Heterogeneous System $GI/GI^{(n)}/\infty$ with Random Customers Capacities

Ekaterina Lisovskaya, Svetlana Moiseeva, Michele Pagano, and Ekaterina Pankratova

Abstract In the paper, we consider a queuing system with n types of customers. We assume that each customer arrives at the queue according to a renewal process and takes a random resource amount, independent of their service time. We write Kolmogorov integro-differential equation, which, in general, cannot be analytically solved. Hence, we look for the solution under the condition of infinitely growing a service time, and we obtain multi-dimensional asymptotic approximations. We show that the n-dimensional probability distribution of the total resource amounts is asymptotically Gaussian, and we look at its accuracy via Kolmogorov distance.

Keywords Renewal arrival process · Different types of servers · Queueing system

1 Introduction

The globalization of modern managed systems sets new tasks at the hardware, structural, and organizational level. Such systems include both global computer and complex socio-economic relations. In addition to the fact that they are highly heterogeneous, they can also comprise a large number of various objects by

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highly connected cooperations. For example, the actively developing conceptions of Internet of Things (IoT), Internet of Everything (IoE), and Internet of Nano Things (IoNT) involve the interaction of both objects and subjects of the social environment [3, 9, 15]. In this regard, an integrated approach is needed to solve multi-dimensional problems of managing complex technical and social objects in a dynamically changing environment.

Cellular networks are transformed from a planned set of large base-stations to an irregular deployment of heterogeneous infrastructure elements. In paper [2], authors developed a tractable, flexible, and accurate model for a heterogeneous cellular network consisting of K level of randomly located base-station, where each level may differ in terms of average transmit power and supported data rate.

It should be noted that the number of publications has been devoted to modeling of wireless communication systems by the resource queueing system [1, 4, 5]. However, the main results were obtained assuming that requests to resources is deterministic. Thus, considering new models of heterogeneous resource queues is currently relevant [7, 8, 12].

Important task of modeling connection networks is cost criterion, which defines the quality of the system operation. A tandem queueing systems with heterogeneous customers is analyzed in the paper [16]. The authors computated the stationary distribution of the system states under the fixed set of the thresholds—the most difficult part of solving the problem of minimizing the cost.

Similarly, in our article, the problem of finding a stationary probability distribution of the total volumes of occupied resources in a heterogeneous queue is solved. The considered heterogeneous resource queue can be applied when analyzing the performance indicators of radio resource separation schemes of next-generation telecommunication [6, 14].

2 Problem Statement

2.1 Mathematical Model

Consider the queueing system (see Fig. 1) with unlimited number and n different servers types, also assume that each customer carries a random capacity (or needed some resource).

Customers arrive in the system according to a renewal arrival process, given by distribution function $A(z)$ of random variable between time point of customers arriving, which has a finite mean and variance (a and σ^2).

Each arriving customer randomly selects its type according to the set of probabilities p_i ($i = 1, \dots, n$), and besides $\sum_{i=1}^{n} p_i = 1$. Further, the customer goes to the conforming server, staying there for a random time with distribution function

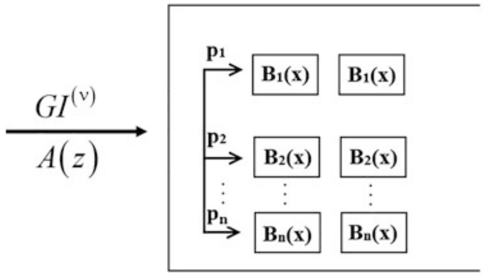

Fig. 1 Queueing system with n servers types

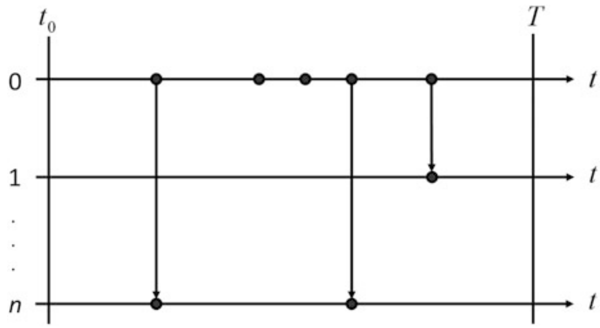

Fig. 2 Dynamic screening of the arrival process

$B_i(x)$, and also taking random resources amount $v_i > 0$ with distribution function $G_i(y)$.

