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Abstract

The modified sine-cosine method is an efficient and powerful mathematical tool in finding exact traveling wave solutions to nonlinear partial differential equations (NLPDEs) with time-dependent coefficients. In this paper, the proposed approach is applied to study a nonhomogeneous generalized form of Benjamin-Bona-Mahony (BBM) equation with time-dependent coefficients. Explicit traveling wave solutions of the equation are obtained under certain constraints on the coefficient functions.

Keywords: Variable-coefficient Benjamin-Bona-Mahony; Modified sine-cosine method

MSC(2000) No.: 47F05, 35B05, 35R11

1. Introduction

Many important phenomena and models can be described by homogeneous nonlinear partial differential equations with constant coefficients. The study of the dynamics of those NLPDEs require the existence of their exact solution. With the development of solitary theory, many powerful methods were established for obtaining the exact solutions of NLPDEs, such as the rational sine-cosine method, the extended tanh-function method, the exp-function method, the Hirota's method, Hirota bilinear method, the tanh-sech method, the first integral method, the

G'/G expansion method, the sub-ODE method and so on. See [Alquran (2012a), Alquran and Al-Khaled (2012), Alquran, Al-Khaled and Ananabeh (2011), Alquran, Ali and Al-Khaled (2012), Alquran and Qawasmeh (2013), Bekir and Unsal (2013), Palacios (2004), Li and Wang (2007), Wazwaz (2007a), Alquran and Al-Khaled (2011a), Alquran and Al-Khaled (2011b), Alquran (2012b), Alquran and Qawasmeh (2014), Wazwaz (2007b) and Qawasmeh (2013)]. Most of the aforementioned methods are related to constant-coefficients NLPDEs. Recently, much efforts have been employed to variable-coefficient nonlinear equations [Yang, Tao and Austin (2010)]. variable-coefficient NLPDEs can describe many nonlinear phenomena more realistically than their constant-coefficient ones. Some of the aforementioned methods have been modified to handle variable-coefficient equations such as the modified sine-cosine method, the modified sech-tanh method and Hirota bilinear method.

The objective of this paper is to use the modified sine-cosine method to study the nonhomogeneous form of Benjamin-Bona-Mahony equation with time-dependent coefficients that reads

$$u_t + \alpha u_{xxt} + \beta(t)u_x + \gamma(t)uu_x = g(t), \tag{1.1}$$

where α is a real constant and $\beta(t), \gamma(t), f(t)$ are functions depending on the variable t only. In the literature, extensive study was done on different types of the BBM equation by different methods [Benjamin, Bona and Mahony (1972), Wazwaz (2005), Alquran (2012a), Alquran and Al-Khaled (2011a), Alquran (2012b), Chen, Lai and Qing (2007) and Abazari (2013)].

2. The Modified Sine-Cosine Method

The modified sine-cosine method "extension of the regular sine-cosine method [Tascan and Bekir (2009), Ali, Soliman and Raslan (2007), Alquran and Qawasmeh (2013) and Alquran and Al-Khaled (2011b)]" admits the use of solutions of the form

$$u(x,t) = A(t)\cos^m(\mu\,\zeta), \quad \zeta = x - c(t), \tag{2.1}$$

and

$$u(x,t) = A(t)\sin^m(\mu\,\zeta), \quad \zeta = x - c(t), \tag{2.2}$$

for some parameters $A(t), \mu, m$ and $c(t)$ to be determined later. Here, μ is the wave number and $c(t)$ is the wave speed being a function of the time t. From (2.1), we have

$$\begin{aligned}
u_t(x,t) &= A'(t)\cos^m\left(\mu\,(x-c(t))\right) + m\,\mu\,A(t)c'(t)\cos^{m-1}\left(\mu\,(x-c(t))\right)\sin\left(\mu\,(x-c(t))\right),\\
u_x(x,t) &= -m\,\mu\,A(t)\cos^{m-1}\left(\mu\,(x-c(t))\right)\sin\left(\mu\,(x-c(t))\right),\\
u_{xxt}(x,t) &= m(m-1)\mu^2\,A'(t)\cos^{m-2}\left(\mu\,(x-c(t))\right) - m^2\mu^2 A'(t)\cos^m\left(\mu\,(x-c(t))\right)\\
&\quad - m^3\mu^3 A(t)c'(t)\cos^{m-1}\left(\mu\,(x-c(t))\right)\sin\left(\mu\,(x-c(t))\right)\\
&\quad + m(m-1)(m-2)\mu^3 A(t)c'(t)\cos^{m-3}\left(\mu\,(x-c(t))\right)\sin\left(\mu\,(x-c(t))\right).
\end{aligned} \tag{2.3}$$

Because of the duality relation between the sine and cosine functions, and without loss of generality, we omit the analysis argument regarding the solution proposed in (2.2). Now, after substituting (2.3) into the original partial differential equation (1.1), a trigonometric equation is obtained with either $\cos^n(\mu\,\zeta)$ or $\cos^n(\mu\,\zeta)\sin(\mu\zeta)$ terms so that the parameter n can be determined by comparing exponents. The fact that the coefficient of $\cos^i(\mu\zeta)$ or $\cos^i(\mu\,\zeta)\sin(\mu\,\zeta)$ must vanish for all powers of i, then produces a system of algebraic equations in the the unknowns $A(t), \mu, m$ and $c(t)$, from which the solution proposed in (2.1) follows immediately.

