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Abstract

The problem of blood flow through an overlapping constriction in arteries is investigated in this work. To account for the non-Newtonian behavior and the peripheral layer, blood has been represented by a two-fluid model, consisting of a core region of suspension of all the erythrocytes assumed to be a Casson fluid and a peripheral layer of plasma (Newtonian fluid). The expression for the flow characteristics, namely, the impedance, the wall shear stress, the shear stress at the stenosis throats and at the critical height of the stenosis has been derived. Moreover, we present some results concerning the dependence of these quantities on the geometrical parameters.

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1. Introduction

Cardiovascular diseases are known to be responsible for over seventy five percent of all deaths and stenosis or arteriosclerosis is one of the frequently occurring diseases. Stenosis is a medical term which means narrowing of any body passage, tube or orifice, Young (1979). It is an abnormal and unnatural growth that develops at various locations of the cardiovascular system under diseased conditions. The etiology of the initiation of stenosis, though not well understood it is strongly believed to be due to deposits of cholesterol and proliferation of connective tissues may be responsible for the disease. Irrespective of the cause it is well established that once the constriction has developed, it brings about significant changes in blood flow, pressure distribution, wall shear stress and the impedance (flow resistance) which occasionally results in serious consequences, [Young (1968), Young and Tsai (1973)].

In the region of narrowing arterial constriction, the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis. With the advent of the discovery that the haemodynamic factors play an important role in the genesis and proliferation of stenosis, since the first investigation of Mann et al. (1938), a large number of researchers [Young (1968), (1979); Young and Tsai (1973); Caro et al. (1978); Shukla et al. (1980); Ahmed and Giddens (1983); Sarkar and Jayaraman (1998); Pralhad and Schultz (2004); Jung et al. (2004); Liu et al. (2004); Srivastava and coworkers (1996), (2009), (2010a,b,c); Mishra et al. (2006); Misra and Verma (2007); Ponalagusamy (2007); Layek et al. (2005), (2009); Joshi et al. (2009); Mekheimer and El-Kot (2008); Tzirtzilakis (2008); Mandal and coworkers (2005), (2007a,b); Politis et al. (2007), (2008); Singh et al. (2010); Medhavi (2011), and many others], have addressed the problem in various contexts.

Blood being a suspension of corpuscles, behaves like a non-Newtonian fluid [Merrill et al. (1965); Charm and Kurland (1974); Hersey et al. (1964)] in small diameter tubes under certain flow conditions, particularly, at low shear rates, Merrill et al. (1965). An examination of viscometric data [Bugliearello et al. (1965); Chein et al. (1965); Rand et al. (1964)] suggests that non-Newtonian behavior of blood increases rapidly, when hematocrit rises above 20%, possibly reaching a maximum at between 40-70%. Merrill et al. (1965) established that the Casson model holds satisfactory in small vessels (of diameter 130-1000 μ m) within certain wall shear stress limits. In addition, Bugliarello and Sevilla (1970), Cokelet (1972) and Thurston (1989) have shown experimentally that for blood flowing through small vessels, there is cell-free plasma (Newtonian viscous fluid) layer and a core region of suspension of all the erythrocytes. Srivastava (2007) concluded that the significance of the peripheral layer increases with decreasing blood vessel diameter.

A survey of the literature on arteriosclerotic development indicates that the studies in the literature have been conducted mainly for single symmetric and non-symmetric stenoses. The stenoses may develop in series (multiple stenoses) or may be of irregular shapes or overlapping. Assuming the pressure variation only along the tube axis, Chakravarty and Mandal (1994) studied the effects of an overlapping stenosis on arterial flow problem of blood. An effort is made in the present work to study the effects of an overlapping stenosis on the flow characteristics of blood taking into account that flowing blood is represented by a two-layered

model. The mathematical model considers a two-fluid (two-layered) model of blood, consisting of a core region of suspension of all the erythrocytes, assumed to be a Casson fluid and a peripheral layer of plasma (Newtonian fluid). The theoretical model used here enables one to observe simultaneous effects of non-Newtonian behavior of blood and peripheral layer on flow characteristics of blood due to presence of an overlapping stenosis in arteries. The arterial wall segment is considered rigid as well as deformable. The wall in the vicinity of the stenosis is usually relatively rigid when stenoses develop in human vasculature. To neglect the entrance, end and special wall effects, the artery length is assumed large enough as compared to its radius.

2. Formulation of the Problem

Consider the axisymmetric flow of blood in a uniform rigid circular artery of radius R with an axisymmetric overlapping stenosis. Blood is assumed to be represented by a two-layered model consisting of a central layer of a Casson fluid of radius R_1 and a peripheral layer of plasma (a Newtonian viscous fluid) of thickness $(R-R_1)$. The stenosis geometry and the shape of the central layer are described in Figures. 1 and 2, respectively, as

$$\frac{(R(z), R_1(z))}{R_0} = (1, \alpha) - \frac{3}{2} \frac{(\delta, \delta_1)}{R_0 L_0^4} \left[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4 \right],$$