Queueing system with such service discipline was considered by the authors in [13]. However, it does not take into account that each customer requires a random amount of resources.

Denote by $\{V_1(t), \ldots, V_n(t)\}$ the each type's customers total capacity in the system at time t. This process is non-Markovian, therefore, we use the dynamic screening method for its investigation.

Let the system be empty at moment t_0, and let us fix any time moment T in the future as shown in Fig. 2. The set of dynamic probabilities $S_1(t), \ldots, S_n(t)$ represents that a customer arriving at time t have the i-type and it will be served at the moment T, i.e. $S_i(t) = p_i(1 - B_i(T - t))$, for $t_0 \leq t \leq T$.

Denote by $\{W_1(t), \ldots, W_n(t)\}$ the each type total customers capacities screened before the moment t. It is easy to prove the property for the probability distribution stochastic processes [10]:

$$P\{V_1(T) < w_1, \ldots, V_n(T) < w_n\} = P\{W_1(T) < w_1, \ldots, W_n(T) < w_n\},\ w_i \geq 0.$$

The above n-dimensional process is non-Markovian, then we will add the residual time from t to the next arrival $z(t)$.

2.2 Kolmogorov Integro-Differential Equation

For the probability distribution of $(n + 1)$-dimensional Markovian process $\{z(t), W_1(t), \ldots, W_n(t)\}$:

$$P(z, w_1, \ldots, w_n, t) = P\{z(t) < z, W_1(t) < w_1, \ldots, W_n(t) < w_n\},$$
$$z, w_1, \ldots, w_n > 0,$$

we can write the following Kolmogorov integro-differential equation:

$$\begin{aligned}\frac{\partial P(z, w_1, \ldots, w_n, t)}{\partial t} = {} & \frac{\partial P(z, w_1, \ldots, w_n, t)}{\partial z} \\ & + \frac{\partial P(0, w_1, \ldots, w_n, t)}{\partial z}(A(z) - 1) + A(z)\sum_{i=1}^{n} S_i(t) \\ & \times \left[\int_0^{w_i} \frac{\partial P(0, w_1, \ldots, w_i - y_i, \ldots, w_n, t)}{\partial z}\right. \\ & \left. \times dG_i(y_i) - \frac{\partial P(0, w_1, \ldots, w_n, t)}{\partial z}\right]. \end{aligned} \quad (1)$$

We define the initial conditions in the form

$$P(z, w_1, \ldots,_n, t_0) = \begin{cases} R(z), & w_1 = \ldots = w_n = 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $R(z) = \frac{1}{a}\int_0^z (1 - A(u))\,du$ is the stationary probability distribution of the values of the random process $z(t)$.

To solve (1), we introduce the partial characteristic function:

$$h(z, v_1, \ldots, v_n, t) = \int_0^\infty e^{jv_1w_1} \ldots \int_0^\infty e^{jv_nw_n} P(z, dw_1, \ldots, dw_n, t),$$

where $j = \sqrt{-1}$ is the imaginary unit. Then, we obtain the following equation:

$$\begin{aligned}\frac{\partial h(z, v_1, \ldots, v_n, t)}{\partial t} = {} & \frac{\partial h(z, v_1, \ldots, v_n, t)}{\partial z} + \frac{\partial h(0, v_1, \ldots, v_n, t)}{\partial z} \\ & \times \left[A(z) - 1 + A(z)\sum_{i=1}^{n} S_i(t)(G_i^*(v_i) - 1)\right], \end{aligned} \quad (2)$$

where

$$G_i^*(v_i) = \int_0^\infty e^{jv_i y} dG_i(y),$$

with the initial condition

$$h(z, v_1, \ldots, v_n, t_0) = R(z). \tag{3}$$

3 Asymptotic Analysis

In general, Eq. (2) cannot be solved analytically, but it is possible to find approximate solutions under suitable asymptotic conditions; in this paper we consider the case that the service times of the different types of customers growth proportionally to each other.

We state and prove the following theorem.

