BAU Journal - Science and Technology

Volume 2 | Issue 2
ISSN: 2706-784X

Article 1

June 2021

UNSTEADY ROTATIONAL MOTION OF A MICROPOLAR FLUID INSIDE A RIGHT CIRCULAR CYLINDER USING STATE SPACE APPROACH

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Abstract

In this work, the unsteady rotational motion of an incompressible micropolar fluid inside a right circular cylinder is investigated. The classical no-slip and no-spin boundary conditions are proposed. The problem is solved analytically using the state space approach together with Laplace transform technique. The inverse Laplace transform is carried out numerically using a numerical technique based on Fourier series expansion. Numerical results for the flow field functions are represented graphically for different values of the time and micropolarity parameters.

Keywords

Micropolar fluid, Rotational motion, State Space approach, Laplace transform, Unsteady flow

1. INTRODUCTION

The study of micropolar fluids proposed by Eringen in 1964 (Eringen, 1964) is a general microcontinuum approach that takes into account the microstructure as well as the macroscopic effects on the fluid motion. The micropolar fluids is the simplest model of the general micromorphic theory introduced by Eringen which is able to accurately describe the behavior of some types of fluids with microstructure such as animal blood, chemical suspensions and liquid crystals. Mathematically, a micropolar fluid flow is described by two independent vectors; the classical velocity vector which represents the translational velocity of the fluid particles and the microrotation vector which characterizes the spin velocity of the microelements of the fluid flow.

Micropolar fluids have found in large number of applications in various fields such as lubrication problems, liquid crystals, colloidal suspensions and polymeric additives (Eringen, 1998). The study of problems involving micropolar fluids has attracted the attention of researchers in wide area of applications.

The steady motion of incompressible micropolar fluid flows have been attempted by many authors. Verma and Sehgal studied the steady motion of an incompressible micropolar fluid confined between two coaxial right circular cylinders rotating about a common axis with constant angular velocity (Verma and Sehgal, 1968). The fluid motion of a micropolar fluid between two concentric cylinders is analyzed for the Couette and Poiseuille flows by Ariman et al (Ariman, Cakmak and Hill, 1967). Calmelet-Eluhu and Majumdar discussed the problem of internal flow of a micropolar fluid inside a circular cylinder in the case of particular flow, namely the longitudinal and torsional flow (Calmelet and Majumdar, 1998). An analytical closed-form solution to the problem of slow motion of a micropolar fluid around a rotating circular cylinder is obtained by Moosaie and Atefi (Moosaie and Atefi, 2008). The steady unidirectional flow of an isothermal, incompressible magneto-micropolar hydrodynamic fluid in an infinitely magnetic insulating circular cylinder was considered by Sherief et al (Sherief, Faltas and El-Sapa, 2017). The interaction between two-rigid spheres moving in a micropolar with slip surfaces was studied by the same authors (Sherief, Faltas and El-Sapa, 2019). El-Sapa considered the gravitational setting slip velocity of a spherical particle in an unbounded micropolar fluid (El-Sapa, 2019). In the case of unsteady flows, the time parameter appears in the partial differential equations governing the fluid motion. The non-steady flow due to the accelerated micropolar fluid-flow past a uniformly rotating circular cylinder was studied by Siddiqui (Siddiqui, 2016). The problem of unsteady rotation of micropolar fluid about a circular cylinder with no-slip boundary conditions was investigated by Ashmawy (Ashmway, 2007). The flow of an electrically conducting thermoelectric micropolar fluid over a suddenly moved heated plate was studied by Ezzat et al (Ezzat, Abbas, El-Bary and Ezzat 2014). The exponential solution of a problem of two-dimensional motion of a micropolar fluid in a half-plane was discussed by El-Bary (El-Bary, 2005).

The state space approach is an analytical technique that can be used to study linear systems of differential equations with time which makes it very useful in solving time dependent problems in fluid dynamics. This technique was used by Devakar and Iyengar (Devakar and Iyengar, 2009) to discuss the Stokes' first problem of a micropolar fluid with no-slip and no-spin conditions. The same authors also used such technique to investigate the Couette and Poiseuille flows of a micropolar fluid between two parallel plates (Devakar and Iyengar, 2011, 2013). The problem of state space approach to magneto hydrodynamic flow of perfectly conducting micropolar fluid with stretch was investigated to obtain the solution of a one-dimensional problem (Ezzat, El-Sapa, 2012). The computational treatment of free convection effects on perfectly conducting viscoelastic fluid was studied by using state space approach (El-Bary, 2005). The thermoelastic MHD theory with memory-dependent heat transfer was discussed using state –space technique to obtain the general solution (Ezzat, El-Bary, 2016). The unsteady free convection flow of micropolar fluid using state space approach was discussed by Helmy (Helmy, 2000).

Slayi and Ashmawy (Slayi, Ashmawy, 2014) studied the problem of unsteady slip flow of a micropolar fluid between parallel plates using state space approach. In 2016, the time dependent slip flow of a micropolar fluid between two parallel plates was studied using the state space approach by Slayi et al (Slayi, Ashmawy and Idriss, 2016). Slayi and Ashmawy employed the same technique to study the problem of unsteady flow of a microstretch fluid with slip conditions (Slayi, Ashmawy, 2018).

In this work, we consider the rotational fluid flow of an incompressible micropolar fluid inside a right circular cylinder using no-slip and no-spin boundary conditions. The state space approach method is employed to get the solution of the problem. In this approach, the governing equations are written in a matrix form after applying the Laplace transform and the analytical expression for the velocity and the micro-rotation are then obtained analytically. The inversion of Laplace transforms of the flow field functions are obtained using a standard numerical inversion technique. The results are presented and discussed graphically.

2. FORMULATION OF THE PROBLEM

The field equations governing the unsteady motion of an incompressible micropolar fluid in the absence of body forces and body couples are given by

Conservation of mass

$$\nabla \cdot \vec{q} = 0 , \tag{2.1}$$

Balance of Momentum

$$\rho \frac{d\vec{q}}{dt} = (\lambda + 2\mu + \kappa)\nabla(\nabla \cdot \vec{q}) - (\mu + \kappa)\nabla \times \nabla \times \vec{q} + \kappa\nabla \times \vec{v} - \nabla p , \tag{2.2}$$

Balance of Moment of Momentum

$$\rho j \frac{d\vec{v}}{dt} = (\alpha + \beta + \gamma)\nabla(\nabla \cdot \vec{v}) - \gamma\nabla \times \nabla \times \vec{v} + \kappa\nabla \times \vec{q} - 2\kappa \vec{v} . \tag{2.3}$$

where, the scalar quantities ρ and j are, respectively, the fluid density and gyration parameters and are assumed to be constants. The vectors $\vec{q}$ and $\vec{v}$ are the velocity and microrotation vectors, respectively. p is the fluid pressure at any point. The material constants (λ, μ, κ) represent the viscosity coefficients and (α, β, γ) represent the gyro-viscosity coefficients.

