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Improved dust acoustic solitary waves in two temperature dust fluids

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Abstract

A theoretical investigation is carried out for contribution of the higher-order nonlinearity to nonlinear dust-acoustic solitary waves (DASWs) in an unmagnetized two types of dust fluids (one cold and the other is hot) in the presence of Bolltzmannian ions and electrons. A KdV equation that contains the lowest-order nonlinearity and dispersion is derived from the lowest order of perturbation and a linear inhomogeneous (KdV-type) equation that accounts for the higher-order nonlinearity and dispersion is obtained. A stationary solution for equations resulting from higher-order perturbation theory has been found using the renormalization method. The effects of hot and cold dust charge grains are found to significantly change the higher-order properties (viz. the amplitude and width) of the DASWs.

Keywords: Dust-acoustic waves; Hot and cold dust charge; KdV-type equation; Higher order nonlinearity

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1. Introduction

The study of the dynamics of dust contaminated plasmas (mixtures of ordinary plasma particles and charged dust grains) has recently received considerable interest due to their occurrence in real charged particle systems, e.g., in interstellar clouds, in interplanetary space, in cometary tails, in ring systems of giant planets (like Saturn F- ring's), in mesospheric noctilucent clouds, as well as in many Earth bound plasma, see for instance Verheest (2000), Shukla and Mamun (2002). Dusty plasma applications range from astrophysics to strongly coupled dusty plasmas and dusty plasma crystals to technology plasma etching and deposition Mendis and Rosenberg (1994), Horonyi (1996), Verheest (1996), Shukla and Mamun (2003).

In dusty plasma, the dust grains may be charged negatively by plasma electron and ion currents or positively by secondary electron emission, UV radiation, or thermionic emission, etc Hornayi (1996), Whipple (1981). From theoretical perspective, Chow et al. (1993) have shown that due to the size effect on secondary emission insulating dust grains with different sizes in space plasmas can have the opposite polarity (smaller ones being positive and larger ones being negative). Therefore, it is so important and interesting to deal with the dust plasma systems if in addition to the electrons and ions, there are two types of dust particles with different masses, charges and temperatures. Consquently, there will be two basic dust-acoustic modes propagating with two different velocities El-Wakil et al. (2006a), Mowafy et al. (2008).

The dust-acoustic modes have also been investigated in the presence of hot dust in unmagnetized plasmas Roychoudhury and Mukherjee (1997), Mahmood and Saleem (2003). In most of the previous investigations of DAW, the dust has been assumed to be cold or hot. Recently, Akhtar et al. (2007), derived the Sagdeevs pseudopotential for dust acoustic waves (DAWs) in an unmagnetized two types of dust fluids (one cold and the other is hot) in the presence of Bolltzmannian ions and electrons, and studied the existence of rarefactive solitons. Investigations of small-amplitude (DAWs) in dusty plasma usually describe the evolution of the wave by the Korteweg--de Vries (KdV) equation. In fact, this equation contains the lowest-order nonlinearity and dispersion, and consequently can only describe a wave restricted to small amplitude. In other words, (the first-order solution would underestimate the amplitude of the solitary wave by as much as 20%.). As the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KdV equation, i.e., the KdV approximation does not apply anymore. Therefore, in order to overcome this deviation, higher-order nonlinear and dispersive effects have to be taken into account El-Labany (1995), El-Wakil et al. (2004a), El-Taibany and Moslem (2005), El-Shewy (2005), El-Shewy et al. (2008), El-Wakil et al. (2006b), El-Wakil et al. (2004b), Attia et al. (2010).

In the present work, our main concern is to examine the effect of both higher-order depressive and nonlinear corrections on the amplitude and width of (DA) solitary waves propagated in an unmagnetized two types of dust fluids (one cold and the other is hot) in the presence of Bolltzmannian ions and electrons. Also, we aim to study the effect of hot and cold dust charge grains on the formations of both the improved rarefactive and compressive solitons. This paper is organized as follows; in Section 2, we present the basic set of fluid equations governing our plasma model which reduced to the well-known KdV equation by employing the reductive perturbation theory. In Section 3, a linear inhomogeneous (KdV-type) equation is obtained. In

Section 4, higher-order solution is obtained by using the renormalization method. Finally, discussions and conclusions are given in Sections 5 and 6, respectively.

2. Basic equations and KdV equation

We consider homogeneous unmagnetized plasma having electrons, singly charged ions, hot and cold dust species. The electrons and ions are assumed to follow the Boltzmann distribution with $T_e, T_i >> T_d$, where T_e and T_i are the temperatures of the electrons and ions, respectively, and T_d is the temperature of hot dust. The hot dust is assumed to be heated adiabatically. The one-dimensional continuity and momentum equations for cold dust, respectively, are,

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_c) = 0, \tag{1.a}$$

$$\frac{\partial u_c}{\partial t} + (u_c \frac{\partial}{\partial x}) u_c - \frac{Z_c}{m_c}\frac{\partial \varphi}{\partial x} = 0. \tag{1.b}$$

Corresponding equations for hot dust particles are:

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_h) = 0, \tag{2.a}$$

$$\frac{\partial u_h}{\partial t} + (u_h \frac{\partial}{\partial x}) u_h - \frac{Z_h}{m_h}\frac{\partial \varphi}{\partial x} + \frac{1}{m_h n_h}\frac{\partial P_h}{\partial x} = 0. \tag{2.b}$$