Theorem 1 *The asymptotic characteristic function of the stationary probability distribution of the process* $\{V_1(t), \ldots, V_n(t)\}$ *has the form*

$$h(v_1, \ldots, v_n) \approx \exp\left\{\lambda \sum_{i=1}^n jv_i a_1^{(i)} p_i b_i + \lambda \sum_{i=1}^n \frac{(jv_i)^2}{2} a_2^{(i)} p_i b_i \right.$$
$$\left. + \kappa \sum_{i=1}^n \sum_{m=1}^n \frac{jv_i jv_m}{2} a_1^{(i)} a_1^{(m)} p_i p_m K_{im} \right\}, \tag{4}$$

where $\lambda = (a)^{-1}$, $\kappa = \lambda^3(\sigma^2 - a^2)$, *(*$a$ *and* σ^2 *being the mean and the variance of the interval time, respectively), and*

$$a_1^{(i)} = \int_0^\infty y dG_i(y), \quad a_2^{(i)} = \int_0^\infty y^2 dG_i(y),$$

$$b_i = \int_0^\infty (1 - B_i(x))dx; \qquad K_{im} = \int_0^\infty (1 - B_i(x))(1 - B_m(x))dx.$$

Proof At first, we prove a secondary statement.

Lemma 1 *The first-order asymptotic characteristic function of the process* $\{z(t), W_1(t), \ldots, W_n(t)\}$ *is given by*

$$h(z, v_1, \ldots, v_n, t) \approx R(z) \exp\left\{\lambda \sum_{i=1}^{n} j v_i a_1^{(i)} \int_{t_0}^{t} S_i(\theta) d\theta\right\}.$$

Proof Let $b_i = bq_i$ for some real values $q_i > 0$ and $b \to \infty$. Put

$$\varepsilon = \frac{1}{bq_i},\ v_i = \varepsilon y_i,\ t\varepsilon = \tau,\ t_0\varepsilon = \tau_0,\ T\varepsilon = \tilde{T},\ S_i(t) = \tilde{S}_i(\tau),$$

$$h(z, v_1, \ldots, v_n, t) = f_1(z, y_1, \ldots, y_n, \tau, \varepsilon).$$

Then, from the expressions (2) and (3), we get

$$\varepsilon \frac{\partial f_1(z, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial \tau} = \frac{\partial f_1(z, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} + \frac{\partial f_1(0, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} \times \left[A(z) - 1 + A(z) \sum_{i=1}^{n} \tilde{S}_i(\tau)(G_i^*(\varepsilon y_i) - 1)\right], \tag{5}$$

with the initial condition

$$f_1(z, y_1, \ldots, y_n, \tau_0, \varepsilon) = R(z).$$

Let $\varepsilon \to 0$; then Eq. (5) becomes:

$$\frac{\partial f_1(z, y_1, \ldots, y_n, \tau)}{\partial z} + \frac{\partial f_1(0, y_1, \ldots, y_n, \tau)}{\partial z}(A(z) - 1) = 0.$$

and hence $f_1(z, y_1, \ldots, y_n, \tau)$ can be expressed as

$$f_1(z, y_1, \ldots, y_n, \tau) = R(z)\Phi_1(y_1, \ldots, y_n, \tau), \tag{6}$$

where $\Phi_1(y_1, \ldots, y_n, \tau)$ is some scalar function, satisfying the condition

$$\Phi_1(y_1, \ldots, y_n, \tau_0) = 1.$$

Now let $z \to \infty$ in (5):

$$\varepsilon \frac{\partial f_1(\infty, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial \tau} = \frac{\partial f_1(0, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} \sum_{i=1}^{n} \tilde{S}_i(\tau)(G_i^*(\varepsilon y_i) - 1).$$

Then, we substitute here the expression (6), take advantage of the Taylor expansion

$$e^{j\varepsilon s} = 1 + j\varepsilon s + O\big(\varepsilon^2\big), \tag{7}$$

divide by ε and perform the limit as $\varepsilon \to 0$. Since $R'(0) = \lambda$, we get the following differential equation:

$$\frac{\partial \Phi_1(y_1, \dots, y_n, \tau)}{\partial \tau} = \Phi_1(y_1, \dots, y_n, \tau)\lambda \sum_{i=1}^{n} \tilde{S}_i(\tau) j y_i a_1^{(i)}. \tag{8}$$

