Applications and Applied Mathematics: An International Journal (AAM)

Volume 5 | Issue 1 Article 18

6-2010

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 5, Issue 1 (June 2010) pp. 239 - 254
(Previously, Vol. 5, No. 1)

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Temperature Profiles in a Disc Brake

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Received: January 5, 2010; Accepted: June 15, 2010

Abstract

The Science of heat transfer allows us to determine the time rate of energy transfer caused by non equilibrium of temperatures. The importance of heat transfer in proper design of Automobiles has long been recognized. In this paper we determined the transient temperature distributions in a disc brake during a single brake application using Finite difference numerical technique. Hyperbolic heat conduction which includes the effect of the finite heat propagation is gaining importance. It is necessary to consider hyperbolic heat conduction in problems involving short time intervals and for very high heat fluxes. Here we considered both parabolic and hyperbolic heat conduction and the results are evaluated numerically, represented graphically and analyzed by comparing the temperature profiles corresponding to parabolic and hyperbolic heat conduction.

Keywords: Heat transfer, Hyperbolic heat conduction, Temperature profile, Heat flux, Automobile

AMS 2000 Classification: 34B07, 34B40

1. Introduction

Heat transfer is one of the important branches of Science and engineering, which deals with the mechanisms responsible for transferring energy from one place to another when a temperature difference exists. The science of heat transfer allows us to determine the time rate of energy transfer caused by non-equilibrium of temperatures.

Transient heat conduction takes place in the heating or cooling of bodies, glass manufacture, brick burning, vulcanization of rubber, starting and stopping of various heat exchangers, power installations etc. Kakac and Yener (1979), Lyknon (1972), Sachenko and Sukomel (1977), Balachandra et al. (1989), Blackwell (1981), Tikhe and Deshmukh (2006), Deshmuch et al. (2009) and others have discussed in detail the methods of solving many important problems in transient heat conduction.

The importance of heat transfer in the proper design of Automobiles has long been recognized. In this paper we determine the transient temperature distributions in a disc brake during a single brake application concerning to parabolic heat conduction and hyperbolic heat conduction, using finite difference numerical technique. We evaluated the results numerically, assuming that a car makes a panic stop from 150 km/h to rest in 10 seconds due to application of a sudden brake. In Figure 2 and in Figure 3 we represented the variation of temperature with times when L is 0.6 cms and 0.7 cms respectively for parabolic case. Corresponding results of hyperbolic heat conduction were represented in Figure 4 and in Figure 5.

2. Parabolic Formulation and Solution of the Problem

In any four wheeler vehicle disc brakes consists of two blocks of frictional material known as pads which are pressed against each side of a rotating annulus, usually made of a ferrous material, see Figure 1.

Figure 1.

In a single brake application, a constant torque acts on each pad and produces a uniform disc deceleration. The rate of heat generation from the friction surfaces decreases linearly with time i.e.,

$$N(1-Mt), \tag{1}$$

where M and N are constants to be determined from the rate of evolution of heat during braking. Newcomb (1960) had shown that for conventional pad materials, the heat generated flows wholly into the brake piece and the problem can be formulated as a linear flow of heat through an infinite slab bounded by parallel planes $x = +L$ and $x = -L$, where the thermal flux through the two plane boundaries decrease linearly with time.

The governing heat conduction equation is

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}. \tag{2}$$

Here, a^2 is the diffusivity of the brake disc and the origin is taken at the midpoint of the thickness of the brake disc. At the planes $x = \pm L$ there is a uniformly distributed thermal flux $N(1\text{-}Mt)$ together with an average loss of heat due to convention which is proportional to the temperature.

The initial and boundary conditions corresponding to our problem are,

$$T(x,0) = 0 \quad \text{for} \quad x \geq 0, \tag{3}$$

$$K\frac{\partial T}{\partial x} = N(1 - Mt) - hT \quad \text{at} \quad x = L, \quad \text{for all} \quad t, \tag{4}$$

and

$$K\frac{\partial T}{\partial x} = -N(1 - Mt) + hT \quad \text{at} \quad x = -L, \quad \text{for all} \quad t, \tag{5}$$

where K is the thermal conductivity and h is the heat transfer coefficient.

