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New Explicit Solutions for Homogeneous Kdv Equations of Third Order by Trigonometric and Hyperbolic Function Methods

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Abstract

In this paper, we study two-component evolutionary systems of the homogeneous KdV equation of the third order types (I) and (II). Trigonometric and hyperbolic function methods such as the sine-cosine method, the rational sine-cosine method, the rational sinh-cosh method, sech-csch method and rational tanh-coth method are used for analytical treatment of these systems. These methods, have the advantage of reducing the nonlinear problem to a system of algebraic equations that can be solved by computerized packages.

Keywords: Wave variables, Trigonometric function method, Hyperbolic function method, Homogeneous KdV equations of third order

MSC 2010: 83C15, 35C08, 35B10

1. Introduction

Applications in physics are modeled by nonlinear partial differential equations (PDEs). To understand these models, many researchers insist on explicit solutions needing powerful methods such as the Hirota bilinear method (A. M. Wazwaz, 2007a and 2008), the rational sine-cosine function method (Huabing Jia and Wei Xu, 2010), the Tanh method (Alquran and Al-Khaled, 2011a, b), the tanh-coth method (A. M. Wazwaz, 2007b), the sine-cosine method (Alquran, 2012), the extended tanh method (Ahmet Bekir, 2008 and Sami S. and Al-Khaled, 2010) and a host of other techniques for their solutions. The results are very interesting classes of solutions:

solitons, kinks and periodic. Solitons are solutions in the form *sech* and $sech^p$, the graph of a soliton is a wave that goes up only. It is not like periodic solutions *sine*, *cosine*, etc, that appears above and below the horizontal. Kink is also considered a soliton, it is in the form *tanh* not $tanh^2$. In kink the limit as $x \to \infty$, is a nonzero constant, unlike solitons where the limit is zero.

In this work, we consider two-component evolutionary system of the homogeneous KdV equations of the third order types (I) and (II) given respectively by

$$
\begin{aligned}
u_t &= u_{xxx} + uu_x + vv_x \\
v_t &= -2v_{xxx} - uv_x
\end{aligned}
\tag{1.1}
$$

and

$$
\begin{aligned}
u_t &= u_{xxx} + 2vu_x + uv_x \\
v_t &= uu_x .
\end{aligned}
\tag{1.2}
$$

Shukri and Al-Khaled (2010), used the extended tanh method to obtain soliton and kink solutions of the above systems.

In what follows we highlight the main features of the trigonometric and the hyperbolic function methods where more details and examples can be found in [(Wazwaz, 2005a, b), (Alquran, 2012) and (Alquran and Al-Khaled, 2011a)].

2. Trigonometric and Hyperbolic Function Methods

Here, we will highlight briefly the main steps of the methods that will be used in this paper. We first unite the independent variables x and t into one wave variable $\zeta = x - ct$ to convert the PDE

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) \tag{2.1}$$

into an ODE

$$Q(u, u', u'', u''', \ldots). \tag{2.2}$$

Equation (2.2) is then integrated as long as all terms contain derivatives.

2.1 The Sine-Cosine Method (Ansatze I)

The sine-cosine algorithm admits the use of the ansatze (Wazwaz, 2010)

$$u(x,t) = \lambda \cos^{\gamma}(\mu\zeta), \quad |\zeta| \leq \frac{\pi}{2\mu} \tag{2.3}$$

and the ansatze,

$$u(x,t) = \lambda \sin^{\gamma}(\mu\zeta), \quad |\zeta| \leq \frac{\pi}{\mu}\ , \tag{2.4}$$

where λ, c, μ and γ are parameters that will be determined. Substituting (2.3) or (2.4) into the reduced ODE gives a polynomial equation of cosine or sine terms. We then collect the coefficients of the resulting triangle functions and setting them to zeros, to get a system of algebraic equations among the unknowns λ, c, μ and γ. The problem is now completely reduced to an algebraic one. Having determined λ, c, μ and γ by algebraic calculations or by using Mathematica, the solutions proposed in (2.3) and in (2.4) follow immediately.

2.2 The Second Rational Sine-Cosine Function Method (Ansatze II)

The second rational sine-cosine functions methods can be expressed in the form (Wazwaz, 2010)

$$u(x,t) = \frac{a_0 + b_0 \sin(\mu\zeta)}{1 + a_1 \sin(\mu\zeta)} \tag{2.5}$$

and

$$u(x,t) = \frac{a_0 + b_0 \cos(\mu\zeta)}{1 + a_1 \cos(\mu\zeta)} \tag{2.6}$$

where a_0, a_1, b_0 , and μ are parameters that will be determined.