$$d \leq z \leq d + L_0,$$

$$= (1, \alpha), \quad \text{otherwise,} \tag{1}$$

where z is the axial coordinate, $R \cong R(z)$ and R_0 are the radius of the tube with and without constriction, respectively; L_0 is the stenosis length, L is the tube length and d indicates the location of stenosis, α is the ratio of the central core radius to the tube radius in the unobstructed region and (δ, δ_1) are the maximum height of the stenosis and the bulging of the interface at two locations in the stenotic region: $z = d+L_0/6$ and $z = d+5L_0/6$. The stenosis height located at $z = d+L_0/2$, called critical height, is $3\delta/4$.
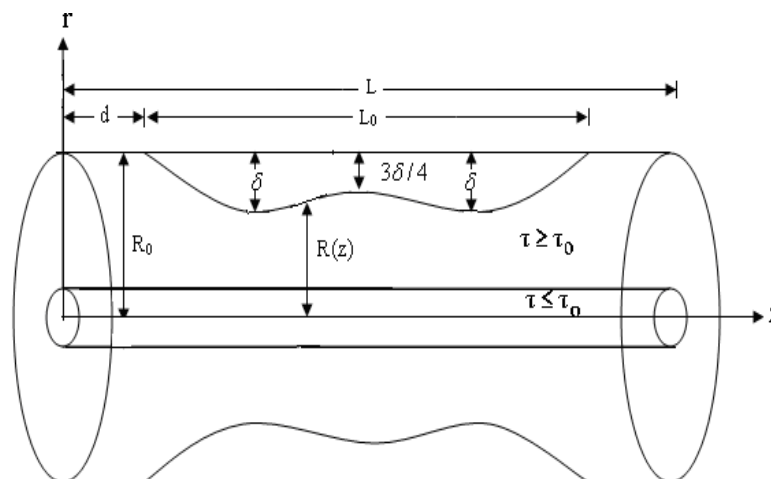
Figure 1. The geometry of an arterial overlapping stenosis
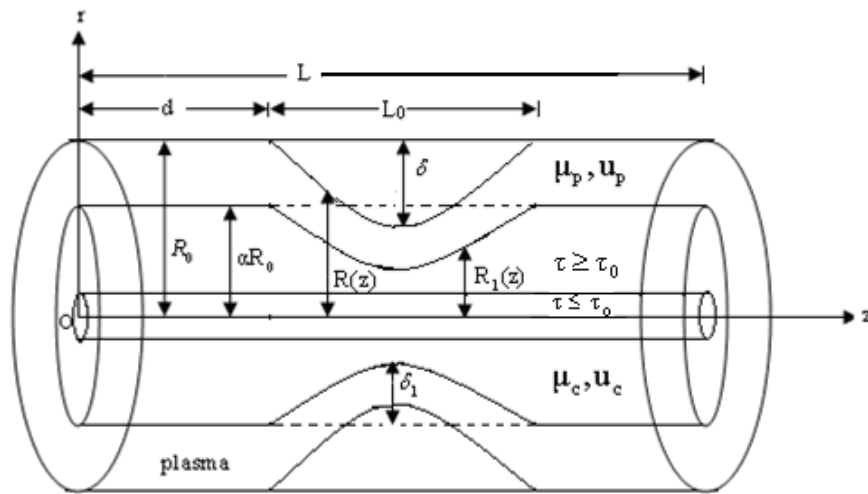
Figure 2. The shape of the central layer

Following the reports of Young (1968) and Srivastava and Rastogi (2009) and considering the axisymmetric, laminar, steady one-dimensional fully developed flow of blood in an artery, the general constitutive equation in the case of a mild stenosis, subject to the additional conditions [Young (1968); Srivastava and Rastogi (2009)]; $\delta/R_0 <<1$, $R_e(2\delta/L_0) <<1$ and $2R_0/L_0 \sim O(1)$, may therefore be written as

$$\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) = 0, \tag{2}$$

where r is the radial coordinate measured normal to the axis of the tube; p, R_e, τ is pressure, Reynolds number, shear stress, respectively.

For a Casson two fluid-model of blood, the shear stress-strain relationship is given (Srivastava and Saxena, 1995) as

$$\tau = \mu_p\left(-\frac{\partial u_p}{\partial r}\right); \quad \tau_0 = 0, \quad R_1 \le r \le R, \tag{3}$$

$$\tau^{1/2} = \tau_0^{1/2} + \mu_c^{1/2}\left(-\frac{\partial u_c}{\partial r}\right); \quad \tau \ge \tau_0, \quad R_p \le r \le R_1, \tag{4}$$

$$\frac{\partial u_p}{\partial r} = 0; \quad \tau \le \tau_0, \quad 0 \le r \le R_p, \tag{5}$$

where τ_0 the yield is stress of the core region, μ_p and μ_c are plasma viscosity in the peripheral and fluid viscosity in the core regions, respectively and R_p is the radius of the plug flow region.

The boundary conditions are

$$u_p = 0 \quad at \quad r = R(z), \quad \text{peripheral layer}, \tag{6}$$

$$\left. \begin{array}{l} u_p = u_c \ \ and \ \ \tau_p = \tau_c \ \ at \ \ r = R_1(z) \\ \\ \\ \tau \ \ is \ \ finite \ \ r = 0 \end{array} \right\} \quad \text{central layer}, \tag{7}$$

where (u_p, τ_p) and (u_c, τ_c) are the (axial velocity of fluid, shearing stress) in the peripheral and central layers, respectively.