3. Generalized Benjamin-Bona-Mahony

Consider the following nonhomogeneous BBM

$$u_t + \alpha u_{xxt} + \beta(t)u_x + \gamma(t)uu_x = g(t). \tag{3.1}$$

First, we use the transformation

$$u(x,t) = w(x,t) + h(t). \tag{3.2}$$

Substituting (3.2) in (3.1) yields

$$w_t + h'(t) + \alpha w_{xxt} + \beta(t)w_x + \gamma(t)ww_x + \gamma(t)h(t)w_x = g(t). \tag{3.3}$$

We require that

$$h'(t) = g(t) \text{ so that } h(t) = \int g(t)dt.$$

Hence, the following homogeneous BBM equation is obtained

$$w_t + \alpha w_{xxt} + k(t)w_x + \gamma(t)ww_x = 0, \tag{3.4}$$

where

$$k(t) = \beta(t) + \gamma(t)\,h(t). \tag{3.5}$$

By substituting the cosine assumption (2.1) and (2.3) into (3.4), and then comparing exponents and collecting coefficients of $\cos^i(\mu\,\zeta)$ or $\cos^i(\mu\,\zeta)\sin(\mu\,\zeta)$ for all i, we have the following algebraic system

$$
\begin{aligned}
0 &= m+2, \\
0 &= (m-1)(m-2)\mu^2\alpha(t)c'(t) - A(t)\gamma(t), \\
0 &= A'(t)(1-m^2\mu^2\alpha(t)), \\
0 &= m(m-1)\mu^2\alpha(t)A'(t), \\
0 &= c'(t) - k(t) - m^2\mu^2\alpha(t)c'(t).
\end{aligned} \tag{3.6}
$$

Solving the above system yields

$$
\begin{aligned}
m &= -2, \\
A'(t) &= 0, \\
c'(t) &= \frac{3k(t)+A(t)\gamma(t)}{3}, \\
\mu &= \pm\frac{\sqrt{A(t)}}{2\sqrt{\alpha}\sqrt{3\dfrac{k(t)}{\gamma(t)}+A(t)}}.
\end{aligned} \tag{3.7}
$$

adducing the following facts (3.7):

1. $A(t)$ must be a constant i.e. $A(t) = A$.

2. $c(t) = \int \frac{3k(t)+A\,\gamma(t)}{3}dt$.

3. Since μ is constant from the cosine assumption, therefore $\frac{k(t)}{\gamma(t)}$ must be constant. Thus, $k(t)$ is a multiple of $\gamma(t)$.

Accordingly, the solution of the BBM (3.1) is

$$
u(x,t) = A\sec^2\left(\frac{\sqrt{A}}{2\sqrt{\alpha}\sqrt{3\dfrac{k(t)}{\gamma(t)}+A}}(x-\int\frac{3k(t)+A\gamma(t)}{3}dt)\right)+\int g(t)dt. \tag{3.8}
$$

4. Discussion

We have discussed in this context several cases of the nonhomogeneous BBM equation based on different selections of the time-dependent coefficients.

Case I: In this case we consider

$$\alpha = 1, \beta(t) = 1, \gamma(t) = 1, g(t) = 0, A = 1.$$

By the constraints on the coefficient functions we find that

$$h(t) = 0, k(t) = 1, \mu = \pm\frac{1}{4}, c(t) = \frac{4t}{3}.$$

Therefore the solution of the BBM is

$$u(x,t) = \sec^2\left(\frac{3x-4t}{12}\right). \tag{4.1}$$

Case II: Consider

$$\alpha = 1, \beta(t) = 2t - t^2, \gamma(t) = t, g(t) = 1, A = 1.$$

We find

$$h(t) = t, k(t) = 2t, \mu = \pm\frac{1}{2\sqrt{7}}, c(t) = \frac{7t^2}{6}.$$

giving the solution:

$$u(x,t) = \sec^2\left(\frac{1}{2\sqrt{7}}(x - \frac{7t^2}{6})\right) + t. \tag{4.2}$$

Case III: Consider

$$\alpha = 1, \beta(t) = 2e^{-t} - 1, \gamma(t) = e^{-t}, g(t) = e^t, A = 1.$$

We find

$$h(t) = e^t, k(t) = 2e^{-t}, \mu = \pm\frac{1}{2\sqrt{7}}, c(t) = \frac{-7e^{-t}}{3}.$$

So that the solution is

$$u(x,t) = \sec^2\left(\frac{1}{2\sqrt{7}}(x + \frac{7e^{-t}}{3})\right) + e^t. \tag{4.3}$$

Case IV: Consider

$$\alpha = 1,\ \beta(t) = 2\sin(t) + 1,\ \gamma(t) = \sin(t),\ g(t) = \csc(t)\cot(t),\ A = 1.$$

We find

$$h(t) = -\csc(t),\ k(t) = 2\sin(t), \mu = \pm\frac{1}{2\sqrt{7}}, c(t) = \frac{-7\cos(t)}{3}.$$

with the solution:

$$u(x,t) = \sec^2\left(\frac{1}{2\sqrt{7}}(x + \frac{7\cos(t)}{3})\right) - \csc(t). \quad (4.4)$$

It should be noted that the type of the time-dependent coefficients "within the constraints given in the previous section", does not deform the physical structure of the BBM equation. This is observed in Figures 1 and 2.