We now consider the unsteady flow of an incompressible micropolar fluid inside a right circular cylinder of radius a due to the rotation of cylindrical boundary about its axis with an angular velocity $\Omega\, \xi(t)$, where Ω is a constant and $\xi(t)$ is a non-dimensional function of time. Initially, the fluid flow is at rest. Working with the cylindrical coordinates (R, φ, z), in view of the geometry of the boundary and type of the fluid flow, it is found that the components of the velocity and the microrotation, respectively, have the following forms

$$\vec{q} = (0, v(R,t), 0) \ and \ \ \vec{v} = (0,0,\omega(R,t)).$$

The equation of continuity (2.1) is satisfied automatically and the equations (2.2) and (2.3) reduce to

$$(\mu + \kappa)\frac{\partial}{\partial R}\left\{\frac{\partial v}{\partial R} + \frac{v}{R}\right\} - \kappa\frac{\partial \omega}{\partial R} - \rho\frac{\partial v}{\partial t} = 0, \tag{2.4}$$

$$\frac{\gamma}{R}\frac{\partial}{\partial R}\left\{R\frac{\partial \omega}{\partial R}\right\} + \frac{\kappa}{R}\frac{\partial}{\partial R}\{Rv\} - 2\kappa\omega - \rho j\frac{\partial \omega}{\partial t} = 0. \tag{2.5}$$

The proposed initial and boundary conditions are given by

$$v(R,0) = 0\,,\ \omega(R,0) = 0 \ \text{at} \ \ t = 0, \tag{2.6}$$

$$v(R,t) = \Omega\, a\, \xi(t) \ \text{on} \ \ R = a, \tag{2.7}$$

$$\omega(R,t) = 0, \text{on} \ \ R = a. \tag{2.8}$$

The spin gradient viscosity j is assumed to be a constant and given by

$$j = \frac{2\gamma}{2\mu+\kappa} \tag{2.9}$$

Introducing the following non-dimensional variables

$$\hat{v} = \frac{v}{\Omega a},\ \hat{\omega} = \frac{\kappa}{\Omega(\mu+\kappa)}\omega,\ \hat{t} = \frac{(\mu+\kappa)}{\rho a^2}t,\ \hat{R} = \frac{R}{a}.$$

Now, we use the above mentioned non-dimensional variables. The hats are dropping for convenience, so the differential equations (2.4) and (2.5) in non-dimensional forms become

$$\frac{\partial}{\partial R}\left\{\frac{\partial v}{\partial R} + \frac{v}{R}\right\} - \frac{\partial \omega}{\partial R} - \frac{\partial v}{\partial t} = 0, \tag{2.10}$$

$$\frac{1}{R}\frac{\partial}{\partial R}\left\{R\frac{\partial \omega}{\partial R}\right\} + \frac{f}{R}\frac{\partial}{\partial R}\{Rv\} - g\omega - h\frac{\partial \omega}{\partial t} = 0, \tag{2.11}$$

where $f = \frac{\kappa^2 a^2}{\gamma(\mu+\kappa)},\ g = \frac{2\kappa a^2}{\gamma},\ h = \frac{2(\mu+\kappa)}{2\mu+\kappa}$.

The non-dimensional boundary conditions (2.7) and (2.8) become

$$v(R,t) = \xi(t), \quad \text{on } R = 1, \tag{2.12}$$

$$\omega(R,t) = 0, \text{on } R = 1. \tag{2.13}$$

3. SOLUTION IN THE LAPLACE DOMAIN

Now, we apply the Laplace transform defined by the formula

$$\bar{F}(y,s) = \int_0^\infty e^{-st}F(y,t)dt \tag{3.1}$$

After applying the above-mentioned Laplace transform, the differential equations (2.10)-(2.11) reduce to

$$\frac{\partial}{\partial R}\left\{\frac{\partial \bar{v}}{\partial R} + \frac{\bar{v}}{R}\right\} - \frac{\partial \bar{\omega}}{\partial R} - s\bar{v} = 0\,, \tag{3.2}$$

$$\frac{1}{R}\frac{\partial}{\partial R}\left\{R\frac{\partial \bar{\omega}}{\partial R}\right\} + \frac{f}{R}\frac{\partial}{\partial R}\{R\bar{v}\} - (g + hs)\bar{\omega} = 0\,. \tag{3.3}$$

The boundary conditions are taking the forms

$$\bar{v}(R,s) = \bar{\xi}(s), \quad \text{on } \ R = 1, \tag{3.4}$$

$$\bar{\omega}(R,s) = 0, \quad \text{on } R = 1. \tag{3.5}$$

Operating on equation (3.3) by $\frac{\partial}{\partial R}$, the differential equations (3.2) and (3.3) can be rewritten in the following forms

$$D^2\bar{v} - \bar{\emptyset} - s\bar{v} = 0\,, \tag{3.6}$$

$$D^2\bar{\emptyset} + f\,D^2\bar{v} - (g + hs)\bar{\emptyset} = 0\,, \tag{3.7}$$

where

$$\bar{\emptyset} = \frac{\partial\bar{\omega}}{\partial R},\quad D^2\bar{v} = \frac{\partial}{\partial R}\left\{\frac{1}{R}\frac{\partial}{\partial R}(R\bar{v})\right\} \text{ and } D^2\bar{\emptyset} = \frac{\partial}{\partial R}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\bar{\emptyset}\right)\right\}.$$

Eliminating $D^2\bar{v}$ between (3.6) and (3.7), we get

$$D^2\bar{\emptyset} + \big(f - (g + hs)\big)\bar{\emptyset} + s\,f\,\bar{v} = 0 \tag{3.8}$$

Then the system of equations can be rewritten as follows

$$D^2\bar{v} - \bar{\emptyset} - s\bar{v} = 0 \tag{3.9}$$

$$D^2\bar{\emptyset} + (f - (g + hs))\bar{\emptyset} + sf\bar{v} = 0 \tag{3.10}$$

To obtain the solution of the differential equations (3.9) and (3.10) subject to the boundary conditions (3.4)-(3.5), we apply the state space approach by first writing the two equations (3.9) and (3.10) in the following matrix form

$$D^2\bar{W}(R,s) = A(s)\bar{W}(R,s)\,, \tag{3.11}$$

where

$$A(s) = \begin{bmatrix} s & 1 \\ -sf & (g - f + hs) \end{bmatrix} \text{ and } \bar{W}(R,s) = \begin{pmatrix} \bar{v}(R,s) \\ \bar{\emptyset}(R,s) \end{pmatrix}. \tag{3.12}$$

The formal solution of the matrix differential equation (3.11) is found to be

$$\overline{W}(R,s) = I_1(R\sqrt{A})\vec{C} + K_1(R\sqrt{A})\vec{D}, \tag{3.13}$$

where $I_1(.)$and $K_1(.)$ are the modified Bessel functions of the first and second kinds of order one. Moreover, $\vec{C}$ and $\vec{D}$are constants.