3. Analysis

An integration of equation (2) using relations (3)-(5), subject to the boundary conditions (6) and (7), yields the expressions for velocities, u_p and u_c as

$$u_p = -\frac{1}{4\mu_p}\frac{dp}{dz}(R^2 - r^2); \qquad R_1 \le r \le R, \tag{8}$$

$$\begin{aligned} u_c = &-\frac{1}{4\mu_p}\frac{dp}{dz}\left\{R^2 - \mu r^2 - (1-\mu)R_1^2\right\} + \frac{\tau_0}{\mu_c}(R_1 - r) \\ \\ &-\frac{4}{3}\left\{\frac{\tau_0}{\mu_c}\left(-\frac{1}{2\mu_c}\frac{dp}{dz}\right)\right\}^{1/2}(R_1^{3/2} - r^{3/2}); \qquad R_p \le r \le R_1, \end{aligned} \tag{9}$$

with $\mu = \mu_c/\mu_p$. The constant plug flow velocity $u_{plug}(= u_c \ for \ 0 \le r \le R_p)$ is determined by evaluating the expression for u_c at $r = R_p$. Under the condition that the radius of the plug flow area is small as compared to the non-plug flow area, i.e., $R_p << R$ [Merill et al. (1965); Bugliarello and Sevilla (1970); Srivastava and Saxena (1995)], the volumetric flow rate, Q is calculated as

$$\begin{aligned} Q = &2\pi\left\{\int_{R_1}^{R} r\, u_p\, dr + \int_{0}^{R_1} r\, u_c\, dr\right\} \\ \\ = &-\frac{\pi}{8\mu_p}\frac{dp}{dz}\left[R^4 - (1-\mu)R_1^4\right] + \frac{\pi\tau_0}{3\mu_c}R_1^3 \\ \\ &-\frac{4\pi}{7}\left\{\frac{\tau_0}{\mu_c}\left(-\frac{1}{2\mu_c}\frac{dp}{dz}\right)R_1^7\right\}^{1/2}. \end{aligned} \tag{10}$$

Using now the fact that the total flux is equal to the fluxes across the two regions (central and peripheral), one determines the relations [Shukla et al. (1980); Srivastava (2007)]: $R_1 = \alpha R$ and $\delta_1 = \alpha \delta$. An application of these relations into equation (10) under the condition [Merill et al. (1965); Srivastava and Saxena (1995)] $\tau_0 << (-R/2)dp/dz$, yields

$$-\frac{dp}{dz} = \frac{8\mu_p Q}{\pi \beta R_0^4} \left\{ \frac{1}{(R/R_0)^4} + \frac{64}{49}\frac{\mu\alpha^7\tau_n}{\beta(R/R_0)} + \frac{16}{7}\left(\frac{\mu\alpha^7\tau_n}{\beta(R/R_0)^5}\right)^{1/2} \right\},$$
(11)

where $\beta = 1 - (1-\mu)\alpha^4$ and $\tau_n = \pi R_0^3 \tau_0 / 4\mu_c Q$, a non-dimensional Casson fluid parameter.

The pressure drop, Δp (= p at $z = 0$ and $-p$ at $z = L$) in the tube of length, L across the stenosis is obtained as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right) dz = \frac{8\mu_p Q}{\pi \beta R_0^4} \psi,$$
(12)

where

$$\psi = \int_0^L \varphi(z)dz = \int_0^d \left[\varphi(z)\right]_{R/R_0=1} dz + \int_d^{d+L_0} \varphi(z)dz + \int_{d+L_0}^L \left[\varphi(z)\right]_{R/R_0} dz,$$

$$\varphi = \frac{1}{(R/R_0)^4} + \frac{64}{49}\frac{\mu\alpha^7\tau_n}{\beta(R/R_0)} + \frac{16}{7}\left(\frac{\mu\alpha^7\tau_n}{\beta(R/R_0)^5}\right)^{1/2}.$$

The computation of the first and the third integrals in the expression for ψ obtained above is straight forward whereas the evaluation of the second integral is almost a formidable task and thus will be evaluated numerically. Following now the definitions given in Srivastava (1996) and Srivastava and Rastogi (2009), the expressions for the impedance (flow resistance), λ the wall shear stress, τ_w the shear stress at stenosis throats, τ_s and the shear stress at stenosis critical height, τ_{crit} are derived in their non-dimensional form as

$$\lambda = \frac{\mu}{\beta}\left[(1 - L_0/L)\left\{1 + \frac{64}{49}\eta + \frac{16}{7}\eta^{1/2}\right\}\right.$$

$$\left. + \frac{1}{L}\int_d^{d+L_0}\left\{\frac{1}{(R/R_0)^4} + \frac{64}{49}\frac{\eta}{(R/R_0)} + \frac{16}{7}\left(\frac{\eta}{(R/R_0)^5}\right)^{1/2}\right\}dz\right],$$
(13)

$$\tau_w = \frac{\mu}{\beta}\left\{\frac{1}{(R/R_0)^3} + \frac{64}{49}\eta + \frac{16}{7}\left(\frac{\eta}{(R/R_0)^3}\right)^{1/2}\right\},$$
(14)

$$\tau_s = \frac{\mu}{\beta}\left\{\frac{1}{\left(1-5\delta/4R_0\right)^3} + \frac{64}{49}\eta + \frac{16}{7}\left(\frac{\eta}{\left(1-5\delta/4R_0\right)^3}\right)^{1/2}\right\}, \tag{15}$$

$$\tau_{crit} = \frac{\mu}{\beta}\left\{\frac{1}{\left(1-3\delta/4R_0\right)^3} + \frac{64}{49}\eta + \frac{16}{7}\left(\frac{\eta}{\left(1-3\delta/4R_0\right)^3}\right)^{1/2}\right\}, \tag{16}$$

where

$$\eta = \mu\alpha^7\tau_n/\beta, \quad \lambda = \bar{\lambda}/\lambda_{0n}, \quad \left(\tau_w, \tau_s, \tau_{crit}\right) = \left(\bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_{crit}\right)/\tau_{0n},$$

$$\bar{\lambda} = \Delta p/Q, \quad \bar{\tau}_w = (-R/2)dp/dz, \quad \bar{\tau}_s = \left[(-R/2)dp/dz\right]_{R/R_0=(1-5\delta/4R_0)},$$

$$\bar{\tau}_{crit} = \left[(-R/2)dp/dz\right]_{R/R_0=(1-3\delta/4R_0)}, \quad \lambda_{0n} = 8\mu_c L/\pi R_0^4, \quad \tau_{0n} = 4\mu_c Q/\pi R_0^3,$$