We obtain the solution of the problem using Finite - Difference method. Replacing the partial derivatives in equation (2) by finite -difference approximations, we get

$$\frac{T_{i,j+1} - T_{i,j}}{\delta t} = a^2 \left\{ \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\delta x)^2} \right\},$$

i.e.,

$$T_{i,j+1} = T_{i,j} + ra^2 (T_{i-1,j} - 2T_{i,j} + T_{i+1,j}), \tag{6}$$

where $r = \dfrac{\delta t}{(\delta x)^2}$ and let $\delta x = 0.1$.

The boundary condition at x = -L, in terms of central differences, can be written as

$$K\left\{\frac{T_{i+1,j} - T_{i-1,j}}{2\delta x}\right\} = -N(1 - Mt) + hT_{i,j},$$

i.e.,

$$T_{i-1,j} = T_{i+1,j} + 2\frac{\delta x}{K}\left\{N(1 - Mt) - hT_{i,j}\right\}. \tag{7}$$

Eliminating $T_{i-1,j}$ from (6) and (7), we get

$$T_{i,j+1} = T_{i,j} + 2ra^2\left\{T_{i+1,j} + \frac{\delta x}{K}(N(1 - Mt) - hT_{i,j}) - T_{i,j}\right\}, \tag{8}$$

and the boundary conditions at $x = L$ is

$$K\left\{\frac{T_{i+1,j} - T_{i-1,j}}{2\delta x}\right\} = N(1 - Mt) - hT_{i,j},$$

i.e.,

$$T_{i+1,j} = T_{i-1,j} + 2\frac{\delta x}{K}\left\{N(1 - Mt) - hT_{i,j}\right\}. \tag{9}$$

Elimination of the 'fictitious' value $T_{i+1,j}$ between (6) and (9) yields

$$T_{i,j+1} = T_{i,j} + 2ra^2\left\{T_{i-1,j} + \frac{\delta x}{K}\left(N(1 - Mt) - hT_{i,j}\right) - T_{i,j}\right\}. \tag{10}$$

This result (10) could have been deduced from the corresponding equation at x = -L because of the symmetry with respect to x = 0. Choosing, r = 1/4, the difference equations (8) and (6), then, become

$$T_{i,j+1} = \frac{a^2}{2}\left\{\left(\frac{2}{a^2} - 1 - \frac{\delta x}{K}h\right)T_{i,j} + T_{i+1,j} + \frac{\delta x}{K}N(1-Mt)\right\}, \tag{11}$$

i = -6, i.e., at x = -L = -0.6 and i = -7, i.e., at x = -L = -0.7,

$$T_{i,j+1} = \frac{a^2}{4}\left\{T_{i-1,j} + (\frac{4}{a^2} - 2)T_{i,j} + T_{i+1,j}\right\}, \tag{12}$$

i = -5, -4,-3,-2,-1, if x = -L = -0.6, and
i = -6, -5, -4,-3,-2,-1, if x = -L= -0.7, i.e., $x \neq -L$ and $x \neq 0$,

and the use of symmetry rather than the equation (10) gives

$$T_{i,j+1} = \frac{a^2}{2}\left\{T_{i-1,j} + (\frac{2}{a^2} - 1)T_{i,j}\right\}, \tag{13}$$

i = 0, i.e., at x = 0.

3. Hyperbolic Formulation and Solution of the Problem

Transient heat transfer problems usually involve the solution of the classical Fourier heat conduction equation, which is of parabolic character, as a consequence, a perturbed heat signal propagates with an infinite velocity through the medium. That is, if an isotropic homogeneous elastic continuum is subjected to a mechanical or thermal disturbance, the effect of the disturbance will be felt instantaneously at distances infinitely far from its source. Such a behavior is physically inadmissible and contradicts the existing theories of heat transport mechanisms.

It seems, therefore reasonable to modify the existing theory of heat conduction. To remove the deficiencies many investigators such as Maxwell (1867), Morse and Feshbach (1953), Chester (1963), Gurtin and Pipkin (1968), Lebon and Lamberamont (1976), Lord and Shulman (1967), Nunziato (1971), Green and Lindsay (1972), have suggested some modifications.