2.3 The Hyperbolic Function Method (Ansatze III)

This method (Wazwaz, 2007) admits the use of the following ansatze

$$u(x,t) = \frac{f(\mu\zeta)}{1 + \lambda f(\mu\zeta)}, \tag{2.7}$$

where $f(\mu\zeta)$ is anyone of the hyperbolic functions. The approach is simply used by applying the equation, setting the coefficients of the resulting hyperbolic functions to zero, and solving the resulting equations to determine the parameters λ and μ.

3. Homogeneous Kdv Equations of Third Order Type (I)

In this section we consider two-component evolutionary system of a homogeneous KdV equations of third order 3 type I

$$u_t = u_{xxx} + uu_x + vv_x,$$

$$v_t = -2v_{xxx} - uv_x. \tag{3.1}$$

Using the wave variable $\zeta = x - ct$ transforms (3.1) into the ODEs

$$0 = cu + u'' + \frac{1}{2}u^2 + \frac{1}{2}v^2 \tag{3.2}$$

and

$$u = c - \frac{2v'''}{v'}. \tag{3.3}$$

Substituting (3.3) into (3.2) gives

$$0 = -2(v')^2 v^{(5)} + 4v'v''v^{(4)} + 4v'(v^{(3)})^2 - 4c(v')^2 v^{(3)}$$

$$-4(v'')^2 v^{(3)} + \frac{3}{2}c^2 (v')^3 + \frac{1}{2}(v')^3 v^2. \tag{3.4}$$

<u>First</u>, we apply Ansatze (II); the second rational sine method. Substitute (2.5) into (3.4) to get

$$0 = a_0^2 + 3c^2 + 8c\mu^2 - 48a_1^2 c\mu^2 + 4\mu^4 + 48a_1\mu^4$$

$$+\sin(\mu\zeta)(2a_0 b_0 + 6a_1 c^2 - 32a_1 c\mu^2 - 16a_1\mu^4)$$

$$+\sin^2(\mu\zeta)(b_0^2 + 3a_1^2 c^2 + 8a_1^2 c\mu^2 + 4a_1^2\mu^4). \tag{3.5}$$

The above equation is satisfied only if the following system of algebraic equations holds

$$0 = a_0^2 + 3c^2 + 8c\mu^2 - 48a_1^2 c\mu^2 + 4\mu^4 + 48a_1\mu^4$$

$$0 = 2a_0 b_0 + 6a_1 c^2 - 32a_1 c\mu^2 - 16a_1\mu^4$$

$$0 = b_0^2 + 3a_1^2 c^2 + 8a_1^2 c\mu^2 + 4a_1^2\mu^4. \tag{3.6}$$

Solving the above system involves numbers one calculations, so we consider some particular cases of c and μ. Therefore,

$$a_0 = \pm 11,\ a_1 = \mp \frac{i\sqrt{5}}{2},\ b_0 = \pm \frac{i\sqrt{5}}{2},\ c = 1,\ \mu = i. \tag{3.7}$$

Thus, the solutions of (3.1) are

$$v_1(x,t) = \pm \frac{11 - \frac{\sqrt{5}}{2}\sinh(x-t)}{1 + \frac{\sqrt{5}}{2}\sinh(x-t)}$$

$$v_2(x,t) = \pm \frac{11 + \frac{\sqrt{5}}{2}\sinh(x-t)}{1 - \frac{\sqrt{5}}{2}\sinh(x-t)}. \tag{3.8}$$

Substituting (3.8) into (3.3) gives that

$$u_1(x,t) = 5 - \frac{6(23 + 5\cosh(2(x-t)))}{3 + 5\cosh(2(x-t)) + 8\sqrt{5}\sinh(x-t)}$$

$$u_2(x,t) = -\frac{123 + 5\cosh(2(x-t)) + 40\sqrt{5}\sinh(x-t)}{2(-2+\sqrt{5}\sinh(x-t))^2}). \tag{3.9}$$