Taking into account the initial condition, the solution of (8) is

$$\Phi_1(y_1, \dots, y_n, \tau) = \exp\left\{\lambda \sum_{i=1}^{n} j y_i a_1^{(i)} \int_{\tau_0}^{\tau} \tilde{S}_i(\theta) d\theta\right\}.$$

By substituting $\Phi_1(y_1, \dots, y_n, \tau)$ from (6), and then we can write

$$\begin{aligned} h(z, v_1, \dots, v_n, t) &= f_1(z, y_1, \dots, y_n, \tau, \varepsilon) \approx f_1(z, y_1, \dots, y_n, \tau) \\ &= R(z)\Phi_1(y_1, \dots, y_n, \tau) \\ &= R(z) \exp\left\{\lambda \sum_{i=1}^{n} j y_i a_1^{(i)} \int_{\tau_0}^{\tau} \tilde{S}_i(\theta) d\theta\right\} \\ &= R(z) \exp\left\{\lambda \sum_{i=1}^{n} j v_i a_1^{(i)} \int_{t_0}^{t} S_i(\theta) d\theta\right\}. \end{aligned}$$

□

Let $h_2(z, v_1, \dots, v_n, t)$ be a solution of the following equation:

$$h(z, v_1, \dots, v_n, t) = h_2(z, v_1, \dots, v_n, t) \exp\left\{\lambda \sum_{i=1}^{n} j v_i a_1^{(i)} \int_{t_0}^{t} S_i(\theta) d\theta\right\}. \tag{9}$$

Substituting this expression into (2) and (3), we get the following equivalent problem:

$$\frac{\partial h_2(z, v_1, \ldots, v_n, t)}{\partial t} + \lambda h_2(z, v_1, \ldots, v_n, t) \sum_{i=1}^{n} j v_i a_1^{(i)} S_i(t)$$
$$= \frac{\partial h_2(z, v_1, \ldots, v_n, t)}{\partial z} + \frac{\partial h_2(0, v_1, \ldots, v_n, t)}{\partial z}$$
$$\times \left[A(z) - 1 + A(z) \sum_{i=1}^{n} S_i(t) \big(G_i^*(v_i) - 1 \big) \right], \tag{10}$$

with the initial condition

$$h_2(z, v_1, \ldots, v_n, t_0) = R(z). \tag{11}$$

By performing the following changes of variable

$$\varepsilon^2 = \frac{1}{bq_i},\ v_i = \varepsilon y_i,\ t\varepsilon = \tau,\ t_0\varepsilon = \tau_0,\ T\varepsilon = \tilde{T},\, S_i(t) = \tilde{S}_i(\tau),$$
$$h_2(z, v_1, \ldots, v_n, t) = f_2(z, y_1, \ldots, y_n, \tau, \varepsilon). \tag{12}$$

In (10) and (11), we get the following problem:

$$\varepsilon^2 \frac{\partial f_2(z, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial \tau} + f_2(z, y_1, \ldots, y_n, \tau, \varepsilon) \lambda \sum_{i=1}^{n} j \varepsilon y_i a_1^{(i)} \tilde{S}_i(\tau)$$
$$= \frac{\partial f_2(z, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} + \frac{\partial f_2(0, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z}$$
$$\times \left[A(z) - 1 + A(z) \sum_{i=1}^{n} \tilde{S}_i(\tau) (G_i^*(\varepsilon y_i) - 1) \right], \tag{13}$$

with the initial condition

$$f_2(z, y_1, \ldots, y_n, \tau_0, \varepsilon) = R(z).$$

As a generalization of the approach used in the previous subsection, the asymptotic solution of this problem

$$f_2(z, y_1, \ldots, y_n, \tau) = \lim_{\varepsilon \to 0} f_2(z, y_1, \ldots, y_n, \tau, \varepsilon).$$

Letting $\varepsilon \to 0$ in (13), we get the following equation:

$$\frac{\partial f_2(z, y_1, \ldots, y_n, \tau)}{\partial z} + \frac{\partial f_2(0, y_1, \ldots, y_n, \tau)}{\partial z}(A(z) - 1) = 0.$$