For adiabatic hot dust, we have

$$P_h = P_{h0}(\frac{n_h}{N_{h0}})^{\gamma}, \tag{2.c}$$

where $\gamma = (N+2)/N$, and N is the number of degrees of freedom. The present work, $N = 1$, $\gamma = 3$ and $P_{h0} = N_{h0} T_h$. The Boltzmann distributed electrons and ions follow the density equations, respectively, as

$$n_e = n_{e0}\exp(e\,\varphi / T_e) \tag{3.a}$$

and

$$n_i = n_{i0}\exp(-e\,\varphi / T_i)\,. \tag{3.b}$$

The Poisson equation can be written as

$$\frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2} = 4\pi e (n_e - n_i + Z_c n_c + Z_h n_h). \tag{4}$$

In equilibrium, we have,

$$n_{i0} = n_{e0} + Z_c\ N_{c0} + Z_h\ N_{h0}. \tag{5}$$

In the earlier equations, φ is the electrostatic potential, Z_h and Z_c are the charge numbers for negatively charged hot and cold dust, respectively. $u_c\ (u_h)$ is the cold (hot) dusty plasma velocity. $n_h, n_c, n_e, n_i, N_{h0}, N_{c0}, n_{e0},$ and n_{i0} are the perturbed and equilibrium number densities of the species, respectively.

According to the general method of reductive perturbation theory for small amplitude, we introduce the slow stretched co-ordinates Kodama and Taniuti (1978):

$$\tau = \varepsilon^{\frac{3}{2}} t, \quad \xi = \varepsilon^{\frac{1}{2}} (x - \lambda t), \tag{6}$$

where ε is a small dimensionless expansion parameter measuring the strength of nonlinearity and λ is the wave speed. All physical quantities appearing in (1) and (2) are expanded as power series in ε about their equilibrium values as:

$$\begin{aligned}
n_c &= N_{c0} + \varepsilon n_{c1} + \varepsilon^2 n_{c2} + \varepsilon^3 n_{c3} + \dots, \\
u_c &= \varepsilon u_{c1} + \varepsilon^2 u_{c2} + \varepsilon^3 u_{c3} + \dots, \\
n_h &= N_{h0} + \varepsilon n_{h1} + \varepsilon^2 n_{h2} + \varepsilon^3 n_{h3} + \dots, \\
u_h &= \varepsilon u_{h1} + \varepsilon^2 u_{h2} + \varepsilon^3 u_{h3} + \dots, \\
\varphi &= \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^3 \varphi_3 + \dots.
\end{aligned} \tag{7}$$

We impose the boundary conditions that as $|\xi| \to \infty,\ n_c = N_{c0}, n_h = N_{h0}, u_c = u_h = 0, \varphi = 0.$

Substituting (6) and (7) into (1-4), and equating coefficients of like powers of ε . Then, from the lowest-order equations in ε , the following results are obtained:

$$\begin{aligned}
n_{c1} &= -\frac{eN_{c0} Z_c}{\lambda^2 m_c} \varphi_1, u_{c1} = -\frac{eZ_c}{\lambda m_c} \varphi_1, \\
n_{h1} &= -\frac{eN_{h0}^2 Z_h}{\lambda^2 m_h N_{h0} - 3p_0} \varphi_1, \\
u_{h1} &= -\frac{e\lambda N_{h0} Z_h}{\lambda^2 m_h N_{h0} - 3p_0} \varphi_1.
\end{aligned} \tag{8}$$

Poisson's equation gives the linear dispersion relation

$$\frac{N_{c0} Z_c^{\,2}}{\lambda^2 m_c} + \frac{N_{h0}^{\,2} Z_h^{\,2}}{\lambda^2 m_h N_{h0} - 3 p_0} - \frac{n_{e0}}{Te} - \frac{n_{i0}}{Ti} = 0. \tag{9}$$

Proceeding to order of ε^2 and by eliminating the second order perturbed quantities n_{c2}, n_{h2}, u_{c2}, u_{h2} and φ_2, we obtain the desired KdV equation for the first-order perturbed potential:

$$\frac{\partial \varphi_1}{\partial \tau} + A\, \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + \frac{B}{2} \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0, \tag{10}$$

where

$$A = \frac{2\left(-\frac{12\pi N_{c0} Z_c^3 e^3}{\lambda^4 m_c^2} - \frac{12\pi N_{h0}^3 \left(m_h N_{h0} \lambda^2 + p_0\right) Z_h^3 e^3}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^3} - \frac{4\pi \left(Ti^{\,2} N_{e0} - Te^2 n_{i0}\right) e^3}{Te^2 Ti^{\,2}}\right)}{\left(\frac{8 e^2 \pi \lambda m_h Z_h^2 N_{h0}^3}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2} + \frac{8 e^2 \pi N_{c0} Z_c^2}{\lambda^3 m_c}\right)},$$

and

$$B = \frac{2}{\frac{8 e^2 \pi \lambda m_h Z_h^2 N_{h0}^3}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2} + \frac{8 e^2 \pi N_{c0} Z_c^2}{\lambda^3 m_c}}.$$