Now, by the second rational cosine method. Substituting (2.6) into (3.4) gives the same system (3.6). Accordingly, two more complex solutions follow and given by

$$v_3(x,t) = \pm \frac{11 - \frac{i\sqrt{5}}{2}\cosh(x-t)}{1 + \frac{i\sqrt{5}}{2}\cosh(x-t)},$$

$$v_4(x,t) = \pm \frac{11 + \frac{i\sqrt{5}}{2}\cosh(x-t)}{1 - \frac{i\sqrt{5}}{2}\cosh(x-t)}. \tag{3.10}$$

Substituting (3.10) into (3.3) gives that

$$u_3(x,t)=\frac{123-40i\sqrt{5}\cosh(x-t)-5\cosh(2(x-t))}{2(-2i+\sqrt{5}\cosh(x-t))^2},$$

$$u_4(x,t)=\frac{123+40i\sqrt{5}\cosh(x-t)-5\cosh(2(x-t))}{2(2i+\sqrt{5}\cosh(x-t))^2}. \tag{3.11}$$

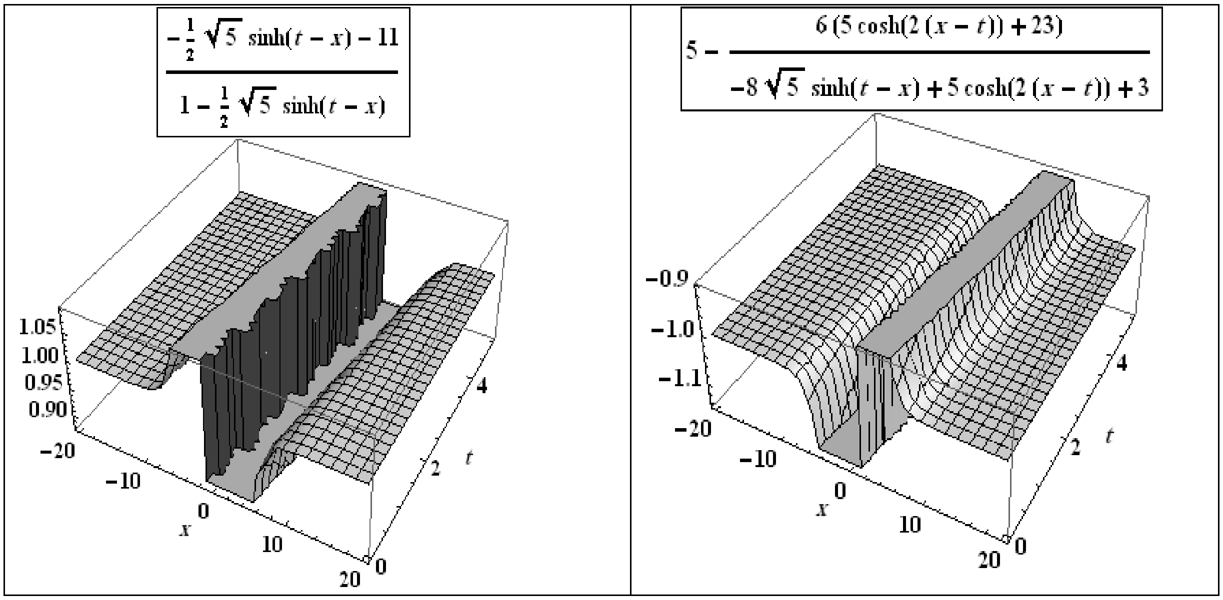

Figure 1. The first obtained soliton − like solution of (3.1) $v(x,t)$ and $u(x,t)$ respectively by using Ansatze II (the second rational sine method)

<u>Second</u>, we apply the cosine method – Ansatze (I). Substituting (2.3) into (3.4) gives

$$\begin{aligned}0 &= -32\mu^4+24\gamma\mu^4+28\gamma^2\mu^4-24\gamma^3\mu^4+4\gamma^4\mu^4\\ &+\cos^2(\mu\zeta)(-16c\mu^2+24c\gamma\mu^2-8c\gamma^2\mu^2+32\mu^4-48\gamma\mu^2+24\gamma^3\mu^4-8\gamma^4\mu^4)\\ &+\cos^4(\mu\zeta)(3c^2+8c\gamma^2\mu^2+4\gamma^4\mu^4)\\ &+\lambda^2\cos^{4+2\gamma}(\mu\zeta).\end{aligned} \tag{3.12}$$