Hence, we can express $f_2(z, y_1, \ldots, y_n, \tau)$ as

$$f_2(z, y_1, \ldots, y_n, \tau) = R(z)\Phi_2(y_1, \ldots, y_n, \tau), \tag{14}$$

where $\Phi_2(y_1, \ldots, y_n, \tau)$ is some scalar function that satisfies the condition

$$\Phi_2(y_1, \ldots, y_n, \tau_0) = 1.$$

The solution $f_2(z, y_1, \ldots, y_n, \tau)$ can be represented in the expansion form

$$f_2(z, y_1, \ldots, y_n, \tau) = \Phi_2(y_1, \ldots, y_n, \tau) \times \left[R(z) + f(z) \sum_{i=1}^{n} j\varepsilon y_i a_1^{(i)} \tilde{S}_i(\tau) \right] + O\big(\varepsilon^2\big), \tag{15}$$

where $f(z)$ is a suitable function, and besides $f(\infty) = const$, let be $f(\infty) = 0$. By substituting the previous expression and the Taylor–Maclaurin expansion (7) in (13), taking into account that $R'(z) = \lambda(1 - A(z))$, it is easy to verify that

$$f(z) = \frac{\kappa}{2} \int_0^z (1 - A(u))\, du + \lambda \int_0^z (R(u) - A(u))\, du.$$

Letting $z \to \infty$ in (13), by the definition of the function $f_2(z, y_1, \ldots, y_n, \tau, \varepsilon)$, we obtain

$$\lim_{z \to \infty} \frac{\partial f_2(z, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} = 0,$$

and, taking into account the expansion

$$e^{j\varepsilon s} = 1 + j\varepsilon s + \frac{(j\varepsilon s)^2}{2} + O\big(\varepsilon^3\big),$$

we can write

$$\varepsilon^2 \frac{\partial f_2(\infty, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial \tau} + f_2(\infty, y_1, \ldots, y_n, \tau, \varepsilon)\lambda \sum_{i=1}^{n} \tilde{S}_i(\tau) j\varepsilon y_i a_1^{(i)}$$
$$= \frac{\partial f_2(0, y_1, \ldots, y_n, \tau, \varepsilon)}{\partial z} \sum_{i=1}^{n} \tilde{S}_i(\tau) \left(j\varepsilon y_i a_1^{(i)} + \frac{(j\varepsilon y_i)^2}{2} a_2^{(i)} \right) + O\big(\varepsilon^3\big).$$

By substituting here the expansion (15) and taking the limit as $z \to \infty$, we get

$$\varepsilon^2 \frac{\partial \Phi_2(y_1, \ldots, y_n, \tau)}{\partial \tau} + \Phi_2(y_1, \ldots, y_n, \tau)\lambda \sum_{i=1}^{n} j\varepsilon y_i a_1^{(i)} \tilde{S}_i(\tau)$$

$$= \Phi_2(y_1, \ldots, y_n, \tau)\lambda \sum_{i=1}^{n} \tilde{S}_i(\tau) \left(j\varepsilon y_i a_1^{(i)} + \frac{(j\varepsilon y_i)^2}{2} a_2^{(i)} \right)$$

$$+ \Phi_2(y_1, \ldots, y_n, \tau) f'(0) \sum_{i=1}^{n} \tilde{S}_i(\tau) j\varepsilon y_i a_1^{(i)}$$

$$\times \sum_{m=1}^{n} \tilde{S}_m(\tau) \left(j\varepsilon y_m a_1^{(m)} + \frac{(j\varepsilon y_m)^2}{2} a_2^{(m)} \right) + O\big(\varepsilon^3\big).$$

After simple remakes, and taking into account that $\kappa = 2f'(0)$, we get the following differential equation for $\Phi_2(y_1, \ldots, y_n, \tau)$:

$$\frac{\partial \Phi_2(y_1, \ldots, y_n, \tau)}{\partial \tau} = \Phi_2(y_1, \ldots, y_n, \tau) \left[\lambda \sum_{i=1}^{n} \frac{(jy_i)^2}{2} a_2^{(i)} \tilde{S}_i(\tau) \right.$$

$$\left. + \kappa \sum_{i=1}^{n} \sum_{m=1}^{n} \frac{jy_i\, jy_m}{2} a_1^{(i)} a_1^{(m)} \tilde{S}_i(\tau) \tilde{S}_m(\tau) \right],$$

whose solution (with the given initial condition) can be expressed as

$$\Phi_2(y_1, \ldots, y_n, \tau) = \exp \left\{ \lambda \sum_{i=1}^{n} \frac{(jy_i)^2}{2} a_2^{(i)} \int_{\tau_0}^{\tau} \tilde{S}_i(\theta) d\theta \right.$$

$$\left. + \kappa \sum_{i=1}^{n} \sum_{m=1}^{n} \frac{jy_i\, jy_m}{2} a_1^{(i)} a_1^{(m)} \int_{\tau_0}^{\tau} \tilde{S}_i(\theta) \tilde{S}_m(\theta) d\theta \right\}.$$

