Woman C.P.A.

Volume 40 | Issue 2 Article 4

4-1978

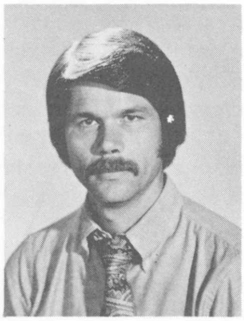
Ratio Estimate: Conceptual Review and a Case Illustration

Joseph G. Van Matre

Loudell O. Ellis

The Ratio Estimate

Conceptual Review and a Case Illustration



Joseph G. Van Matre, Ph.D., is Associate Professor and Acting Chairman of the Department of Business Administration of the University of Alabama in Birmingham. He is President of the Alabama Chapter of the American Statistical Association and a member of the American Institute of Decision Sciences.

Loudell O. Ellis, CPA, CMA, Ph.D., is Professor of Accounting at University of Alabama in Birmingham. She is a member of the Advisory Group to the Commission of IRS, of the AICPA Educator's Advisory Group, and holds memberships in AWSCPA, ASWA, AAA, NAA and AICPA.

A perusal of the recent accounting literature indicates that statistical sampling remains a topic of significant interest to the profession. Why the continuing interest? As we know, the accountant as a decision-maker utilizes imperfect (i.e. sample) information, and such information normally arises through either statistical or judgemental sampling. The principal advantage of statistical sampling is that the auditor is permitted to evaluate objectively the precision of the sample estimates and to demonstrate the basis for confidence in the tests. Judgmental sampling does not afford such objectivity.[1]

In light of the above, one is not surprised to find that an increasing number of accountants are users of statistical sampling techniques. However, the majority of users are employing only the more elementary procedures. For example, in a recent survey of practicing Certified Public Accountants, it was found that fewer than one-half of the users utilized stratified sampling and barely one-fourth use ratio estimation. Nonetheless, applications of the latter appear to be successful, since 94 percent of users consider the technique useful.[2]

Why then is the ratio estimate not more widely used? In many instances it offers striking gains in precision over the more common mean-per-unit estimate, and its application is straightforward and requires only the simple random sampling procedure. Insufficient understanding may be the major reason for lack of use.[3] This paper will review the concept of the ratio estimate. Problem characteristics of when it is superior (more precise) than the mean-per-unit estimate and a case illustration are also given. The reader with a basic knowledge of simple random sampling and the estimation concepts of precision and reliability is sufficiently prepared to understand and apply the ratio estimate.[4] Statistical sampling concepts presented in this paper are applicable to the auditors' decisions in a variety of areas, including inventory valuations and bad debt estimations. Many accountants will find that the ratio estimate is appropriate for use in complying with SEC requirements for reporting replacement costs.

The Mean-Per-Unit and Ratio Estimators

Suppose the accountant desires to estimate the total value of some population, the definition of the population being, of course, a prerogative of the accountant (e.g., expense account vouchers for May in excess of $100.00). Then, a random sample of size n could be drawn from the population,and a point estimate of the population total would be given by (see Table 1 for notation):

$$\hat{T}_1 = N\bar{Y}$$

If precision limits were desired, they would be given by:

$$\hat{T}_1 \pm U \times SE(\hat{T}_1)$$

where U is the standard normal deviate corresponding to the reliability deemed appropriate by the accountant. The standard error of the estimate is given by:

$$SE(\hat{T}_1) = \sqrt{N^2 \times \frac{s^2}{n} \times \frac{N-n}{N}}$$

where the sample variance, s^2, is defined:

$$s^2 = \frac{\Sigma(Y_i-\bar{Y})^2}{n-1}$$

When employing the ratio estimator, the accountant obtains *two* values from each item selected in the sample: (1) a value of the primary variable, Y, whose population total, Ty, is the quantity to be estimated, and (2) a value of the auxiliary variable, X, which is a variable correlated with Y and whose population total, Tx, is known. For example, Y could be the valuation of an inventory item under LIFO and X the valuation of the same item under FIFO. Then, the desired quantity is an estimate of total inventory value under LIFO; we assume that the total valuation under FIFO is known and that the item valuations under LIFO (Y) and FIFO (X) would be correlated.