Tisza (1947) predicted the possibility of extremely small heat propagation rates (second sound) in liquid helium - II. Chester (1963) discussed the possibility of existence of second sound in solids. The experiments on sodium helium by Ackerman et al. (1966) and by Mc Nelly (1970) on sodium fluoride, have shown that second sound occurs in solids also. The second sound effect indicates that wave type mechanism rather than usual diffusion process can transport heat. All these researches lead to the reformulation of existing Fourier heat conduction equation into a damped wave type equation, which is hyperbolic.

Morse and Feshbach (1953) postulated that the governing transient heat conduction must depend upon the velocity of the propagation of heat 'C'. They assumed that the following equation

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\nabla^2 T,$$

which is hyperbolic, must be the correct governing differential equation for heat conduction problems.

In some cases, the effect of the finite speed of propagation is negligible. However this effect is considerable and important even at the ordinary temperature when the elapsed time during a transient is small. If 'C' is very large, the above equation reduces to classical Fourier heat conduction equation. Ackerman and Guyer (1968) had given the estimation for 'C'. According to them it is 1/√3 times the velocity of dilatational wave 'C_1' which is given by $C_1^2=(\lambda+2\mu)/\rho$, where λ and μ are Lame's constants. Sharma and Siddu (1986), Sharma and Kaur (2008), Nowinski (1978), Chandrasekarayya (1980, 1984, and 1996) and others have done considerable work in this field.

The governing hyperbolic heat conduction equation is

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}=a^2\frac{\partial^2 T}{\partial x^2}\ . \tag{14}$$

Here, a^2 is the thermal diffusivity of the brake disc and C is velocity of heat propagation.

At the planes $x = \pm L$ there is a uniformly distributed Thermal flux N (1 - M t) together with an average loss of heat due to convection which is proportional to the temperature. Therefore, the initial and boundary conditions corresponding to hyperbolic formulation are

$$T(x,0)=0 \text{ and } \left.\frac{\partial T}{\partial t}\right|_{t=0}=0 \quad \text{for} \quad x\ge 0, \tag{15}$$

$$K\frac{\partial T}{\partial x}=N(1-Mt)-hT \quad \text{at} \quad x=L,\ \text{for all} \quad t, \tag{16}$$

and

$$K\frac{\partial T}{\partial x}=-N(1-Mt)+hT \quad \text{at} \quad x=-L,\ \text{for all} \quad t, \tag{17}$$

where K is the thermal conductivity and h is the heat transfer coefficient.

We obtain the solution of the problem using Finite - difference method. Replacing partial derivatives in equation (14) by finite -difference approximations, we get

$$\frac{1}{C^2}\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{(\delta t)^2}+\frac{T_{i,j+1}-T_{i,j}}{\delta t}=a^2\left\{\frac{T_{i-1,j}-2T_{i,j}+T_{i+1,j}}{(\delta x)^2}\right\},$$

i.e.,

$$T_{i,j+1}=\frac{2+\delta tC^2}{1+\delta tC^2}T_{i,j}+\frac{r^2a^2C^2}{1+\delta tC^2}(T_{i-1,j}-2T_{i,j}+T_{i+1,j})-\frac{1}{1+\delta tC^2}T_{i,j-1},\ \ j=1,2,3,\text{---},399, \tag{18}$$

where $r=\dfrac{\delta t}{\delta x}$ and let $\delta x=0.1$.

The initial conditions at $t = 0$ in terms of central differences can be written as $T_{i,j}=0$ and

$$\frac{T_{i,j+1}-T_{i,j-1}}{2\delta t}=0,\text{ i.e., }T_{i,j-1}=T_{i,j+1}. \tag{19}$$

Eliminating $T_{i,j-1}$ from (18) and (19), we get

$$T_{i,j+1}=T_{i,j}+\frac{r^2a^2C^2}{2+\delta tC^2}\left\{T_{i-1,j}-2T_{i,j}+T_{i+1,j}\right\},\ \ j=0. \tag{20}$$

