difference then find the average of the first difference and according to the data I = ,148.6557 , then by rounding I by using table (6) I = 100, then the number of the interval $m = 229$. Now define each linguistic term represented by a fuzzy set where $1 \le i \le m$. as follow:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + \ldots\ldots + 0/u_{229}$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + \ldots\ldots + 0/u_{229}$$
$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + \ldots\ldots + 0/u_{229}$$
$$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + \ldots\ldots + 0/u_{229}$$
$$\vdots$$
$$A_{229} = 0/u_1 + 0/u_2 + 0/u_3 + \ldots\ldots + 0/u_{227} + 0.5/u_{228} + 1/u_{229}.$$

Then fuzzify the historical enrollments shown in Table 1, where fuzzy set A_i denotes a linguistic value of the enrollments represented by a fuzzy set, and $1 \le i \le 229$. Establish fuzzy logical relationships based on the fuzzified enrollments where the fuzzy logical relationship

$A_j \to A_q$ denotes " if the fuzzified enrollments of year n-1 is A_i, then the fuzzified enrollments of year n is A_q " as shown in Table 2.

In the following, we use the mean square error (MSE) to compare the forecasting results of different forecasting methods where (MSE) is:

$$MSE = \frac{\sum_{i=1}^{n}\left(Actual\ Enrollment\ _i - Forecasted\ Enrollment\ _i\right)^2}{n}.$$

6. Conclusion

In this paper we have proposed a new method for forecasting fuzzy time series. Historical enrollments of the University of Alabama are used in this study to illustrate the forecasting process. From Table 4 we can see that the proposed method presents better forecasting results and can get a higher forecasting accuracy rate for forecasting enrollments than the existing methods. Table 5 shows that our method produce a forecasting with MSE = 699 which is the smaller MSE than that of the existing methods.

REFERENCE

Allahviranloo, T., Lotfi, F. M. and Firozja, A. (December 2007). Fuzzy Efficiency Measure with Fuzzy Production Possibility Set, AAM, Vol. 2, No. 2, pp. 152 – 166.

Chen, C. D. and Chen, S. M. (October, 2009). A New Method to Forecast the TAIEX Based on Fuzzy Time Series, IEEE International Conference on Systems, Man and Cybernetics San Antonio, TX, USA, pp. 3450-3455.

Chen, S. M. and Hsu, C. C. (2004).A New Method to Forecast Enrollments Using Fuzzy Time Series, International Journal of Applied Science and Engineering, No. 2, Vol. 3, pp. 234-244.

Chen, S. M. and Chung, N. Yi (2006). Forecasting Enrollments of Students by Using Fuzzy Time Series and Genetic Algorithms, Information and Management Sciences, No.3, Vol.17, pp. 1-17.

Duru, O. and Yoshida S. (2009).Comparative analysis of fuzzy time series and judgmental forecasting : an empirical study of forecasting dry bulk shipping index.

Jilani, T. A., Burney, S. M. and Ardil, C. (2007). Multivariate High Order Fuzzy Time Series Forecasting For Car Road Accidents, World Academy of Science, Engineering and Technology, Vol. 25, pp. 288-293.

Jilani, T. A., Burney, S. M. Ardil, C. (2007). Fuzzy Metric Approach For Fuzzy Time Series Forecasting based on Frequency Density Based Partitioning, World Academy of Science, Engineering and Technology, Vol. 34, pp.1-6.

Lee, M. H., Efendi, R., and Ismail, Z. (2009). Modified Weighted For Enrollment Forecasting Based on Fuzzy Time Series, MATEMATEKA, NO.1, Vol 25, pp. 67-78.

Memmedli, M. and Ozdemir, O. (2010). An Application of Fuzzy Time Series to Improve ISE Forecasting, WSEAS TRANSACTIONS on mathematics. Issue 1, Vol. 9,pp. 12-21.

Nasser, A.T. Mikaeilv Narsis, K.A. and Rasol, S.M. (June, 2008). Signed Decomposition of Fully Fuzzy Linear Systems, AAM, Vol. 3, Issue 1.

Şah, M. and Degtiarev, Y.K. (January, 2005). Forecasting Enrollment Model Based on First-Order Fuzzy Time Series, World Academy of Science, Engineering and Technology. Vol. 1, pp. 375-378, The Board of Trustees The University of Alabama.

Table 1. The historical enrollments of the University of Alabama

Year	Enrolment	Year	Enrolment	Year	Enrollment
1948	8916	1969	13035	1990	19328
1949	7974	1970	13017	1991	19366
1950	6293	1971	13055	1992	18804
1951	5269	1972	13563	1993	18909
1952	5172	1973	13867	1994	18707
1953	5652	1974	14696	1995	18561
1954	6111	1975	15460	1996	17572
1955	7038	1976	15311	1997	17877
1956	7112	1977	15603	1998	17929
1957	7032	1978	15861	1999	18267
1958	7089	1979	16807	2000	18859
1959	7407	1980	16919	2001	18735
1960	7848	1981	16388	2002	19181
1961	8257	1982	15433	2003	19828
1962	8560	1983	15497	2004	20512
1963	8879	1984	15145	2005	20969
1964	9724	1985	15163	2006	21835
1965	10938	1986	15984	2007	23878
1966	11975	1987	16859	2008	25580
1967	12251	1988	18150	2009	27052
1968	12816	1989	18970		