The ratio estimate of the total is given by:

$$\hat{T}_2 = \frac{\bar{Y}}{\bar{X}} \times T_X,$$

and the standard error of this estimate is:

$$SE(\hat{T}_2) = \sqrt{\frac{N(N-n)}{n(n-1)} \times \Sigma \left[Y_i - \left(\frac{\bar{Y}}{\bar{X}}\right) X_i\right]^2}.$$

The ratio estimated is so named because the ratio of Y to X is used to "adjust" the known population total of X to obtain an estimate of the population total of Y.

We reiterate that the sampling procedure to obtain either T^1 or T^2 is identical: simple random. Further, the computational burden of the latter is not substantially greater, especially if electronic calculators or, perferably, computers are utilized when n is large.

N	population size
n	sample size
Y_i	value of the primary variable for the ith observation
X_i	value of the auxiliary variable for the ith observation
$\bar{Y}$	sample mean of the primary variable
$\hat{T}_1$	mean-per-unit estimate of total (Y)
$\hat{T}_2$	ratio estimate of total (Y)
$SE(\hat{T}_1)$	standard error of $\hat{T}_1$
$SE(\hat{T}_2)$	standard error of $\hat{T}_2$
T_X	population total of the auxiliary variable
fpc	finite population correction

Table I
Symbolic Notation to be Employed

Comparing Precision

Since on occasion the accountant may have choice of estimators to employ, which should be chosen? One should choose the estimate with the greater precision; guidelines are discussed below which indicate the proper choice. Before comparing precision of the two estimates, however, it should be pointed out that ratio estimation can lead to biased estimates, while the mean-per-unit estimate always yields an unbiased estimate. (i.e., the expectation of T^1 is Ty.) However, the bias associated with the ratio estimate decreases with increasing sample size,[5] and will be negligible with the sample sizes generally required in auditing[6] and certain management accounting applications.

To ascertain which estimate would be more precise without resorting to complex rules, one may follow the suggestion of Cochran:

> *When we are trying to decide what kind of estimate to use, a graph in which Y^i is plotted against Xi is helpful. If this graph shows the relation is a straight line through the origin and if the variance of the points Yi about the line seem to increase proportionally to Xi, the ratio estimate will be hard to beat.*[7]

In other words, plot the sample observations (if desired using a small preliminary sample) as shown in the scatter diagram of Figure 1. If a line passing through the center of the data intersects the origin and if the dispersion of the points about the line increases from left to right, the ratio estimate is indicated as the proper choice.

When the auxiliary and primary variables are measurements made on the same item at different points in time or using different processes (e.g. the value of an inventory item under LIFO and FIFO), the selection of the ratio estimate is indicated if the correlation coefficient between X and Y is greater than 0.5. The correlation coefficient is easily computed on electronic calculators or using a "canned" computer program. With experience one will likely recognize estimation problems calling for the ratio estimate.

The gain in precision through use of the ratio estimate may be striking or marginal. Arkin gives an example where the width of the confidence interval using the mean-per-unit is approximately nine times the width of the confidence interval for Ty using the ratio estimate.[8] In other problems, as in the following example, the gains may be slight.

