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Mathematical Modeling, a Small Step in a Right Direction

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Abstract

Models developed by mathematicians/statisticians based on criterion such as goodness of fit often leads to a “best” model only for the data utilized. Moreover the parameters in such models often do not have physical interpretations and as such their validity cannot be checked by other means. This article makes argument against modeling processes that do not incorporate information from discipline related to the origin of data and presents an example to demonstrate benefits of doing so.

Keywords: Mathematical Modeling, Goodness of Fit, Meaningful Interpretation

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1. Introduction

Is there any right way to do mathematical modeling? After forty years in academia this is my partial answer to this question. First, I have noticed models that mathematicians develop do not always earn the appreciation of the practitioners mainly because practitioners consider a model good if: (a) is general, (b) incorporates rules of their field of application, and (c) is understandable to them in their own jargon. For example, for more than two decades I followed the progress made in modeling seismic records of earthquakes and even personally tried to construct few myself. I even tried to learn some geophysics and seismology to help the matter. During that period, many well-known earth scientists I talked to expressed their concern about models mathematicians have developed. They generally believed that in many of the models developed by mathematicians, the emphasis has been mathematics and often little or no attempt was made to tie together the mathematical concepts with geophysical facts. They viewed this approach of modeling essentially a mathematical exercise with a bit of geological or geophysical justification. To make a distinction, we may call this geo-mathematics. In contrast they expressed desire to see a geological problem being investigated with mathematical tools, where mathematics is of purely secondary interest. In other words, the objective is to derive models with physical significance (e.g. models whose parameters have physical interpretation), not to produce elegant mathematics, though that may indeed occur. We may refer to this as mathematical geology. I learned that constructing models following their advice may lead to more appropriate models whose validity may be verified with methods other than goodness of fit. Additionally such models may include fewer unknown parameters. Going through this process I also learned that there are generally two major attitudes towards mathematical modeling applied to the disciplines such as seismology. In one, modeling is carried out solely based on goodness of fit.

Regression and time series modeling are examples of the tools people often use for this approach. In the second approach, the deterministic models developed by experts in the field are either analyzed further or extension is made to them by considering, for example, time-dependent solutions, spatial patterns or by adding random variation to account for nondeterministic factors or those not being completely understood. From the present literature it seems that the latter has gained a great popularity in recent decades. In this approach the critical problem is to incorporate random variation into an established deterministic formulation in a way that is physically meaningful and mathematically tractable.

The following describes an application of this approach for developing a model for seismic records from earthquakes. For detailed information regarding the deterministic forms used see Noubary (1999).

2. An Example

Suppose that the seismogram is written in its usual forms as

$$y(t) = Z(t,\theta) + x(t), \tag{1}$$

where $Z(t,\theta)$ is the signal (here the source time function), θ is the vector of unknown parameters and $x(t)$ is the noise. Let $Z(\omega,\theta)$ denote the Fourier transform of $Z(t,\theta)$. For earthquakes what is usually given is the functional form of $|Z(\omega,\theta)|$, without phase information.

For displacement measurements, common forms proposed by seismologists, include the following well-known ω-square (*j* =1) and ω-cube (*j* =2) models;

$$|Z(\omega,\theta)| = h\left[1+\left(\frac{\omega}{k}\right)^2\right]^{\frac{\omega+j}{2}}, \qquad j = 1,2. \tag{2}$$

Here ω denotes the frequency, *k* is known as corner frequency (1/*k* is proportional to the duration of earthquake) and *h* is a dimensionless constant with a certain physical meaning. A time domain version for (2) is

$$Z(t,\theta) = ct^j \exp(\kappa t), \qquad j = 1,2. \tag{3}$$

These are deterministic models. To derive stochastic model, I followed the approach that suggests including a random variation to model (3). For this, in each case I regarded the displacement (in this case the radial displacement function) as the complementary solution to a stochastic difference equation. In other words, I constructed a stochastic difference equation whose complementary solution had the same form as the source time function (3). For ω-squared and ω-cubed models, the resulting stochastic difference equation take the form

$$(1-\exp(-k)B)^j\, y(t) = e(t), \qquad j = 2,3, \tag{4}$$

where *B* represents the backwards shift operator.

Note that in this model the characteristic polynomial has *j* equal roots with *j* = 2 for ω-squared and *j* =3 for ω-cubed models, respectively. The difference equation (4) represents a special class of j^{th} order autoregressive processes with coefficients determined by the single parameter *k*.

Tjϕstheim (1975a,b) has found empirical evidence for unconstrained third-order autoregressive models as an appropriate model for a majority of observed P-wave records for earthquakes and used the coefficients of the fitted models for discriminating earthquakes from underground nuclear explosions, a problem that was of great concern during the Cold War. In fact, he checked autoregressive models of different orders and selected an arbitrary third-order autoregressive models of the form

$$y(t) + \alpha y(t-1) + \beta y(t-2) + \gamma y(t-3) = x(t). \tag{5}$$

He, then, picked the "best" model based on a statistical goodness of fit. It is interesting to compare his findings with the one-parameter model (4). This helps to see some of the shortcomings of the models obtained solely based on goodness of fit. Tjϕstheim has fitted (5) to large numbers of records from the NORSAR array, producing summary coefficients by averaging. For instance, for short-period seismic noise, he has found [from Tables 4 and 5, in Tjϕstheim (1975a)] that

(a) -1.69, +0.97, -0.18 (from 34 samples) and
(b) -1.80, +1.09, -0.22 (from 29 samples).

The coefficients of the model (4) fitted to the records sampled at the rate of 10 observations per second are, respectively:

-3exp(-*k*/10), 3exp(-2*k*/10), -exp(-3*k*/10).

For the above data, these coefficients are, respectively:

(a) -1.69, 0.96, -0.18 for *k* = 5.716, and
(b) -1.81, 1.09, -0.22 for *k* = 5.047.

This suggests that the model (4) with only one parameter would also fit the same data. Additionally the validity of model (4) can be verified by comparing the estimated values of *k* with direct measurements obtained by seismologists.

Tjφstheim (1975a) also found that the most pronounced difference in autoregressive structure between earthquakes and underground nuclear explosions was that, on average, the autoregressive coefficients for explosions had a lower absolute value than the corresponding coefficients for earthquakes. This can be directly inferred from model (4), by using the fact that *k* is a time constant and earthquakes usually have a longer duration than underground nuclear explosions. Also Bungum and Tjφstheim (1976) have found that just the first coefficient of Tjφstheim's autoregressive models is needed in discrimination between earthquakes and underground nuclear explosions. This is again predictable from model (4). In short, these observations confirm the superiority of the one-parameter model (4) over models obtained using goodness of fit.

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