3. Linear inhomogeneous KdV-type equation

In this section we start by writing down the second-order perturbed quantities n_{c2}, u_{c2}, n_{h2} and u_{h2} in terms of Ω_1 and Ω_2 as:

$$n_{c2} = \frac{N_{c0} Z_c e}{2\lambda^4 m_c^{\,2}} \left(2\lambda m_c A \varphi_1^{\,2} + 4\lambda m_c \frac{B}{2} \frac{\partial^2 \varphi_1}{\partial \xi^2} + 3 Z_c e \varphi_1^{\,2} - 2\lambda^2 m_c \varphi_2 \right),$$

$$u_{c2} = \frac{Z_c e}{2\lambda^3 m_c^{\,2}} \left(\lambda m_c A \varphi_1^{\,2} + 2\lambda m_c \frac{B}{2} \frac{\partial^2 \varphi_1}{\partial \xi^2} + Z_c e \varphi_1^{\,2} - 2\lambda^2 m_c \varphi_2 \right),$$

$$n_{h2} = \frac{Z_h e\, N_{h0}}{2\,(-\lambda^6 m_h^{\ 3} + 9T_h m_h^{\ 2}\lambda^4 - 27T_h^{\ 2} m_h \lambda^2 + 27T_h^{\ 3})}$$
$$(-3\lambda^2 Z_h e m_h \varphi_1^{\ 2} - 2\lambda^3 m_h^{\ 2} A\varphi_1^{\ 2} - 4\lambda^3 m_h^{\ 2}\frac{B}{2}\frac{\partial^2\varphi_1}{\partial\xi^2} + 6\lambda m_h T_h A\varphi_1^{\ 2}$$
$$+12\lambda m_h T_h \frac{B}{2}\frac{\partial^2\varphi_1}{\partial\xi^2} + 2m_h^{\ 2}\lambda^4\varphi_2 - 12m_h\lambda^2 T_h\varphi_2 + 18T_h^{\ 2}\varphi_2 - 3T_h Z_h e\varphi_1^{\ 2}),$$
$$u_{h2} = \frac{eZ_h}{2(-\lambda^6 m_h^{\ 3} + 9T_h m_h^{\ 2}\lambda^4 - 27T_h^{\ 2} m_h\lambda^2 + 27T_h^{\ 3})}$$
$$(18T_h^{\ 2}\frac{B}{2}\frac{\partial^2\varphi_1}{\partial\xi^2} + 18T_h^{\ 2}\lambda\varphi_2 + 9T_h^{\ 2}A\varphi_1^{\ 2} - 9T_h Z_h e\lambda\varphi_1^{\ 2} - 12T_h m_h\lambda^3\varphi_2 \qquad (11)$$
$$+2m_h^{\ 2}\lambda^5\varphi_2 - Z_h e\lambda^3 m_h\varphi_1^{\ 2} - m_h^{\ 2}\lambda^4 A\varphi_1^{\ 2} - 2m_h^{\ 2}\lambda^4\frac{B}{2}\frac{\partial^2\varphi_1}{\partial\xi^2}).$$

If we now go to the higher order in ε , we obtain the following equations

$$\frac{\partial}{\partial\tau}n_{c2} - \frac{Z_c e}{\lambda m_c}\varphi_1\frac{\partial}{\partial\xi}n_{c2} - (\frac{Z_c e}{\lambda m_c}n_{c2} + \frac{N_{c0}Z_c e}{\lambda^2 m_c}u_{c2})\frac{\partial\varphi_1}{\partial\xi} + N_{c0}\frac{\partial}{\partial\xi}u_{c3} - \lambda\frac{\partial}{\partial\xi}n_{c3} - \frac{N_{c0}Z_c e}{\lambda^2 m_c}\varphi_1\frac{\partial}{\partial\xi}u_{c2} = 0, \qquad (12.a)$$

$$\frac{\partial}{\partial\tau}u_{c2} - \frac{Z_c e}{\lambda m_c}u_{c2}\frac{\partial\varphi_1}{\partial\xi} - \frac{Z_c e}{\lambda m_c}\varphi_1\frac{\partial}{\partial\xi}u_{c2} - \frac{Z_c e}{m_c}\frac{\partial\varphi_3}{\partial\xi} - \lambda\frac{\partial}{\partial\xi}u_{c3} = 0, \qquad (12.b)$$

$$\frac{\partial}{\partial\tau}n_{h2} + \frac{\lambda Z_h e}{3T_h - m_h\lambda^2}\varphi_1\frac{\partial}{\partial\xi}n_{h2} + (\frac{\lambda Z_h e}{3T_h - m_h\lambda^2}n_{h2} + \frac{Z_h e\, N_{h0}}{3T_h - m_h\lambda^2}u_{h2})\frac{\partial\varphi_1}{\partial\xi} + n_{h0}\frac{\partial}{\partial\xi}u_{h3} + \frac{Z_h e\, N_{h0}}{3T_h - m_h\lambda^2}\varphi_1\frac{\partial}{\partial\xi}u_{h2} - \lambda\frac{\partial}{\partial\xi}n_{h3} = 0, \qquad (12.c)$$