λ_{0n}, τ_{0n} are the impedance and shear stress in a normal (no stenosis) artery for a Newtonian fluid of viscosity, μ_c and $(\bar{\lambda}, \bar{\tau}_w, \bar{\tau}_s, \bar{\tau}_{crit})$ are (impedance, wall shear stress, wall shear stress at the stenosis throats, and shear stress at stenosis critical height) in their dimensional form.
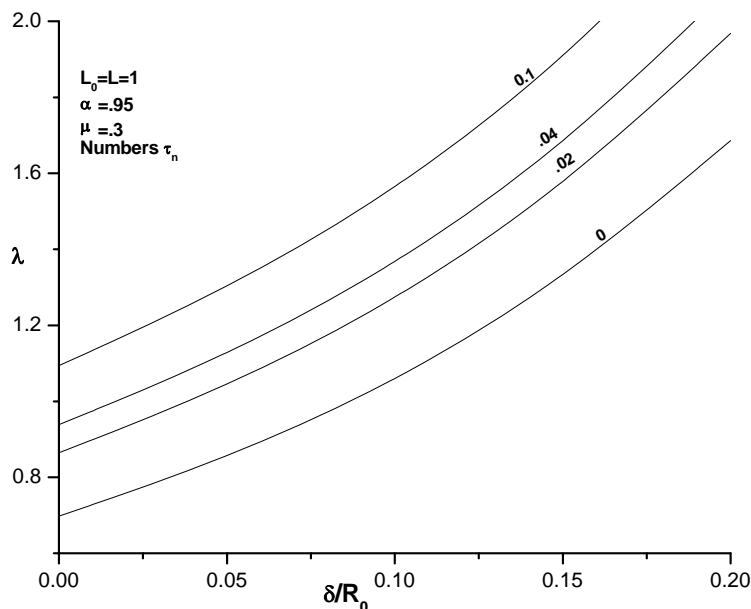

Fig.3 Variation of impedance, λ with δ/R$_0$ for different τ$_n$.

It is worth mentioning here that for $\mu = 1$ and $\alpha = 1$, the results obtained in equations (13)-(16) reduce the corresponding results of one-fluid model analysis of Casson fluid. For $\tau_0 = 0$, the results obtained in equations (13)-(16) yield the results for a two-fluid model of Newtonian fluid.

In particular for $\mu = 1$, it leads to a single-layered analysis of a Newtonian fluid. The single-layered Newtonian fluid results may also be derived by setting $\mu = 1$, $\alpha = 1$ and $\tau_0 = 0$ in equations (13)-(16).

4. Results and Discussion

Computer codes are developed to evaluate the analytical results obtained in equations (13)-(16) in order to observe the sensitivity of the blood flow characteristics due to the presence of the overlapping stenosis on the geometrical parameters. The values of the latter are selected [Young (1968); Merill et al. (1965); Srivastava and Saxena (1995); Srivastava and Rastogi (2009)] as: $L_0(cm) = 1$; $L(cm) = 1, 2, 5$; $\mu = 0.1, 0.2, 0.3, 1$; $\alpha = 1, 0.98, 0.95, 0.90$, $\delta / R_0 = 0, 0.05, 0.10, 0.15, 0.20$. We point out that the present study corresponds to single-layered Casson fluid, to two-layered Newtonian fluid and single-layered Newtonian fluid for parameter values $\mu = 1, \tau_n = 0$, and $\mu = 1, \tau_n = 0$, respectively.

For a given value of α and μ, the impedance (flow resistance), λ increases with increasing non-Newtonian behavior (τ_n) of blood for a given stenosis height and also increases with stenosis height for a given value of τ_n (Figure 3). The impedance, λ increases with increasing peripheral layer viscosity, μ for any given value of α and τ_n (Figure 4). We observe that for any given parameters, τ_n and μ, the flow characteristic, λ decreases with decreasing values of the parameter α (Figure 5). Blood flow characteristic, λ also increases with increasing stenosis length for any given values of other parameters (Figure 6).
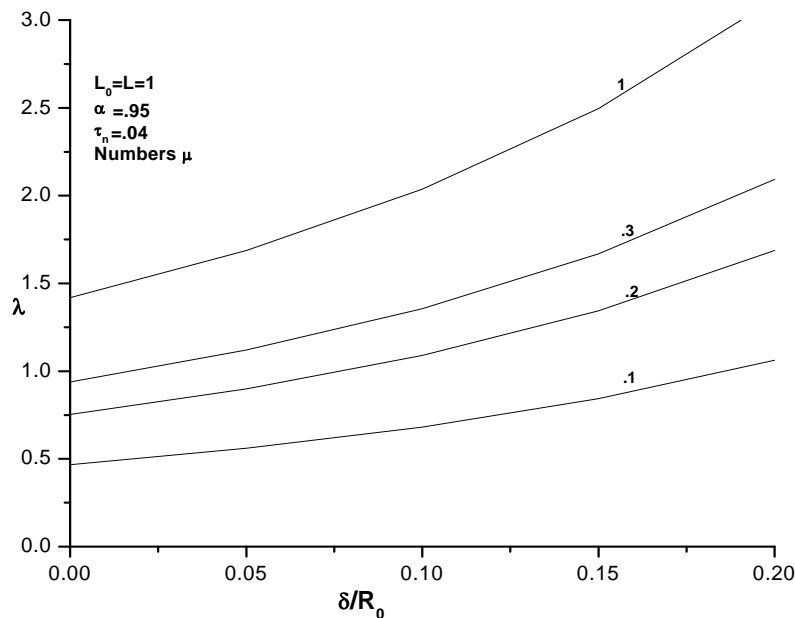