Since the fluid is considered inside the cylinder and the flow field is bounded as $R \to 0$, so we have$\vec{D} = 0$. Therefore, equation (3.13) reduces to

$$\bar{W}(R,s) = I_1(R\sqrt{A})\vec{C}, \text{ where } \vec{C} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{3.14}$$

It can be easily seen that the characteristic equation of the matrix $A(s)$ is

$$\lambda^2 - (g - f + (h+1)s)\lambda + s(g + hs) = 0 \tag{3.15}$$

The roots of this equation, namely λ_1 and λ_2, are given by

$$\lambda_1 = \frac{1}{2}\left[\{g - f + (h+1)s\} + \sqrt{\{g - f + (h+1)s\}^2 - 4s(g+hs)}\right]$$

$$\lambda_2 = \frac{1}{2}\left[\{g - f + (h+1)s\} - \sqrt{\{g - f + (h+1)s\}^2 - 4s(g+hs)}\right]$$

The power series expansion of $I_1(R\sqrt{A})$ is given by (Martin, Olivares and Sotomayor, 2017)

$$I_1(R\sqrt{A}) = \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(k+2)} [R\sqrt{A}/2]^{2k+1} \tag{3.16}$$

Using Cayley-Hamilton theorem, we can express A^2 and higher orders of A in terms of the matrix A and the identity matrix I. Therefore, the series expansion in the right hand side of the expression (3.16) can be written in the following form (Gordon, 2007)

$$I_1\left(R\sqrt{A}\right) = L(R,s) = a_0 I + a_1 A\,, \tag{3.17}$$

where $L(R,s)$ is a matrix of order two, and a_0 and a_1 are coefficients depending on R and s. By using Cayley-Hamilton theorem, the characteristic roots λ_1 and λ_2 of the matrix A must satisfy equation (3.17), thus we get the following system of linear equations

$$I_1(R\sqrt{\lambda_1}) = a_0 + a_1\lambda_1, \tag{3.18}$$

$$I_1(R\sqrt{\lambda_2}) = a_0 + a_1\lambda_2. \tag{3.19}$$

After solving the above system, we get

$$a_0 = \frac{\lambda_1 I_1\left(R\sqrt{\lambda_2}\right) - \lambda_2 I_1\left(R\sqrt{\lambda_1}\right)}{(\lambda_1 - \lambda_2)} \tag{3.20}$$

$$a_1 = \frac{I_1\left(R\sqrt{\lambda_1}\right) - I_1\left(R\sqrt{\lambda_2}\right)}{(\lambda_1 - \lambda_2)} \tag{3.21}$$

The elements $L_{ij}\,, i,j = 1,2$ of the matrix $L(R,s)$ can be determined after inserting the matrix A given by (3.12) into equation (3.17) to get

$$L_{11} = a_0 + a_1 s,\; L_{12} = a_1,\; L_{21} = -a_1 s f,\; L_{22} = a_0 + a_1(g - f + hs). \tag{3.22}$$

With these, the solution (3.14) is obtained in the form

$$\overline{W}(R,s) = L(R,s)\vec{C}\,. \tag{3.23}$$

Hence, the solution takes the following form

$$\bar{v}(R,s) = c_1 L_{11}(R,s) + c_2 L_{21}(R,s), \tag{3.24}$$

$$\bar{\phi}(R,s) = c_1 L_{12}(R,s) + c_2 L_{22}(R,s). \tag{3.25}$$

To find the microrotation $\overline{\omega}(R,s)$, we integrate equation (3.25) to obtain

$$\overline{\omega}(R,s)=c_1M_1(R,s)+c_2M_2(R,s), \qquad (3.26)$$

where

$$M_1(R,s)=\int \mathrm{L}_{12}(\mathrm{R},\mathrm{s})dR\ ,\ \ M_2(R,s)=\int \mathrm{L}_{22}(\mathrm{R},\mathrm{s})dR$$

Thus, we arrive at

$$\overline{\omega}(\mathrm{R},\mathrm{s})=\frac{c_1}{\lambda_1-\lambda_2}\left[\frac{1}{\sqrt{\lambda_1}}I_0\left(R\sqrt{\lambda_1}\right)-\frac{1}{\sqrt{\lambda_2}}I_0\left(R\sqrt{\lambda_2}\right)\right]+\frac{c_2}{\lambda_1-\lambda_2}\left[\frac{\lambda_1}{\sqrt{\lambda_2}}I_0\left(R\sqrt{\lambda_2}\right)-\frac{\lambda_2}{\sqrt{\lambda_1}}I_0\left(R\sqrt{\lambda_1}\right)+(g-f+hs)\left(\frac{1}{\sqrt{\lambda_1}}I_0\left(R\sqrt{\lambda_1}\right)-\frac{1}{\sqrt{\lambda_2}}I_0\left(R\sqrt{\lambda_2}\right)\right)\right] \qquad (3.27)$$

Applying the imposed boundary conditions (3.4)-(3.5), we get

$$c_1=\frac{-M_1(1,s)M_2(1,s)\bar{\xi}(s)}{M_1(1,s)\left(M_1(1,s)L_{21}^1-L_{11}^1M_2(1,s)\right)},$$

$$c_2=\frac{M_1(1,s)\bar{\xi}(s)}{M_1(1,s)L_{21}^1-L_{11}^1M_2(1,s)},$$

where L_{ij}^1's are the values of L_{ij}'s evaluated at $R=1$.