The above equation is satisfied only if the following system of algebraic equations holds

$$0=-8+24\gamma+7\gamma^2-6\gamma^3+\gamma^4,$$

$$0=\lambda^2-16c\mu^2+24c\gamma\mu^2-8c\gamma^2\mu^2+32\mu^4-48\gamma\mu^2+24\gamma^3\mu^4-8\gamma^4\mu^4,$$

$$0=3c^2+8c\gamma^2\mu^2+4\gamma^4\mu^4,$$

$$2 = 4 + 2\gamma,$$
$$c = 1. \tag{3.13}$$

Solving the above system yields the following two cases:

$$\lambda = \pm 6i, \ \mu = \pm \frac{i}{\sqrt{2}}, \ c = 1, \gamma = -1.$$
$$\lambda = \pm 6i\sqrt{5}, \ \mu = \pm i\sqrt{\frac{3}{2}}, \ c = 1, \gamma = -1. \tag{3.14}$$

Thus, the solutions of (3.1) in complex form are

$$v_1(x,t) = \pm 6i \ sech\left(\frac{x-t}{\sqrt{2}}\right),$$

$$v_2(x,t) = \pm 6i\sqrt{5} sech\left(\sqrt{\frac{3}{2}}(x-t)\right). \tag{3.15}$$

Substituting (3.15) into (3.3) yields

$$u_1(x,t) = 6\, sech^2\left(\frac{x-t}{\sqrt{2}}\right),$$

$$u_2(x,t) = -2 + 18\, sech^2\left(\sqrt{\frac{3}{2}}(x-t)\right). \tag{3.16}$$

Now, we solve (3.4) by using the sine method. Substituting (2.4) into (3.4) gives the same system (3.13). Thus, two more solutions follow and given by:

$$v_3(x,t) = \pm 6\, csch\left(\frac{x-t}{\sqrt{2}}\right),$$

$$v_4(x,t) = \pm 6\sqrt{5}\, csch\left(\sqrt{\frac{3}{2}}(x-t)\right). \tag{3.17}$$

Substituting (3.17) into (3.3) gives

$$u_3(x,t) = -6\, csch^2\left(\frac{x-t}{\sqrt{2}}\right),$$

$$u_4(x,t) = -2 - 18\,\mathrm{csch}^2\left(\sqrt{\frac{3}{2}}(x-t)\right). \tag{3.18}$$

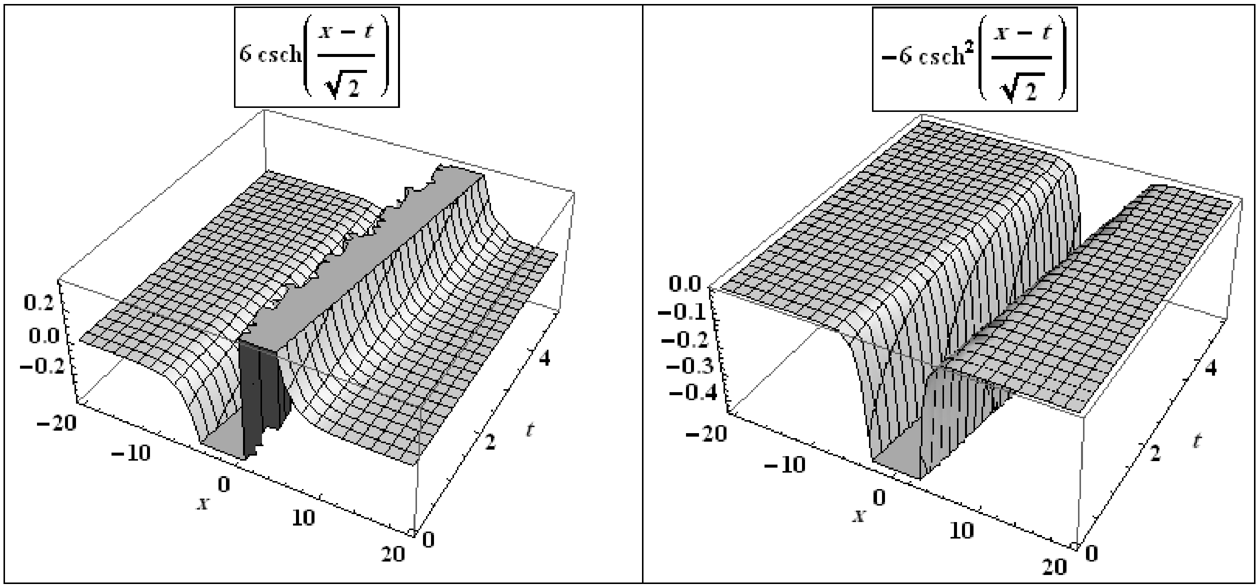

Figure 2. The second obtained like − soliton solution of (3.1) $v(x,t)$ and $u(x,t)$ respectively by using Ansatze I (the sine method)

