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Syed T. Mohyud-Din
COMSATS Institute of Information Technology

Mustafa Inc
Fırat University

Ebru Cavlak
Fırat University

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On Numerical Solutions of Two-Dimensional Boussinesq Equations by Using Adomian Decomposition and He's Homotopy Perturbation Method

Syed Tauseef Mohyud-Din
Department of Mathematics
COMSATS Institute of Information Technology
Islamabad, Pakistan
syedtauseefs@hotmail.com

Mustafa Inc and Ebru Cavlak
Department of Mathematics
Fırat University
23119 Elazığ, Türkiye
minc@firat.edu.tr; ebrucavlak@hotmail.com.tr

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Abstract

In this paper, we obtain the approximate solution for 2-dimensional Boussinesq equation with initial condition by Adomian's decomposition and homotopy perturbation methods and numerical results are compared with exact solutions.

Keywords: Boussinesq equations, Adomian's polynomials, nonlinear problems, homotopy perturbation, error estimates

MSC 2000 #: 65 N 10

1. Introduction

One of the equation describing the propagation of long waves on shallow water is the Boussinesq one which first appeared. The author was the first to explain scientifically the effect of the existence of solitary waves or solitons discovered in 1834 by Scott-Russell. Boussinesq equation can be written in the form

$$u_{tt} = -\alpha u_{xxxx} + u_{xx} + \beta(u^2)_{xx},$$

where $u(x,t)$ is an elevation of the free surface of fluid and the coefficients $\alpha, \beta = const \in R.$ The Boussinesq equation was proposed earlier than Korteweg-de Vries one, but the mathematical theory for it is not as complete as for the latter one.

The one-dimensional in space Boussinesq equation and its generalization

$$u_{tt} = -\alpha u_{xxxx} + u_{xx} + \beta(f(u))_{xx}$$

have been studied in many papers. The two-dimensional version of the generalized Boussinesq equation

$$u_{tt} - u_{xx} - \varepsilon^2 \left[u_{yy} - \alpha u_{xxxx} + \left(\frac{u^{m+1}}{m+1} \right)_{xx} \right] = 0, \alpha = \pm 1$$

has been proposed by Ablowitz et al. (1997). This equation is motivated by considerations underlying the derivation of the Kadomtsev-Petviashvili type equations and models slow transverse variations balanced by longitudinal dispersion and weak nonlinearity.

In this study, we also consider the 2-dimensional Boussinesq equation

$$u_{tt} - u_{xx} + 3(u^2)_{xx} + u_{xxxx} = 0, \tag{1}$$

subject to initial conditions

$$u(x,0) = \frac{c}{2} \sec h^2 \left[\frac{\sqrt{c}}{2}(x+1) \right]. \tag{2}$$

2. The Adomian Decomposition Method (ADM)

The ADM was first introduced by Adomian in the beginning of 1980's. The method is useful obtaining both a closed form and the explicit solution and numerical approximations of linear or nonlinear differential equations and it is also quite straighforward to write computer codes. This method has been applied to obtain a formal solution to a wide class of stochastic and deterministic problems in science and engineering involving algebraic, differential, integro-differential, differential delay, integral and partial differential equations Lesnic et al. (1999) and Dehghan (2004). The convergence of ADM for partial differential equations was presented by Cherruault (1990). Application and convergence of this method for nonlinear partial differential equations are found in Ngarhasta et al. (2002) and Hashim et al. (2006).

In general, it is necessary to contruct the solution of the problems in the form of a decomposition series solution. In the simplest case, the solution can be developed as a Taylor series expansion about the function not the point at which the initial condition and integration right hand side function of the problem are determined the first term u_0 of the decomposition series for $n \geq 0$. The sum of the $u_0, u_1, u_2, \ldots$ terms are simply the decomposition series Adomian (1989), Adomain (1994), Adomain (1998), and Dehghan (2004).

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{3}$$

Suppose that the differential equation operator including both linear and nonlinear terms, can be formed as

$$Lu + Ru + Nu = F(x,t), \tag{4}$$

with initial condition

$$u(x,0) = g(x), \tag{5}$$

where L is the higher-order derivative which is assumed to be invertible, R is a linear differential operator of order less than L, N is the nonlinear term and $F(x,t)$ is a source term. We next apply the inverse operator L^{-1} to both sides of equation (4) and using the given condition (5) to obtain

$$u(x,t) = g(x) + f(x,t) - L^{-1}(Ru) - L^{-1}(Nu), \tag{6}$$

where the function $f(x,t)$ represents the terms arising from integrating the source term $F(x,t)$ and from using the given conditions, all are assumed to be prescribed. The nonlinear term can be written as El-Sayed (2002), Inc (2006), and Inc (2007).