$$\frac{\partial}{\partial\tau}u_{h2} + \frac{\lambda Z_h e}{3T_h - m_h\lambda^2}\varphi_1\frac{\partial}{\partial\xi}u_{h2} - \lambda\frac{\partial}{\partial\xi}u_{h3} + 3\frac{T_h}{N_{h0}m_h}\frac{\partial}{\partial\xi}n_{h3} - \frac{Z_h e}{m_h}\frac{\partial\varphi_3}{\partial\xi} + 3\,\frac{T_h Z_h e}{N_{h0}m_h\left(3T_h - m_h\lambda^2\right)}(n_{h2}\frac{\partial\varphi_1}{\partial\xi} + \varphi_1\frac{\partial}{\partial\xi}n_{h2}) + \frac{\lambda Z_h e}{3T_h - m_h\lambda^2}u_{h2}\frac{\partial\varphi_1}{\partial\xi} = 0, \qquad (12.d)$$

$$\frac{\partial^2\varphi_2}{\partial\xi^2} - 4\frac{\pi e^2 n_{e0}}{T_e}\varphi_3 - 4\,\frac{\pi e^2 n_{i0}}{T_i}\varphi_3 - 4\pi e Z_h\, n_{h3} - 4\pi e Z_c\, n_{c3} = 0\,. \qquad (12.e)$$

Eliminating n_{c3}, u_{c3}, n_{h3}, u_{h3} and φ_3 from (12), and with the aid of both (8) and (10), a linear inhomogeneous equation for the second-order perturbed potential φ_2 can be obtained

$$\widehat{L}(\varphi_1)\varphi_2 = \frac{\partial \varphi_2}{\partial \tau} + A\,\frac{\partial(\varphi_1\varphi_2)}{\partial \xi} + \frac{B}{2}\frac{\partial^3 \varphi_2}{\partial \xi^3} - S(\varphi_1) = 0, \tag{13}$$

where

$$S(\varphi_1) = A_1\frac{\partial \varphi_1^3}{\partial \xi} + A_2\left(\frac{\partial}{\partial \xi}\varphi_1\frac{\partial^2 \varphi_1}{\partial \xi^2}\right) + A_3\varphi_1\left(\frac{\partial^3 \varphi_1}{\partial \xi^3}\right) - A_4\frac{\partial^5 \varphi_1}{\partial \xi^5}, \tag{14}$$

where the coefficients A_i (where $i = 1, 2, ..., 4$) are listed as:

$$\begin{aligned}
A_1 = {} & \left(4\left(m_c m_h N_{h0}^3 Z_h^2 \lambda^4 + N_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2 Z_c^2\right)\right)^{-1} \\
& \lambda^3 m_c\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2\left(\frac{3e^2\left(5m_h^2 N_{h0}^2\lambda^4 + 30 m_h N_{h0} p_0 \lambda^2 + 9p_0^2\right) Z_h^4 N_{h0}^4}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^5}\right. \\
& + \left(\frac{20Ae\lambda m_h\left(m_h N_{h0}\lambda^2 + 3p_0\right) Z_h^3 N_{h0}^4}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^4} + \frac{6A^2 m_h\left(m_h N_{h0}\lambda^2 + p_0\right) Z_h^2 N_{h0}^3}{\left(\lambda^2 m_h N_{h0} - 3p_0\right)^3}\right. \\
& \left. - \frac{e^2 n_{e0}}{T_e^3} - \frac{e^2 n_{i0}}{T_i^3} + \frac{N_{c0} Z_c^2\left(6A^2\lambda^2 m_c^2 + 20Ae\lambda Z_c m_c + 15e^2 Z_c^2\right)}{\lambda^6 m_c^3}\right),
\end{aligned}$$

$$\begin{aligned}
A_2 = {} & \left(16\pi\left(e m_c m_h N_{h0}^3 Z_h^2 \lambda^4 + eN_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2 Z_c^2\right)^2\right)^{-1} \\
& \lambda\left(4em_c^2 m_h N_{h0}^4\left(m_h N_{h0}\lambda^2 + 3p_0\right) Z_h^3 \lambda^6\right. \\
& + 9Am_c^2 m_h N_{h0}^3\left(\lambda^2 m_h N_{h0} - 3p_0\right)\left(m_h N_{h0}\lambda^2 + p_0\right) Z_h^2 \lambda^5 \\
& \left. + N_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^4 Z_c^2\left(9A\lambda m_c + 4eZ_c\right)\right), \\
A_3 = {} & \left(8\pi\left(e m_c m_h N_{h0}^3 Z_h^2 \lambda^4 + eN_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2 Z_c^2\right)^2\right)^{-1} \\
& \lambda\left(4em_c^2 m_h N_{h0}^4\left(m_h N_{h0}\lambda^2 + 3p_0\right) Z_h^3 \lambda^6\right. \\
& + 3Am_c^2 m_h N_{h0}^3\left(\lambda^2 m_h N_{h0} - 3p_0\right)\left(m_h N_{h0}\lambda^2 + p_0\right) Z_h^2 \lambda^5 \\
& \left. + N_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^4 Z_c^2\left(3A\lambda m_c + 4eZ_c\right)\right),
\end{aligned}$$

and

$$A_4 = (128e^4\pi^2\left(m_c m_h N_{h0}^3 Z_h^2 \lambda^4 + N_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^2 Z_c^2\right)^3)^{-1}$$
$$3\lambda^5 m_c^2\left(\lambda^2 m_h N_{h0} - 3p_0\right)^3\left(m_c m_h N_{h0}^3\left(m_h N_{h0}\lambda^2 + p_0\right)Z_h^2\lambda^4 + N_{c0}\left(\lambda^2 m_h N_{h0} - 3p_0\right)^3 Z_c^2\right).$$

In summary, we have reduced the basic set of fluid (1-4) to a nonlinear KdV equation (10) for φ_1 and a linear inhomogeneous differential equation (13) for φ_2; for which the source term is described by a known function φ_1 .