<u>Third</u>, we apply Ansatze (III) with $f(\zeta) = sech(\zeta)$. Substituting (2.7) into (3.4) gives

$$\begin{aligned}0 &= 1 + 3c^2\lambda^2 - 8c\lambda^2\mu^2 + 48c\mu^2 + 48\mu^4 + 4c\lambda^2\mu^4 \\ &\quad + \cosh(\mu\zeta)(6\lambda c^2 + 32c\lambda\mu^2 - 16\lambda\mu^4) + \cosh^2(\mu\zeta)(3c^2 - 8c\mu^2 + 4\mu^4).\end{aligned} \tag{3.19}$$

The above equation is satisfied only if the following system of algebraic equations holds

$$\begin{aligned}0 &= 1 + 3c^2\lambda^2 - 8c\lambda^2\mu^2 + 48c\mu^2 + 48\mu^4 + 4c\lambda^2\mu^4, \\ 0 &= 6\lambda c^2 + 32c\lambda\mu^2 - 16\lambda\mu^4, \\ 0 &= 3c^2 - 8c\mu^2 + 4\mu^4.\end{aligned} \tag{3.20}$$

Solving the above system yields

$$c = \frac{-i}{6},\ \mu = \pm\frac{(-1)^{\frac{3}{4}}}{2\sqrt{3}},\ \lambda = 0, \qquad c = \frac{i}{6},\ \mu = \pm\frac{(-1)^{\frac{1}{4}}}{2\sqrt{3}},\ \lambda = 0,$$

$$c=\frac{-i}{6\sqrt{5}},\ \mu=\pm\frac{(-1)^{\frac{3}{4}}}{2\sqrt[4]{5}},\lambda=0,\quad c=\frac{i}{6\sqrt{5}},\ \mu=\pm\frac{(-1)^{\frac{1}{4}}}{2\sqrt[4]{5}},\lambda=0. \tag{3.21}$$

Thus, the solutions of (3.1) are

$$v_1(x,t)=\mathrm{sech}\left(\frac{(-1)^{\frac{3}{4}}\zeta}{2\sqrt{3}}\right),\ v_2(x,t)=\mathrm{sech}\left(\frac{(-1)^{\frac{1}{4}}\zeta}{2\sqrt{3}}\right),$$

$$v_3(x,t)=\mathrm{sech}\left(\frac{(-1)^{\frac{3}{4}}\zeta}{2\sqrt[4]{5}}\right),\ v_4(x,t)=\mathrm{sech}\left(\frac{(-1)^{\frac{1}{4}}\zeta}{2\sqrt[4]{5}}\right). \tag{3.22}$$

Substituting (3.22) into (3.3) gives

$$u_1(x,t)=6\,\mathrm{sech}^2\left(\frac{x-t}{\sqrt{2}}\right)$$

$$u_2(x,t)=-2+18\,\mathrm{sech}^2\left(\sqrt{\frac{3}{2}}(x-t)\right). \tag{3.23}$$

It is noticed that the above obtained solutions $u(x,t)$ are the same solutions obtained earlier by using different Ansatze, see Eq. (3.18).

When we use the csch instead of sech in Ansatze (III), we get the same system (3.20) except that the first equation is replaced by

$$1+3c^2\lambda^2-8c\lambda^2\mu^2-48c\mu^2-48\mu^4+4c\lambda^2\mu^4=0. \tag{3.24}$$

Solving the new system yields

$$c=-\frac{1}{6},\ \mu=\pm\frac{i}{2\sqrt{3}},\ \lambda=0,\ \ c=\frac{1}{6},\ \mu=\pm\frac{1}{2\sqrt{3}},\ \lambda=0,$$

$$c=-\frac{1}{6\sqrt{5}},\mu=\pm\frac{i}{2\sqrt[4]{5}},\ \lambda=0,\ \ c=\frac{1}{6\sqrt{5}},\ \mu=\pm\frac{1}{2\sqrt[4]{5}},\lambda=0. \tag{3.25}$$