4. THE NUMERICAL INVERSION OF LAPLACE TRANSFORM

The numerical inversion technique developed by Honig and Hirdes (Honig and Hirdes, 1984) has been employed to obtain the invesion of the Laplace transform of the field functions.Using this method, the inverse Laplace transform of the function $\overline{\mathrm{F}}(\mathrm{s})$is approximated by

$$F(t)=\frac{e^{bt}}{T}\left[\frac{1}{2}\bar{F}\ (\mathrm{b})+\mathrm{Re}\left(\sum_{k=1}^{N}\bar{F}\left(b+\frac{ik\pi}{T}\right)\exp\left(\frac{ik\pi t}{T}\right)\right)\right], \qquad 0<t<2T,$$

where N is sufficiently large integer chosen such that,

$$e^{bt}\mathrm{Re}\left[\bar{F}\ \left(\mathrm{b}+\frac{iN\pi}{T}\right)\exp\left(\frac{iN\pi t}{T}\right)\right]<\varepsilon$$

where ε is a small positive number that corresponds to the degree of accuracy required. The parameter b is a positive free parameter that must be greater than real parts of all singularities of $\bar{F}(s)$.

5. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we implement the results obtained using the state space approach with the numerical inversion method of Laplace transform numerically for some cases. The velocity component $v(R,t)$, the microrotation $\omega(R,t)$ and the function $\emptyset(R,t)$ are represented graphically for different values of the time parameter and micropolarity coefficient. Three different cases has been considered; flow due to a constant sudden motion, flow due to a sine oscillatory motion and flow due to a cosine oscillatory motion. The numerical value of the angular frequency σ is taken one during all numerical calculations.

Case 1

In this case, we suppose flow due to the constant motion

$$\xi(t) = H(t), \tag{5.1}$$

where $H(t)$ is the Heaviside unit step function defined by

$$H(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Case 2

Here, the non-dimensional velocity is given by

$$\xi(t) = \sin \sigma t, \tag{5.2}$$

where σ is the frequency of oscillation.

Case 3

Here, the non-dimensional velocity is given by

$$\xi(t) = \cos \sigma t. \tag{5.3}$$

In Figs.1 and 2, we study the variation of the velocity $v(R,t)$ and the microrotation $\omega(R,t)$ with distance for different times, respectively, when the flow is due to a constant motion; we can conclude that the values of the velocity increase with time, but the values of the microrotation decrease. The effect of micropolarity parameter on the velocity $v(R,t)$, the microrotation $\omega(R,t)$ and the function $\emptyset(\mathrm{R},\mathrm{t})$ is discussed. We can see that this parameter has a clear effect on the values of microrotation $\omega(R,t)$in Fig. 4, while does not affect the velocity $v(R,t)$ and the function $\emptyset(\mathrm{R},\mathrm{t})$ in Figs. 3 and 5. From Figs. 6 and 7, we observe that the velocity $v(R,t)$ increases with time and the microrotation $\omega(R,t)$ decreases, in the case of sine oscillatory motion. Figs. 8, 9 and 10, represent, respectively, the effect of micropolarity parameter on the velocity $v(R,t)$, microrotation $\omega(R,t)$and the function $\emptyset(\mathrm{R},\mathrm{t})$.We can see that this parameter has an increasing effect on the microrotation $\omega(R,t)$ but there is no appreciable change on the velocity $v(R,t)$and the function $\emptyset(\mathrm{R},\mathrm{t})$. For the third case, we represent the variation of the velocity, microrotation, respectively for different values of time. Finally, in Figs. 13-15, the variation of the velocity $v(R,t)$, the microrotation $\omega(R,t)$ and the function $\emptyset(\mathrm{R},\mathrm{t})$ are represented for different values of the micropolarity parameter. It is found that the effect of this parameter is similar to the first and second cases.

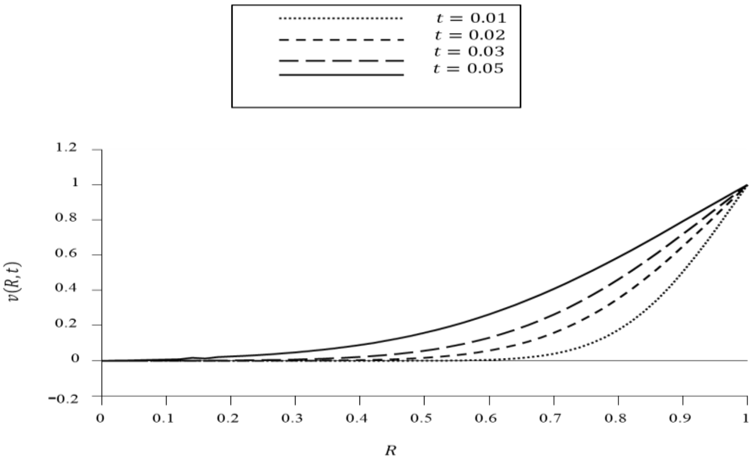

Fig.1: Variation of velocity $v(R,t)$ versus distance for $\kappa = 1$ for case 1.

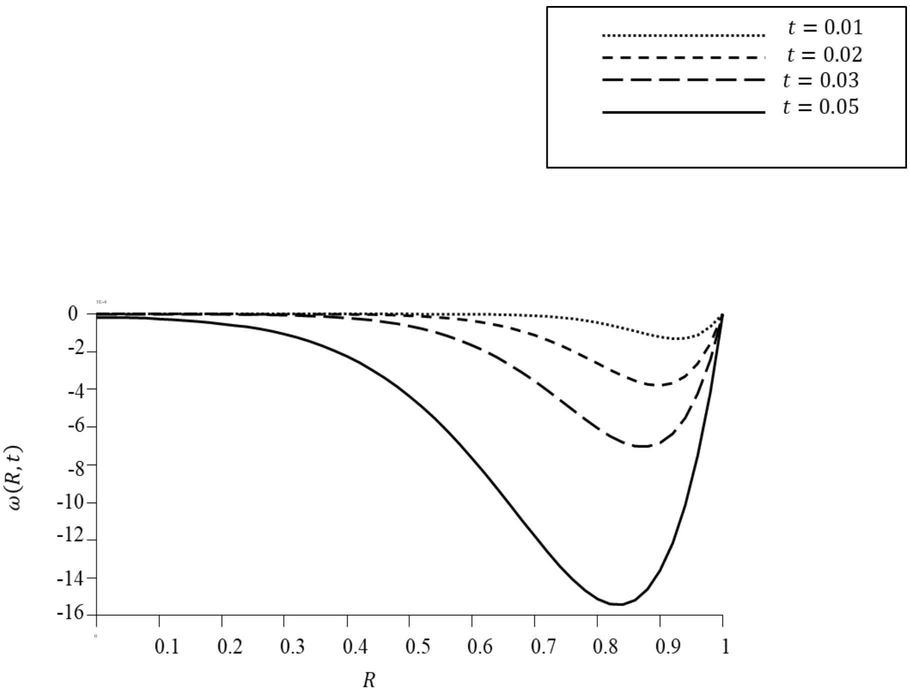

Fig.2: Variation of microrotation $\omega(R,t)$ versus distance for $\kappa = 1$ for case 1.