Table 2. Fuzzified enrollments of the University of Alabama

Year	Act. Enr.	Fuz. En	Year	Act. Enr.	Fuz. En	Year	Act. Enr.	Fuz. En	Year	Act. Enr.	Fuz. En.
1948	8916	A_{40}	1964	9724	A_{48}	1980	16919	A_{120}	1996	17572	A_{126}
1949	7974	A_{30}	1965	10938	A_{60}	1981	16388	A_{114}	1997	17877	A_{129}
1950	6293	A_{13}	1966	11975	A_{70}	1982	15433	A_{105}	1998	17929	A_{130}
1951	5269	A_{3}	1967	12251	A_{73}	1983	15497	A_{105}	1999	18267	A_{137}
1952	5172	A_{7}	1968	12816	A_{79}	1984	15145	A_{102}	2000	18859	A_{139}
1953	5652	A_{7}	1969	13035	A_{81}	1985	15163	A_{102}	2001	18735	A_{138}
1954	6111	A_{12}	1970	13017	A_{81}	1986	15984	A_{110}	2002	19181	A_{142}
1955	7038	A_{21}	1971	13055	A_{81}	1987	16859	A_{119}	2003	19828	A_{149}
1956	7112	A_{22}	1972	13563	A_{86}	1988	18150	A_{132}	2004	20512	A_{156}
1957	7032	A_{21}	1973	13867	A_{89}	1989	18970	A_{140}	2005	20969	A_{160}
1958	7089	A_{21}	1974	14696	A_{97}	1990	19328	A_{144}	2006	21835	A_{169}
1959	7407	A_{25}	1975	15460	A_{105}	1991	19366	A_{144}	2007	23878	A_{189}
1960	7848	A_{29}	1976	15311	A_{104}	1992	18804	A_{139}	2008	25580	A_{206}
1961	8257	A_{33}	1977	15603	A_{107}	1993	18909	A_{140}	2009	27052	A_{221}
1962	8560	A_{36}	1978	15861	A_{109}	1994	18707	A_{138}			
1963	8879	A_{39}	1979	16807	A_{119}	1995	18561	A_{136}			

Table 3. Forecasted enrollments of the University of Alabama

Year	Actual Enrolment	Forecasted Enrollment	Year	Actual Enrolment	Forecasted Enrollment
1948	8916	-----	1979	16807	16850
1949	7974	7950	1980	16919	16950
1950	6293	6250	1981	16388	16350
1951	5269	5250	1982	15433	15450
1952	5172	5150	1983	15497	15450
1953	5652	5625	1984	15145	15150
1954	6111	6150	1985	15163	15150
1955	7038	7050	1986	15984	15975
1956	7112	7150	1987	16859	16850
1957	7032	7050	1988	18150	18150
1958	7089	7050	1989	18970	18975
1959	7407	7450	1990	19328	19350
1960	7848	7850	1991	19366	19350
1961	8257	8250	1992	18804	18850
1962	8560	8550	1993	18909	18950
1963	8879	8850	1994	18707	18750
1964	9724	9750	1995	18561	18550
1965	10938	10950	1996	17572	17550
1966	11975	11950	1997	17877	17850
1967	12251	12250	1998	17929	17950
1968	12816	12850	1999	18267	18650
1969	13035	13050	2000	18859	18850
1970	13017	13050	2001	18735	18750
1971	13055	13050	2002	19181	19150
1972	13563	13550	2003	19828	19850
1973	13867	13850	2004	20512	20550
1974	14696	14650	2005	20969	20950
1975	15460	15450	2006	21835	21850
1976	15311	15317	2007	23878	23850
1977	15603	15650	2008	25580	25550
1978	15861	15825	2009	27052	27050

Table 4. A comparison of the forecasting results of different forecasting methods

year	Actual Enrollment	Huarng (Chen and Hsu 2004)	Chen'c (Chen and Hsu 2004)	Chen& Chia-Ching (Chen and Hsu 2004)	Jilani, Burney,& Ardil (2007)	The proposed method
1971	13055				13579	------
1972	13563	14000		13750	13798	13550
1973	13867	14000		13875	13798	13850
1974	14696	14000	14500	14750	14452	14650
1975	15460	15500	15500	15375	15373	15450
1976	15311	15500	15500	15312.5	15373	15317
1977	15603	16000	15500	15625	15623	15650
1978	15861	16000	15500	15812.5	15883	15825
1979	16807	16000	16500	16833.5	17079	16850
1980	16919	17500	16500	16833.5	17079	16950
1981	16388	16000	16500	16416.2	16497	16350
1982	15433	16000	15500	15375	15373	15450
1983	15497	16000	15500	15375	15373	15450
1984	15145	15500	15500	15125	15024	15150
1985	15163	16000	15500	15125	15024	15150
1986	15984	16000	15500	15937.5	15883	15975
1987	16859	16000	16500	16833.5	17079	16850
1988	18150	17500	18500	18250	17991	18150
1989	18970	19000	18500	18875	18802	18975
1990	19328	19000	19500	19250	18994	19350
1991	19337	19500	19500	19250	18994	19350
1992	18876	19000	18500	18875	18916	18850
MSE		226611	86694	5353	41426	699

Table 5. A comparison of the MSE of the forecasted enrollment for different forecasting methods

Method	MSE
Shyi-Ming Chen and Chia-Ching(Chen and Hsu 2004)	5353
Lee, Efendi and Zuhaimy(Lee et al.2009)	16248.7
Shyi-Ming Chen and Nien-Yi Chung (Chen and Chung, 2006).	35324
Huarng (Chen and Hsu 2004).	226611
Jilani, Burney& Ardil (2007).	41426
The proposed method	699

Table 6. Base mapping table [Huarng (200 I b)]

Range	**Base**
0.1-1.0	0.1
1.1-10	1
11-100	10
101-1000	100
1001-10000	1000