Therefore, the following solutions follow

$$v_1(x,t) = \pm i\ \csc\left(\frac{\zeta}{2\sqrt{3}}\right), \quad v_2(x,t) = \pm csch\left(\frac{\zeta}{2\sqrt{3}}\right)$$

$$v_3(x,t) = \pm i\ \csc\left(\frac{\zeta}{2\sqrt[4]{5}}\right), \quad v_4(x,t) = \pm csch\left(\frac{\zeta}{2\sqrt[4]{5}}\right), \ \ \zeta = x - ct. \tag{3.26}$$

Substituting (3.26) into (3.3) gives that

$$u_1(x,t) = -\csc^2\left(\frac{\zeta}{2\sqrt{3}}\right), \quad u_2(x,t) = -csch^2\left(\frac{\zeta}{2\sqrt{3}}\right),$$

$$u_3(x,t) = \frac{1 - 9\ \csc^2\left(\frac{\zeta}{2\sqrt{3}}\right)}{3\sqrt{5}}, \quad u_4(x,t) = -\frac{1 + 9csch^2\left(\frac{\zeta}{2\sqrt{3}}\right)}{3\sqrt{5}}, \ \ \zeta = x - ct. \tag{3.27}$$

4. Homogeneous Kdv Equations of Third Order Type (II)

In this section we consider two-component evolutionary system of a homogeneous KdV equations of third order type (II)

$$u_t = u_{xxx} + 2vu_x + uv_x$$

$$v_t = uu_x. \tag{4.1}$$

Using the wave variable $\zeta = x - ct$ transforms (4.1) into the ODEs

$$cu_x + u''' + 2(uv)_x - uv_x = 0 \tag{4.2}$$

and

$$v = \frac{-u^2}{2c}. \tag{4.3}$$

Substituting (4.3) into (4.2), and then integrating and setting the constant of integration to zero yields

$$c^2u + cu'' - \frac{2}{3}u^3 = 0. \tag{4.4}$$

<u>**First**</u>, we solve (4.4) by using the cosine method; Ansatze (I). Substituting (2.3) into (4.4) gives

$$
\begin{aligned}
0 &= 3c\gamma\mu^2 - 3c^2\gamma^2\mu^2 \\
&+\cos^2(\mu\zeta)(-3c^2 + 3c\gamma^2\mu^2) \\
&+2\lambda^2\cos^{2+2\gamma}(\mu\zeta).
\end{aligned} \tag{4.5}
$$

The above equation is satisfied only if the following system of algebraic equations holds

$$
\begin{aligned}
0 &= 3c\gamma\mu^2 - 3c^2\gamma^2\mu^2 + 2\lambda^2 \\
0 &= -3c^2 + 3c\gamma^2\mu^2 \\
0 &= 2 + 2\gamma.
\end{aligned} \tag{4.6}
$$

Solving the above system gives

$$\lambda = \pm\sqrt{3}c,\ \ \mu = \pm\sqrt{c},\ \ \gamma = -1,$$

where $c \neq 0$ is an arbitrary constant. Thus, the solution of system (4.1) is

$$u_1(x,t) = \pm\sqrt{3}\,c\ \sec(\sqrt{c}\ (x-ct)) \tag{4.7}$$

and

$$v_1(x,t) = -\frac{3c}{2}\sec^2(\sqrt{c}\ (x-ct)). \tag{4.8}$$

Now, we use the sine method. Substituting (2.4) into (4.4) gives the same system (4.6), therefore, one more solution follows