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{7}$$

where A_n are the Adomian polynomials. These polynomials are defined as

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{n=0}^{\infty} \lambda^k u_k(x,t) \right) \right]_{k=0}, \; n = 0,1,2, \ldots. \tag{8}$$

For example,

$$A_0 = N(u_0),$$

$$
\begin{aligned}
A_1 &= u_1 N'(u_0), \\
A_2 &= u_2 N'(u_0) + \frac{1}{2} u_1 N''(u_0), \\
A_3 &= u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{6} u_1^3 N'''(u_0),
\end{aligned}
\tag{9}
$$

and so on, the other polynomials can be constructed in a similar way, Wazwaz (2002). As indicated before, Adomian method defines the solution u by an infinite series of components given by equation (4) and the components $u_0, u_1, u_2, \ldots$ are usually recurrently determined. Thus, the formal recursive relation is defined by

$$
\begin{cases}
u_0(x,t) = g(x) + f(x,t), \\
u_{n+1}(x,t) = -L^{-1}(Ru_n) - L^{-1}(Nu_n), \; n \geq 0,
\end{cases}
\tag{10}
$$

which are obtained all components of u. As a result, the terms of the series $u_0, u_1, u_2, \ldots$ are identified and the exact solution may be entirely determined by using the approximation

$$
u(x,t) = \lim_{n\to\infty} \varphi_n(x,t),
\tag{11}
$$

where

$$
\varphi_n(x,t) = \sum_{k=0}^{n-1} u_k(x,t),
\tag{12}
$$

or

$$
\begin{cases}
\varphi_0 = u_0, \\
\varphi_1 = u_0 + u_1, \\
\varphi_2 = u_0 + u_1 + u_2, \\
\vdots \\
\varphi_n = u_0 + u_1 + u_2 + \ldots + u_{n-1}, n \geq 0.
\end{cases}
\tag{13}
$$

Equation (1) can be rewritten in an operator form

$$
L_t u - L_x u + 3(u^2)_{xx} + L_{xx} u = 0,
\tag{14}
$$

where the linear differential operators L_t, L_x and L_{xx} are given by $\partial^2/\partial t^2$, $\partial^2/\partial x^2$ and $\partial^4/\partial x^4$, respectively. Assuming the inverse of the operator L^{-1} exists and it can conveniently be taken as the definite integral with respect to t from 0 to t, that is, $L^{-1} = \int_0^t \int_0^t (.) dtdt.$ The decomposition method suggests that the unknown functions u be decomposed by an infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad (15)$$

and the nonlinear terms $A_n(x,t) = (u^2)_{xx} = \sum_{n=0}^{\infty} A_n$. In here, A_n are the so-called Adomian polynomials and these polynomials can be calculated as

$$\begin{aligned} A_0 &= 3(u_0^2)_{xx}, \\ A_1 &= 3(2u_0 u_1)_{xx}, \\ A_2 &= 3(2u_0 u_2 + u_1^2)_{xx}, \\ &\vdots \end{aligned} \quad (16)$$

Thus, applying the inverse operator L^{-1} to (14) yields

$$L^{-1} L_t u = L^{-1}\left(L_x u - 3(u^2)_{xx} - L_{xx} u\right). \quad (17)$$

Therefore, equations (1) are transformed into a set of recursive relations given by

$$\begin{cases} u_0(x,t) = u(x,0), \\ u_1(x,t) = t\, u_t(x,0) + L^{-1}\left(L_x u_0 + A_0 - L_{xx} u_0\right) \\ u_{n+1} = L^{-1}\left(L_x u_k + A_k + L_{xx} u_k\right), k \geq 0, \end{cases} \quad (18)$$

3. The Homotopy Perturbation Method

The homotopy perturbation method (HPM) was first proposed by He (1998), He (2004), He (2004), He (2004), He (2005), He (2005), He (2006). The HPM does not depend on a small parameter in the equation. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$ which is considered as a " small parameter".

The HPM was successfully applied to nonlinear oscillators with discontinuities He (2004) and bifurcation of nonlinear problems He (2004). In He (2004), a comparison of HPM and homotopy analysis method was made, revealing that the former is more powerful than the latter. The HPM was proposed to search for limit cycles or bifurcation curves of nonlinear equations He (2005). In He (2005), heuristical example was given to illustrate the basic idea of the HPM. Also this method was applied to solve boundary value problems He (2006) and heat radiation equations Ganji et al. (2006) and Noor et al.