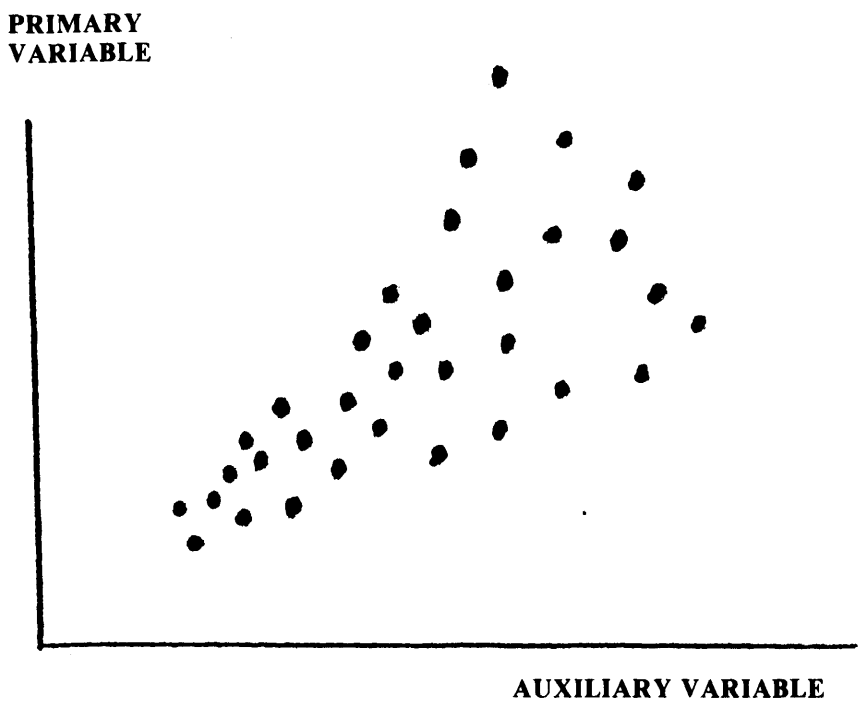


FIGURE 1: Scatter Diagram of Data Where Ratio Estimate Would be Optimal

Many accountants will find that the ratio estimate is appropriate for complying with SEC requirements for reporting replacement costs.

Case Illustration

In 1975, the current owners of a finance company, say Beta Finance, sued the previous owners in an attempt to recover a portion of the purchase price. One of the major elements of the case was a determination of the total liquidating value of the company's portfolio of loans. The plaintiff's attorney sought assistance from one of the authors in this paper in developing a sampling plan and statistical estimate that would be acceptable in a court of law.

The total number of accounts in the loan portfolio of Beta Finance was 5262; the total book value was over $12 million with individual accounts having a book value ranging from a few dollars to $449,468.50. However, two account categories (having 146 accounts) were exempted from study. Examination of the remaining accounts suggested creation of two strata with a book value of: (1) more than $7500 and (2) $7500 or less. The first strata, consisting of 93 accounts, would be subjected to a 100% verification, and thus no sampling error would be involved in this segment of the investigation. The second strata then constituted the population to be sampled. Table 2 illustrates the analysis thus far.

The liquidating value of the loans was to be determined by a financial expert. Of course the process of pulling the individual loans, examining various facets thereof, and establishing a liquidating value is quite laborious; for this reason a census of the entire loan portfolio was impractical. Based on a preliminary sample of thirty observations, consideration of time constraints, and the attorney's rather liberal precision requirements, a sample of 191 observations was randomly selected from the population of 5023. The financial expert then established a liquidating value for each (Yi); the book value (Xi) was readily available.

Prior to the acquisition of the data, the estimation problem was thought to be an ideal application of the ratio estimate. A priori, it was thought that book and liquidating values would be highly correlated with a linear relationship through the origin. However, the portfolio was of such dubious and varied quality that the correlation was quite low.

The computation of both the mean-per-unit and ratio estimates and their standard errors was quite straightforward and easy using a "canned" program. The data was punched (some two hundred cards most of which, however, used only a few columns) in less than an hour, and processed using the SAS package.[9] Computer time cost was less than two dollars, and yielded the various summations necessary for "plugging in" (e.g. Y, Y^2, etc.). The resultant computations yielded:

$$\hat{T}_1 = 561{,}024$$

$$\hat{T}_2 = 542{,}379$$

$$SE(\hat{T}_1) = 176{,}863$$

$$SE(\hat{T}_2) = 166{,}895$$

Using either point estimate, the estimated liquidation value was far below the stated book value of over $9 million. While this estimate appears suspicious, it was later determined that customers with accounts having a book value of nearly $8 million had made no payments in at least sixteen months. The financial expert noted in his report:

> *The company did not alternate the men with the dealers and close personal relationships developed between the dealers and the credit check men responsible for rejections or acceptance of the notes offered by the dealers. Original judgment on the passing of credit on these remaining*

	Number of Accounts	Total Book Value
Entire Portfolio	5262	$12,010,568.24
Less: Excluded Accounts	146	378,611.32
Portfolio of Current Interest	5116	$11,631,956.92
Less: Accounts with book value 7500	93	2,441,857.61
Population to be sampled	5023	$9,190,099.31

TABLE 2: Description of the Sampling Problem for Beta Finance Portfolio

loans was very bad. The value of the properties was not determined before the loan was made, the courthouse records were not checked to determine title status or what was against the property, bad credit reports were ignored and verification of work completed was nil...The accounts have very poor pay records, little equity exists in the security and in most cases the security is gone, the bulk of the accounts being paid on are collections by attorneys and collection agencies.