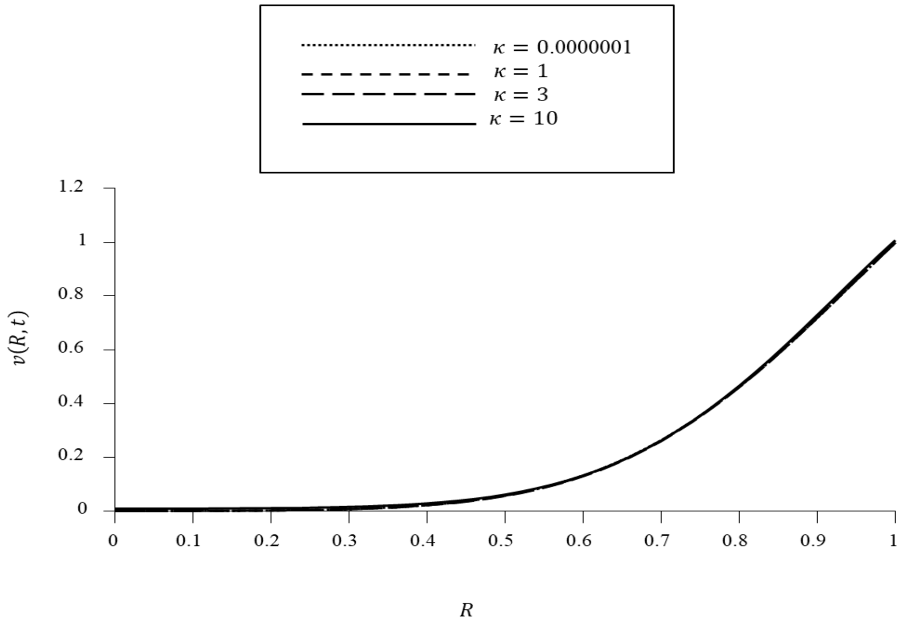

Fig.3: Variation of velocity $v(R,t)$ versus distance for $t = 0.03$ for case 1.

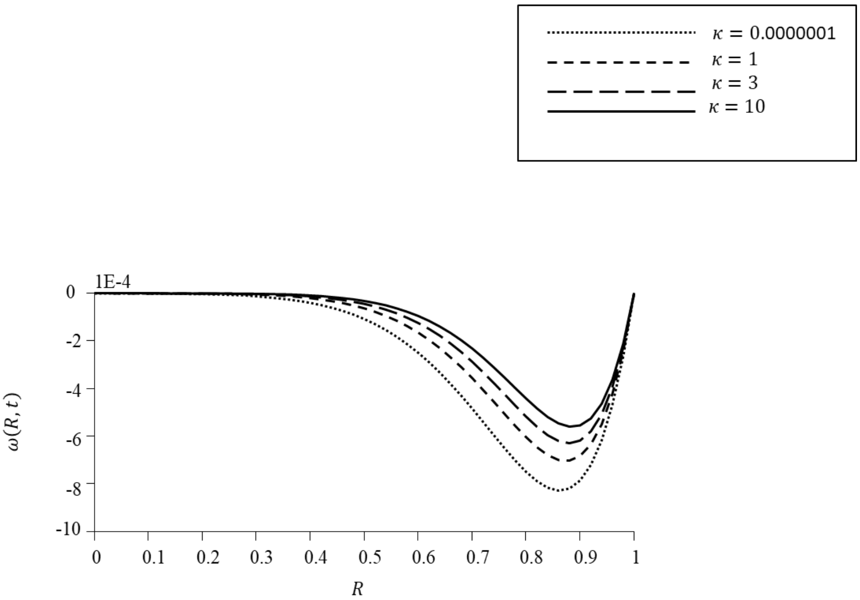

Fig.4: Variation of microrotation $\omega(R,t)$ versus distance for $t = 0.03$ for case 1.

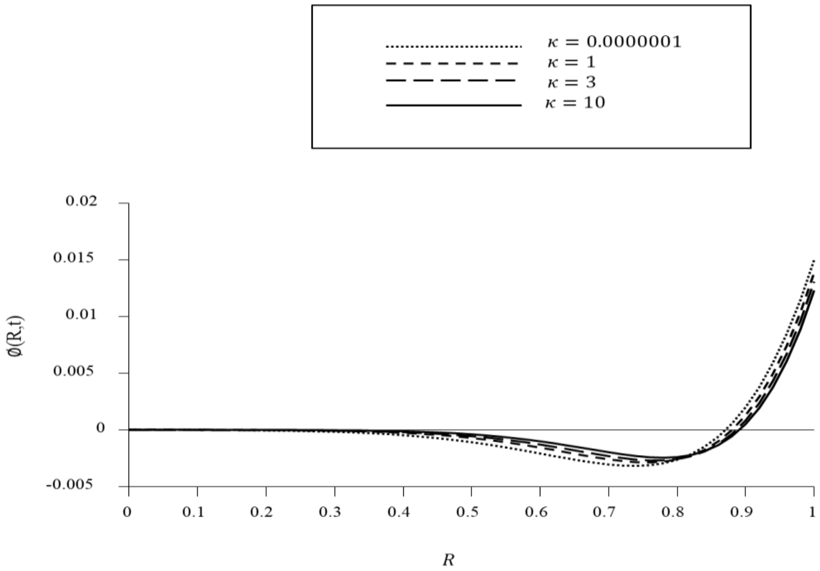

Fig.5: Variation of function $\emptyset(\mathrm{R}, \mathrm{t})$ versus distance for $t = 0.03$ for case 1.