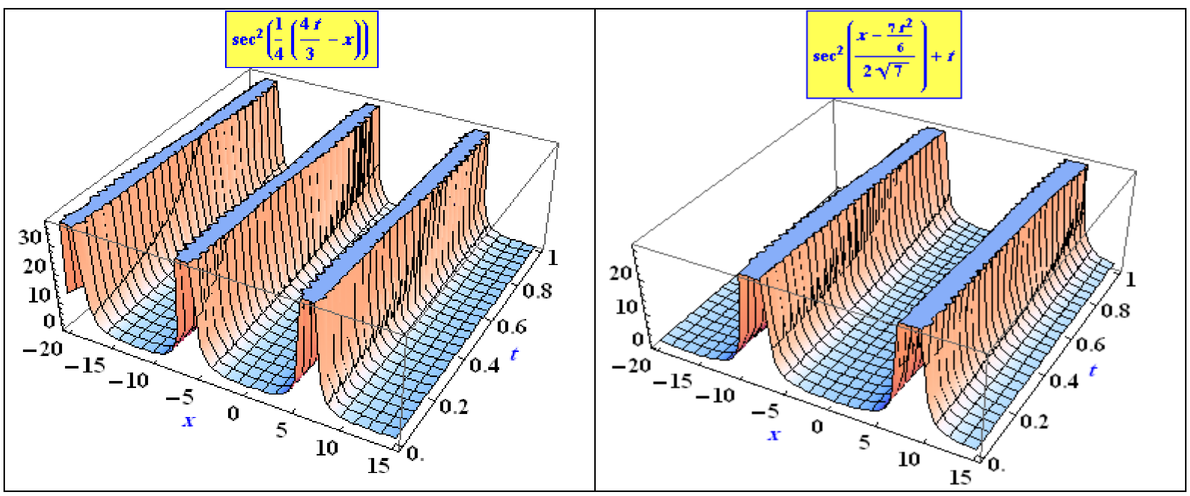

Figure 1. Plots of solutions to the BBM obtained in Case I and II respectively

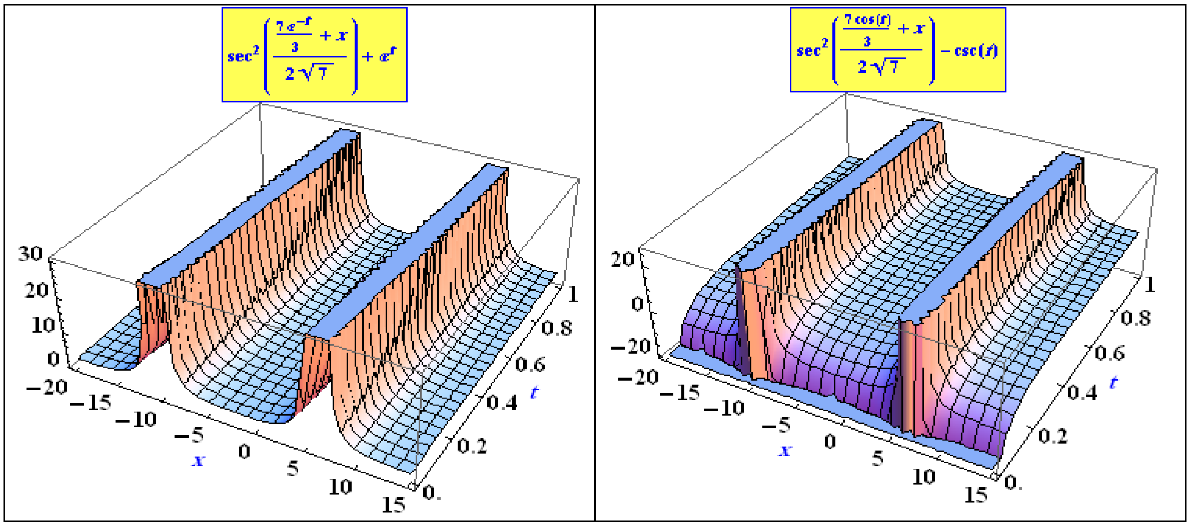

Figure 2. Plots of solutions to the BBM obtained in Case III and IV respectively

5. Conclusion

In summary, the modified sine-cosine method with symbolic computation is employed for a reliable treatment of the proposed BBM with time-dependent variable coefficients. Solitary wave solutions to this model are obtained under certain constraints on the coefficient functions and geometric illustrations of the physical structure of the nonhomogeneous BBM has been addressed.

The modified version of the well-known sine-cosine method is effective and powerful in handling variable-coefficient nonlinear equations while other existing methods may not be so handy for the same study.

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