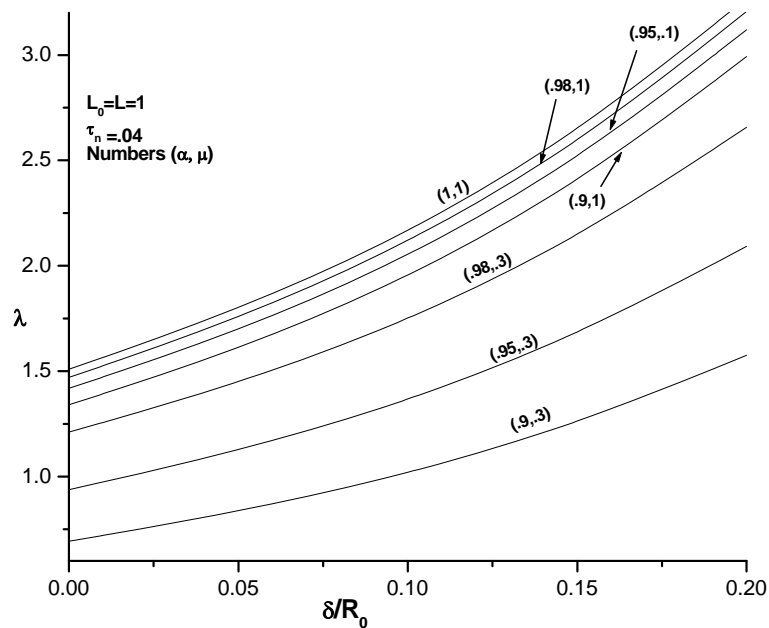**Fig.4 Variation of impedance, λ with δ/R_0 for different μ.**

Fig.5 Variation of impedance, λ with δ/R_0 for different μ.
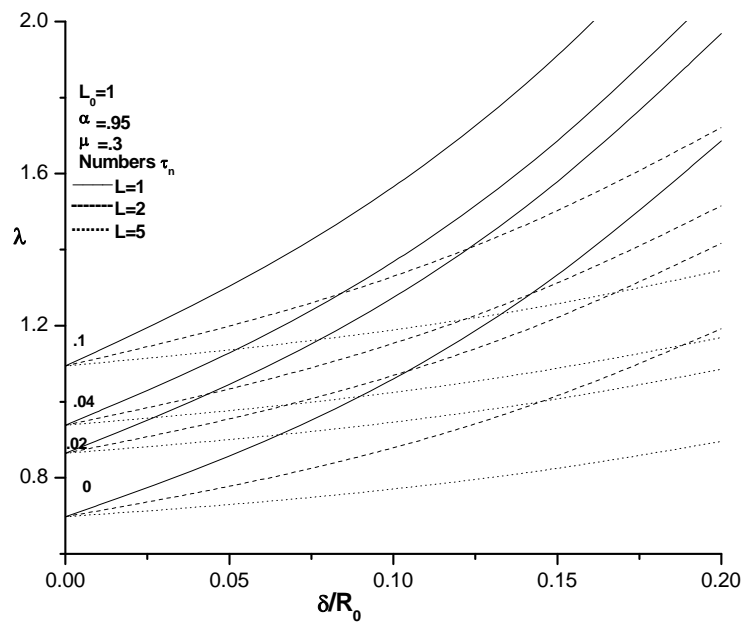

Fig.6 Variation of impedance, λ with δ/R_0 for different L.
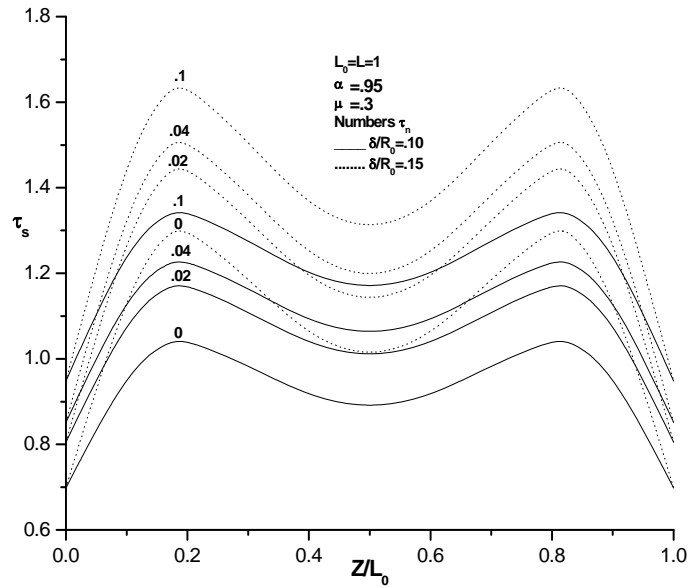
Fig.7 Wall shear stress distribution in stenotic region, τ_w for different δ/R_0 and τ_n.