$$u_2(x,t) = \pm\sqrt{3}\,c\ \csc(\sqrt{c}\ (x-ct)) \tag{4.9}$$

and

$$v_2(x,t) = -\frac{3c}{2}\csc^2(\sqrt{c}\ (x-ct)). \tag{4.10}$$

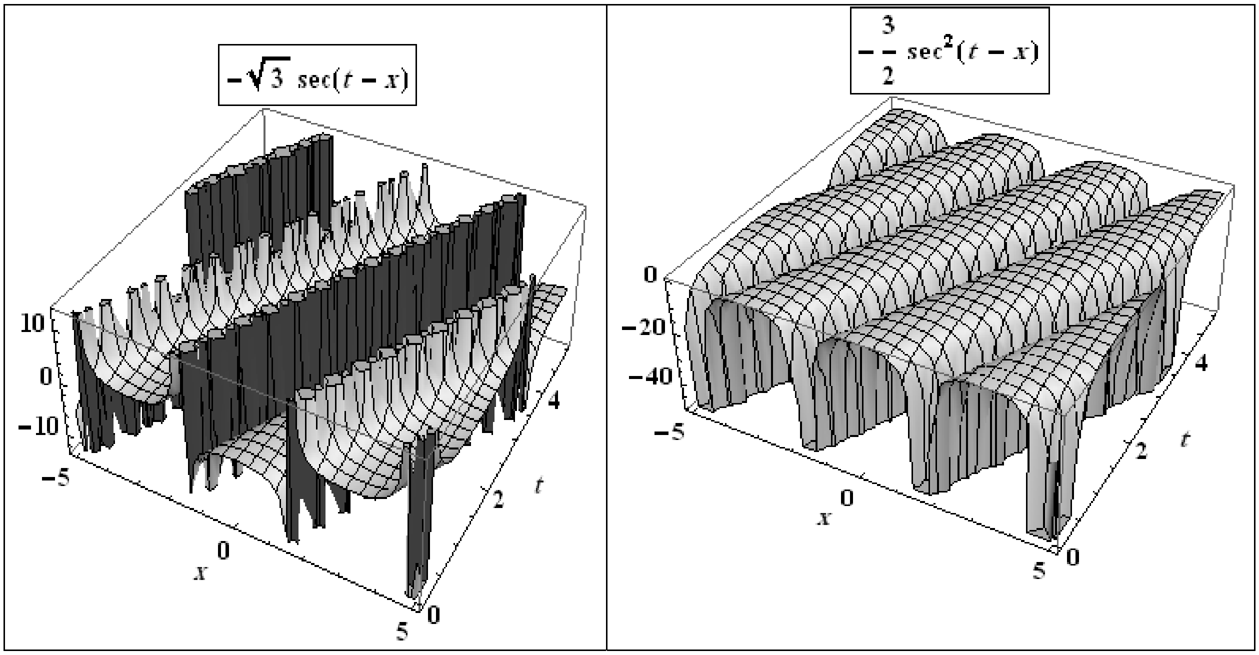

Figure 3. The first obtained periodic solution of (4.1) $u(x,t)$ and $v(x,t)$ respectively by using Ansatze I (The cosine method)

Second, we apply Ansatze (III) with $f(\zeta) = sech(\zeta)$. Substituting (2.7) into (4.4) gives

$$0 = -2 + 3c^2\lambda^2 - 6c\mu^2 + \cosh(\mu\zeta)(6\lambda c^2 - 3c\lambda\mu^2) + \cosh^2(\mu\zeta)(3c^2 + 3c\mu^2). \tag{4.11}$$

The above equation is satisfied only if the following system of algebraic equations holds

$$\begin{aligned} 0 &= -2 + 3c^2\lambda^2 - 6c\mu^2 \\ 0 &= 6\lambda c^2 - 3c\lambda\mu^2 \\ 0 &= 3c^2 + 3c\mu^2. \end{aligned} \tag{4.12}$$

Solving the above system gives that

$$\lambda = 0,\ \mu = \pm\frac{i}{\sqrt[4]{3}}, c = \frac{1}{\sqrt{3}}, \qquad \lambda = 0, \mu = \pm\frac{1}{\sqrt[4]{3}},\ c = \frac{-1}{\sqrt{3}}. \tag{4.13}$$

Thus, the solutions of (4.1) are

$$u_1(x,t) = \sec\left(\frac{x - \frac{1}{\sqrt{3}}t}{\sqrt[4]{3}}\right), \quad u_2(x,t) = sech\left(\frac{x + \frac{1}{\sqrt{3}}t}{\sqrt[4]{3}}\right), \tag{4.14}$$

and

$$v_1(x,t) = -\frac{\sqrt{3}}{2}\sec^2\left(\frac{x - \frac{1}{\sqrt{3}}t}{\sqrt[4]{3}}\right), \quad v_2(x,t) = \frac{\sqrt{3}}{2}sech^2\left(\frac{x + \frac{1}{\sqrt{3}}t}{\sqrt[4]{3}}\right). \tag{4.15}$$