When implementing the HPM, we get the explicit solutions of the two-dimensional parabolic equation without using any transformation method. The method presented here is also simple to use for obtaining numerical solution of the equations without using any discretization techniques. Furthermore, we will show that considerably better approximations related to the accuracy level can be obtained.

To illustrate the basic ideas of this method, we consider the following nonlinear differential equation:

$$A(u)-f(r)=0,\quad r\in\Omega, \tag{19}$$

with the boundary conditions of

$$B(u,\partial u/\partial n)=0,\quad r\in\Gamma, \tag{20}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ on denotes differentiation along the normal vector drawn outwards from Ω.

Generally speaking, the operator A can be divided into two parts which are L and N, where L is linear, but N is nonlinear. Equation (10) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0. \tag{21}$$

By the homotopy technique, we construct a homotopy $H:\Omega\times 0,1]\rightarrow R$ which satisfies:

$$H(V,p)=(1-p)[L(V,r)-L(u_0,r)]+p[A(V,r)-f(r)]=0,\quad p\in[0,1],\quad r\in\Omega, \tag{22}$$

or

$$H(V,p)=L(V,r)-L(u_0,r)+p\,L(u_0,r)+p[N(V,r)-f(r)]=0, \tag{23}$$

where $p\in[0,1]$ is an embedding parameter, u_0 is an initial approximation of equation (1), which satisfies the boundary conditions. Obviously, from equations (21) and (22) we will have:

$$H(V,0)=L(V,r)-L(u_0,r)=0, \tag{24}$$

$$H(V,1)=A(V,r)-f(r)=0. \tag{25}$$

Changing process of p from zero to unity is just that $V(r,p)$ changes from $u_0(r)$ to $u(r)$. In topology, this is called deformation, and $L(V,r)-L(u_0,r)$ and $A(V,r)-f(r)$ are called homotopy.

According to the HPM, we can first use the embedding parameter p as a " small parameter" , and assume that the solution of equation (13) and (14) can be written as a power series in p:

$$V = V_0 + pV_1 + p^2 V_2 + \cdots. \tag{26}$$

Setting $p = 1$ results in the approximate solution of equation (1):

$$u = \lim_{p \to 1} V = V_0 + V_1 + V_2 + \cdots. \tag{27}$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantage of the traditional perturbation techniques. The series (26) is convergent for most cases He (1998, 2004, 2005, 2006).

To investigate the traveling wave solution of equation (1), we first construct a homotopy as follows:

$$(1-p)(\ddot{Y} - \ddot{u}_0) + p(\ddot{Y} - Y^{''} + 3(Y^2)^{''} + Y^{(IV)}) = 0, \tag{28}$$

where " primes" denote differentiation with respect to x, and " dot" denotes differentiation with respect to t. Substituting equation (14) and arranging the coefficients of p powers we have

$$\begin{aligned} &\ddot{Y}_0 + p\ddot{Y}_1 + p_2^2\ddot{Y} + p_3^3\ddot{Y} + p_4^4\ddot{Y} - \ddot{u}_0 + p\ddot{u}_0 - pY_0^{''} - p^2Y_1^{''} - p^3Y_2^{''} - p^4Y_3^{''} - p^5Y_4^{''} \\ &\quad + 3p^2Y_1^{2''} + 6p^2Y_1^{''}Y_0^{''} + 3pY_0^{2''} + 6p^4Y_1^{''}Y_2^{''} + 6p^3Y_0^{''}Y_2^{''} \\ &\qquad + 3p^5Y_2^{2''} + 6p^5Y_1^{''}Y_3^{''} + 6p^4Y_0^{''}Y_3^{''} + 6p^5Y_0^{''}Y_4^{''} \\ &\qquad\quad + pY_0^{(IV)} + p^2Y_1^{(IV)} + p^3Y_2^{(IV)} + p^4Y_3^{'(IV)} + p^5Y_4^{'(IV)} + \ldots = 0. \end{aligned} \tag{3}$$