The boundary condition at $x = -L$ in terms of central differences, can be written as

$$K\left\{\frac{T_{i+1,j}-T_{i-1,j}}{2\delta x}\right\}=-N(1-Mt)+hT_{i,j},$$

i,e.,

$$T_{i-1,j}=T_{i+1,j}+2\frac{\delta x}{K}\left\{N(1-Mt)-hT_{i,j}\right\}. \tag{21}$$

Eliminating $T_{i-1,j}$ from (18), (20) and (21), we get

$$T_{i,j+1}=T_{i,j}+\frac{2r^2a^2C^2}{2+\delta tC^2}\left\{T_{i+1,j}-T_{i,j}+\frac{\delta x}{K}\{N(1-Mt)-hT_{i,j}\}\right\},$$

$j = 0$, i.e, $t = 0$, $i = -6$ if $x = -L = -0.6$ and $i = -7$ if $x = -L = -0.7$, (22) and

$$T_{i,j+1} = \frac{2+\delta t C^2}{1+\delta t C^2} T_{i,j} + \frac{2r^2 a^2 C^2}{1+\delta t C^2} \left\{ T_{i+1,j} - T_{i,j} + \frac{\delta x}{K} \{ N(1-Mt) - hT_{i,j} \} \right\}$$

$$- \frac{1}{1+\delta t C^2} T_{i,j-1}, \quad i = -6 \text{ if } x = -L = -0.6 \text{ and } i = -7 \text{ if } x = -L = -0.7, \text{ and } j = 1,2,3, ---,399, \tag{23}$$

and the boundary condition at $x = L$ is

$$K \left\{ \frac{T_{i+1,j} - T_{i-1,j}}{2\delta x} \right\} = N(1-Mt) - hT_{i,j},$$

i.e.,

$$T_{i+1,j} = T_{i-1,j} + 2\frac{\delta x}{K} \left\{ N(1-Mt) - hT_{i,j} \right\}. \tag{24}$$

Elimination of the 'fictitious' value $T_{i+1,j}$ between (18), (20) and (24) yields

$$T_{i,j+1} = T_{i,j} + \frac{2r^2 a^2 C^2}{2+\delta t C^2} \left\{ T_{i-1,j} - T_{i,j} + \frac{\delta x}{K} \{ N(1-Mt) - hT_{i,j} \} \right\}, \tag{25}$$

$t = 0$ i.e., $j = 0$ and $i = 6$ if $x = L = 0.6$ and $i = 7$ if $x = L = 0.7$,

and

$$T_{i,j+1} = \frac{2+\delta t C^2}{1+\delta t C^2} T_{i,j} + \frac{2r^2 a^2 C^2}{1+\delta t C^2} \left\{ T_{i-1,j} - T_{i,j} + \frac{\delta x}{K} \{ N(1-Mt) - hT_{i,j} \} \right\}$$

$$- \frac{1}{1+\delta t C^2} T_{i,j-1}, \quad i = 6 \text{ if } x = L = 0.6 \text{ and } i = 7 \text{ if } x = L = 0.7, \; j = 1,2,3, ---,399. \tag{26}$$

These results (25) and (26) could have been deduced from the corresponding equations at x = -L because of symmetry with respect to x = 0.

Choosing r = 1/4, the difference equations (22), (23), (20) and (18), then become

$$T_{i,j+1} = \frac{a^2C^2}{8(2+\delta tC^2)}\left\{\left(\frac{8(2+\delta tC^2)}{a^2C^2} - 1 - \frac{\delta x}{K}h\right)T_{i,j} + T_{i+1,j} + \frac{\delta x}{K}N(1-Mt)\right\},$$

j = 0, i.e., t = 0, i = -6 if x = -L= -0.6 and i = -7 if x = -L= -0.7. (27)

$$T_{i,j+1} = \frac{a^2C^2}{8(1+\delta tC^2)}\left\{\left(\frac{8(2+\delta tC^2)}{a^2C^2} - 1 - \frac{\delta x}{K}h\right)T_{i,j} + T_{i+1,j} + \frac{\delta x}{K}N(1-Mt)\right\} - \frac{1}{1+\delta tC^2}T_{i,j-1},$$