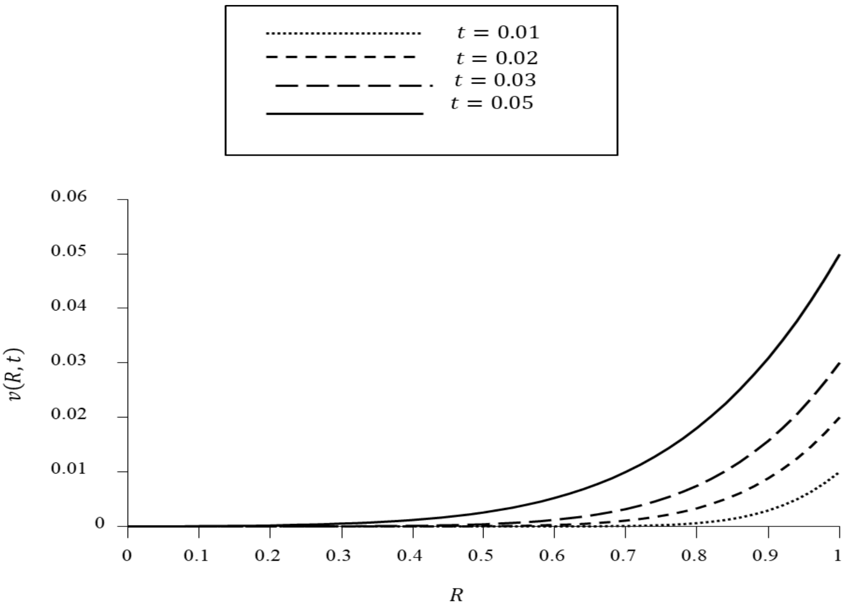

Fig.6: Variation of velocity $v(R, t)$ versus distance for $\kappa = 1$ for case 2.

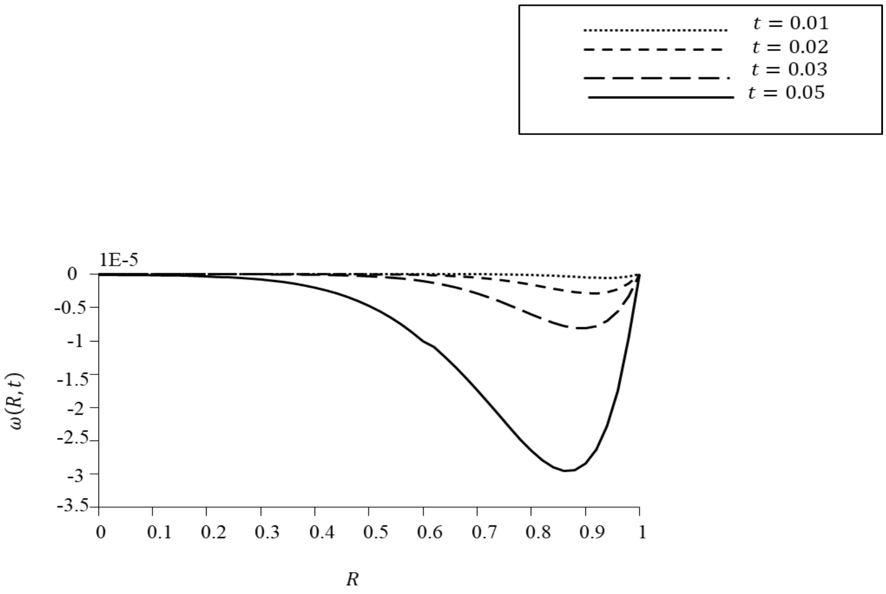

Fig.7: Variation of microrotation $\omega(R,t)$ versus distance for $\kappa = 1$ for case 2

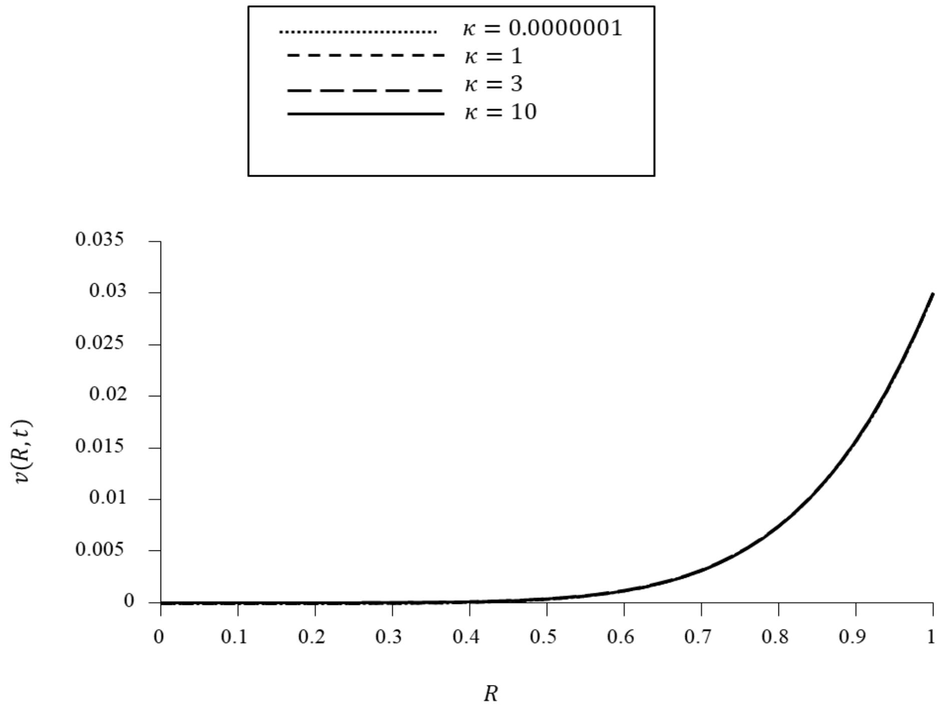

Fig.8: Variation of velocity $v(R,t)$ versus distance for $t = 0.03$ for case 2.

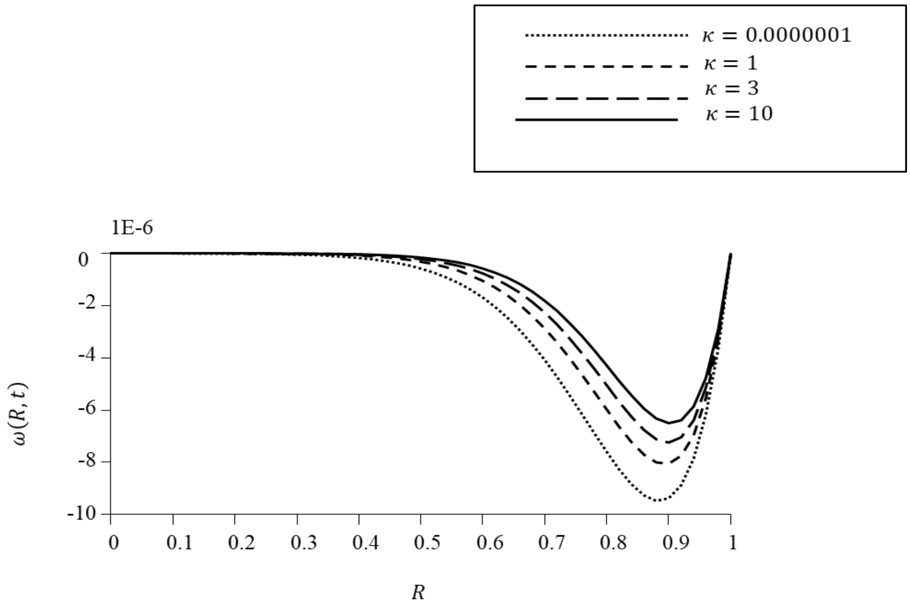

Fig.9: Variation of microrotation $\omega(R,t)$ versus distance for $t = 0.03$ for case 2.