It was reassuring to the attorney, though not of statistical import, to note that if the sample were used to estimate the *book* value of the portfolio (using the mean-per-unit estimator NX), the point estimate was $9,505,876. This point estimate is in error by 3.44 percent from the known book value.

As indicated earlier, the ratio estimate in this instance is only marginally better than the mean-per-unit estimate because of the weak correlation between X and Y. Although the standard errors seem rather large, the precision was adequate for the attorneys' purpose. In personal correspondence from one of the attorneys:

...the principal case...was settled by their payment of $1,000,000.00 cash.

The work you performed in conjunction with the (financial) expert in Atlanta provided us with sufficient fire power to enable us to stand firm in our demand...Without your help, we may not have been able to secure a reliable estimate of the value...

Conclusions

The ratio estimate can be used to estimate population totals (or means), often gaining in precision as compared to the mean-per-unit estimate. The sampling procedure, simple random, is the same for either estimate. In the ratio method the value of an auxiliary variable, as well as the primary variable, are obtained for each sample observation. The population total of the auxiliary variable X must be known. In practice X is usually the value of Y at a previous time or under a different process when a census was taken. The gain in precision is largely dependent upon the degree of correlation between X and Y.

The ratio estimate offers one further advantage: it may be employed when the total number of items in the population (N) is unknown. This property of the ratio estimate has considerable utility in accounting applications.[10] In contrast, the mean-per-unit estimate, $T^1 = NY$, requires knowledge of N.

Statistical sampling continues to gain popularity in the accounting profession primarily because of its objectivity. It is more readily defensible than judgmental sampling, an important consideration in the legal arena.[11]

FOOTNOTES

[1]See, for example: James K. Loebbecke and John Neter, "Statistical Sampling in Confirming Receivables," *The Journal of Accountancy*, July, 1973, P. 47; Kenneth W. Stringer, "Some Basic Concepts of Statistical Sampling in Auditing," *The Journal of Accountancy*, November, 1961, p. 65; and James P. Bedingfield, "The Current State of Statistical Sampling in Auditing," *The Journal of Accountancy*, December, 1975, p. 53.

[2]Bedingfield, *op. cit.*, p. 52.

[3]*Ibid.*, p. 54.

[4]For a review of basic sampling principles, consult: Herbert Arkin, *Handbook of Sampling for Auditing and Accounting*, McGraw-Hill, 1963.

[5]William G. Cochran, *Sampling Techniques*, John Wiley & Sons, 1963, p. 160.

[6]Robert S. Kaplan, "Statistical Sampling in Auditing with Auxiliary Information Estimators," *Journal of Accounting Research*, Autumn, 1973, p. 242.

[7]Cochran, *op. cit.*, p. 167.

[8]Arkin, *op. cit.*, pp. 219-225.

[9]Jolayne Service, *A User's Guide to the Statistical Analysis System*, Student Supply Stores, North Carolina State University, August 1972.

[10]For a discussion of such applications with appropriate formulas, the interested reader should consult: William Mendenhall, Lymann Ott, and Richard L. Scheaffer, *Elementary Survey Sampling*, Wadsworth, 1971.

[11]See: Ronald M. Copeland and Ted D. Englebrecht, "Statistical Sampling: An Uncertain Defense Against Legal Liability," *CPA Journal*, November, 1975. Their article provides some valuable insights into auditing, sampling, and the law. Advantages and courtroom presentation of the interval estimate can be found in: Leo Katz, "Presentation of a Confidence Interval as Evidence in a Legal Proceeding," *The American Statistician*, November, 1975; and a primer on the role of an expert witness can be found in: Joseph G. Van Matre and William N. Clark, "The Statistician as Expert Witness," *The American Statistician*, February, 1976.