$i = -6$, if $x = L = -0.6$ and $i = -7$, if $x = L = -0.7$, $j = 1, 2, 3, \cdots, 399$,

(28)

$$T_{i,j+1} = \frac{a^2C^2}{16(2+\delta tC^2)}\left\{T_{i-1,j} + \left(\frac{16(2+\delta tC^2)}{a^2C^2} - 2\right)T_{i,j} + T_{i+1,j}\right\},$$

i = -5, -4,-3,-2,-1 if x = -0.6 and i = -6, -5, -4,-3,-2,-1 if x = -0.7, j = 0, i.e., t = 0 (29)

and

$$T_{i,j+1} = \frac{a^2C^2}{16(1+\delta tC^2)}\left\{T_{i-1,j} + \left(\frac{16(2+\delta tC^2)}{a^2C^2} - 2\right)T_{i,j} + T_{i+1,j}\right\} - \frac{1}{1+\delta tC^2}T_{i,j-1}$$

i = -5, -4,-3,-2,-1 if x = -0.6 and i = -6, -5, -4,-3,-2,-1 if x = -0.7 and j =1,2,3, …, 399, (30)

respectively, and the use of symmetry rather than the equations (25) and (26) gives

$$T_{i,j+1} = \frac{a^2C^2}{8(2+\delta tC^2)}\left\{T_{i-1,j} + \left(\frac{8(2+\delta tC^2)}{a^2C^2} - 1\right)T_{i,j}\right\}, \tag{31}$$

i = 0, i.e. x = 0 and j = 0, i.e., t = 0,

and

$$T_{i,j+1} = \frac{a^2C^2}{8(2+\delta tC^2)}\left\{T_{i-1,j} + \left(\frac{8(2+\delta tC^2)}{a^2C^2} - 1\right)T_{i,j}\right\} - \frac{1}{1+\delta tC^2}\ T_{i,j\text{-}1},$$

i = 0, i.e., x = 0, j = 1,2,3, …, 399, (32)

respectively.

4. Numerical Evaluation

The results obtained are evaluated corresponding to a braking from 150 *kmph* to rest in 10 seconds with the following values of the parameters involved.

$a^2 = 0.123\ cm^2/\sec$, $h = 0.0013\ cal/cm^2\ {}^0\text{c}\sec$

$k = 0.115\ cal/cm\ sec\ {}^0\text{c}$, $N = 54.2\ cal/cm^2\ sec$

$M = 1/\tau$, where τ is the time of a single brake application (10 *seconds*)

ρ is density = 7.83 gm/cm^3, E is Young's modulus = $2\times10^{12}\ dynes/cm^2$

σ is Poisson's ratio = 0.3, $\lambda = 1153846 \times 10^6\ dynes/cm^2$

$\mu = 769230 \times 10^6\ dynes/cm^2$

$$C_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} = 586382.9425\ cm/sec$$

$$C = \frac{C_1}{\sqrt{3}} = 338548.3496\ cm/sec$$

$\delta x = 0.1$ and $\delta t = 0.0025$ to determine the parabolic temperature distribution

$\delta x = 0.1$ and $\delta t = 0.025$ to determine the hyperbolic temperature distribution

Since the diffusivity of pads (nearly $a^2 = 10^{-3} cm^2/\sec$) is so small compared to that of the disc (i.e., $a^2 = 0.123 cm^2/\sec$), all the heat flow occurs within the disc.

We analyze the results and to study the importance of hyperbolic heat conduction, we considered the following two cases.

Case 1

Here we evaluated the results by considering a brake disc of thickness 2*L*, where *L* = 0.6 cms. The temperature distribution at various positions and at particular times corresponding to parabolic heat conduction are shown in Table 1, while the corresponding results concerning to hyperbolic heat conduction are shown in Table 2. Also we represented these results graphically in Figures 2 and 4.

Case 2

Here, we evaluated the results corresponding to break disc of thickness 2*L*, where *L* is 0.7cms to study the effect of the size of the disc on the surface temperatures. The temperature distribution at various positions and at particular times corresponding to parabolic heat conduction are shown in Table 3, corresponding results of hyperbolic heat conduction are shown in Table 4. These results were also shown graphically in Figures 3 and 5.