In order to obtain the unknownsof $Y_i(x,t)$, $i = 1,2,3,4$, we must construct and solve the following system which includes five equations with five unknowns, considering the initial condition of $Y(x,0) = u(x,0)$ and having the initial approximations of equation (1):

$$\begin{aligned} &p^0 : \ddot{Y}_0 - \ddot{u}_0 = 0, \\ &p^1 : \ddot{Y}_1 + \ddot{u}_0 - Y_0^{''} + 3Y_0^{2''} + Y_0^{'v} = 0, \\ &p^2 : \ddot{Y}_2 - Y_1^{''} + 3Y_1^{2''} + 6Y_1^{''}Y_0^{''} + Y_1^{'v} = 0, \\ &p^3 : \ddot{Y}_3 - Y_2^{''} + 6Y_0^{''}Y_2^{''} + Y_2^{(IV)} = 0, \\ &p^4 : \ddot{Y}_4 - Y_3^{''} + 6Y_1^{''}Y_2^{''} + 6Y_0^{''}Y_3^{''} + Y_3^{'v} = 0, \\ &p^5 : \ddot{Y}_5 - Y_4^{''} + 3Y_2^{2''} + 6Y_1^{''}Y_3^{''} + 6Y_0^{''}Y_4^{''} + Y_4^{'v} = 0, \\ &\vdots \end{aligned} \tag{4}$$

4. Test the Example

In this section, we present the 2-dimensional Boussinesq equation with analytical solutions to show the efficiency of methods described in the previous section.

We shall consider equation (1) with the following initial conditions. These gives the exact solution

$$u(x,t) = \frac{c}{2}\operatorname{sec}h^2\left[\frac{\sqrt{c}}{2}x + \frac{\sqrt{c}}{2}\sqrt{1+ct}\right].$$

First we apply the ADM to equation (1). To construct the correction functional, it is sufficient to use Eqs.(16) and (18).

$$u_0 = \frac{c}{2}\sec h^2\left[\frac{\sqrt{c}}{2}(x+1)\right],$$

$$u_1 = -\frac{1}{4}c^{5/2}\sec h^2\left[\frac{\sqrt{c}}{2}(x+1)\right]\tan h\left[\frac{\sqrt{c}}{2}(x+1)\right]t + ...,$$

$$u_2 = \frac{1}{8}c^2 t\sec h^4\left[\frac{\sqrt{c}}{2}(x+1)\right](-2(1+c)t + (1+c)t\cosh[\sqrt{c}(x+1)] - \sqrt{c}\sinh[\sqrt{c}(x+1)])$$

$$u_3 = \frac{1}{1536}(c^3(1+c)t^3\sec h^8\left[\frac{\sqrt{c}}{2}(x+1)\right](40t - 440ct + 15(1+33c)t\cosh[\sqrt{c}(x+1)]$$

$$-24(3c+1)t\cosh[2\sqrt{c}(x+1)] + t\cosh[3\sqrt{c}(x+1)] + ct\cosh[3\sqrt{c}(x+1)] + \dots$$

$$\vdots \tag{35}$$

and so on, in this manner the other components of the decomposition series (15) were obtained of which u was evaluated to have the following expansions:

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$= \frac{1}{2}\operatorname{sec}h^2\left[\frac{\sqrt{c}}{2}(x+1)\right] - \frac{1}{8}(-1+c)c^2t^2(-2+\cosh[\sqrt{c}(x+1)]\operatorname{sec}h^4\left[\frac{\sqrt{c}}{2}(x+1)\right]$$

$$-\frac{1}{1280}((-1+c)^2c^5t^6(-140 + 157\cosh[\sqrt{c}(x+1)]) - 26\cosh[2\sqrt{c}(x+1)]$$

$$+\cosh[3\sqrt{c}(x+1)]\operatorname{sec}h^{10}\left[\frac{\sqrt{c}}{2}(x+1)\right] + \dots.$$

We now apply the HPM to equation (1), we obtained in succession $u_1, u_{2,} u_3, \ldots$ etc. by using equation (30) as

$$Y_0 = \frac{c}{2}\operatorname{sec}h^2\left[\frac{\sqrt{c}}{2}(x+1)\right],$$

$$Y_1 = -\frac{1}{8}(-1+c)c^2t^2(-2+\cosh[\sqrt{c}(x+1)]\operatorname{sec}h^4\left[\frac{\sqrt{c}}{2}(x+1)\right]$$

$$Y_2 = \frac{1}{30720}((-1+c)c^3t^4(-475-6125c-9480c^2-3360c^2t^2+3360c^3t^2$$
$$-2(215+385c+1884c^3t^2-6c^2(855+\ldots,$$

$$Y_3 = -\frac{1}{82575360}((-1+c)c^4t^6(-148512-4557504c-75455688c^2$$
$$-151881912c^3-82486656c^4-336960c^2t^2-\ldots,$$

$\vdots$

and so on, in the same manner the other components can be obtained using the Mathematica package.