The wall shear stress in the stenotic region τ_w increases with increasing non-Newtonian behavior of blood, (τ_n) as well as with increasing peripheral layer viscosity, μ (Figures 7 and 8). We point out that τ_w decreases with decreasing values of the parameter α. At an
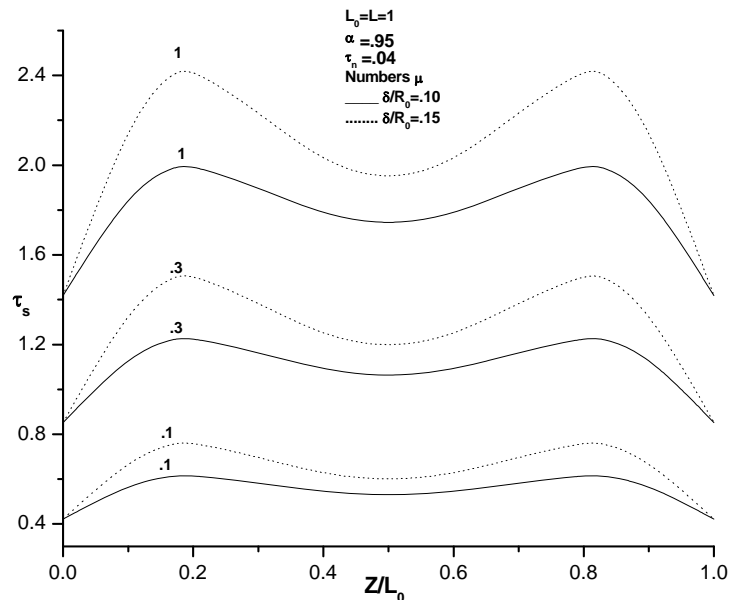

Fig.8 Wall shear stress distribution in stenotic region, τ_w for different δ/R_0 and μ.

axial distance in the stenotic region τ_w decreases with the parameter, α but increases with the parameter, δ/R_0 (Figure 9). In both the one and two-layered models, the wall shear stress in the stenotic region, τ_w rapidly increases from its approached value at $z = 0$ to its
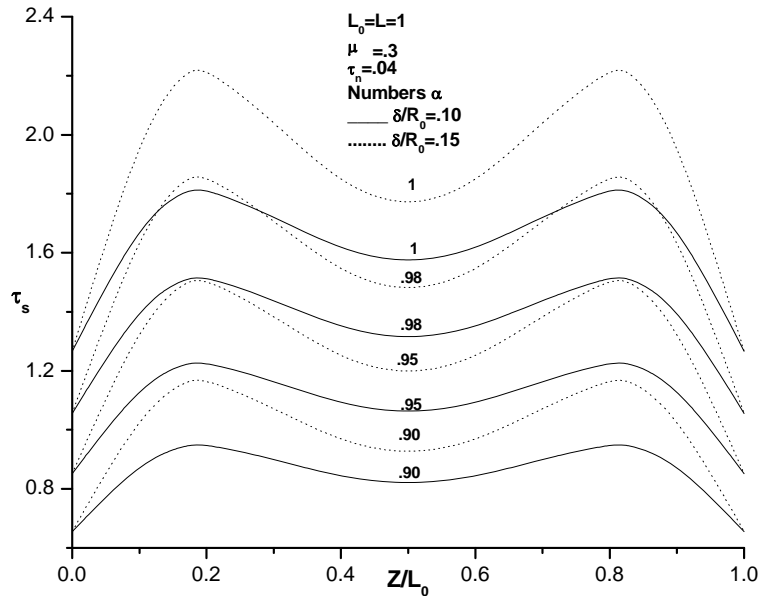

Fig.9 Wall shear stress distribution in stenotic region, τ_w for different δ/R_0 and α.
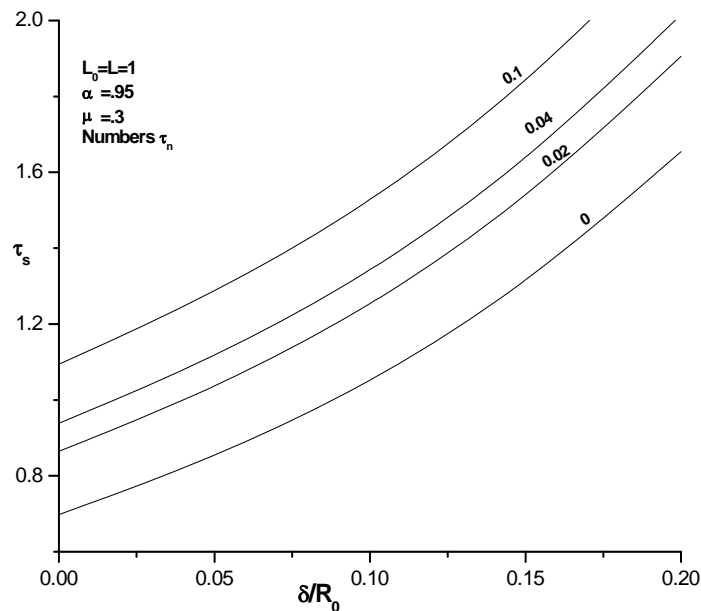