From the figures, it can be observed that temperature will go on decrease as we move away from the surface plane $x = L$ and $x = -L$ of the disc towards the plane of symmetry $x = 0$ of the disc. It is also observed that the maximum temperature is at the points on the surface plane $x = L$ and $x = -L$ of the disc and it is minimum at the points on plane $x = 0$ of the disc, which is in full agreement with the physical nature of the problem. Also we can observe that after 10 seconds, there won't be any variation in the temperature and the temperature distribution will be uniform throughout the disc.

By comparing the temperature distributions corresponding to parabolic and hyperbolic heat conduction, we can observe that at points on the surface of the disc, the hyperbolic temperatures are greater than that of parabolic temperatures, while at the points nearer to the plane of symmetry $x = 0$ of the disc, the hyperbolic temperatures are less than that of the parabolic temperatures. This is due to the finiteness of heat propagation velocity in hyperbolic heat conduction. The results corresponding to hyperbolic heat conduction are more general and accurate.

It is also observed that after application of the sudden brakes, as time goes on, the maximum temperature at the points on the surface of the disc of thickness $2L$, where L is 0.6 cms are higher than that of the maximum temperatures at the corresponding points on the surface of the disc of thickness $2L$, where L is 0.7 cms. Therefore, while designing the brake disc, care must be taken regarding the size of the disc, so that it can bear the maximum temperatures that will occur at the points on the surface of the disc due to high friction when brakes are applied suddenly.

Table 1. Parabolic Temperature Distribution when $L = 0.6$ *cms*

	i = -6 x = -0.6	i = -5 x = -0.5	i = -4 x = -0.4	i = -3 x = -0.3	i = -2 x = -0.2	i = -1 x = -0.1	i = -0 x = -0.0
t = 0.2	80.1604	42.4043	19.7725	8.1159	2.9518	1.0104	0.5381
t = 1	175.0457	136.2878	104.6497	80.2102	62.9037	52.5996	49.1802
t = 2	249.1454	214.5144	185.7121	163.0237	146.6662	136.7919	133.4906
t = 3	311.3962	281.1507	255.9183	235.9906	221.5945	212.8922	209.9806
t = 4	363.7836	337.9218	316.2763	299.1388	286.7356	279.2292	276.7162
t = 5	406.4073	384.9175	366.8506	352.497	342.0827	335.7694	333.6543
t = 6	439.2935	422.1645	407.6667	396.0898	387.6594	382.5368	380.8186
t = 8	475.961	467.5184	460.1301	454.0847	449.6064	446.8552	445.9274
t = 10	473.9897	474.1873	473.8711	473.3278	472.7804	472.388	472.2465

Table 2. Hyperbolic Temperature Distribution when $L = 0.6$ *cms*

	i = -6 x = -0.6	i = -5 x = -0.5	i = -4 x = -0.4	i = -3 x = -0.3	i = -2 x = -0.2	i = -1 x = -0.1	i = -0 x = -0.0
t = 0.2	81.3262	43.2605	19.8759	7.6612	2.3816	0.5696	0.1805
t = 1	175.0956	136.3721	104.694	80.1659	62.758	52.3752	48.9265
t = 2	248.9376	214. 3626	185.6028	162.9441	146.6058	136.7419	133.4438
t = 3	311.1852	280.9879	255.7957	235.8999	221.5268	212.8383	209.9314
t = 4	363.6025	337.7805	316.1687	299.0579	286.6743	279.1794	276.6703
t = 5	406.2677	384 8112	366.7721	352.4408	342.0429	335.7396	333.6277
t = 6	439.2051	422.1039	407.6294	396.0712	387.6545	382.5401	380.8245
t = 8	475.9865	467.5657	460.1957	454.1646	449.6964	446.9515	446.0258
t = 10	474.1259	474.3455	474.0471	473.5175	472.9798	472.5934	472.454