Table 1. Error between the ADM using $6-$terms and exact solutions of $u(x,t)$ for $c=1$.

t/x	20	25	30	35	40
0.1	-1.13536×10^{-11}	-7.65003×10^{-1}	-5.15455×10^{-1}	-3.47311×10^{-1}	-2.34016×1^{-2}
0.2	-4.54354×10^{-11}	-3.06141×10^{-13}	-2.06276×10^{-1}	-1.38988×10^{-1}	-9.36493×10^{-1}
0.3	-1.02534×10^{-111}	-6.90868×10^{-13}	-4.65504×10^{-1}	-3.13654×10^{-1}	-2.11338×10^{-1}
0.4	-1.83324×10^{-1}	-1.23523×10^{-12}	-8.32291×10^{-1}	-5.60793×10^{-1}	-3.77859×10^{-1}
0.5	-2.88913×10^{-1}	-1.94668×10^{-12}	-1.31166×10^{-1}	-8.83791×10^{-1}	-5.95494×10^{-1}
0.6	-4.20881×10^{-1}	-2.83588×10^{-12}	-1.91081×10^{-1}	-1.28749×10^{-1}	-8.67501×10^{-1}
0.7	-5.81331×10^{-1}	-3.91698×10^{-12}	-2.63924×10^{-1}	-1.77831×10^{-1}	-1.19821×10^{-1}
0.8	-7.72935×10^{-1}	-5.20801×10^{-12}	-3.50912×10^{-1}	-2.36443×10^{-1}	-1.59314×10^{-1}
0.9	-9.98996×10^{-1}	-6.73118×10^{-12}	$-4.53544\times10^{-}$	-3.05595×10^{-1}	-2.05908×10^{-1}
1.0	$-1.26352\times10^{-}$	-8.51353×10^{-12}	-5.73637×10^{-1}	-3.86513×10^{-1}	-2.60431×10^{-1}

Table 2. Error between the ADM using $6-$terms and exact solutions of $u(x,t)$ for $c = 2$.

t/x	20	25	30	35	40
0.1	-6.87025×10^{-15}	-5.83508×10^{-18}	-4.95588×10^{-21}	-4.20916×10^{-24}	-3.57495×10^{-27}
0.2	-2.85325×10^{-14}	-2.42334×10^{-17}	-2.05821×10^{-20}	-1.74808×10^{-23}	-1.48469×10^{-26}
0.3	-6.64626×10^{-14}	-5.64484×10^{-17}	-4.79431×10^{-20}	-4.07193×10^{-23}	-3.45839×10^{-26}
0.4	-1.22546×10^{-13}	-1.04082×10^{-16}	-8.83993×10^{-20}	-7.50798×10^{-23}	-6.37672×10^{-26}
0.5	-1.99512×10^{-13}	-1.69451×10^{-16}	-1.43918×10^{-19}	-1.22234×10^{-22}	-1.03816×10^{-25}
0.6	-3.01246×10^{-13}	-2.55856×10^{-16}	-2.17305×10^{-19}	-1.84563×10^{-22}	-1.56754×10^{-25}
0.7	-4.33107×10^{-13}	-3.67849×10^{-16}	-3.12423×10^{-19}	-2.65349×10^{-22}	-2.25368×10^{-25}
0.8	-6.02288×10^{-13}	-5.11538×10^{-16}	-4.34463×10^{-19}	-3.69001×10^{-22}	-3.13402×10^{-25}
0.9	-8.182872×10^{-13}	-6.94979×10^{-16}	-5.90264×10^{-19}	-5.01326×10^{-22}	-4.25789×10^{-25}
1.0	-1.09342×10^{-16}	-9.28671×10^{-16}	-7.77844×10^{-19}	-6.69901×10^{-22}	-5.68964×10^{-25}

Table 3. Error between the HPM using $6-$terms and exact solutions of $u(x,t)$ for $c=1$.