Fig.10 Variation of shear stress at the stenosis throats, τ_s with δ/R_0 for different τ_n.

peak value in the upstream of the first stenosis throat at $z/L_0 = 1/6$, it then decreases steeply in the downstream of the first throat to its magnitude at the critical height $(0.75\,\delta/R_0)$ of the stenosis at $z/L_0 = 1/2$. The shear stress in the stenotic region, τ_w further
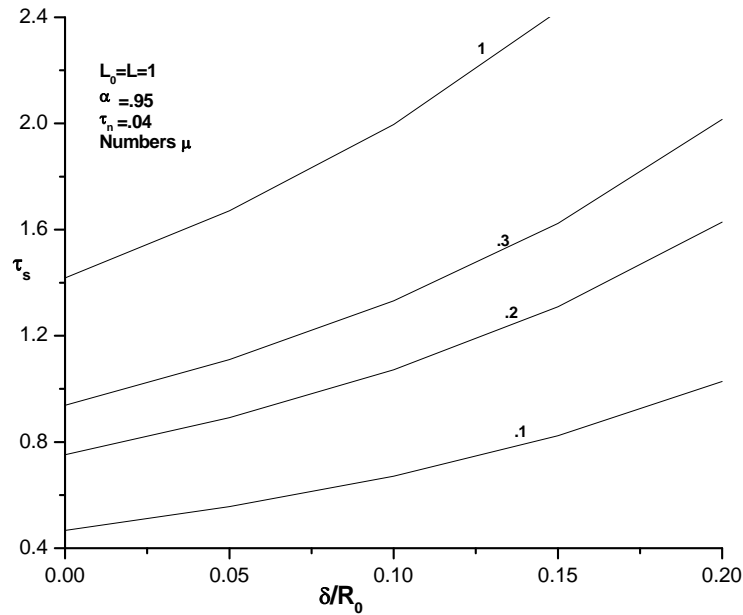

Fig.11 Variation of shear stress at the stenosis throats, τ_s with δ/R_0 for different μ.
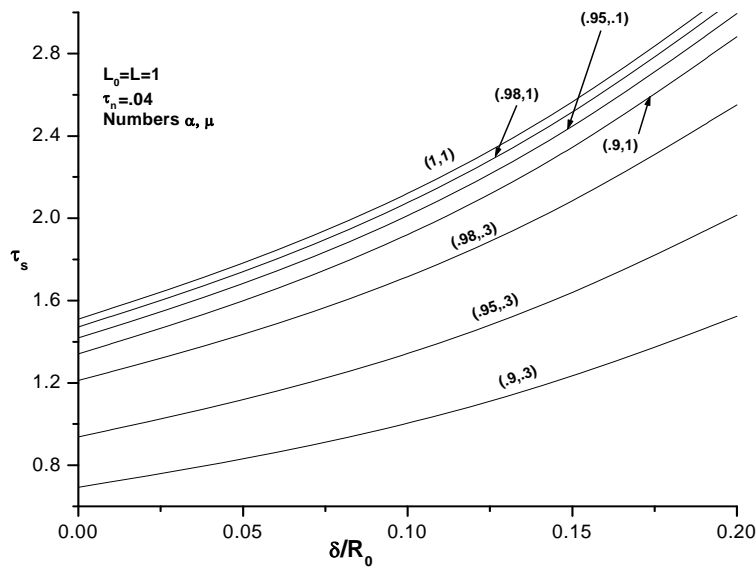

Fig.12 Variation of shear stress at the stenosis throats, τ_s with δ/R_0 for different α.

increases steeply in the upstream of the second stenosis throat and attains its peak magnitude (with the same value as at the first throat of the stenosis) at the second throat of the stenosis at z/L_0 =5/6, it then decreases rapidly to the same magnitude as it's
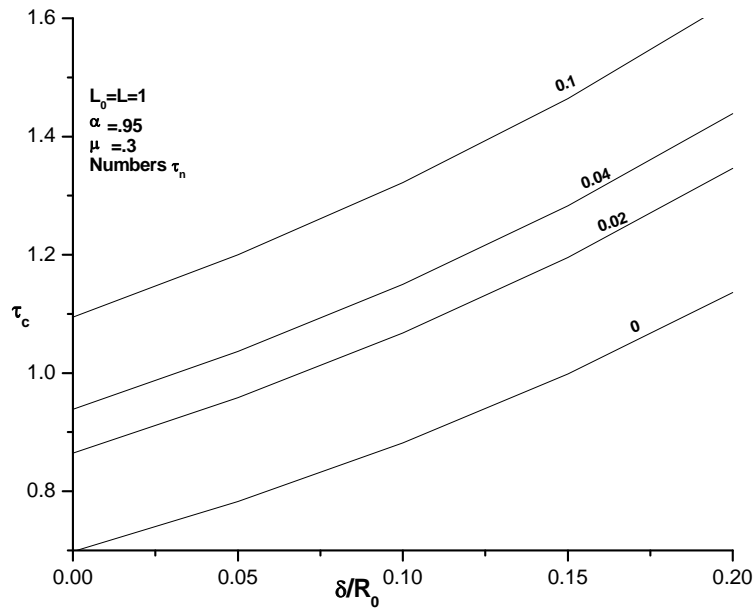
**Fig.13 Variation of shear stress at the stenosis critical
height, τ_c with δ/R_0 for different τ_n.**
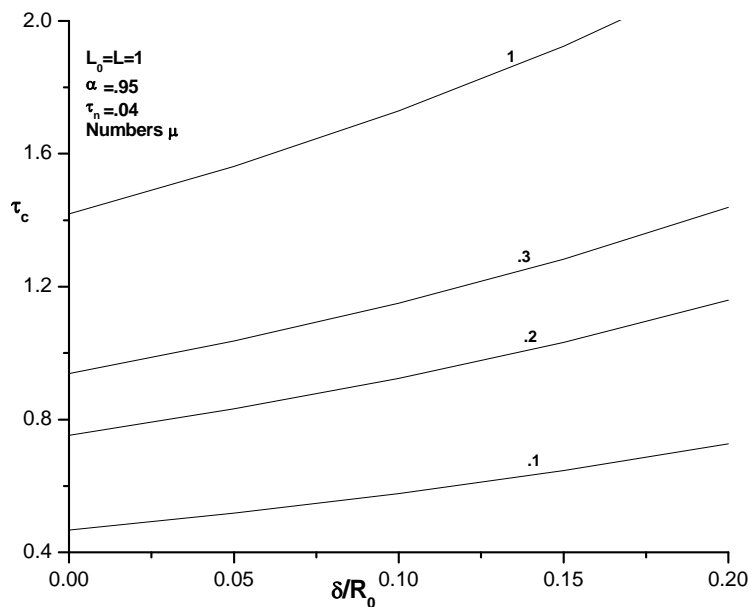
**Fig.14 Variation of shear stress at the stenosis critical height,
τ_c with δ/R_0 for different μ.**