Table 3. Parabolic Temperature Distribution when $L = 0.7$ *cms*

	i = -7 x = -0.7	i = -6 x = -0.6	i = -5 x = -0.5	i = -4 x = -0.4	i = -3 x = -0.3	i = -2 x = -0.2	i = -1 x = -0.1	i = -0 x = -0.0
t = 0.2	80.1604	42.4042	19.7719	8.1127	2.9366	0.9448	0.2849	0.1378
t = 1	173.3239	134.3094	101.8686	75.9717	56.3835	42.7600	34.7553	32.1177
t = 2	237.6756	202.5635	172.3438	147.2933	127.6204	113.4732	104.9487	102.1014
t = 3	288.5543	257.8335	231.2374	209.0686	191.5733	178.9416	171.3087	168.7555
t = 4	330.6314	304.3348	281.4756	262.358	247.2308	236.2875	229.6663	227.4500
t = 5	364.3279	342.4492	323.3289	307.2707	294.5232	285.2797	279.6783	277.8020
t = 6	389.6976	372.227	356.838	343.8335	333.4617	325.9154	321.3325	319.7957
t = 7	406.7623	393.6897	382.0235	372.0659	364.0645	358.2116	354.6449	353.4468
t = 10	408.3163	408.3779	407.8294	406.9709	406.0483	405.2527	404.7207	404.5342

Table 4. Hyperbolic Temperature Distribution when $L = 0.7$ *cms*

	i = -7 x = -0.7	i = -6 x = -0.6	i = -5 x = -0.5	i = -4 x = -0.4	i = -3 x = -0.3	i = -2 x = -0.2	i = -1 x = -0.1	i = -0 x = -0.0
t = 0.2	81.3262	43.2605	19.8759	7.6612	2.3816	0.5621	0.0902	0.0150
t = 1	173.5068	134.5222	102.0258	76.0109	56.2758	42.5132	34.4101	31.7369
t = 2	237.5028	202.4449	172.2589	147.2249	127.5560	113.4062	104.8778	102.0287
t = 3	288.3293	257.6603	231.1076	208.9735	191.5045	178.8912	171.2692	168.7195
t = 4	330.4171	304.1662	281.3465	262.2616	247.1602	236.2355	229.6254	227.4129
t = 5	364.1387	342.3002	323.2147	307.1855	294.4608	285.2338	279.6424	277.7694
t = 6	389.5392	372.1046	356.7467	343.7681	333.4166	325.8849	321.3109	319.7770
t = 7	406.6374	393.598	381.9603	372.0263	364.0434	358.2039	354.6452	353.4498
t = 10	408.2834	408.3776	407.8565	407.0204	406.1151	405.3318	404.8072	404.6232

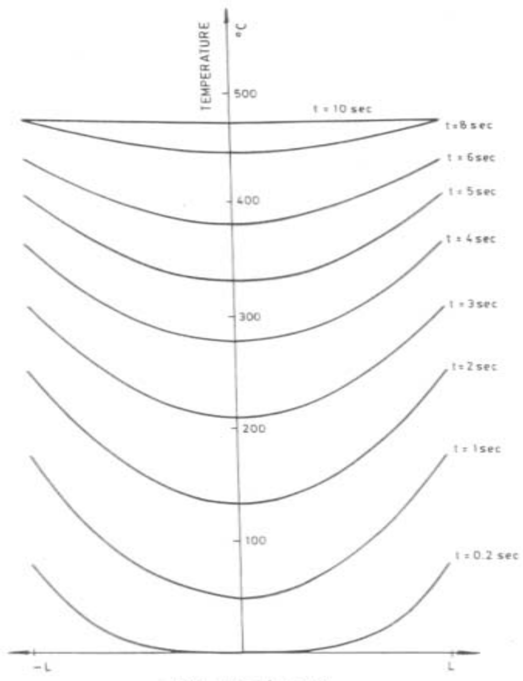

Figure 2. The parabolic temperature distribution with distance for different times when l= 0.6 cms