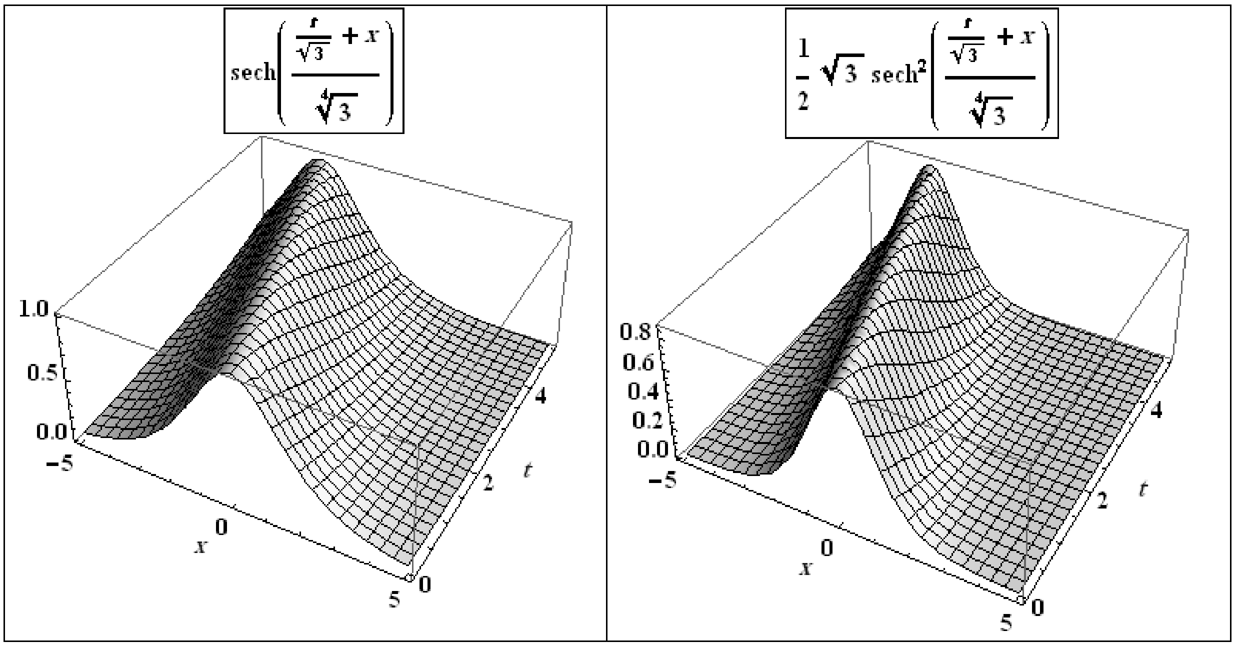

Figure 4. The second obtained Kink soliton solution of (4.1) $u(x,t)$ and $v(x,t)$, respectively, by using Ansatze III ($f(\zeta) = sech(\zeta)$).

When we use the c*sch* instead of s*ech*, we get the same system (4.12) but the first equation is replaced by

$$-2 - 3c^2 + 3c^2\lambda^2 + 3c\mu^2 = 0. \tag{4.16}$$

Solving the new system yields

$$\lambda = 0,\ \mu = \pm\frac{(-1)^{\frac{1}{4}}}{\sqrt[4]{3}}, c = \frac{-i}{\sqrt{3}}, \qquad \lambda = 0, \mu = \pm\frac{(-1)^{\frac{3}{4}}}{\sqrt[4]{3}}, c = \frac{i}{\sqrt{3}}. \tag{4.17}$$

Therefore, two more complex solutions follow and given by

$$u_3(x,t) = \pm \operatorname{csch}\left(\frac{(-1)^{\frac{1}{4}}}{\sqrt[4]{3}}(x + \frac{i}{\sqrt{3}}t)\right), \quad u_4(x,t) = \pm \operatorname{csch}\left(\frac{(-1)^{\frac{3}{4}}}{\sqrt[4]{3}}(x - \frac{i}{\sqrt{3}}t)\right), \tag{4.18}$$

and

$$v_3(x,t) = \frac{i\sqrt{3}}{2}\operatorname{csch}^2\left(\frac{(-1)^{\frac{1}{4}}}{\sqrt[4]{3}}(x + \frac{i}{\sqrt{3}}t)\right), v_4(x,t) = -\frac{i\sqrt{3}}{2}\operatorname{csch}^2\left(\frac{(-1)^{\frac{3}{4}}}{\sqrt[4]{3}}(x - \frac{i}{\sqrt{3}}t)\right). \tag{4.19}$$

5. Conclusions

In this work we used trigonometric-function method and hyperbolic-function method to handle some nonlinear evolution systems. The simplified form of these methods were applied to establish soliton and periodic solutions to such evolution equations. The methods are applicable to several types of equations, easy to use, and may provide us a straightforward, effective and alternative mathematical tool for generating solutions.

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