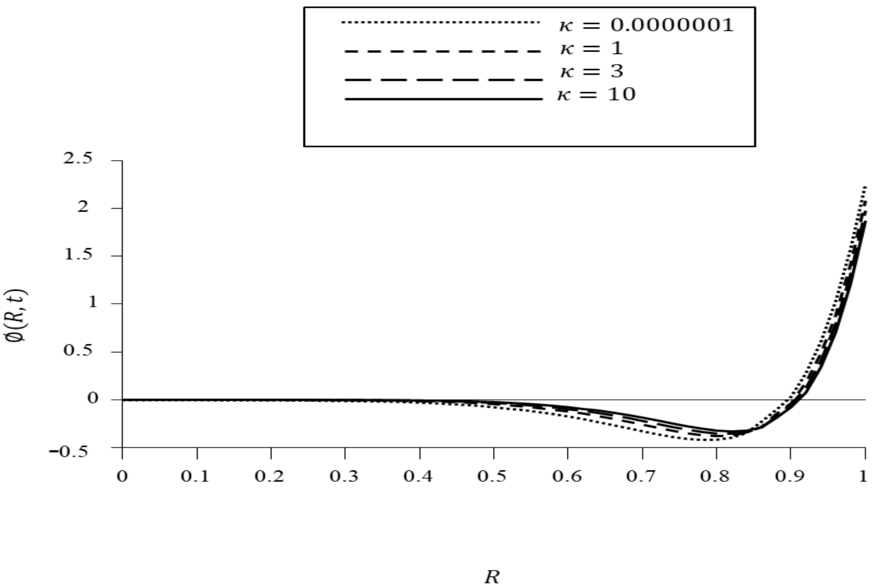

Fig.10: Variation of function $\emptyset(R,t)$versus distance for $t = 0.03$ for case 2.

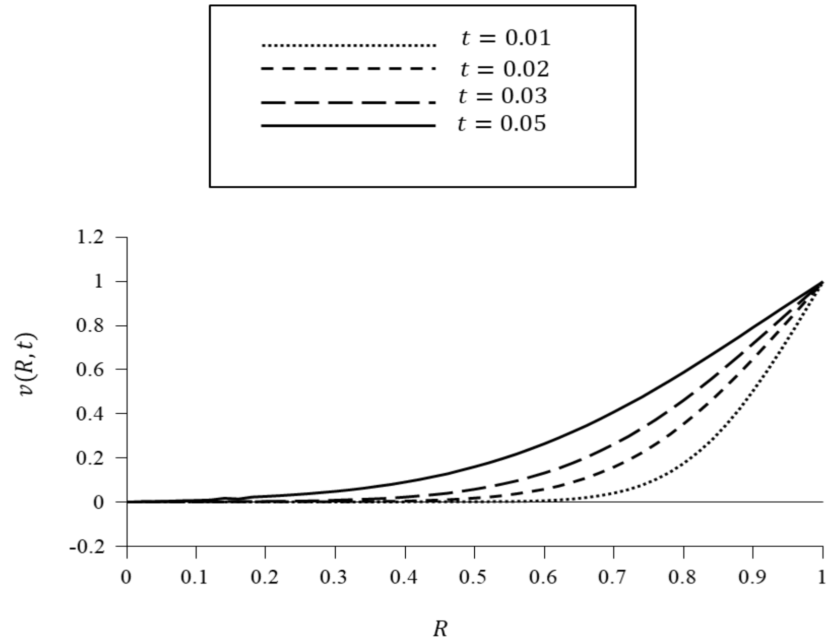

Fig.11: Variation of velocity $v(R, t)$ versus distance for $\kappa = 1$ for case 3.

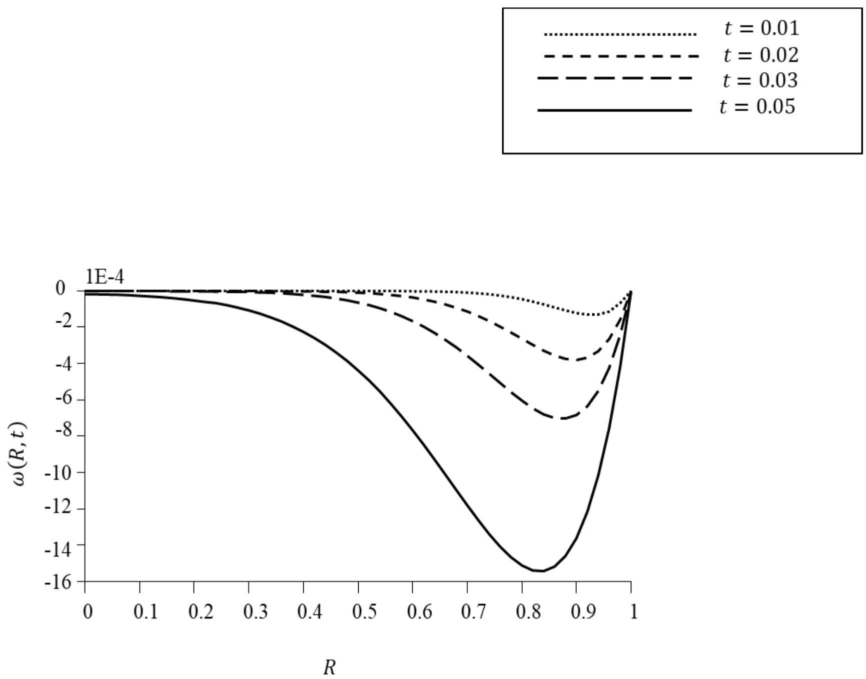

Fig.12: Variation of microrotation $\omega(R, t)$ versus distance for $\kappa = 1$ for case 3.