approached value (at $z = 0$) at the end point of the constriction profile at $z/L_0 = 1$ (Figures 7-9). We point out here that at any axial distance in the stenotic region, the shear stress, τ_w assumes

lower magnitude in two-fluid model than its corresponding value in the one- fluid model for any given set parameters. We notice that the flow characteristics, τ_w assumes significantly lower magnitude at critical height of the stenosis than at stenosis throats. The nature of the variations of the shearing stress at throats τ_s with respect to any parameter is similar to that of the flow resistance, λ (Figures 3 and 10). We point out that the variations in shear stress at the stenosis critical height (i.e., at $z/L_0 = 0.5$), τ_c is similar to that of the impedance, λ and shear stress at stenosis throats, τ_s. However, the flow shear stress at stenosis critical height, τ_c assumes significantly lower magnitude than its corresponding value of the shear stress at the stenosis two throats, τ_s (Figures 10 and 15).
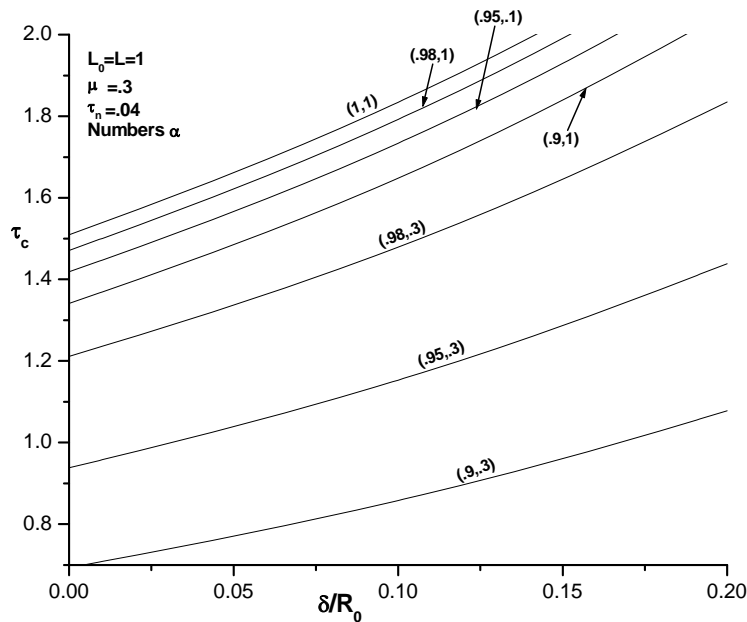
Fig.15 Variation of shear stress at the stenosis critical height,
τ_c **with** δ/R_0 **for different** α.

The Casson fluid seems to be more sensitive to the stenosis than Newtonian fluid. The condition $\delta/R_0 \ll 1$, limits the usefulness of the present study to very early stages of vessel constrictions, which enables one to use the fully developed flow equations and leads to the locally Poiseuille like flow and closed form solutions. Use of the parameter, δ/R_0 is restricted to the values up to 0.15 as beyond this value a separation in the flow may occurs even at a relatively small Reynolds number [Young (1968); Srivastava (1995)].

5. Conclusions

To observe the effects of non-Newtonian behavior and the peripheral layer on flow characteristics of blood, a two-layered model of blood, assuming that blood in the central region is represented by a Casson fluid and a peripheral layer of plasma, has been applied to blood flow through an overlapping stenosis in a narrow artery. The flow characteristics (impedance, wall shear stress in the stenotic region, shear stress at stenosis throats and at the critical height of the

stenosis) increase with non-Newtonian behavior of blood as well as with stenosis size (height and length). The flow characteristics assume lower magnitude in a two-layered model than its corresponding value in a one-layered model. This concludes that the peripheral layer helps in the functioning of the diseased artery. The shear stress at the two stenosis throats assumes the same magnitude. The shear stress at the critical height of the stenosis assumes significantly smaller value than at stenosis throats. The nature of variations in the flow characteristics is similar with respect to any given parameter.

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