4. Stationary Solution

To obtain a stationary solution from (10) and (13), we adopt the renormalization method introduced by Kodama and Taniuti (1978), and El- Labany (1995) to eliminate the secular behavior up to the second-order potential. According to this method, (10) can be added to (13) to yield

$$K(\varphi_1) + \sum_{n\geq 2}\varepsilon^n \widehat{L}(\varphi_1)\varphi_n = \sum_{n\geq 2}\varepsilon^n S_n, \tag{15}$$

where S_2 represents the right-hand side of (13). Adding $\sum_{n\geq 1}\varepsilon^n \delta\Theta \frac{\partial\varphi_n}{\partial\xi}$ to both sides of (15), where $\delta\Theta = \varepsilon\Theta_1 + \varepsilon^2\Theta_2 + \varepsilon^3\Theta_3 + \ldots$, with coefficients to be determined later, Θ_n are determined successively to cancel out the resonant term in S_n. Then, (10) and (13) may be written as

$$\frac{\partial\widehat{\varphi}_1}{\partial\tau} + A\widehat{\varphi}_1\frac{\partial\widehat{\varphi}_1}{\partial\xi} + \frac{1}{2}B\frac{\partial^3\widehat{\varphi}_1}{\partial\xi^3} + \delta\Theta\frac{\partial\widehat{\varphi}_1}{\partial\xi} = 0, \tag{16}$$

$$\frac{\partial\widehat{\varphi}_2}{\partial\tau} + A\frac{\partial}{\partial\xi}(\widehat{\varphi}_1\widehat{\varphi}_2) + \frac{1}{2}B\frac{\partial^3\widehat{\varphi}_2}{\partial\xi^3} + \delta\Theta\frac{\partial\widehat{\varphi}_2}{\partial\xi} = S_2(\widehat{\varphi}_1) + \Theta_1\frac{\partial\widehat{\varphi}_1}{\partial\xi}. \tag{17}$$

The parameter $\delta\Theta$ in (15) and (16) can be determined from the conditions that the resonant terms in $S_2(\widehat{\varphi}_1)$ may be cancelled out by the terms $\delta\Theta(\partial\widehat{\varphi}_1/\partial\xi)$ in (16) see for instance El-Labany (1995).

Let us introduce the variable,

$$\eta = \xi - (\Theta + \delta\Theta)\tau, \tag{18}$$

where the parameter Θ is related to the Mach number $M = \Theta/C_d$ by

$$\Theta + \delta\Theta \equiv M - 1 = \Delta M,$$

with M being the soliton velocity and C_d is the dust-acoustic velocity.

Integrating equations (16) and (17) with respect to the new variable η and using the appropriate vanishing boundary conditions for $\widehat{\varphi}_1(\eta)$ and $\widehat{\varphi}_2(\eta)$ and their derivatives up to second order as $|\eta| \to \infty$, one gets

$$\frac{d^2\widehat{\varphi}_1}{d\eta^2} + B^{-1}(A\widehat{\varphi}_1 - 2\Theta)\widehat{\varphi}_1 = 0\,, \tag{19}$$

$$\frac{d^2\widehat{\varphi}_2}{d\eta^2} + 2B^{-1}(A\widehat{\varphi}_1 - \Theta)\widehat{\varphi}_2 = 2B^{-1}\int_{-\infty}^{\eta}[S_2(\widehat{\varphi}_1) + \Theta_1\frac{d\widehat{\varphi}_1}{d\eta}]d\eta\,. \tag{20}$$

The one-soliton solution of (19) admits

$$\widehat{\varphi}_1 = \varphi_{1m}\,\mathrm{sech}^2\ (D\eta), \tag{21}$$

where the soliton amplitude φ_{1m} and the soliton width D^{-1} are given

$$\varphi_{1m} = \frac{3\Theta}{A}, \qquad D^{-1} = \sqrt{\frac{2B}{\Theta}}. \tag{22}$$

Substituting (21) in (19), then the source term of (19) becomes

$$2B^{-1}\int_{-\infty}^{\eta}[S_2(\widehat{\varphi}_1) + \Theta_1\frac{d\widehat{\varphi}_1}{d\eta}]d\eta = 2B^{-1}(\Theta_1 - 16D^4A_4)\varphi_{1m}\,\mathrm{sech}^2\ (D\eta) + \frac{2\varphi_{1m}^2\Theta}{B^2}\alpha_1\,\mathrm{sech}^4\ (D\eta) + \frac{\varphi_{1m}^2\Theta}{B^2}\alpha_2\,\mathrm{sech}^6\ (D\eta), \tag{23}$$

with

$$\alpha_1 = 2A_2 + A_3 + \frac{10A}{B}A_4,$$
$$\alpha_2 = \frac{2B}{A}A_1 - 6A_2 - 2A_3 - \frac{20A}{B}A_4.$$