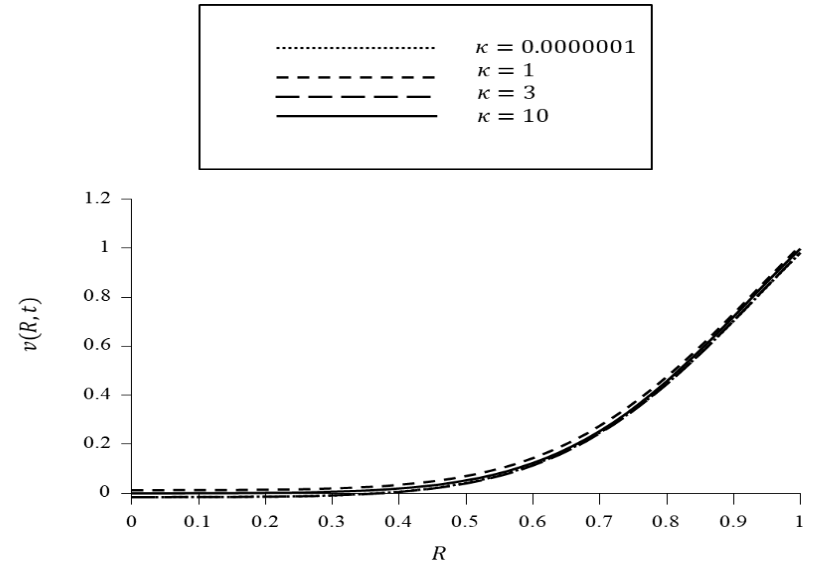

Fig.13: Variation of velocity $v(R,t)$ versus distance for $t = 0.03$ for case 3.

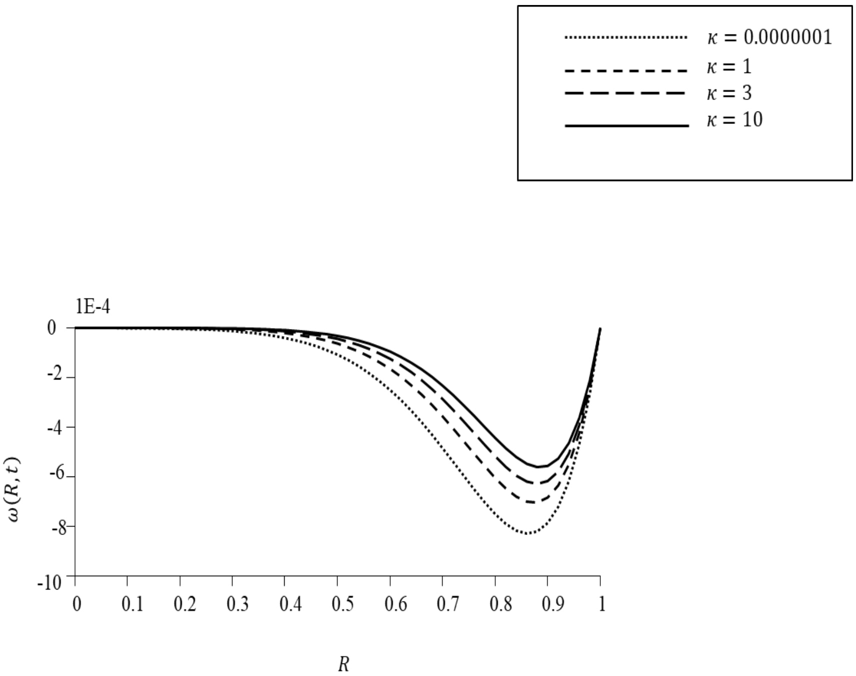

Fig.14: Variation of microrotation $\omega(R,t)$ versus distance for $t = 0.03$ for case 3.

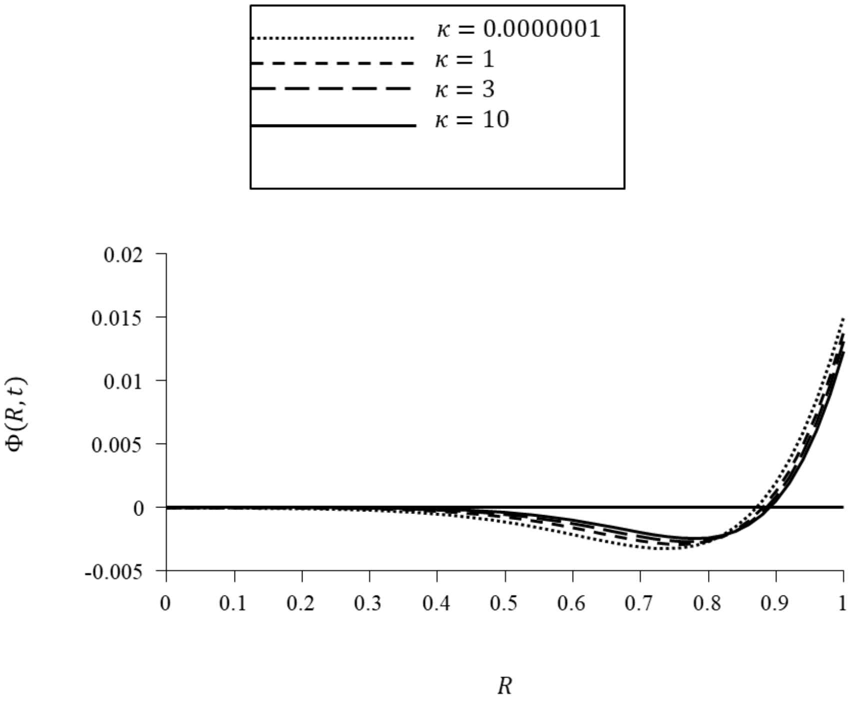

Fig.15: Variation of function $\phi(R,t)$ versus distance for $t = 0.03$ for case 3.

6. CONCLUSIONS

The state space approach method is applied to study the unsteady rotational flow of an incompressible micropolar fluid inside a circular cylinder. The motion is generated by rotating the cylinder about its axis with a time dependent angular velocity. The solution of the problem is obtained analytically in Laplace domain. The Laplace transform is inverted numerically by using a standard numerical inversion technique. Three different cases are discussed. In the first case, the flow is generated by moving the cylinder suddenly with a constant angular velocity, the second and the third cases are representing the sine and cosine oscillations, respectively. It is concluded that the increase in the time parameter increases the values of the velocity and decreases the values of the microrotation. In addition, the micropolarity parameter has an increasing effect on the microrotation only while its effect on the velocity $v(R,t)$ and on the function $\emptyset(R,t)$ is negligible. Moreover, it can be observed, as expected, that far from the cylinder boundary, the values of the velocity and microrotation approach zero.

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