Substituting this expression into (14) and performing the inverse substitutions of (12) and (9), we get the following expression for the asymptotic characteristic function of the process $\{z(t), W_1(t), \ldots, W_n(t)\}$:

$$h(z, v_1, \ldots, v_n, t) \approx R(z) \exp\left\{\lambda \sum_{i=1}^{n} jv_i a_1^{(i)} \int_{t_0}^{t} S_i(\theta)d\theta \right.$$
$$+ \lambda \sum_{i=1}^{n} \frac{(jv_i)^2}{2} a_2^{(i)} \int_{t_0}^{t} S_i(\theta)d\theta$$
$$\left. +\kappa \sum_{i=1}^{n} \sum_{m=1}^{n} \frac{jv_i jv_m}{2} a_1^{(i)} a_1^{(m)} \int_{t_0}^{t} S_i(\theta) S_m(\theta) d\theta \right\},$$

For $z \to \infty$, $t = T$ and $t_0 \to -\infty$ we get the characteristic function of the process $\{V_1(t), \ldots, V_n(t)\}$ in the steady state regime

$$h(v_1, \ldots, v_n) \approx \exp\left\{\lambda \sum_{i=1}^{n} jv_i a_1^{(i)} p_i b_i \right.$$
$$\left. +\lambda \sum_{i=1}^{n} \frac{(jv_i)^2}{2} a_2^{(i)} p_i b_i + \kappa \sum_{i=1}^{n} \sum_{m=1}^{n} \frac{jv_i jv_m}{2} a_1^{(i)} a_1^{(m)} p_i p_m K_{im} \right\}.$$

□

The structure of this characteristic function implies that the n-dimensional process $\{V_1(t), \ldots, V_n(t)\}$ is asymptotically Gaussian with mean

$$\mathbf{a} = \lambda \left[a_1^{(1)} p_1 b_1 \ a_1^{(2)} p_2 b_2 \ \ldots \ a_1^{(n)} p_n b_n \right]$$

and covariance matrix

$$\mathbf{K} = \left[\lambda \mathbf{K}^{(1)} + \kappa \mathbf{K}^{(2)} \right],$$

where

$$\mathbf{K}^{(1)} = \begin{bmatrix} a_2^{(1)} p_1 b_1 & 0 & \ldots & 0 \\ 0 & a_2^{(2)} p_2 b_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & a_2^{(n)} p_n b_n \end{bmatrix},$$

$$\mathbf{K}^{(2)} = \begin{bmatrix} a_1^{(1)}a_1^{(1)}p_1p_1K_{11} & a_1^{(1)}a_1^{(2)}p_1p_2K_{12} & \dots & a_1^{(1)}a_1^{(n)}p_1p_nK_{1n} \\ a_1^{(2)}a_1^{(1)}p_2p_1K_{21} & a_1^{(2)}a_1^{(2)}p_2p_2K_{22} & \dots & a_1^{(2)}a_1^{(n)}p_2p_nK_{2n} \\ \dots & \dots & \dots & \dots \\ a_1^{(n)}a_1^{(1)}p_np_1K_{n1} & a_1^{(n)}a_1^{(2)}p_np_2K_{n2} & \dots & a_1^{(n)}a_1^{(n)}p_np_nK_{nn} \end{bmatrix}.$$

4 Simulation Results

The result (4) was obtained under the asymptotic condition for an unlimited increase of the service time ($b_i \to \infty$). We conducted several simulation experiments [11], changing all the systems parameters (i.e., the laws that characterize the incoming flow, the service time, and the customers resource, as well as the probabilities p_i), in order to investigate their practical applicability. Since the different values of the source data show similar results, for example, we present only one of them.