t/x	20	25	30	35	40
0.1	-7.22418×10⁻¹¹	-4.86762×10⁻¹³	-3.27977×10⁻¹⁵	-2.20989×10⁻¹⁷	-1.48902×10⁻¹⁹
0.2	-1.38051×10⁻¹⁰	-9.30179×10⁻¹³	-6.26749×10⁻¹⁵	-4.22301×10⁻¹⁷	-2.84544×10⁻¹⁹
0.3	-1.98351×10⁻¹⁰	-1.33648×10⁻¹²	-9.00512×10⁻¹⁵	-6.06761×10⁻¹⁷	-4.08832×10⁻¹⁹
0.4	-2.53882×10⁻¹⁰	-1.71064×10⁻¹²	-1.15262×10⁻¹⁴	-7.76631×10⁻¹⁷	-5.23289×10⁻¹⁹
0.5	-3.05244×10⁻¹⁰	-2.05674×10⁻¹²	-1.38581×10⁻¹⁴	-9.33747×10⁻¹⁷	-6.29154×10⁻¹⁹
0.6	-3.52932×10⁻¹⁰	-2.37803×10⁻¹²	-1.60231×10⁻¹⁴	-1.07963×10⁻¹⁶	-7.27446×10⁻¹⁹
0.7	-3.97359×10⁻¹⁰	-2.67738×10⁻¹²	-1.80401×10⁻¹⁴	-1.21553×10⁻¹⁶	-8.19017×10⁻¹⁹
0.8	-4.38873×10⁻¹⁰	-2.95711×10⁻¹²	-1.99248×10⁻¹⁴	-1.34252×10⁻¹⁶	-9.04586×10⁻¹⁹
0.9	-4.77772×10⁻¹⁰	-3.21921×10⁻¹²	-2.16908×10⁻¹⁴	-1.46152×10⁻¹⁶	-9.84762×10⁻¹⁹
1.0	-5.14311×10⁻¹⁰	-3.46539×10⁻¹²	-2.33496×10⁻¹⁴	-1.57329×10⁻¹⁶	-1.06007×10⁻¹⁹

Table 4. Error between the HPM using $6-$terms and exact solutions of $u(x,t)$ for $c = 2$.

t/x	20	25	30	35	40
0.1	-5.88421×10⁻¹⁴	-4.99761×10⁻¹⁷	-4.24461×10⁻²⁰	-3.60505×10⁻²³	-3.06186×10⁻²⁶
0.2	-9.53984×10⁻¹⁴	-8.10243×10⁻¹⁷	-6.88161×10⁻²⁰	-5.84472×10⁻²³	-4.96407×10⁻²⁶
0.3	-1.13251×10⁻¹⁴	-9.61872×10⁻¹⁷	-8.16943×10⁻²⁰	-6.93851×10⁻²³	-5.89305×10⁻²⁶
0.4	-1.15063×10⁻¹⁴	-9.77259×10⁻¹⁷	-8.30012×10⁻²⁰	-7.04501×10⁻²³	-5.98732×10⁻²⁶
0.5	-1.03015×10⁻¹⁴	-8.74932×10⁻¹⁷	-7.43102×10⁻²⁰	-6.31136×10⁻²³	-5.36041×10⁻²⁶
0.6	-7.90375×10⁻¹⁴	-6.71286×10⁻¹⁷	-5.70141×10⁻²⁰	-4.84235×10⁻²³	-4.11273×10⁻²⁶
0.7	-4.49351×10⁻¹⁴	-3.81645×10⁻¹⁷	-3.24141×10⁻²⁰	-2.75301×10⁻²³	-2.33821×10⁻²⁶
0.8	-2.45251×10⁻¹⁵	-2.08298×10⁻¹⁷	-1.76913×10⁻²⁰	-1.50257×10⁻²³	-1.27617×10⁻²⁶
0.9	4.66915×10⁻¹⁴	3.96563×10⁻¹⁷	3.36811×10⁻²⁰	2.80662×10⁻²³	2.42961×10⁻²⁶
1.0	1.00792×10⁻¹³	8.56056×10⁻¹⁷	7.27071×10⁻²⁰	6.15752×10⁻²³	5.24475×10⁻²⁶

5. Conclusion

Thus, we have illustrated how Adomian decomposition method and homotopy perturbation method can be used to solve of Boussinesq equation. The accuracy of the numerical solutions investigated that the methods is well suited for the solution of the nonlinear equations. The results of numerical example is presented and only few terms are required to obtain accurate solutions. In Table 1 and 2 shows that the error, between the exact value of u and the approximation of u. The errors obtained by using the approximate solution given in using only few terms iterations of the decomposition method. In Table 3 and 4 show that the exact and numerical solutions are for only few terms by using Homotopy perturbation method.

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