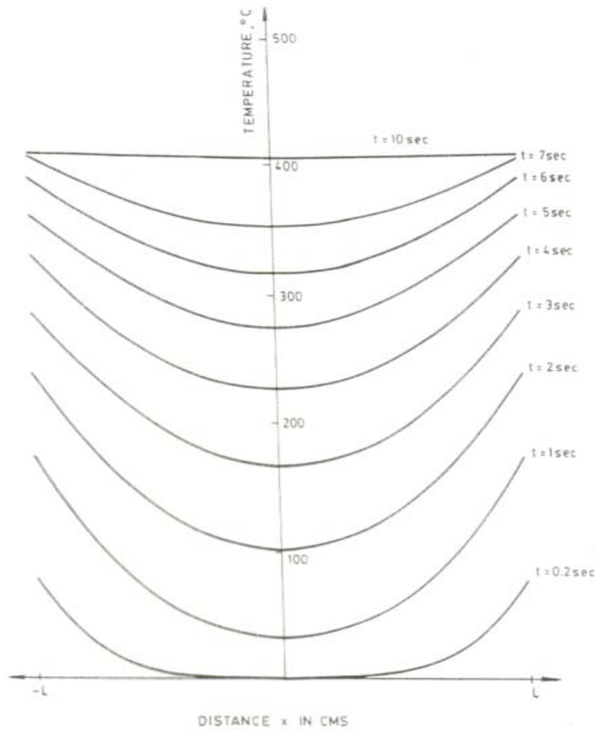

Figure 3. The parabolic temperature distribution with distance for different times when $l = 0.7$ cms

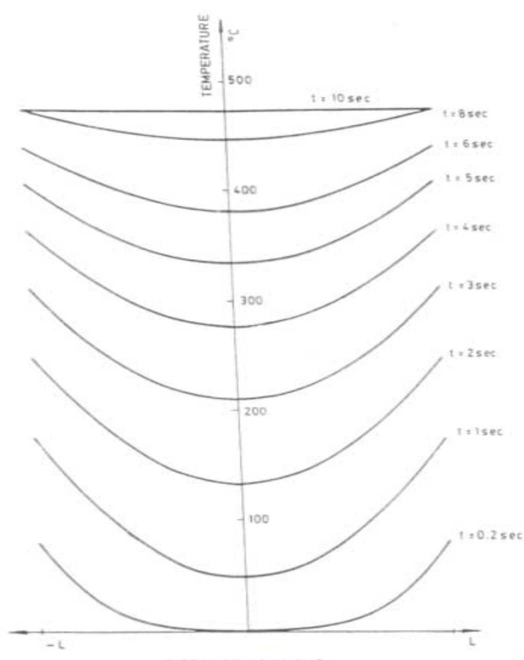

Figure 4. The hyperbolic temprerature distribution with distance for different times when $L = 0.6$ *cms*

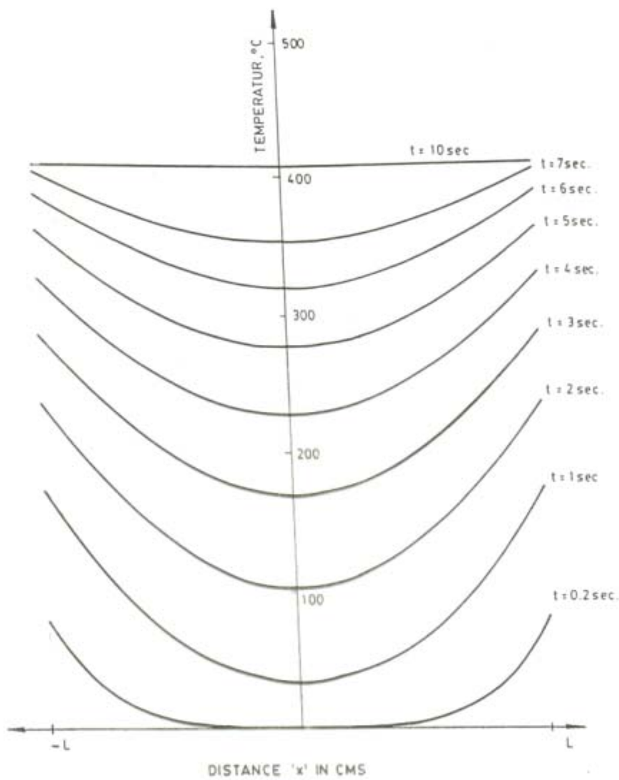

Figure 5. The hyperbolic temperature distribution with distance for different times when l = 0.7 *cms*

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