In order to cancel out the resonant terms in $S(\widehat{\varphi}_1)$, the value of $\delta\Theta$ should be

$$\Theta_1 = 16D^4A_4. \tag{24}$$

and for (23), we introduce the independent variable

$$\Psi = \tanh(\eta D), \tag{25}$$

to recast (23) into

$$\frac{d}{d\Psi}\left[\left(1-\Psi^2\right)\frac{d\widehat{\varphi}_2}{d\Psi}\right]+\left[3(3+1)-\frac{2^2}{1-\Psi^2}\right]\widehat{\varphi}_2 = \alpha_3(1-\Psi^2)+\alpha_4(1-\Psi^2)^2, \tag{26}$$

where

$$\alpha_3 = \frac{4\varphi_{1m}^2}{B}\alpha_1,$$
$$\alpha_4 = \frac{2\varphi_{1m}^2}{B}\alpha_2.$$

Note that the two independent solutions of homogeneous part of (26) can be represented in terms associated Legendre function of first and second kind,

$$P_3^2 = 15\Psi(1-\Psi^2), \tag{27}$$

$$Q_3^2 = \frac{15}{2}\Psi(1-\Psi^2)\ln\left(\frac{1+\Psi}{1-\Psi}\right)+2(1-\Psi^2)^{-1}-15(1-\Psi^2)^2+5, \tag{28}$$

the complementary solution of (26) admits the following form

$$\widehat{\varphi}_{2c} = C_1(15\Psi(1-\Psi^2))+C_2(\frac{15}{2}\Psi(1-\Psi^2)\ln\left(\frac{1+\Psi}{1-\Psi}\right)+2(1-\Psi^2)^{-1}-15(1-\Psi^2)+5). \tag{29}$$

By applying the method of variation of parameters, the particular solution of (26) takes the form

$$\widehat{\varphi}_{2p} = L_1(\Psi)P_3^2(\Psi)+L_2(\Psi)Q_3^2(\Psi), \tag{30}$$

where $L_1(\Psi)$ and $L_2(\Psi)$ given by

$$L_1(\Psi) = -\int\frac{Q_3^2(\Psi)T(\Psi)}{(1-\Psi^2)W(P_3^2(\Psi),Q_3^2(\Psi))}d\Psi,$$

$$L_2(\Psi) = \int\frac{P_3^2(\Psi)T(\Psi)}{(1-\Psi^2)W(P_3^2(\Psi),Q_3^2(\Psi))}d\mu,$$

with

$$T(\Psi)=\alpha_3(1-\Psi^2)+\alpha_4(1-\Psi^2)^2,$$

$$W(P_3^2,Q_3^2)=P_3^2\frac{dQ_3^2}{d\Psi}-Q_3^2\frac{dP_3^2}{d\Psi}=\frac{120}{1-\Psi^2}.$$

Using P_3^2 and Q_3^2 , then $L_1(\Psi)$ and $L_2(\Psi)$ reduced to

$$L_1(\Psi)=\frac{1}{32}(1-\Psi^2)^3(\frac{1}{3}\alpha_3+\frac{1}{4}\alpha_4(1-\Psi^2))\ln(\frac{1+\Psi}{1-\Psi})+(\alpha_5)\Psi+(\alpha_6)\Psi^3+(\alpha_7)\Psi^5+\frac{1}{64}\Psi^7,$$
$$L_2(\Psi)=-\frac{1}{16}(1-\Psi^2)^3(\frac{1}{3}\alpha_3+\frac{1}{4}(1-\Psi^2)\alpha_4).$$

with

$$\alpha_5=(-\frac{1}{48}+\frac{1}{15})\alpha_3+(-\frac{1}{64}+\frac{1}{15})\alpha_4,$$
$$\alpha_6=(-\frac{1}{18})\alpha_3+(-\frac{33}{360}+\frac{1}{64})\alpha_4,$$
$$\alpha_7=(\frac{1}{48})\alpha_3+(-\frac{3}{320}+\frac{1}{15})\alpha_4.$$

The particular solution is given by

$$\widehat{\varphi}_{2p}=\frac{\varphi_{1m}^2D^2}{\Theta}(1-\Psi^2)[\alpha_8-\alpha_2(1-\Psi^2)], \tag{31}$$

where

$$\alpha_8=\frac{6BA_1}{A}-\frac{10}{3}A_2-\frac{4}{3}A_3-\frac{20A}{3B}A_4.$$

In (23) the first term is the secular one, which can be eliminated by renormalization the amplitude. Also the boundary conditions $\widehat{\varphi}_2=0$ as $\eta\to\infty$ produce $C_2=0.$ Thus, only the particular solution contributes to $\widehat{\varphi}_2$.

Expressing (31) in terms of the old variable η the solution of (26) is given by

$$\widehat{\varphi}_{2p}=\frac{\varphi_{1m}^2D^2}{\Theta}\operatorname{sech}^2(D\eta)\left[\alpha_{10}+\alpha_2\tanh^2(D\eta)\right]. \tag{32}$$

The stationary soliton solution for DA waves is given by

$$\widehat{\varphi}(\eta) = \widehat{\varphi}_1(\eta) + \widehat{\varphi}_2(\eta)$$
$$= (\varphi_{1m} + \frac{\varphi_{1m}^2 D^2}{\Omega}\alpha_{11})\,\text{sech}^2\ (D\eta) - \alpha_2 \frac{\varphi_{1m}^2 D^2}{\Omega}\text{sech}^4(D\eta). \tag{33}$$

where

$$\alpha_{10} = \frac{3BA_1}{A} - \frac{1}{2}A_2 + \frac{2}{3}A_3 + \frac{10A}{3B}A_4,$$
$$\alpha_{11} = \alpha_{10} + \alpha_2,$$
$$\Theta = B^2[(1 + \frac{16A_4\Delta M}{B^2})^{\frac{1}{2}} - 1]/(8A_4).$$

and the soliton width is given by

$$D^{-1} = (\frac{2B}{\Delta M})^{\frac{1}{2}}(1 + \frac{2A_4\,\Delta M}{B^2}).$$