Thus, we assume that the arrival renewal process is characterized by a uniform distribution of the interval time in the [0.5, 1.5], corresponding to a fundamental rate of arrivals $\lambda = 1$ customers per time unit. The remaining distribution laws and their parameters are presented in Table 1, according to the customers type.

We compared the asymptotic distributions with the empiric ones by Kolmogorov distance:

$$\Delta = \sup_x |F_{em}(x) - F_{as}(x)|,$$

where $F_{em}(x)$ is the distribution function built on the basis of simulation results, and $F_{as}(x)$ is the Gaussian approximation given by (4).

Table 2 shows the results for the marginal distributions of the total resource amount for each customers types (Δ^1 and Δ^2, respectively) and for two-dimensional distributions (Δ).

As expected, the asymptotic results become more precise when the service time parameter b increases. This conclusion is also confirmed by Figs. 3 and 4, which compare the asymptotic approximations with the empirical histograms for the total resource amount of each type of customers for two different values of b.

Table 1 Types of customers and their distribution laws

Type	Distribution laws		
	Probability	Service time	Resources
First	$p_1 = 0.7$	Gamma ($0.5b$, 0.5)	Exponential (2)
Second	$p_2 = 0.3$	Gamma ($1.5b$, 1.5)	Exponential (1)

Table 2 Kolmogorov distance

b	10	20	50	100	200
Δ^1	0.136	0.072	0.035	0.024	0.017
Δ^2	0.041	0.027	0.020	0.014	0.010
Δ	0.136	0.072	0.035	0.024	0.017

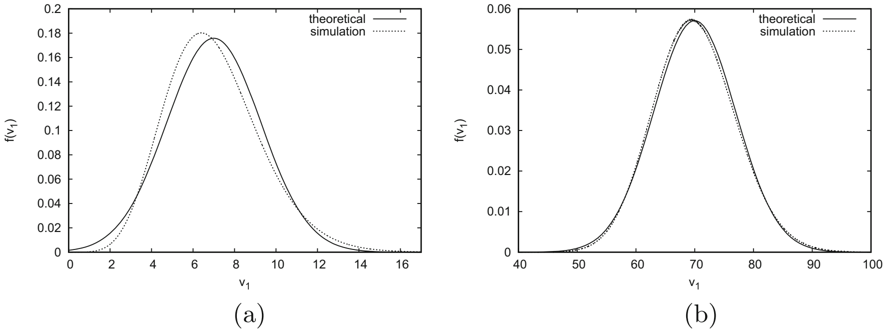

Fig. 3 Distributions of the total resource amount for the first type of customers. (**a**) $b = 20$. (**b**) $b = 200$

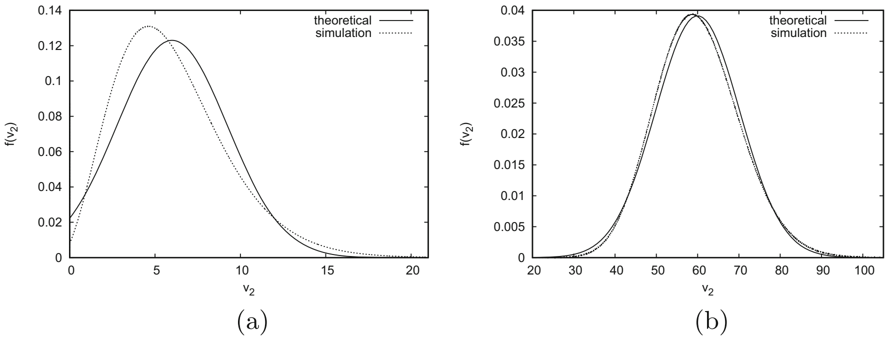

Fig. 4 Distributions of the total resource amount for the second type of customers. (**a**) $b = 20$. (**b**) $b = 200$

5 Conclusion

In this work we considered a queue with n customers types under the assumption that arrival points correspond to a renewal process and each customer occupies a random resource amount. At first we constructed the system of Kolmogorov differential equations, which in the general case cannot be solved analytically. Hence, we obtained the approximations of probability distributions in case of infinitely growing service time by asymptotic analysis method, and we noticed that the n-dimensional probability distribution of the total resource amount is asymptotically Gaussian.

Finally, by discrete-event simulation we tested the approximation reliability, by considering the Kolmogorov distance as accuracy measure.

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