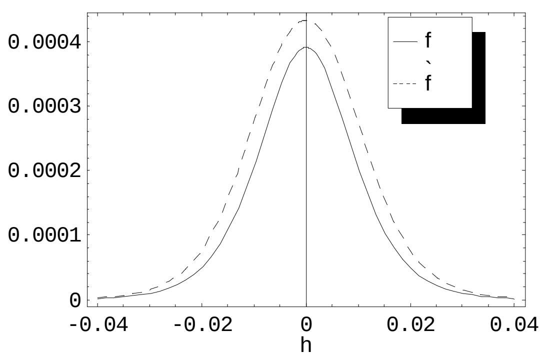

Figure 1. The comparison between the lowest order potential $\varphi(\eta)$ and higher-order correction $\hat{\varphi}(\eta)$ of the compressive soliton with respect to η for, $m_h = m_c = 10^{14} m_i$, $N_{c0} = 6$, $N_{h0} = 5$, $n_{e0} = 10$, $n_{i0} = 10010$, $T_i = T_e = 1eV$, $Z_c = 1000$, $Z_h = 1000$ and $\Theta = 0.4$.

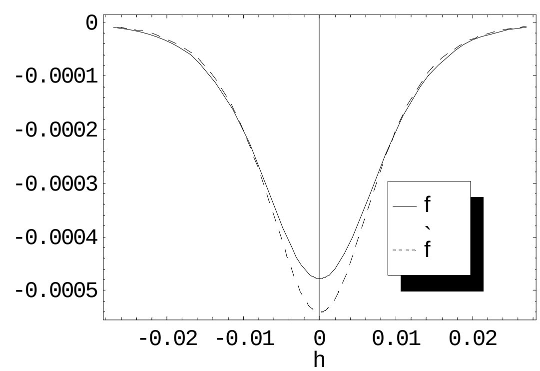

Figure 2. The comparison between the lowest order potential $\varphi(\eta)$ and higher-order correction $\hat{\varphi}(\eta)$ of the rarefactive soliton with respect to η for $N_{c0} = 6$, $N_{h0} = 11$, $n_{e0} = 10$, $n_{i0} = 10010$, $m_h = m_c = 10^{14} m_i$, $T_i = T_e = 1eV$, $Z_c = 1000$, $Z_h = 1000$, and $\Theta = 0.4$.

5. Numerical Results and Discussion

Nonlinear DAWs in a homogeneous unmagnetized plasma having electrons, singly charged ions, hot and cold dust grains have been investigated. We have assumed that the effect of gravity in the system is neglected ($r_d << 1$) as well as there are no neutrals. Saturn F-ring's are one of the space plasma observations that satisfy our conditions: (i) there are no neutrals, (ii) the ratio between the inter-grain distances between dust particles to plasma Debye radius is less than one, (iii) coupling parameter Γ is less than one, and (iv) r_d is smaller than $1\mu m$. Hence, numerical studies have been made using plasma parameters close to those values corresponding to Saturn F-ring's i.e., the equilibrium electron and dust densities are $n_{e0} = 10cm^{-3}, N_{h0} = 10cm^{-3}$ and dust charge and mass are taken as $Z_h = 10^2 - 10^3$, $m_h = m_c = 10^{12} m_i$, respectively, as given in Refs. Akhtar et al. (2007), Shukla and Mendis (1997), Farid et al. (2001). Generally speaking, the present system supports compressive and rarefactive soliton i.e. the amplitude of KdV solitons can be positive ($A > 0$) or negative ($A < 0$). Since our objectives was to study the effect of higher-order nonlinearity on the formation of solitary waves, we specifically elucidated to what extent the higher-order solution modifies the soliton amplitude. The comparison between the lowest order potential $\varphi(\eta)$ and higher-order correction $\hat{\varphi}(\eta)$ of the compressive (rarefactive) solitons were depicted in Figures (1, 2).

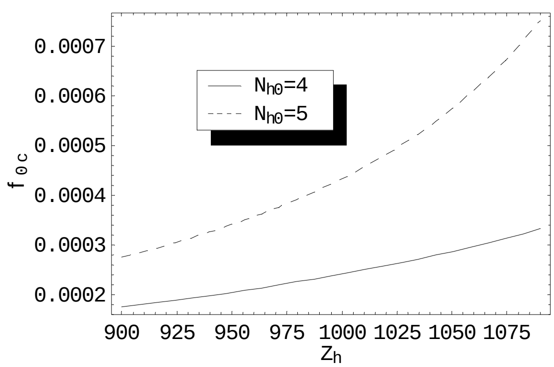

Figure 3. The variation of the amplitude φ_{0c} of the compressive soliton for higher-order correction with Z_h for different values of N_{h0} for $N_{c0} = 6, n_{e0} = 10, n_{i0} = 10010$, $m_h = m_c = 10^{14} m_i$, $T_i = T_e = 1eV$, $Z_c = 1000$ and $\Theta = 0.4$.

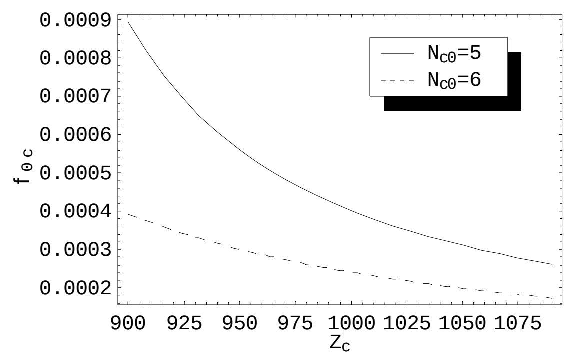

Figure 4. The variation of the amplitude φ_{0c} of the compressive soliton for higher-order correction with Z_c for different values of N_{c0} for $N_{h0} = 4, n_{e0} = 10, n_{i0} = 10010$, $m_h = m_c = 10^{14} m_i$, $T_i = T_e = 1eV$, $Z_h = 1000$ and $\Theta = 0.4$.

It is found that, the higher-order correction increasing the soliton amplitude for compressive and rarefactive solitons. On the other hand, it is important to study effects of the dusty plasma density $n_h(n_c)$ and the charge numbers for negatively charged hot and cold dusty grains on the existence of improved solitary waves. It is obvious from Figures (3, 4) that compressive soliton amplitude increases with enhancing of N_{h0}, Z_h and decreases with N_{c0}, Z_c. Finally, the rarefactive soliton amplitude increases with N_{c0}, Z_c and decreases with N_{h0}, Z_h as shown in Figures 5 and 6.

These results show that the presence of cold (hot) dusty plasma density n_c (n_h) and the charge numbers for negatively charged cold (hot) dust Z_c (Z_h) as well as the higher-order correction modify significantly the properties of dust-acoustic solitary waves. The results presented here should be useful in understanding salient features of localized electrostatic perturbations in space and laboratory plasmas.

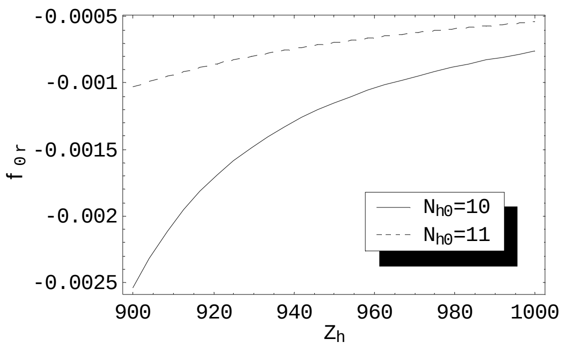

Figure 5. The variation of the amplitude φ_{0r} of the rarefactive soliton for higher-order correction with Z_h for different values of N_{h0} for $N_{c0} = 6, n_{e0} = 10, n_{i0} = 10010$, $m_h = m_c = 10^{14} m_i$, $T_i = T_e = 1eV$, $Z_c = 1000$ and $\Theta = 0.4$.

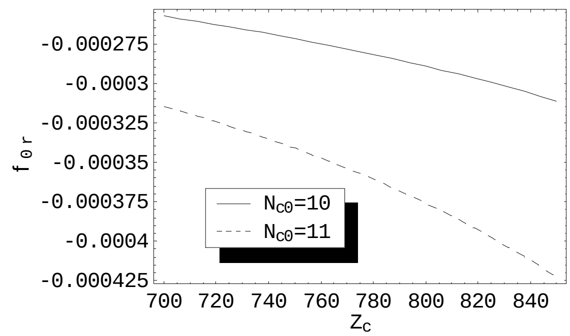

Figure 6. The variation of the amplitude φ_{0r} of the rarefactive soliton for higher-order correction with Z_c for different values of N_{c0} for $N_{h0} = 10, n_{e0} = 10, n_{i0} = 10010$, $m_h = m_c = 10^{14} m_i$, $T_i = T_e = 1eV$, $Z_h = 1000$ and $\Theta = 0.4$.

6. Conclusions

In this work, we have investigated the properties of nonlinear DAWs in a unmagnetized plasma having electrons, singly charged ions, hot and cold dust grains. The reductive perturbation method has been used to reduce the basic set of fluid equations to the well-known KdV equation; however, as the wave amplitude increases, the width and velocity of a soliton deviate from the prediction of the KdV equation. In particular, we study the next-order of perturbation theory, a linear inhomogeneous (KdV-type) equation that accounts for the higher-order nonlinearity and dispersion is obtained. A stationary solution for equations resulting from higher-order perturbation theory has been found using the renormalization method. The consideration of higher-order approximation was found to increase the amplitude of DAWs solitons. In other word, we have shown that the presence of cold (hot) dusty plasma density n_c (n_h) and the charge numbers for negatively charged cold (hot) dust Z_c (Z_h) do not only modify the basic properties of the improved dust acoustic solitary potential structures, but also causes two different potential profiles, namely compressive and rarefactive pulses.

We inject to that the analytical model demonstrated here can provide a useful basis for the interpretation of recent observations of solitary wave in dusty plasma environments. For example, the results presented may be applicable to dusty plasma existing in Saturn F-ring's region.

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